CMPE 492

A.TAYLAN CEMGIL

OPTIONS PRICING AND STOCHASTIC VOLATILITY

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INTRODUCTION

Option is a crucial phenomenon for investors to make profitable moves in their trades. They need to know at which price they should trade stocks, i.e underlying securities. Since stock prices modeled with times series, we can not exactly know the future prices of a stock. However, we can fairly try to guess the price of it. This is solely stock view of the story. In this paper, we are interested in option pricing which means that we will focus our attention on one step before the stock trading.

An investor does not want to make a prospective move that can result in loss. To prevent any loss to some extent, he needs a kind of an insurance policy. At this point, options comes into play. In other words, options were invented for the purpose of hedging. Let us explain this in more detail with an example. Suppose that you are an investor and you realize that underlying value of a technology stock is going upward. Naturally, you desire to take the advantage of this trend. Yet, you can not completely be sure that stock will end up in favorable position. Therefore, you want to be able to minimize your any downside while preserving your full upside in a cost-efficient way.

As we mentioned before, an investor tries to reduce his cost and loss. One way of achieving this can be options, we said. However, options are not free and he has to make a payment to for this handy tool. Payment takes place between two parties, namely, *Option Holder* and *Option Writer*. Holder makes payment at a price upon which he and writer make an agreement. This agreement should be made in such a way that holder will not be paying more than enough amount of money. At this point, our role, in this paper, begins.

After we make a brief why options pricing is in our interest and who can make advantageous of them, we might provide the reader with an insight how we should approach to this problem. Options pricing require some several parameters and a model into which we plug these parameters. Since 1973, there have been several models that were employed by various scientists such as Black-Scholes, Heston etc. In following chapters, we will give some details and some results about several models. Our final goal might be to find an volatility parameter which is one of the most important factors having impact on options pricing. Yet, to understand these concepts completely, we shall provide you with the terminology in the next section.

TERMINOLOGY

First of all, let us explain the basic terms which we will be utilizing in this report.

1.1 Options

Options are derivative contracts giving the holder the right of buying or selling shares of a stock at a price (strike or exercise) upon which both parties agree. While option holder is not obligated to exercise the option, option writer is required to buy or sell underlying security. Options can give the holder the advantage of leverage. If we are to elaborate on this, we can consider 2 types of options, namely, *Call* and *Put* Options.

1.1.1 Call Option

A call option gives the holder the right of buying underlying security at a specific price in a time interval determined. A call option can give the holder the leverage if it satisfies the following simple condition:

Current market value of underlying security > Strike price

1.1.2 Put Option

A put option gives the holder the right of selling underlying security at a specific price in a time interval determined. A put option can give the holder the leverage if it satisfies the following simple condition:

Current market value of underlying security < Strike price

1.2 Intrinsic Value

When there is a difference between the underlying security price and strike price then intrinsic value comes out. For a *call option* intrinsic value is as below, and vice-versa for a *put option*:

Underlying security price - Strike price

1.3 Premium

Premium is the cost the holder pays to the writer to acquire the right provided by the option. Premium value consists of two components which are intrinsic value and time value.

1.4 Time Value

Time value refers to the price difference between premium and intrinsic value. For instance, if buyer pays the seller $7,00 for a share of stock($700,00 if contract) and intrinsic value of the stock is $5,00 then time value will be $2,00. Furthermore, buyers tend to pay the sellers more premium for more remaining time until the expiration date of an option. This leads us to conclude that time increases the likelihood that the stock will be at more profitable position.

2 Factor of Options Pricing

It's time to turn our focus on the factors having impact on option pricing. There are mainly six parameters that influence the process of determining option pricing.

2.1 Underlying Price

Current market value of underlying security is the most impactful factor on option pricing. Its impact on option types can be summarized in the table given below:

Underlying Price	Call Price	Put Price
↑	↑	↓
↓	↓	↑

2.2 Expected Volatility

Volatility is to exhibit how much and how fast the price of underlying security has been changing. Historical volatility is deduced from the actual data, in other words, actual price changes of underlying. If we train the past data and try to find a possible future volatility, it is called Implied Volatility. Relation between expected volatility and option price is, generally, like below:

Underlying Price	Call Price
↑	↑

2.3 Strike Price

We mentioned that intrinsic value is the difference between underlying price and strike price. Therefore, we can consider strike price as a threshold signifying if option is in-the-money,i.e intrinsic value is positive.

2.4 Time Until Expiration

The more time until the expiration date the more likelihood that option is further in-the-money. As a consequence of this situation, time value is larger compared to the case of shorter expiration date. And , also, if there is high volatility in price of underlying security then we can expect to have larger time value since there will be fluctuations in the underlying price.

