

Develop Turing machines for two of the following problems using the simulator provided (<https://schaetztc.github.io/tursi/>). If your student ID mod 3 = m and student ID mod 5 = n, then you need to solve the m-th problem from the first group and the n-th problem from the second group.

1. Design Turing machines for the following languages: (5)

0. The set of strings with an equal number of 0's and 1's

1.  $\{a^n b^n c^n \mid n \geq 1\}$

2.  $\{ww^R \mid w \text{ is any string of 0's and 1's}\}$

2. Design Turing machines to perform the following operations (the leftmost bit is LSB): (5)

0. Increment (add 1). The tape initially contains  $wc$  where  $w$  is a binary number. Your TM should add 1 to  $w$  and write the sum to the right of the  $c$ .

1. Decrement (subtract 1). The tape initially contains  $wc$  where  $w$  is a binary number. Your TM should subtract 1 from  $w$  and write the result to the right of the  $c$ .

2. Two's complement. The tape initially contains  $wc$  where  $w$  is a binary number. Your TM should compute 2's complement of  $w$  and write the result to the right of the  $c$ .

3. Logical AND. The tape initially contains  $w_1cw_2c$  where  $w_1$  and  $w_2$  are binary numbers. Your TM should compute logical AND of  $w_1$  and  $w_2$  and write the result to the right of the second  $c$ .

4. Logical OR. The tape initially contains  $w_1cw_2c$  where  $w_1$  and  $w_2$  are binary numbers. Your TM should compute logical OR of  $w_1$  and  $w_2$  and write the result to the right of the second  $c$ .

3. Show that the halting problem, the set of  $(M, w)$  pairs such that  $M$  halts when given input  $w$  is RE but not recursive. (5)

4. a) Suppose A and B are two problems in NP. Explain whether the following two statements are true or false:

i) If B is an NP-complete problem and we find a polynomial time reduction from A to B, then the problem A is also NP-complete.

ii) If B is an NP-complete problem and we find a polynomial time reduction from B to A, then the problem A is also NP-complete.

b) Explain why if you find a polynomial time algorithm for an NP-complete problem, it implies  $P=NP$ . (5)

Submission instructions:

Submit the two \*.tm files and the solutions to problems 3 and 4 in a zipped folder, named using your student ID, through moodle.