Lab 02 Report

Circuit Theory and Eletronic's Fundamentals

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March 23, 2021

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1 Introduction

With this project, we aimed to better understand RC circuits, and study how they respond under various scenarios. In order to achieve our goals, we focused our attention in the circuit shown bellow

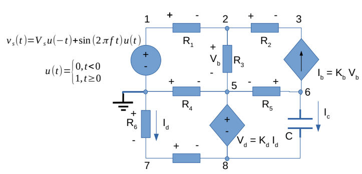

Figure 1: Studied Circuit.

We then began by analysing the circuit by the now familiar nodal analysis method and compared it to the results obtained with the software *ngspice*, when the voltage source outputted a constant voltage. Following that, we than turned our attention, to analysing the system under a sinusoidal voltage input. Thus the system was studied at the instant where the voltage was changed, and then the general response of the system was obtained both theoretically and through simulations, by adding the natural and forced solution of the system for a given driving frequency. At last, we concluded our survey, by examining the response of the system for a wide range of driving frequencies, and plotting both the magnitude and phase of the voltages obtained at the terminals of the capacitor. At every step, the theoretical results were compared with the data obtained from simulating the circuit.

2 Theoretical Analysis

Similar to what was done on the simulation analysis, we began by determining the voltages in all the nodes, and currents in all the branches, for $t < 0$. That amounts to $v_s = V_s$, and therefore, to there being no changes in the currents in all the branches for that time interval, and thus, the capacitor behaving as a open circuit. To do this analysis, we used the nodal method to obtain all the equations necessary, and then used Octave to get the numerical results. Octave was also used during the rest of this Theoretical Analysis.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{R_1} & \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & \frac{-1}{R2} & \frac{-1}{R_3} & 0 & 0 & 0 \\ 0 & \frac{1}{R_2}+K_b & \frac{-1}{R_2} & -K_b & 0 & 0 & 0 \\ \frac{-1}{R1} & \frac{1}{R_1} & 0 & \frac{1}{R_4} & 0 & \frac{1}{R_6} & 0 \\ 0 & 0 & 0 & 1 & 0 & \frac{K_d}{R_6} & -1 \\ 0 & K_b & 0 & \frac{-1}{R_5}-K_b & \frac{1}{R_5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{R_6}-\frac{1}{R_7} & \frac{1}{R_7} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{pmatrix} = \begin{pmatrix} V_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (1)$$

V_i	Value [V]
V_1	5.134559 V
V_2	4.871563 V
V_3	4.321260 V
V_4	4.908647 V
V_5	5.722227 V
V_6	-1.876518 V
V_7	-2.800822 V

Table 1: Nodal analysis results for $t < 0$ s

We then determined the equivalent resistance R_{eq} as seen from the capacitor terminals. This is achieved by making $V_s = 0$ and replacing the capacitor by a voltage source $V_x = V(6) - V(8)$, where $V(6)$ and $V(8)$ are the previously obtained voltages on node 6 and node 8, respectively. Then, we ran nodal analysis to determine the current I_x supplied by the voltage source V_x. This way, it was possible to compute $R_eq = \frac{V_x}{I_x}$. (explicar porque é que é possível fazer isso) We obtained the following results:

$$\begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & \frac{-1}{R2} & \frac{-1}{R_3} & 0 & 0 & 0 & 0 \\ \frac{1}{R_2}+K_b & \frac{-1}{R_2} & -K_b & 0 & 0 & 0 & 0 \\ \frac{1}{R_1} & 0 & \frac{1}{R_4} & 0 & \frac{1}{R_6} & 0 & 0 \\ 0 & 0 & 1 & 0 & \frac{K_d}{R_6} & -1 & 0 \\ K_b & 0 & \frac{-1}{R_5}-K_b & \frac{1}{R_5} & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & \frac{-1}{R_6}-\frac{1}{R_7} & \frac{1}{R_7} & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ I_x \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_x \end{pmatrix} \quad (2)$$