2.5 Interest Rate

Interest rate is the cost of holding money. Therefore if there is high interest rates then call option price will be higher and vice-versa. Summary table given below:

Interest Rate	Call Price	Put Price
↑	↑	↓
↓	↓	↑

2.6 Dividends

If the underlying dividend increases then call price decreases and put prices decreases. Relation is summarized below:

Dividends	Call Price	Put Price
↑	↓	↑
↓	↑	↓

3 METHODS & MODELS

Plotting historical data is a handy method to try fitting a model to data. By doing this, one can make generalization to adapt the new coming inputs. Therefore, we use some models to find a best theoretical output value given that we have sufficient features to be used. Hence, option traders can consider this theoretical value to make more profitable move. Therefore, there are several models generated for that purpose. Very first, and probably, most well-known option pricing model is Black-Scholes.

3.1 Black-Scholes

Formula is given:

$$C = SN(d_1) - N(d_2)Ke^{-rt}$$

$$d_1 = \frac{\ln(\frac{S}{K}) + (r + \frac{s^2}{2})t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

Variables are explained below:

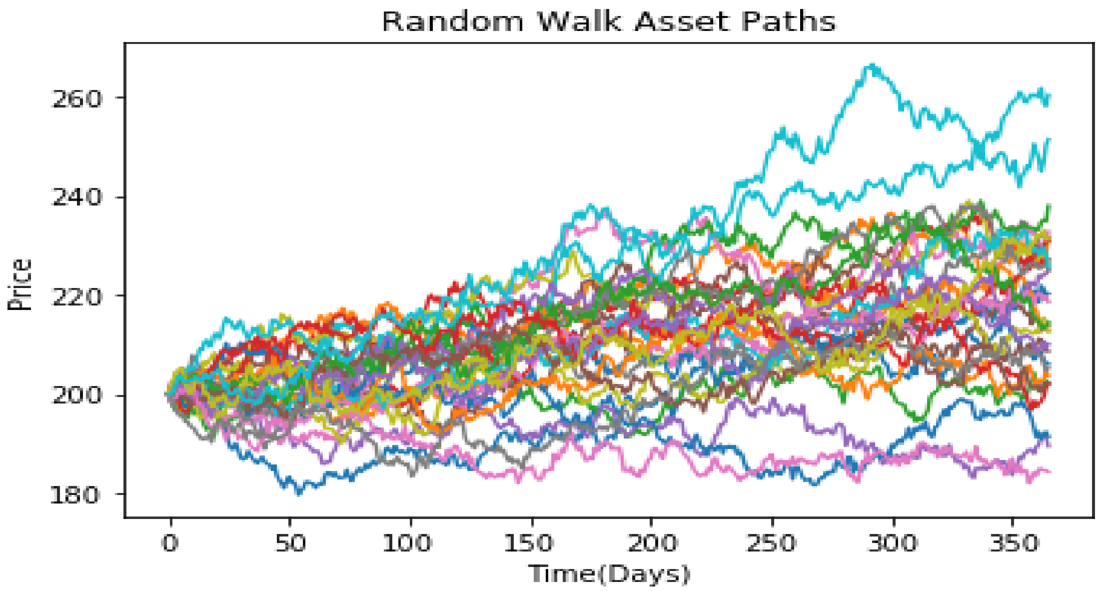
- C = Call Premium
- S = Current stock price
- t = time until expiration
- K = Option strike price
- r = risk-free interest rate
- N = Cumulative standard normal distribution
- σ: Standard deviation

Model Assumptions: Dividends,r and σ are constant whereas, in reality, this assumption might not hold. Since volatility fluctuates over the life of an option.

3.2 Monte Carlo Method for Option Pricing

$$S_T = S_t e^{(r - \frac{\sigma^2}{2})(T-t) + \sigma\sqrt{T-t}\epsilon}$$

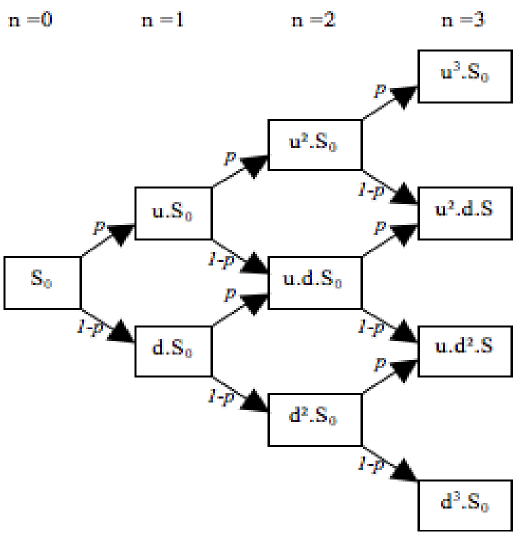
- r is risk free interest rate.
- σ is volatility, the annualized standard deviation of a stock's returns.
- (T-t) gives the annualized time to maturity.(in years).
- S_t The price of the underlying asset.
- ϵ is a random value from standard normal distribution$(\mu = 0, \sigma = 1)$.



3.3 Binomial Option Pricing

Black-Scholes is one of the most popular models used for options pricing, yet can not be applied to every type of options to obtain more fair price. Binomial option pricing model is another well-known approach to specify fair option prices.

