

Interval Estimation Problems

1. The production manager of a frozen-foods packaging plant must maintain strict controls over the package weights in order to satisfy state licensing requirements. She now wishes to estimate the mean weight of restaurant packs of frozen corn at a confidence level of 95%. A simple random sample of 100 packs reveals a mean weight of 10.1 pounds. The packaging machine tolerances are such that the process standard deviation is 0.50 pounds. Establish the 95% confidence limits.
2. An operations analyst for an airline company has been asked to develop a fairly accurate estimate of the average refueling and baggage handling time at a foreign airport. The estimate is to be used to arrange flight schedules so that the airline has a very high percentage of “on-time” departures. Suppose the analyst randomly samples refueling and baggage handling times on 36 flights and finds they average 24.2 minutes. The population standard deviation is 4.2 minutes.
 - a) Establish a 90% confidence interval estimate. Interpret the meaning of this interval.
 - b) Find the 99.7% confidence interval estimate using the same data.
 - c) Compare the widths of the two intervals. What do you conclude about the precision when the confidence level increases?
3. The personnel manager of a small company wants to estimate the mean number of days of absence per year per employee. It can be assumed that this variable follows a normal distribution with a standard deviation of 2.5 days per employee. His sample is the record of absenteeism for a random selection of 40 employees taken from work records over the past several years. The data reveals an average of 5 days absences per employee for this sample of 40 employees.
 - a) Establish a 90% confidence interval for the mean number of absent days per employee.
 - b) What sample dimension will be necessary if the personnel manager wants the difference between the parameter and the estimate not to be larger than 0.5 days with a probability of 0.99?

(2nd Exam 23rd June 2009)
4. The Texas Chemicals Company has developed a new liquid solution that is supposed to increase the useful lifetime of auto batteries. In a test on 16 randomly selected batteries, the average lifetime was increased by 11.00 months with a standard deviation of 6.00 months. Assuming that the increase in lifetimes is a normal random variable, establish a 99% confidence estimate of the true increase in lifetime. What increase in months of lifetime can reasonably be expected from the use of this solution?
5. The Chamber of Commerce in a college town is anxious to estimate the average amount of money that university students spend in at the local economy in any given year. If the amount is high enough, they will promote advertising that should appeal to students. The researcher would have to specify values for the confidence level and the error margin, EM, in this situation. Suppose these were set at 95% confidence and $EM = \pm \$50.00$. The standard deviation of the amount in dollars spend by individual students is known from previous studies and is \$256.00.
 - a) How many students should be surveyed?

- b) Suppose that a Chamber of Commerce official wanted lower error margin - assume the manager wants an estimate for the mean amount spend by these students with a maximum error of only \$25.00. How many students should be surveyed to provide an estimate that is twice as precise as the first one?
- c) Suppose now we want to increase confidence level to 99%, while the allowable error is unchanged, in the estimate of the average amount spend by the university students. What is the required sample size?
6. Consider a confidence interval, with confidence level λ , for the mean of normal distribution with known variance σ^2 , based on a random sample of n observations. How does the width of the interval change
- as n is increased, keeping σ^2 and λ fixed?
 - as σ^2 is increased, keeping n and λ fixed?
 - as λ is decreased, keeping n and σ^2 fixed?
- The standard deviation of the lifetime of a certain type of electrical component is 144 hours. How large a sample of the components must be taken to be (a) 95% (b) 99% confident that the error margin in the estimated mean lifetime of such components will not exceed (i) 15 hours, (ii) 20 hours?
7. A political candidate has retained a polling agency to determine her voter strength. At the time of its most recent poll, the agency found that 225 of 400 registered voters would vote for this candidate. Assuming the agency took a random sample, what percent of the electorate favored the candidate at the time of this poll? Answer the question using a 95% confidence interval.
8. The personnel manager of a large firm wants to find the percentage of employee absences per day in order to determine a standard against which to gauge periods of excessive absenteeism. His sample is the record of absenteeism for a random selection of 500 employee-days taken from work records over the past several years. The data reveals 30 absences for the sample of 500 employee-days.
- Establish a 90% confidence interval for the percentage of employee absences per day.
 - What sample will be necessary for the personnel manager to estimate the true percentage of absenteeism for his firm to within 1% with 90% confidence?
9. The advertising manager for a manufacturer of self-teaching language cassettes wants to know the market share (percentage of total consumer sales) for the company's language cassettes. The company has allocated \$3000 for this project, and the manager wants an estimate to within $\pm 2\%$ of its actual market share. Overhead expenses for the project are \$500, and each consumer interview costs \$1. A confidence of 95% is required. Can their market share be estimated from a random sample of language cassette customers within the required precision for \$3000?
10. From the results of a random sample of 100 university students the following confidence interval was constructed for the percentage of students favourable to abortion practices:] 54%; 66%[. What is the confidence level associated to the previous interval?
11. Teletex Communications is closing a plant in Dallas and offers to move its employees to a new facility in Honolulu. However, an employee union is also insisting on a cost-of-living adjustment for food. To determine the cost difference, the Teletex personnel manager has obtained published data describing the prices for a random selection of foods, including milk, sugar, peanut butter, orange juice, and ground beef.
- The data are summarized in the table. The personnel manager wishes to develop an interval estimate of the difference in average costs, so that she can work within an acceptable range to

