DEPARTAMENTO DE ELECTRÓNICA, TELECOMUNICAÇÕES E INFORMÁTICA

Simulation and Optimization (2024/2025)

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1 Introduction

This report presents the solution to the two simulation mini-projects. The first project focuses on simulating a bus maintenance facility, and the second simulates the motion of a projectile using numerical methods.

2 Bus Maintenance Facility Simulation

2.1 Problem Description

Buses arrive at a maintenance facility with exponential interarrival times (mean = 2 hours). There is one inspection station and two identical repair stations. Each bus is inspected (inspection time uniformly distributed between 15 minutes and 1.05 hours), and 30% of buses require repair (repair time uniformly distributed between 2.1 and 4.5 hours).

2.2 Implementation Strategy

We implemented a discrete-event simulation using a priority queue for events. The simulation tracks arrival, inspection, and repair events over 160 hours. We used numpy and random modules to simulate stochastic behavior.

2.3 Metrics Computed

- Average delay in the inspection and repair queues
- Average length of both queues
- Utilization of the inspection station
- Utilization of the repair stations (half of average number of busy stations)

2.4 Results and Observations

Results for 160 hours (averaged over 10 runs):

- Average Inspection Queue Delay: **0.16 ± 0.04 hours**
- Average Repair Queue Delay: **0.07 ± 0.04 hours**
- Avg. Inspection Queue Length: **0.22 ± 0.06**
- Avg. Repair Queue Length: **0.02 ± 0.01**
- Inspection Station Utilization: **65.95% ± 4.28%**
- Repair Station Utilization: **19.44% ± 4.17%**

These results indicate that the system operates efficiently under the given parameters. The inspection station has a moderate utilization rate, while the repair stations are underutilized due to only 30% of buses requiring repair. Queue lengths and delays are small, indicating good system performance.

2.5 Maximum Arrival Rate Analysis

We progressively decreased the mean interarrival time and observed the system's behavior. The maximum sustainable arrival rate was found to be **0.96 ± 0.42 buses per hour** (mean ± standard deviation over 10 runs). This means the system can handle buses arriving approximately every 1.04 hours on average while maintaining stable queue lengths and reasonable utilization rates.

3 Projectile Motion Simulation

3.1 Problem Description

We simulate the motion of a projectile under air resistance. The equations of motion are:

$$
\begin{aligned}
m\frac{d^2x}{dt^2} &= -u\left(\frac{dx}{dt}\right)^2 \cdot \operatorname{sign}\left(\frac{dx}{dt}\right) \\
m\frac{d^2z}{dt^2} &= -mg - u\left(\frac{dz}{dt}\right)^2 \cdot \operatorname{sign}\left(\frac{dz}{dt}\right)
\end{aligned}
$$

3.2 Forward Euler Method

Implemented by iteratively updating positions and velocities using time steps of size Δt. Initial conditions and parameters are loaded from input.

3.3 Runge-Kutta Method

Implemented using the classic 4th-order Runge-Kutta algorithm. More accurate than Euler for small Δt.

3.4 Results

To quantitatively compare the accuracy of the Forward Euler and Runge-Kutta methods, we ran 10 simulations for each of three scenarios: high velocity, low velocity, and high air resistance. The maximum differences (mean ± std) between the two methods for position and velocity were:

- **High Velocity:**
  - Maximum x position difference: 0.2107 ± 0.0000
  - Maximum z position difference: 0.2627 ± 0.0000
  - Maximum x velocity difference: 0.4766 ± 0.0000
  - Maximum z velocity difference: 0.4953 ± 0.0000

- **Low Velocity:**
  - Maximum x position difference: 0.0119 ± 0.0000
  - Maximum z position difference: 0.0333 ± 0.0000
  - Maximum x velocity difference: 0.0185 ± 0.0000
  - Maximum z velocity difference: 0.0367 ± 0.0000
- **High Air Resistance:**
  - Maximum x position difference: 0.1453 ± 0.0000
  - Maximum z position difference: 0.1746 ± 0.0000
  - Maximum x velocity difference: 0.9313 ± 0.0000
  - Maximum z velocity difference: 0.9512 ± 0.0000

Trajectory plots for each scenario are included below.