American options differs from European options in a way that they can be exercised before the expiration date. Hence, binomial model will give us the ease of considering option life step-by-step. Implementation of this approach is given below.



$$p = \frac{e^{rt/n} - d}{u - d}$$

$$u = e^{\sigma\sqrt{t/n}}$$

$$d = e^{-\sigma\sqrt{t/n}}$$

```
import matplotlib.patches as mpatches
def BinomialOption(S0, K, r, sigma, T, N=10):
    '''
    S0: Initial value of security
    K: Strike Price
    r: Risk-free interest rate
    sigma: Underlying volatility
    T: Expiration date
    '''
    #calculate delta T
    dT = float(T) / N
    # up and down factors
    u = np.exp(sigma * np.sqrt(dT))
    d = 1.0 / u
    #init array
    callPrices =  np.asarray([0.0 for i in range(N + 1)])
    putPrices =  np.asarray([0.0 for i in range(N + 1)])
    #security price array
    sec = np.asarray([(S0 * u**j * d**(N - j)) for j in range(N + 1)])
    #The probability of up and down.
    p = (np.exp(r * dT) - d)/ (u - d)
    q = 1.0 - p
    # Compute the leaves
    callPrices[:] = np.maximum(sec-K, 0.0)
    putPrices[:] = np.maximum(K-sec, 0.0)
    #calculate backward the option prices
    for i in range(N-1, -1, -1):
        callPrices[:-1]=np.exp(-r * dT) * (p * callPrices[1:] + q * callPrices[:-1]
        putPrices[:-1]=np.exp(-r * dT) * (p * putPrices[1:] + q * putPrices[:-1])
        sec[:]=sec[:]*u
        callPrices[:]=np.maximum(callPrices[:],sec[:]-K)
        putPrices[:]=np.maximum(putPrices[:],K-sec[:])
    # option price
    return callPrices[0],putPrices[0]
```

3.4 Heston Model

In finance, the Heston model, named after Steven Heston, is a mathematical model describing the evolution of the volatility of an underlying asset. In this model, we don't have a constant or deterministic volatility. It follows a random process. It is widely used since it fits to real-world case more than Black-Scholes or Binomial does. Differential equation of basic Heston Model is given below:

$$dS_t = \mu S_t dt + \sqrt{v_t} S_t d{W_t}^S$$

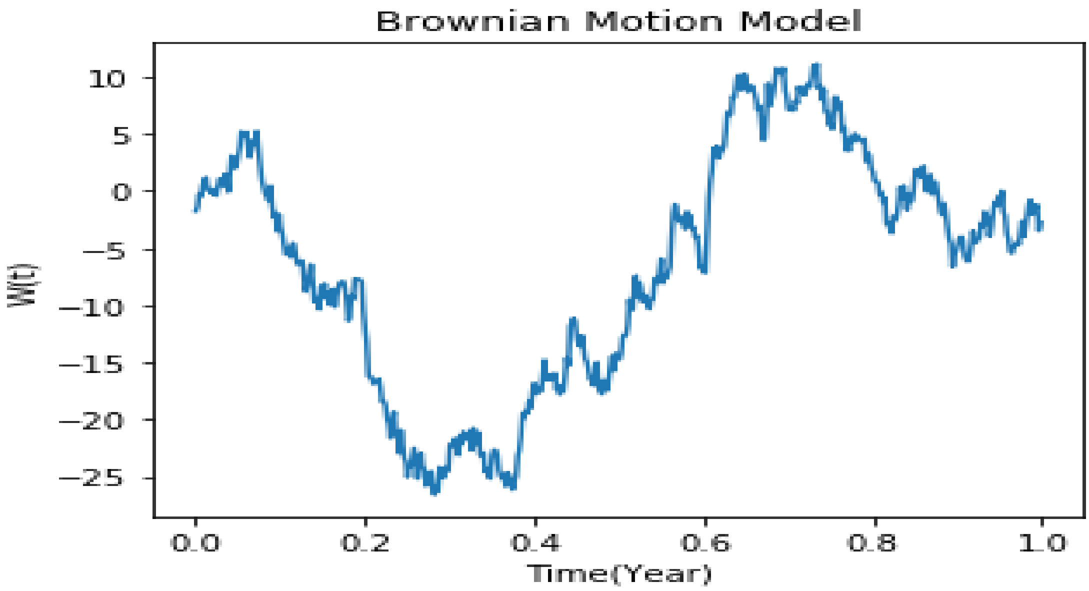
$$dv_t = \kappa(\theta - v_t)dt + \xi\sqrt{v_t} d{W_t}^v$$

dW_t^S, dW_t^v are Wiener processes with correlation ρ, or equivalently, with covariance ρdt. The parameters are the following:

- μ : the rate of return of the asset.
- θ : long run average price variance.
- κ : the rate at which v_t reverts to θ.
- ξ : the vol of vol.

Feller Condition: $2\kappa\theta > \xi^2 \longrightarrow v_t > 0$

To understand this formula and go further in advance approach, we should review the *Brownian Motion Model*. Its definition and simulation are given in the next section to provide the reader with more insight into the Heston Model.