With these results, we determined the equivalent differential equation for v_6, and solved it using V_x as an initial condition. This way, we obtained the natural solution $v_{6n}(t)$, for the node voltage v_6, in the interval $[0, 20]$ms.

$$8.523048798e^{-0.32100971396t}$$

The next step to obtain the full solution for $v_6(t)$ is to obtain the forced solution $v_{6f}(t)$, in that same time interval. With that objective in mind, we started by replacing the capacitor for its equivalent impedance $Z_c = \frac{1}{j\omega C}$ That way, we can obtain equations that are similar to (fazer

Name	Value [V or mA or kOhm or ms]
V_2	0.000000 V
V_3	0.000000 V
V_5	0.000000 V
V_6	8.523049 V
V_7	-0.000000 V
V_8	-0.000000 V
I_x	2.833761 mA
R_{eq}	3.007681 mA
τ	3.115171 ms

Table 2: Nodal analysis resutls for $t < 0$s and $V_s = 0$V

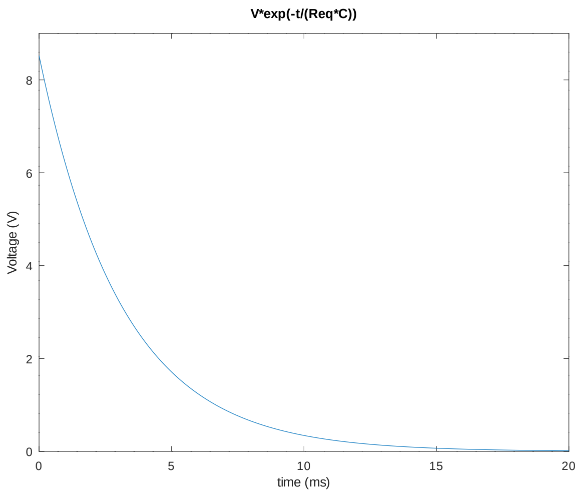

Figure 2: Natural solution

referência às primeiras equações da análise), by again running a nodal analysis of the circuit. By making $V_s = 0$, and solving this system of equations, we obtain phasor voltages for all the nodes. In this case, $f = 1$ kHz.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{R_1} & \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & \frac{-1}{R2} & \frac{-1}{R_3} & 0 & 0 & 0 \\ 0 & \frac{1}{R_2}+K_b & \frac{-1}{R_2} & -K_b & 0 & 0 & 0 \\ \frac{-1}{R1} & \frac{1}{R_1} & 0 & \frac{1}{R_4} & 0 & \frac{1}{R_6} & 0 \\ 0 & 0 & 0 & 1 & 0 & \frac{K_d}{R_6} & -1 \\ 0 & K_b & 0 & \frac{-1}{R_5}-K_b & \frac{1}{R_5}+jC\omega & 0 & -jC\omega \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{R_6}-\frac{1}{R_7} & \frac{1}{R_7} \end{pmatrix} \begin{pmatrix} \tilde{V_1} \\ \tilde{V_2} \\ \tilde{V_3} \\ \tilde{V_5} \\ \tilde{V_6} \\ \tilde{V_7} \\ \tilde{V_8} \end{pmatrix} = \begin{pmatrix} V_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{3}$$

Name	**Value [V]**
$abs(V_1)$	1.000000 V
$abs(V_2)$	0.948779 V
$abs(V_3)$	0.841603 V
$abs(V_5)$	0.956002 V
$abs(V_6)$	0.547734 V
$abs(V_7)$	0.365468 V
$abs(V_8)$	0.545484 V

