Fibonacci series in R

Laurent Gatto[1]

October 18, 2012

[1] lg390@cam.ac.uk – http://proteome.sysbiol.cam.ac.uk/lgatto

Fibonacci sequence[2]

$$0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots$$

$$f[n] = f[n-1] + f[n-2]$$

How to vectorize?

Exercise: write a function, `fibonnaci(n)` that returns the n^{th} element of the sequence. Assume that `fibonnaci(1) == 0`, `fibonacci(2) == 1`.

Exercise: use `fibonacci()` to estimate the golden ratio.

[2] Slide from *Scientific programming with R* , MPhil in Computational Biology, Cambridge

Fibonacci sequence

```
> fibonacci <- function(n) {
+     .fib <- function(n) {
+         if (n == 0)
+             return(0)
+         if (n < 3)
+             return(1)
+         ans <- c(1, 1, rep(0, n - 2))
+         for (i in 3:n) ans[i] <- ans[i - 1] + ans[i - 2]
+         return(ans[n])
+     }
+     if (length(n) == 1)
+         return(.fib(n))
+     sapply(n, .fib)
+ }
```

Fibonacci sequence

Using $F(n) = \frac{\varphi^n - (1-\varphi)^n}{\sqrt{5}} = \frac{\varphi^n - (-\varphi)^{-n}}{\sqrt{5}}$, where $\varphi = \frac{1+\sqrt{5}}{2}$ is the golden ratio.

```
> fibdirect <- function(n) {
+     stopifnot(n >= 0)
+     phi <- (1 + sqrt(5))/2
+     round((phi^n - (1 - phi)^n)/sqrt(5))
+ }
```

Fibonacci sequence

```
> x <- 1:100
> all.equal(fibonacci(x), fibdirect(x))

[1] TRUE
```

```
> library("rbenchmark")
> benchmark(fibonacci(x), fibdirect(x),
+           columns=c("test", "replications",
+             "elapsed", "relative"),
+           order = "relative", replications = 100)

          test replications elapsed relative
2 fibdirect(x)          100   0.007      1.0
1 fibonacci(x)          100   2.406    343.7
```

Golden ratio

We have $\lim_{x \to \infty} \frac{F(n+1)}{F(n)} = \varphi$

```
> all.equal((1 + sqrt(5))/2, fibonacci(10)/fibonacci(9))

[1] "Mean relative difference: 0.0002391"

> all.equal((1 + sqrt(5))/2, fibonacci(20)/fibonacci(19))

[1] "Mean relative difference: 1.581e-08"

> all.equal((1 + sqrt(5))/2, fibonacci(30)/fibonacci(29))

[1] TRUE
```

Golden ratio

```
> phi <- (1 + sqrt(5))/2
> plot(fibdirect(2:20)/fibdirect(1:19), type = "b")
> abline(h = phi, lty = "dotted", col = "red")
> plot(fibdirect(19:27)/fibdirect(18:26), type = "b")
> abline(h = phi, lty = "dotted", col = "red")
```