3.4.1 Matrix Representation of Heston Model

$$\begin{bmatrix} dS_t \\ dv_t \end{bmatrix} = \begin{bmatrix} \mu S_t \\ \kappa(\theta - v_t) \end{bmatrix} d_t + \begin{bmatrix} \sqrt{v} S_t & 0 \\ 0 & \xi\sqrt{v_t} \end{bmatrix} \begin{bmatrix} dW_t{}^S \\ dW_t{}^v \end{bmatrix}$$

$$\begin{bmatrix} dS_t \\ dv_t \end{bmatrix} = \begin{bmatrix} \mu S_t \\ \kappa(\theta - v_t) \end{bmatrix} d_t + \begin{bmatrix} \sqrt{v} S_t & 0 \\ 0 & \xi\sqrt{v_t} \end{bmatrix} \begin{bmatrix} c1 & c2 \\ 0 & c3 \end{bmatrix} \begin{bmatrix} dW_t{}^S \\ dW_t{}^v \end{bmatrix}$$

$$\begin{bmatrix} dS_t \\ dv_t \end{bmatrix} = \begin{bmatrix} \mu S_t \\ \kappa(\theta - v_t) \end{bmatrix} d_t + \begin{bmatrix} c1\sqrt{v} S_t & c2\sqrt{v} S_t \\ 0 & c3\xi\sqrt{v_t} \end{bmatrix} \begin{bmatrix} dW_t{}^S \\ dW_t{}^v \end{bmatrix}$$

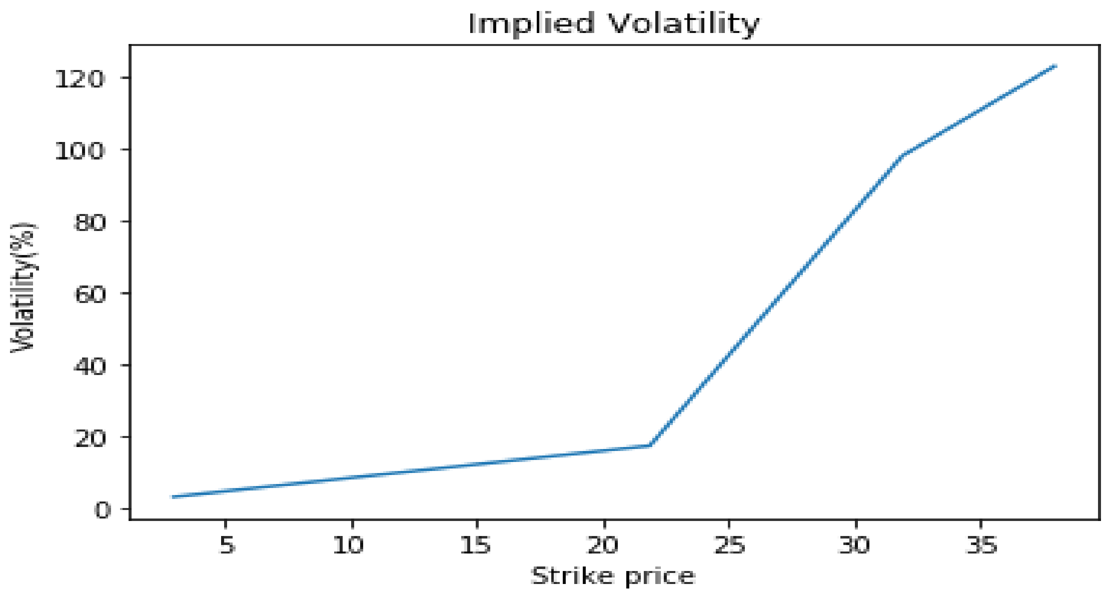
4 IMPLIED VOLATILITY

Implied volatility is one of the significant parameter to price the options. Options will tend to have higher premiums with high levels of implied volatility, and vice versa. Implied volatility tries to guess the future value of an option,and, current underlying value takes this guess into consideration. Therefore, implied volatility is a very important factor to which investors pay attention. Also note that higher implied volatility means large price swing. In other words, it does not guarantee that price will be high.

4.1 Black-Scholes Implied Volatility

We used Black-Scholes Implied Volatility in our project. To obtain the volatility, we utilized the famous Newton-Raphson method in a naive way that we tried to approximate a fair value to option price, at the same time, update volatility. The code snippet used for the implementation of this approach is given below:

```
def find_vol(target_value, call_put, S, K, T, r):
    MAX_IT= 500
    EPSILON = 0.000001
    sigma = 0.5
    for i in range(0, MAX_IT):
        price = bs_price(call_put, S, K, T, r, sigma)
        vega = bs_vega(call_put, S, K, T, r, sigma)
        diff = target_value - price
        if (abs(diff) < EPSILON):
            return sigma
        sigma = sigma + diff/vega # f(x) / f'(x)
    return sigma
```