Table 3: Nodal analysis resutls for $t < 0$s and $V_s = 0$V

To finally obtain the forced solution $v_{6f}(t)$, we converted the phasors to real time solutions of the form $v_{6f}(t) = A\,cos(\omega t + \phi)$: the norm of the phasor corresponding to the amplitude A of the solution and the the argument corresponding to the phase ϕ. We obtained the following solution for $v_{6f}(t)$ on time interval $[0, 20]$ ms:

$$0.54773358245 \sin(6.28318530717958623199592693708847t - 2.9865432432)$$

The total solution $v_6(t)$ is computed by summing the two solutions obtained beforehand, the natural solution $v_{6n}(t)$ and the forced solution $v_{6f}(t)$.

$$0.54773358245 \sin(6.28318530717958623199592693708847t - 2.9865432432) + 8.523048798e^{-0.32100971396t}$$

We finally explored the frequency responses (transfer function) $v_c(f) = v_6(f) - v_8(f)$ to the frequency f of the voltage source.

$$\frac{-2.58398734354048 \cdot 10^{72} if + 2.69716941607415 \cdot 10^{71}}{4.73705055795528 \cdot 10^{72} if + 2.4201725243886 \cdot 10^{71}}$$

$$\frac{3.92318858461668 \cdot 10^{56} if + 4.0173358562871 \cdot 10^{71}}{4.73705055795528 \cdot 10^{72} if + 2.4201725243886 \cdot 10^{71}}$$

The following plots were obtained:

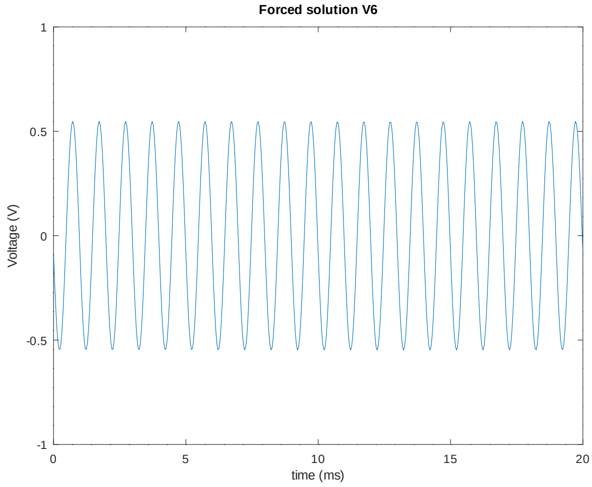

Figure 3: Forced solution $t > 0$s

3 Simulation Analysis

Using the ngspice program, we began by analysing the circuit for $t < 0$, the currents and voltages at each node are summarized in the table below.

Then we proceeded to calculate the tensions V_6 and V_8, for time $t = 0s$, with an applied tension of $0V$, and where the capacitor was substituted by a independent voltage source which imposes a voltage equal to $V_6 - V_8$. This is indeed, necessary to establish the boundary conditions for the system, for $t > 0s$, since in the instant $t = 0s$ the applied tension is $0v$, thus by performing the operating point analyses for the conditions described above, the initial values of the aforementioned tensions may be obtained. The capacitor is replaced by a voltage source because at that instant for our purposes the tension at each of its terminals is constant since we are holding time still. The results that were obtained are shown bellow

Using the results listed above, we then simulated the natural response of the circuit, imposing that the initial voltage at node 6 be equal to what we obtained beforehand. To accomplish it, we set the independent voltage source to $0V$ and ,as was expected, the capacitor discharged, following a negative exponential model, as is shown

We then, repeated the same process but this time for the general solution, consisting on the natural and the forced solutions, with the same initial conditions as before. We expected to observe a sinusoidal signal of the same frequency of the applied tension, supper imposed with the previous exponential solution, which as we can see was obtained

At last, we simulated the systems response to several different driving frequencies ranging from 0.1Hz to 1MHz, and the resultant magnitude and phase of the ensuing voltages