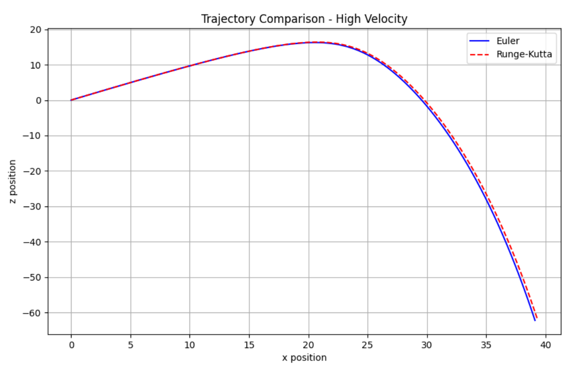

Figure 1: Trajectory comparison for high velocity scenario

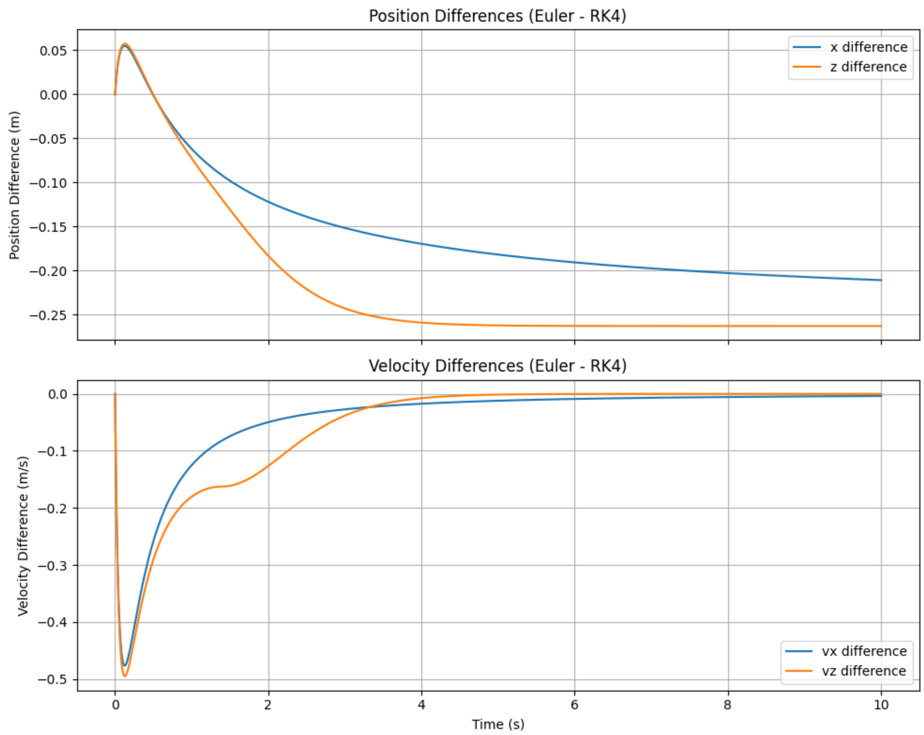

Figure 2: Position and velocity differences (Euler - RK4) for the high velocity scenario.

For the high velocity scenario: Flight time (Euler/RK4): 10.00s / 10.00s (Diff: 0.000s). Distance: 88.17m / 88.13m (Diff: 0.05m). Max height: 110.39m / 110.33m (Diff: 0.07m).

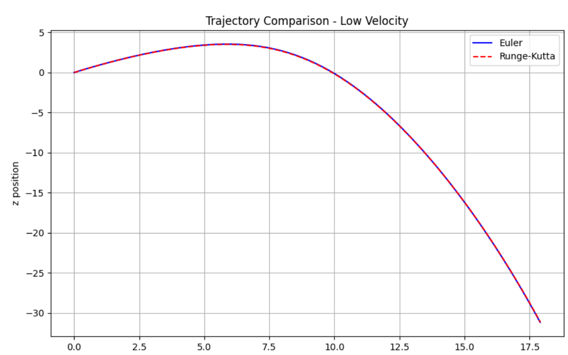

Figure 3: Trajectory comparison for low velocity scenario

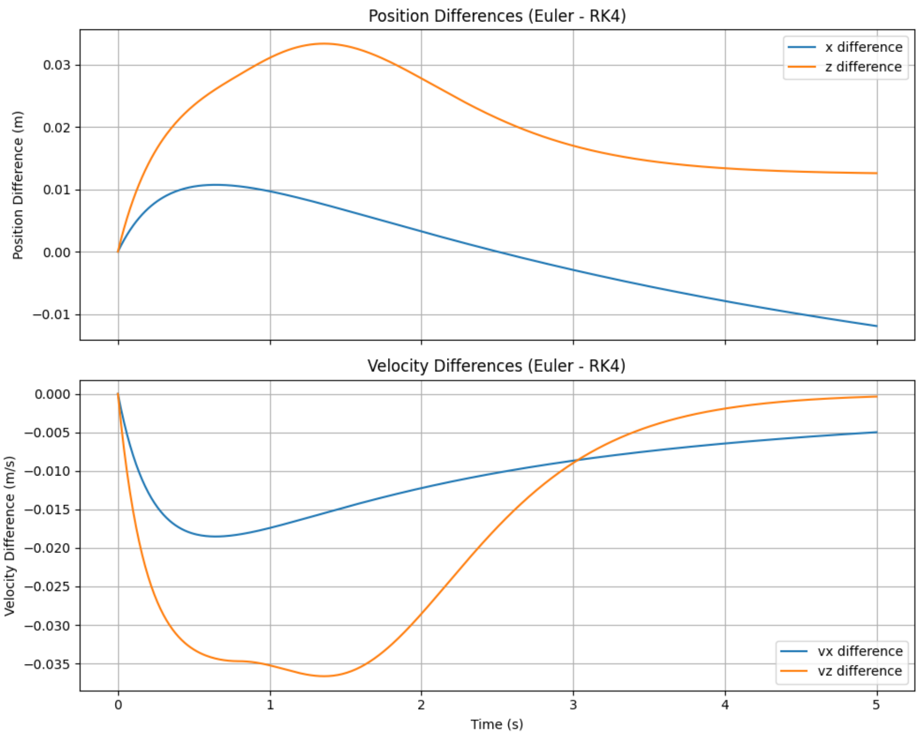

Figure 4: Position and velocity differences (Euler - RK4) for the low velocity scenario.