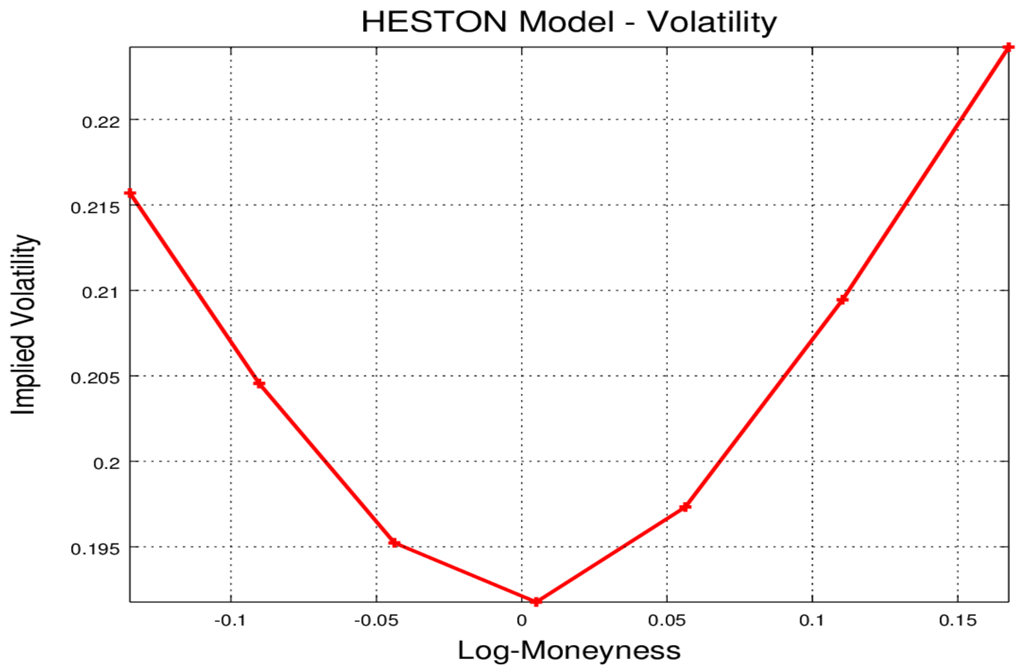


4.2 Heston Volatility Smile

We can use Heston model to deduce the implied volatility for a given option. By using the given parameters and the formulas given for Heston Model, we obtained a plot called *Heston Volatility Smile*

Parameters:

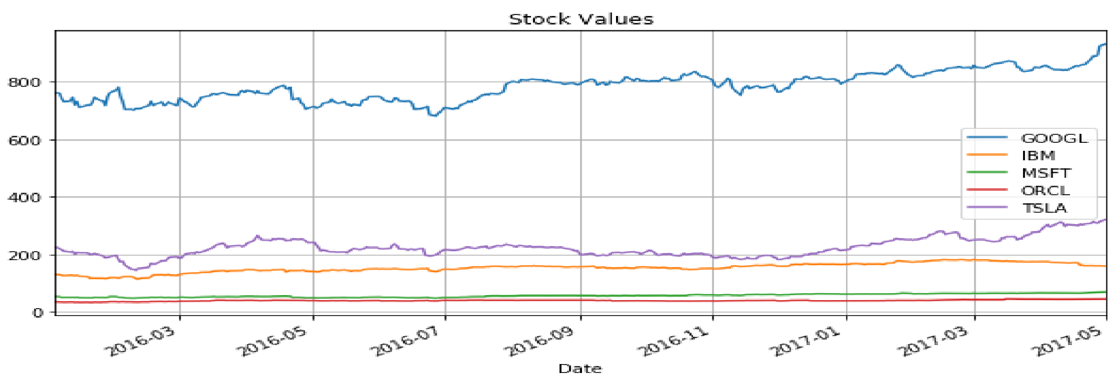
- S0 = 100;
- r = 0.02;
- V0 = 0.04;
- eta = 0.7;
- theta = 0.06;
- kappa = 1.5;
- strike = 85:5:115;
- T = 0.25;
- M = 2000; % Number of paths.
- N = 250; % Number of time steps per path