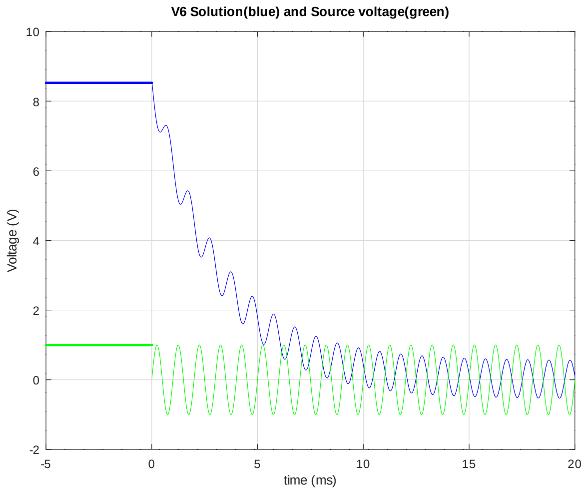

Figure 4: Forced solution $t > 0$s

(V_6, V_S, V_C) where plotted, the first the magnitude was expressed in decibels, the second in degrees, and in both cases the x scale is logarithmic. The graphs obtained follow

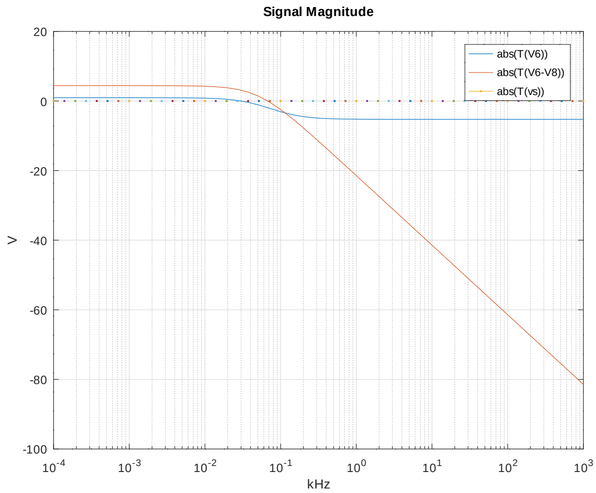

Figure 5: Forced solution $t > 0$s

4 Conclusion

With this paper, we were able to see how the capacitor behaves when a constant voltage is supplied ans well as time varying, in the matter at hand, a sinusoidal signal. As we stated before the theoretical predictions made, were in close agreement with the simulation results obtained. Any differences obtained may be attributed to numerical errors or approximations committed while carrying out the maths, since this deviations were never greater than 1% of the results obtained analytically. As stated before, the operating point analysis supported the results attained using nodal analysis. The simulated natural response closely resembled the exponential solution derived from first principles, and so did the general solution as we saw from the resemblance of both graphs. At last, the transfer function derived in the last section, was a close match to the plot obtained from *ngspice*, allowing us to verify that high frequencies tend to have significantly lower voltage magnitudes than lower frequencies at the terminals of the capacitor, thus as said before the system behaves as low pass filter.

