Example knitr document: estimating π

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1 Introduction

This is an example document created using the knitr system (http://yihui.name/knitr/). knitr is a tool for combining both LaTeX documentation and R code within the same file, similar to Sweave. For this document, the master file is estimatek.Rnw. This is processed by knitr in R , which runs the R code to generate textual/graphical output, and also creates a LaTeX document. The LaTeX document is then typeset to create the pdf document. On recent machines, once knitr is installed, you should be able to generate the pdf using:

```
require(knitr)
knit2pdf("estimatek.Rnw")
```

knitr is newer than Sweave, and is more flexible. Both estimatek.Rnw and estimatek.pdf are available from:
http://proteome.sysbiol.cam.ac.uk/lgatto/teaching/files/estimate.zip

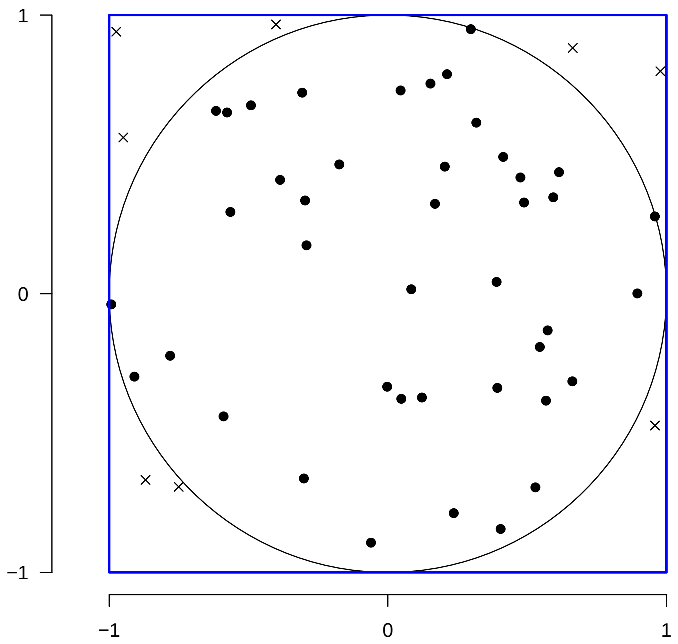
2 Task: estimate the value of π

Our task is to estimate the value of π by simulating darts being thrown at a dartboard. Imagine that the person throwing the darts is not very good, and randomly throws each dart so that it falls uniformly within a square of side length $2r$, with the dartboard of radius r centred within that square. If the player throws n darts, and d of them hit the dartboard, then for large enough n, the ratio d/n should approximate the ratio of the area of the dartboard to the enclosing square, $\pi r^2/4r^2 \equiv \pi/4$. From this, we can estimate $\pi \approx 4d/n$.

We start with an example, using R to draw both the dartboard and the surrounding square, together with $n = 50$ darts. The radius of the dartboard here is 1 unit, although the value is not important.

```
r <- 1
n <- 50
par(las = 1)
plot(NA, xlim = c(-r, r), ylim = c(-r, r), asp = 1, bty = "n", xaxt = "n", yaxt = "n",
    xlab = "", ylab = "")
axis(1, at = c(-r, 0, r))
axis(2, at = c(-r, 0, r))
symbols(x = 0, y = 0, circles = r, inch = F, add = T)
x <- runif(n, -r, r)
y <- runif(n, -r, r)
inside <- (x^2 + y^2) < r^2
```

```r
d <- sum(inside)
points(x, y, pch = ifelse(inside, 19, 4))
rect(-r, -r, r, r, border = "blue", lwd = 2)
```



A dart is drawn as a filled circle if it falls within the dartboard, else it is drawn as a cross. In this case the number of darts within the circle is 42, and so the estimated value is $\pi \approx 3.36$.

The estimate of π should improve as we increase the number of darts thrown at the dartboard. To verify this, we write a short function that, given the number of darts to throw, n, returns an estimate of π.

```r
estimate.pi <- function(n = 1000) {
    ## Return an estimate of PI using dartboard method with N trials.
    r <- 1  ## radius of dartboard
    x <- runif(n, min = -r, max = r)
    y <- runif(n, min = -r, max = r)
    l <- sqrt(x^2 + y^2)
    d <- sum(l < r)
    4 * d/n
}
```