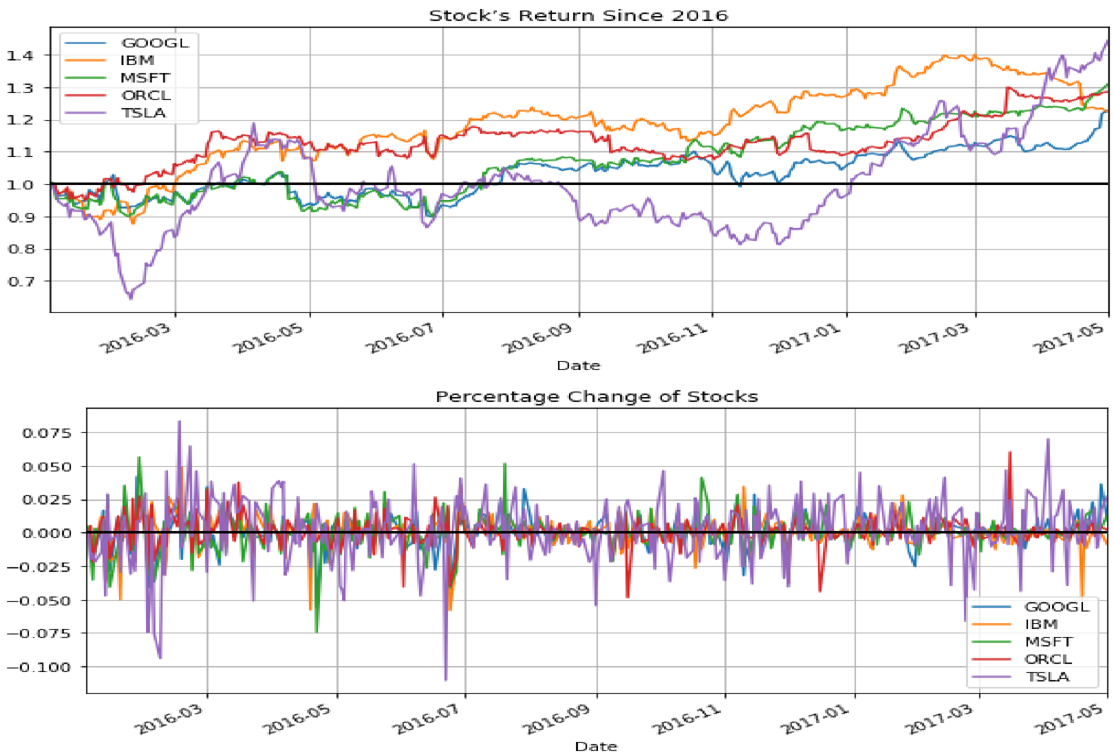


4.3 Historical Data & Stock Values

We mentioned in previous sections, our primary goal is to find an implied volatility. Again, as we know stock prices follow time series , it might be impossible to denote future values, precisely. However, investors, at least, want to be provided with some parameters to obtain as much fair price as they can predict. To satisfy this need, first step might be to analyze the historical data. Therefore, from now on, I will utilize some useful libraries of python to get and parse the data of some companies like Google ,Tesla etc. Yet, my main goal will be determining an implied volatility and comparing those values with the ones coming with Yahoo API.

4.3.1 Visualization of Stock Values





4.3.2 Tesla Stock

Now, it's time to continue with a stock with which we will play. We will focus our attention on finding an implied volatility in a given period. As the title denotes, we will analyze Tesla Stock since we can see that it has a considerable volatility at the last graphic.

Yahoo API provides us with the data associated with tickers such as options, historical data etc. Code for generating plot given below can be found my Github page. [8]

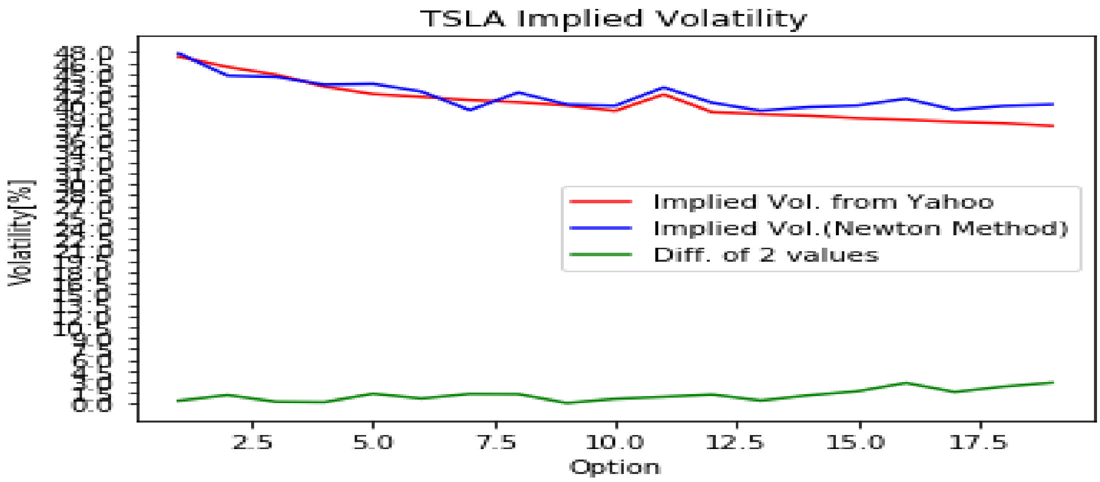


Figure 1: TSLA implied vol. vs API vol. and their differences

When we look closer the difference line on Figure 1, we can observe that the difference is about 1.5% = 0.015 which is good to be achieved by using only a naive approach, that is, *Newton-Raphson* method.

4.3.3 Google Stock

In previous part, we plot the implied volatility of TSLA stock which shows remarkable dispersion over the time. However, there are some stocks that are showing almost stable trend over the time. Our candidate is now GOOGL. We try to deduce the implied volatility for it.