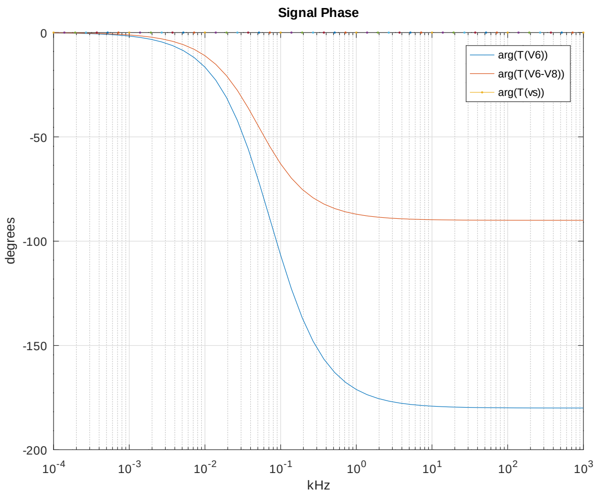

Figure 6: Forced solution $t > 0$s

Name	Value [A or V]
@r1[i]	-2.58478e-04
@r2[i]	2.705006e-04
@r3[i]	-1.20225e-05
@r4[i]	-1.17739e-03
@r5[i]	2.705006e-04
@r6[i]	9.189098e-04
@r7[i]	9.189098e-04
@c[i]	0.000000e+00
v(1)	5.134559e+00
v(2)	4.871563e+00
v(3)	4.321260e+00
v(4)	-1.87652e+00
v(5)	4.908647e+00
v(6)	5.722227e+00
v(7)	-1.87652e+00
v(8)	-2.80082e+00

Table 4: Operating point. A variable preceded by @ is of type *current* and expressed in mA; other variables are of type *voltage* and expressed in Volt.

Name	Value [A or V]
@r1[i]	2.032426e-18
@r2[i]	-2.12696e-18
@r3[i]	9.453332e-20
@r4[i]	-4.26077e-19
@r5[i]	2.833760e-03
@r6[i]	0.000000e+00
@r7[i]	-1.76599e-18
v(1)	0.000000e+00
v(2)	2.067951e-15
v(3)	6.395007e-15
v(4)	0.000000e+00
v(5)	1.776357e-15
v(6)	8.523047e+00
v(7)	0.000000e+00
v(8)	1.776357e-15

Table 5: Operating point for $V_s = 0$V and $t = 0$s. A variable preceded by @ is of type *current* and expressed in mA; other variables are of type *voltage* and expressed in Volt.