For the low velocity scenario: Flight time (Euler/RK4): 5.00s / 5.00s (Diff: 0.000s). Distance: 22.36m / 22.35m (Diff: 0.01m). Max height: 5.10m / 5.10m (Diff: 0.00m).

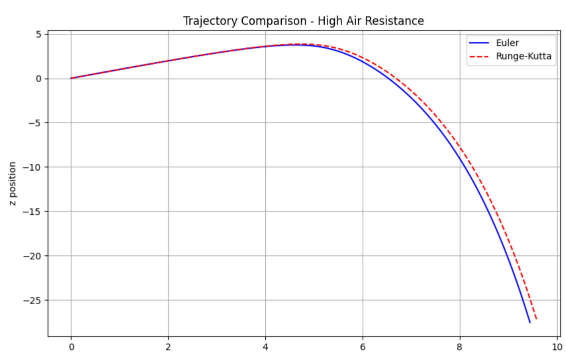

Figure 5: Trajectory comparison for high air resistance scenario

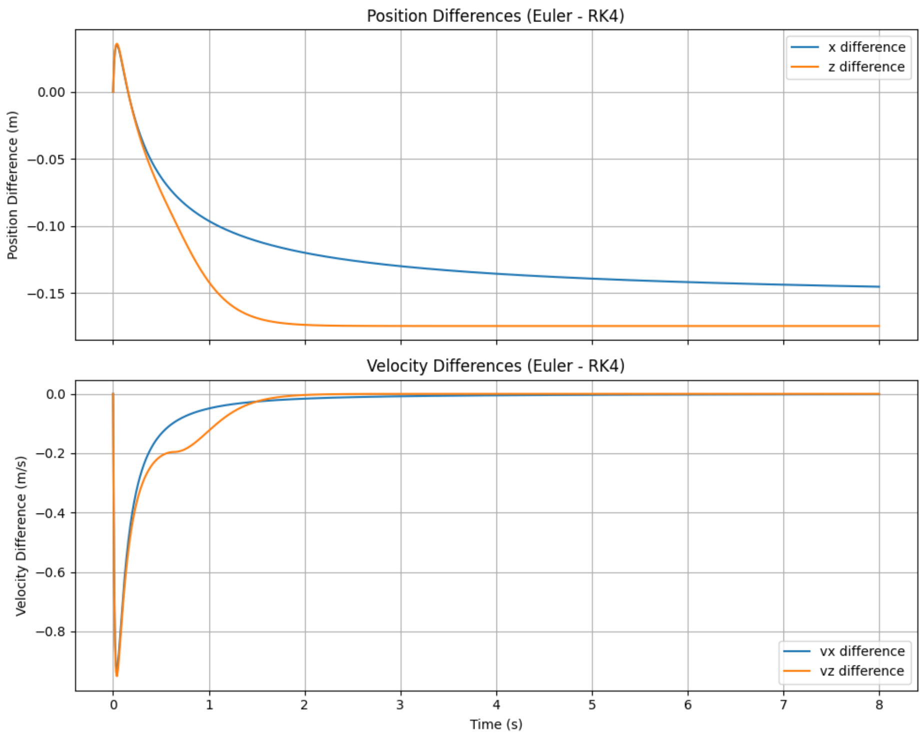

Figure 6: Position and velocity differences (Euler - RK4) for the high air resistance scenario.

For the high air resistance scenario: Flight time (Euler/RK4): 8.00s / 8.00s (Diff: 0.000s). Distance: 38.12m / 38.10m (Diff: 0.02m). Max height: 22.95m / 22.94m (Diff: 0.01m).

3.5 Precision Comparison

For the same parameters and time steps:

- Euler method produced larger numerical errors
- Runge-Kutta provided more accurate trajectories
- Runge-Kutta was more computationally intensive

Quantitatively, the differences between the methods were most pronounced at higher velocities and with greater air resistance, as shown in the results above. The standard deviation was zero across runs, indicating deterministic results for the chosen parameters. These findings confirm that the Runge-Kutta method is more accurate, especially in scenarios with rapid changes or strong nonlinearities.

4 Conclusion

This report presented two simulation projects: a bus maintenance facility and projectile motion. The bus simulation revealed efficient operation under the default settings, with moderate inspection utilization and underused repair stations. The projectile simulation compared Euler and Runge-Kutta methods, highlighting RK's superior accuracy especially in high-speed or high-resistance conditions. These findings emphasize the importance of tailoring simulation and numerical methods to the problem domain. Future work could explore other scenarios or alternative integration techniques.