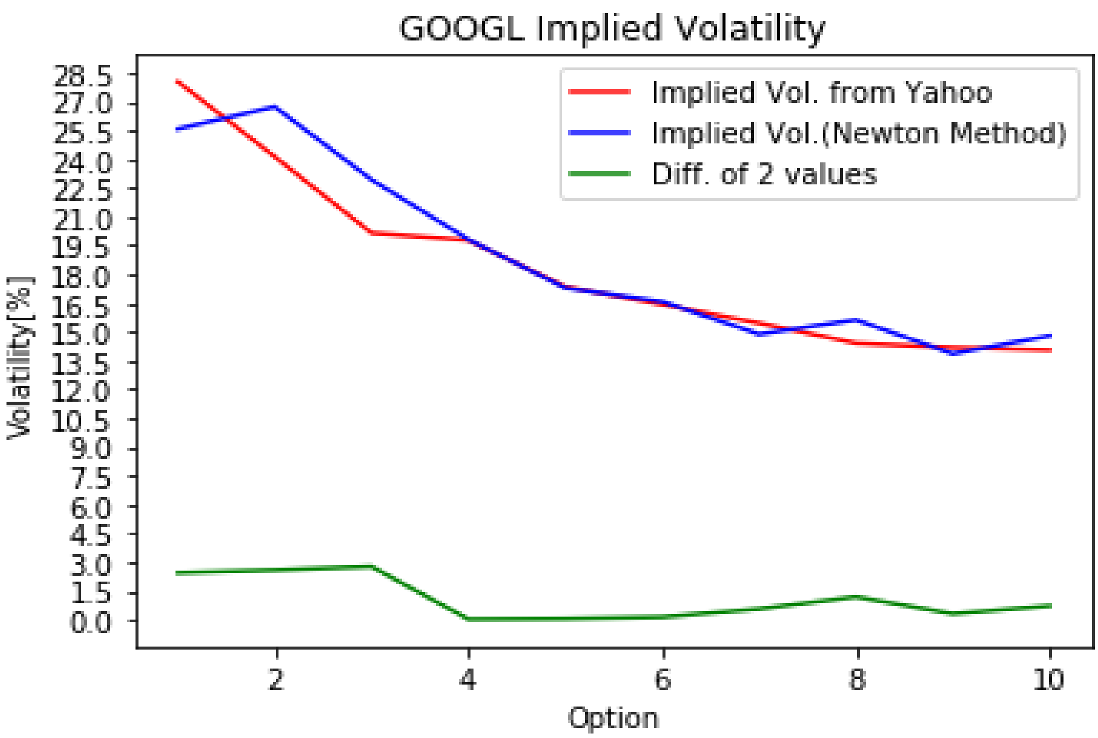


Figure 2: GOOGL implied vol. vs API vol. and their differences

Now, when we consider Figure 2, we can denote that the difference line is averagely less than 1.5%. Intuitively, someone can make the assumption that some tickers show less dramatic changes over time. This is generally because of the fact that parameters affecting the stock does not fluctuate greatly over time horizon.

4.4 A Small Application

I decided to make a small GUI that shows the graphic of implied volatility of a stock with the difference line. Users can select a ticker name from the drop-down and observe the results below immediately. Screen-shots from the application is given below:

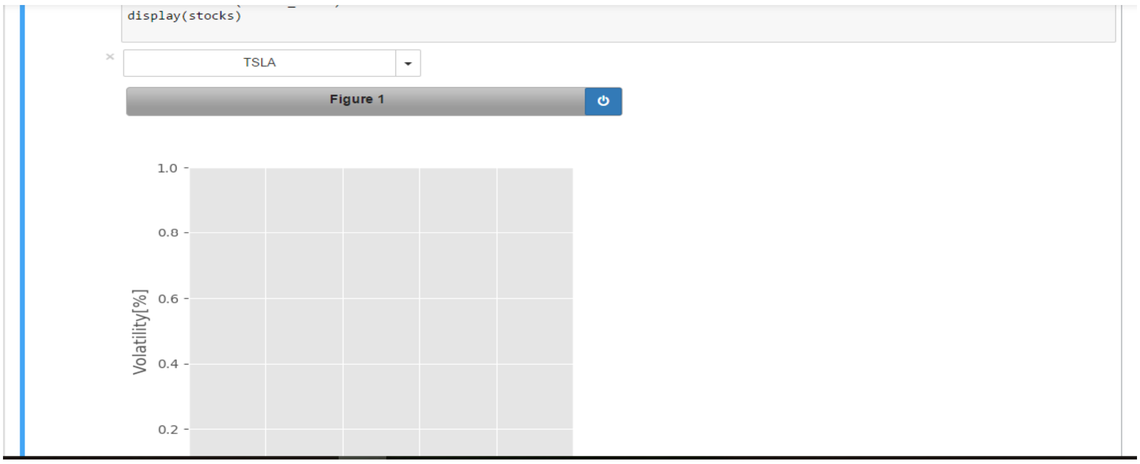


Figure 3: GUI of the simple application

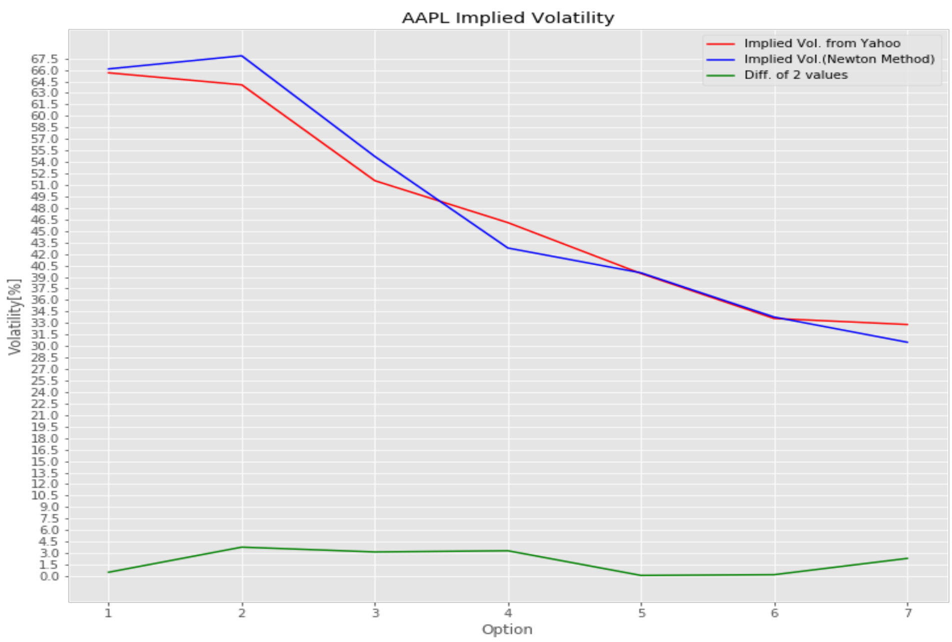


Figure 4: AAPL is selected from the drop-down

5 Related/Future Work

Since finance is a field arousing interest to many people, there are lots of studies made for *Options Pricing.* For my own work, I utilized Newton-Raphson which is a parametric method. Some people insist on using Artificial Neural Network or Support Vector Regression fall under non-parametric categories.

Table 4: Performance of various model when market condition is out of the money market

Models	Data-Points	TE	ME	MSE	RMSE	NRMSE
BS	3589	2.9702×10^{6}	8.2759×10^{2}	1.1528×10^{6}	1073.7	275.08
ε-SVR	3589	7.3363×10^{4}	10.0006	3.6523×10^{3}	60.4344	.4125
ε-SSVR	3589	3.7030×10^{4}	10.3176	3.6367×10^{3}	60.3050	.4117

Figure 5: Support Vector Regression used to estimate option prices [5]

I have made my experiments in out of the money situation, therefore I put above the table related the out of moneyness. We can see that Smooth ϵ-insensitive Support Vector Regression gives the best error rate. In my own work error rate is about 0.015, yet, comparing those values directly is not reasonable since I did not use the same dataset.

Future work I would like to make on this field can be using Support Vector Regression. My approach is to analyze last 10 days volatility and try to guess if tomorrow volatility would be higher than that volatility.

6 Conclusion

In this study, fair price of an option is predicted using Black Scholes Model, Binomial Model and Heston Model. Implied volatility is predicted by using Newton-Raphson. The empirical results show good error rates even though I used a naive approach to calculate implied volatility.

REFERENCES

[1] Hedging and Pricing Options, <http://cs229.stanford.edu/proj2009/KolindHarrisPrzybytkowski.pdf> [Accessed December 10, 2009]

[2] An Algorithmic Introduction to Numerical Simulation of Stochastic Differential Equations,<http://www-sop.inria.fr/members/Olivier.Faugeras/MVA/Articles_11/higham:01.pdf> [Accessed 23 October 2013].

[3] An Analysis of the Heston Stochastic Volatility Model, <https://arxiv.org/ftp/arxiv/papers/1502/1502.02963.pdf> [Accessed December 2014].

[4] Options Pricing-Investopedia, <http://www.investopedia.com/university/options-pricing/>

[5] Forecasting the Price of Call Option Using Support Vector Regression IOSR-JM, Dec,2014., Neetu Verma, Namita Srivastava, Sujoy Das, <http://iosrjournals.org/iosr-jm/papers/Vol10-issue6/Version-6/F010663843.pdf>

[6] Wim Schoutens,Lévy Processes in Finance,Katholieke Universiteit Leuven, Belgium, <http://onlinelibrary.wiley.com/book/10.1002/0470870230>

[7] Jeremy Andre, Alfred W., Shanbin Zhao. Volatility Forecasting using SVM,2007, <http://cs229.stanford.edu/proj2007/AndreWechselbergerZhao-VolatilityForecasting.pdf>

[8] <https://github.com/enesozi/Option-Pricing-Stochastic-Volatility>