negotiate an adjustment for cost-of-living with the company. Estimate the mean cost difference at 90% confidence.

	Honolulu	Dallas
Mean cost of food items	$\bar{x}_1 = \$2842$	$\bar{x}_2 = \$2323$
Known standard deviation	$\sigma_1 = \$337$	$\sigma_2 = \$218$
Sample sizes used	$n_1 = 31$	$n_2 = 36$

12. a) On a particular day a random sample of 12 tins of peas is taken from the output of a canning factory, and their contents are weighed. The mean and standard deviation of weight for the sample are 301.8 gm and 1.8 gm respectively. Find the 99% confidence limits for the mean weight of peas in tins produced by the factory on the day in question.
 b) On the following day a further random sample of 12 tins is taken, and the mean and standard deviation of contents for this sample are 302.1 gm and 1.6 gm respectively. Assuming that the variance of the weights is the same on two days, show that a 95% confidence interval for the difference between mean weights on the two days includes zero.
13. A consumer test group wants to determine the difference in gasoline mileage of cars using regular unleaded gas and cars using premium unleaded gas. Researchers for the group divided a fleet of 100 of the same make of cars in half and tested each car on one tank of gas. Fifty of the cars were filled with regular unleaded gas, and 50 were filled with premium unleaded gas. The sample average and the standard deviation were 21.45 mpg and 3.46 mpg, respectively, for the regular gasoline group and were 24.6 mpg and 2.99 mpg, respectively, for the premium gasoline group. Construct a 95% confidence interval to estimate the difference in the mean gas mileage between the cars using regular gasoline and the cars using premium gasoline.
14. Sue Spencer was recently promoted to quality-control manager of Space Population Systems, Inc.. She had been on the new job less than one week when her boss asked her to explain why some customers appeared to be receiving a higher proportion of defective castings than others. Sue suspected a difference in the proportion of defectives by shift, and she knew that the production from each shift went to a different location. So, she had a quality-control inspector randomly sample 320 castings produced on a day shift and 200 produced on the swing shift. The results were 45 defectives in the day-shift sample and in the swing-shift sample. Estimate the difference in the proportion of defectives using (a) a 95% level of confidence and (b) an 80% level of confidence. What answer does this provide to Sue for her boss?
15. The time to repair a machine follows a normal distribution with mean μ and variance σ^2 . From a sample with dimension $n=9$, a value of the corrected standard deviation of 0.3 hours was obtained.
 a) Construct a 95% confidence interval for the population variance.
 b) What is the probability of the variance σ^2 being included in the previous interval? Why?
 c) The following interval for the population mean μ was obtained:

$$I_{\lambda, \mu}^* =]0,814; 1,186[$$

 Which is the confidence level associated with this interval?
 d) If a decrease in the sample dimension occurs simultaneously with a decrease in the confidence level, what will happen to the precision level of the confidence interval estimated in c)? Justify adequately your answer. (1st Exam 31st May 2008)
16. The manager of a bottling plant is anxious to reduce the variability in net weight of fruit bottled. Over a long period the standard deviation has been 15.2 gm. A new machine is introduced, and the net weights (in grams) in randomly selected bottles (all of the same nominal weight) are
 987, 966, 955, 977, 981, 967, 975, 980, 953, 972

Would you report to the manager that the new machine has a better performance? Answer to the problem using a 95% confidence interval for σ^2 .