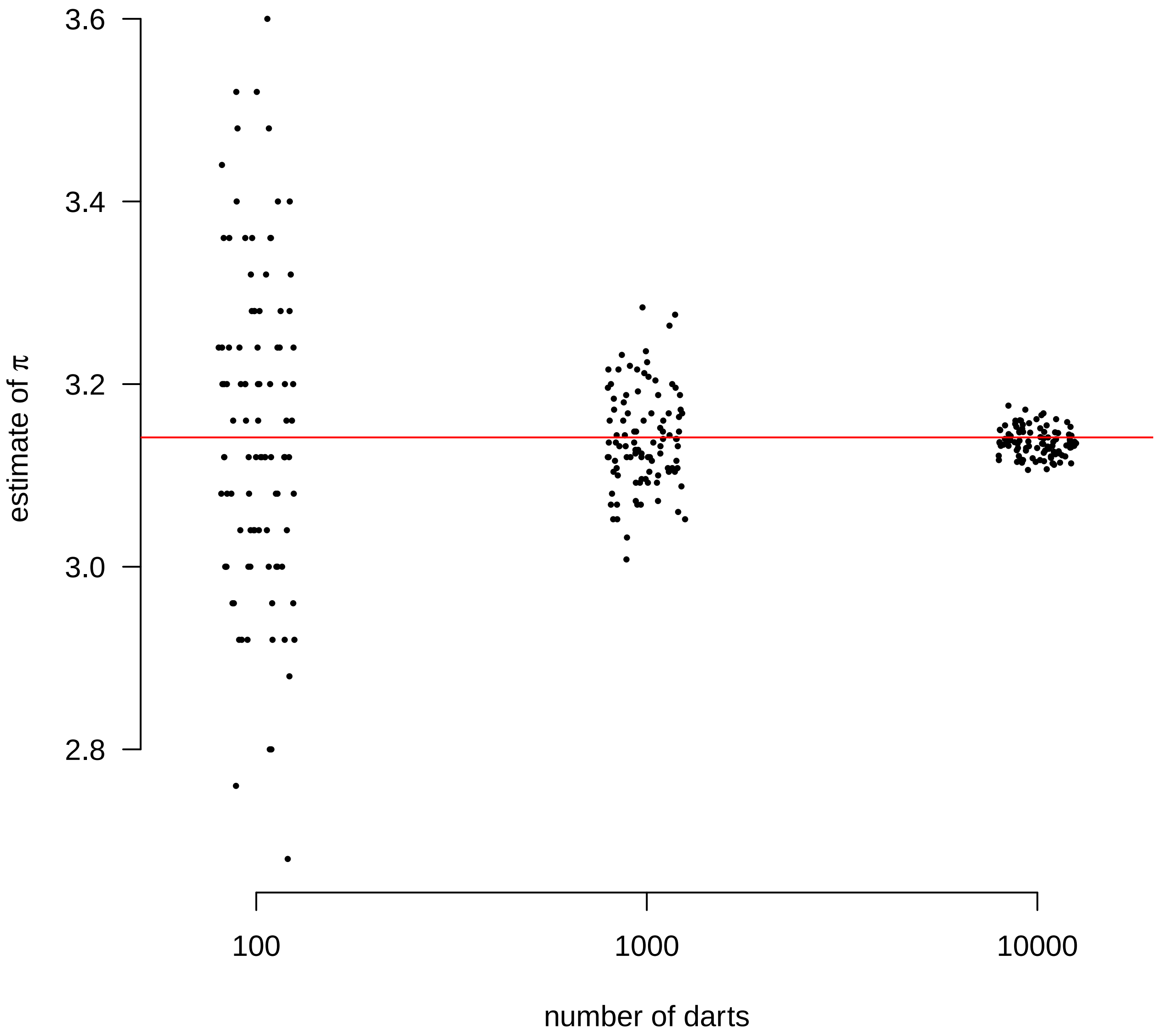
We can then test the procedure a few times, using the default number of darts, 1000:

```r
replicate(9, estimate.pi())

## [1] 3.076 3.104 3.176 3.240 3.140 3.144 3.196 3.128 3.180
```

Finally, for a given value of n, we can show 99 estimates of π, as clearly the estimate will vary from run to run. In the following plot, we compare the estimates of π for three different values of n:

```r
ns <- 10^c(2, 3, 4)
res <- lapply(ns, function(n) replicate(99, estimate.pi(n)))
par(las = 1, bty = "n")
stripchart(res, method = "jitter", group.names = ns, xlab = "number of darts",
    ylab = expression(paste("estimate of ", pi)), vert = TRUE, pch = 20, cex = 0.5)
abline(h = pi, col = "red")
```



As the number of darts increases, the estimate of π gradually converges onto the actual value of π (shown by the solid red line).