

# FORMULA SHEET:-

$$[\sin^{-1}(x)]' = \frac{1}{\sqrt{1-x^2}}$$

$$\operatorname{cosec}^{-1}x = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$[\cos^{-1}(x)]' = \frac{-1}{\sqrt{1-x^2}}$$

$$\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}}$$

$$[\tan^{-1}(x)]' = \frac{1}{1+x^2}$$

$$\cot^{-1}x = \frac{-1}{1+x^2}$$

$$(\sin x)' = \cos x$$

$$(\operatorname{cosec} x)' \Rightarrow -\operatorname{cosec} x \cot x$$

$$(\cos x)' = -\sin x$$

$$\tan 90 = \infty$$

$$(\tan x)' = \sec^2 x$$

$$\cot^2 x = \operatorname{cosec}^2 x - 1$$

$$\cos^2 x = 1 - \sin^2 x$$

$$(e^x)' = e^x (1)$$

Integration by parts:-

$x^{10}$  (algebraic).

ILATE:-

$$\int f(x) \cdot g(x) dx \Rightarrow f(x) \int g(x) dx - \int [f(x)]' \int g(x) dx \quad \frac{dx}{dx}$$

LIATE:-

$$UV - \int v du$$



## Halfangle identities

$$\sin \frac{t}{2} = \sqrt{\frac{1 - \cos t}{2}}$$

$$\sin^2 t = \frac{1 - \cos 2t}{2}$$

$$(\sin 2t)^2 = \frac{1 - \cos 4t}{2}$$

$$\int \frac{1}{x\sqrt{x^2-1}} dx = \tan^{-1}(\sqrt{x^2-1}) + C$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n = e^x$$



Domain of all  
 $(f(x)) = \mathbb{R}$  if polynomial  
 $(g(x))$  without  $\sqrt{\quad}$

In multivariable funts i.e.

$f(x, y)$  : double variable

$f(x, y, z)$  : triple variable.

a linear funt is always a plane structure & a non-linear structure is a 3D object / Surface.

o Taking multivariable funts;  
 Limits.

▷ either isolate  $x$  (1 variable).

e.g.  $z = f(x, y)$ .

$$\frac{df}{dx} / f_x (x, y) = \lim_{h \rightarrow 0} \frac{f(x+h, y) - f(x, y)}{h}$$

▷ or isolate  $y$  (other variable).

$$\frac{df}{dy} / f_y (x, y) = \lim_{h \rightarrow 0} \frac{f(x, y+h) - f(x, y)}{h}$$

o Taking partial derivative in multivariable funts.

e.g.  $f(x, y) = xy^2 + x + y$ .

$$\frac{df}{dx} = \frac{df}{dx} (xy^2) + \frac{df}{dx} (x) + \frac{df}{dx} (y)$$

w.r.t  $x$

∴ treat  $y$  as a const.  $\Rightarrow y^2(1) + 1 + (y)'(1)$

$$y^2 + 1 + 0 \Rightarrow y^2 + 1 \text{ Ans!}$$

$$\frac{df}{dy} \Rightarrow \frac{d}{dy} (xy^2) + \frac{d}{dy} (x) + \frac{d}{dy} (y)$$

w.r.t  $y$   
 treat  $x$  as const.

$$2yx + 1 \text{ Ans!}$$



## Chapter no. 4.

### 4.1

#### o Extreme Values:-

Let a funct be defined at an interval  $(a, b)$

$$y = f(x) \quad (a, b)$$

then  $x_0$  any value from the domain/interval will give to be minima or maxima of the funct.

#### Minima.

if  $(x_0) \in (a, b)$  &

$$f(x_0) \leq f(x) \therefore$$

$x_0$ ; it is the minima.

#### Maxima.

if  $(x_0) \in (a, b)$  &

$$f(x_0) \geq f(x) \therefore$$

$x_0$ , is the maxima.

→ minima & maxima are the extreme values of a funct and when described in subdomain called as local & when described in whole domain called as global.

#### → Steps:-

- (i) Find endpoints :  $f(a)$  &  $f(b)$
- (ii) Find Critical point  $f'(x) = 0$ .
- (iii) Compare all points.  $f(a), f(b), f(C.P.)$ .

\* to describe if the minima or maxima is absolute; closed interval (or continuous funct) is needed.

\* In modulus functs  $f'(x) = 0$  cannot give the critical point rather the critical point is equal to  $f(a)$  and  $a$  is the common const found in the interval of both +ve & -ve values of the modulus funct.

\* for Q questions if  $f'(x) = 0$  cannot be solved.  $f(\text{critical point}) = 0$ .



if  $P(x) > f(x)$

$q(x)$  given. Polynomial & disc of  $q(x)$  is neg.  $\therefore$

if  $f'(x)$  is positive and not undefined anywhere.

$e^x$  ( $\mathbb{D} \in \mathbb{R}$ ).

$\ln$  ( $\mathbb{D} \in \mathbb{R}^+$ )

$\sin^{-1}$  ( $\mathbb{D} \in [-1, 1]$ )

4.3:-

Finding interval at which the fnc is increasing or decreasing.

0 steps to solve / find increasing or decreasing.

- (i) take first derivative of the fnc.
- (ii) let  $f'(x) = 0$  [to usually get two end points]
- (iii) make intervals <sup>open</sup> from  $[-\infty - (\text{critical point}) - +\infty]$  to check from each interval.
- (iv) let value from each interval and if  $f'(a) > 0 \therefore$  increasing  
 $f'(a) < 0 \therefore$  decreasing.

Eg:-  $f'(x) = \frac{(x-2)(x+4)}{(x+1)(x-3)}$   $\mathbb{D} = \mathbb{R} - \{-1, 3\}$

$$0 = x^2 + 4x - 2x - 8 \Rightarrow x^2 + 2x - 8 \Rightarrow (x-2)(x+4) = 0$$

$x = 2$  &  $x = -4$ . however the restrictions of domain apply.

$(-\infty, -4)$   $(-4, -1)$   $(-1, 2)$   $(2, 3)$   $\Rightarrow$  are the intervals to be checked.

\* interval breaking by domain is highly important.

\* Critical point a point at which the first derivative = 0 or undefined. however critical point can not be an interior part of the domain.



44.

## Concavity of Function:-

Concave up

concave down.

- i) set the intervals from domain & the critical point.
- ii) find the point of inflexion.
- iii) Put the det values from each interval into the second derivative. ( $y''$ )

if  $y''(a) > 0$   
concave up

if  $y''(a) < 0$   
concave down.

Point of inflexion.

(2nd derivative put equal to 0).  
When the second order derivative is equal to zero;

It is known as point of inflexion; a point where the concavity is changing and thus cannot be noted.

Undefined form  $1^\infty, \frac{\infty}{\infty}, \frac{0}{0}, 0^0, \infty - \infty$

\* L'Hopital need a fraction to be executed.

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

\* limit can be pass a common limit.

$$\ln(0) = -\infty$$

$$\log_a b = \frac{\ln b}{\ln a}$$

$\int = \text{---}$  (prove?)  
simply find derivative of non integral size.

\* Amorphous Silicon  $\rightarrow$  main semiconductor!