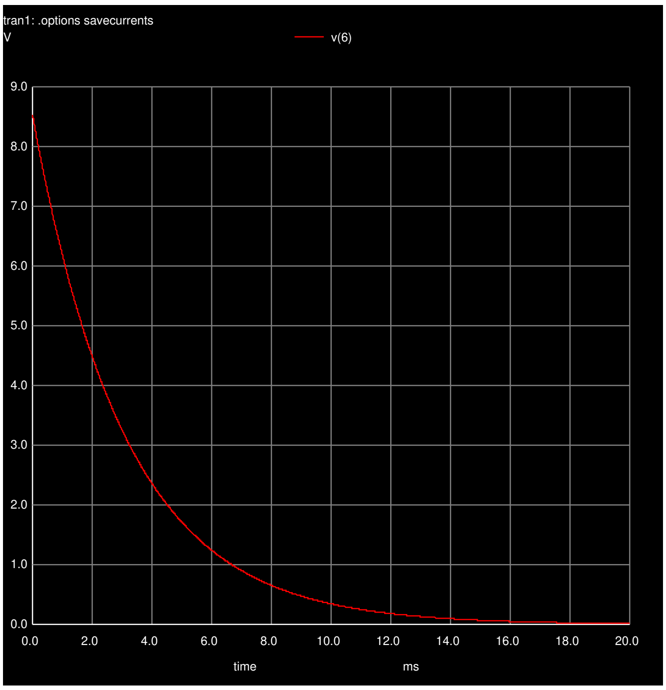

Figure 7: Natural solution for $t > 0$ s

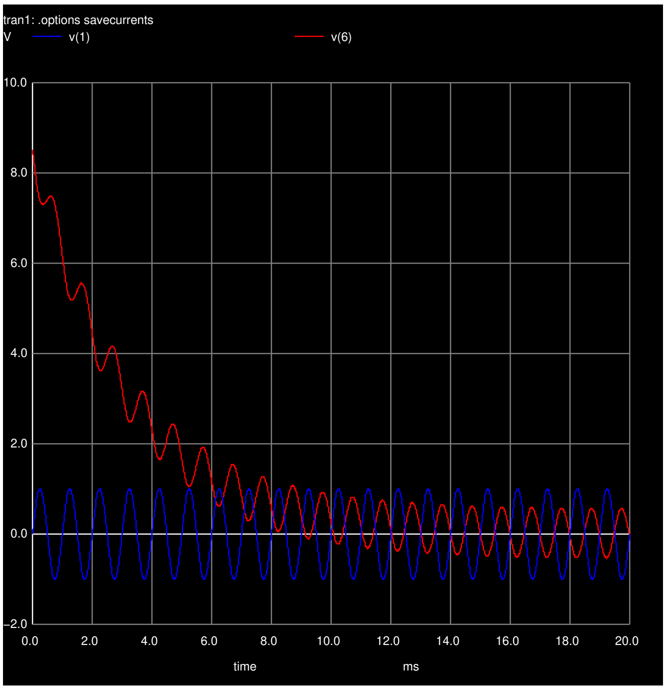

Figure 8: Full solution for $t > 0$ s

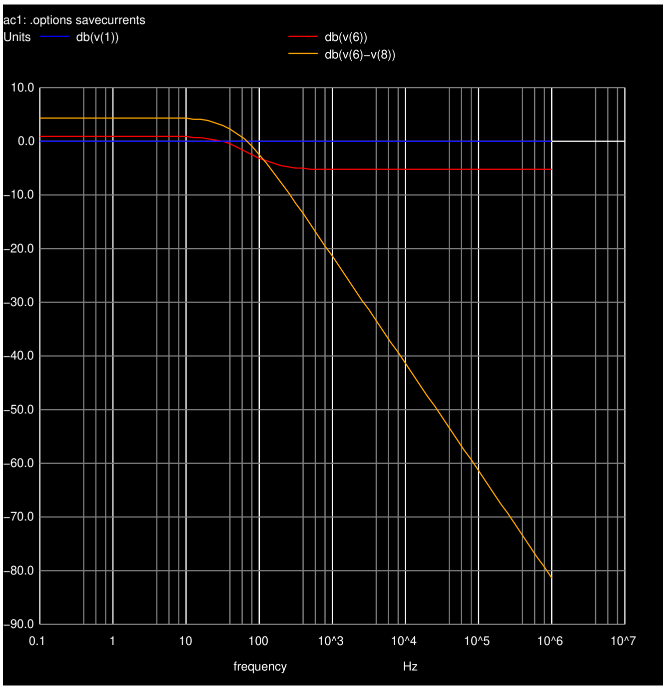

Figure 9: Circuit's amplitude response

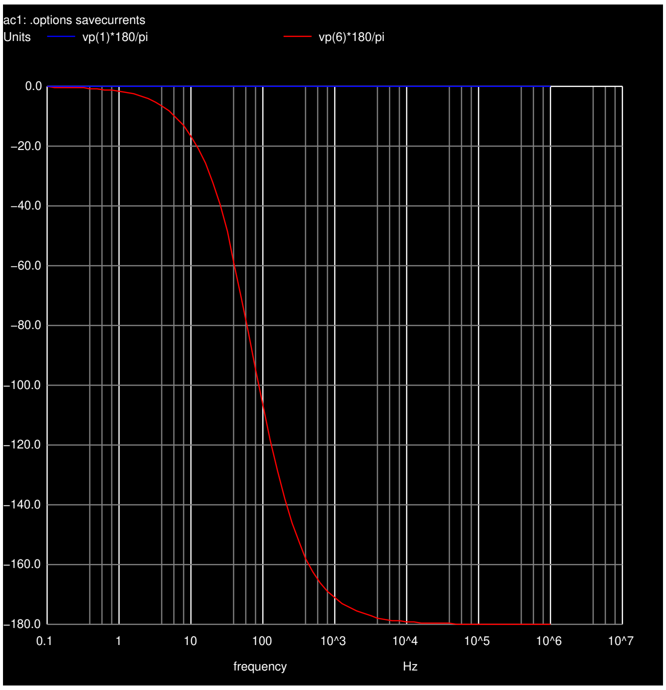

Figure 10: Circuit's phase response