17. In a study about the age distribution of university management students, a sample of dimension 25 was collected. The results are presented in the following table:

Table 1

		Statistic	Std. Error
Idade	Mean	21,40	a
95% Confidence Interval for Mean		Lower Bound	b
		Upper Bound	c
5% Trimmed Mean		21,23	
Median		21,00	
Variance		5,667	
Std. Deviation		2,380	
Minimum		18	
Maximum		28	
Range		10	
Interquartile Range		3	
Skewness		1,193	,464
Kurtosis		1,233	,902

Complete Table 1, by calculating the values of **a**, **b** and **c**, which are missing. Interpret the meaning of the estimated confidence interval.

(1st Test 14th April 2007)

18. A random sample of 10 dairy cows is taken from a large herd at Farm A, and the weekly milk yield, in kg, is recorded for each cow in the sample. Similar measurements are made for a random sample of 15 cows from a large herd at Farm B. The sample mean and sample variance for Farm A are 142 kg and 440 kg², whereas the sample mean and sample variance for the sample of cows from Farm B are 169 kg and 811.24 kg² respectively.
- Find a 95% confidence interval for the variance of milk yield at Farm A.
 - Repeat part a) for Farm B and discuss carefully, without doing any further formal analysis, whether the variances at the two farms could be equal.
 - Confirm the discussion made in part b), using a 95% confidence interval for σ_1^2 / σ_2^2 .

19. Eight school children, chosen at random from the first year of a large school, were given, without prior warning, a mathematical task, and the time taken (in minutes) by each child to complete the task was recorded. The following day the children were instructed how to perform such tasks efficiently, and a week later they were tested again on a similar task. Once again, the time taken to complete the task was recorded for each child and the results were as follows.

Time taken (minutes)									
Child	1	2	3	4	5	6	7	8	
Before instruction	26	20	17	21	23	24	21	18	
After instruction	19	14	13	16	19	18	16	17	

Find a 90% confidence interval for the mean difference between times before and after instruction, for first year children.

20. A company has two factories, one in a rural county and another in an urban county. The urban county factory workers demand a transport allowance because, according to them, they spend more time travelling everyday between their homes and the factory. The company decided to investigate whether these workers were right and collected two samples of the travelling times from workers' home to the factories, obtaining the following results:

	Urban Factory	Rural Factory
Number of observations	360	450
Average transport time (minutes)	45	35
Variance of transport time (minutes ²)	1260	855

Construct a 95% confidence interval for the difference between the travelling mean times of the workers of the two factories and interpret the result. (1st Test 17th March 2010)

21. The quantity of Duque wine per bottle (measured in centilitres) is a random variable with normal distribution with unknown mean and variance. A random sample of 25 bottles of Duque wine gave a value of 0,0625 centilitres² for the corrected sample variance.

- a) Estimate the population standard deviation of the quantity of Duque wine in the bottles through a 90% confidence level.

In the past several consumers have complained about the quantity of Duque wine in the bottles, which, according to them, is less than 75 centilitres. The quality control manager decided to prove that the information on the bottles is correct and showed the following confidence interval, constructed from a random sample of 25 bottles:

$$]CI_{\lambda}[\mu =]74.8968; 75.1032[$$

- b) Which value would you propose as an estimate for the mean quantity of Duque wine in the bottles?
- c) Which is the error margin of the confidence interval calculated by the quality control manager?
- d) Determine the confidence level λ used by the quality control manager to get the interval above.
- e) The consumers association said: "We believe that only half of the Duque bottles contain a quantity of wine within the limits of the above interval". Comment about the truth of this statement.

(1st Test 19th March 2011)

22. A company selling cosmetics and bath products wants to estimate the proportion of potential clients interested in buying a new bath gel. The company assumes this bath gel can be consumed by most people, so the population of potential buyers is quite large.

- a) Define the population and the parameter being studied.
- b) Calculate the appropriate sample dimension, assuming that the company wants, for the sample results, a confidence level of 95% and an error margin not bigger than 2%.
- c) Suppose now that, based on the sample with the dimension calculated in question b), 1441 individuals declared they would consume regularly this new bath gel. Construct a 99% confidence interval for the true population proportion of consumers of the new bath gel and interpret the result. If you did not answer question b) consider $n=2402$.
- d) Compare the error margin for the interval constructed in question c) with the conditions defined in b).
- e) Which is the best estimate for the population variance? Why?

(1st Test 24th March 2012)

23. A transport company states that its Express bus trip takes on average 120 minutes from Lisbon to Faro with a variance not higher than 4 minutes². A costumers committee questions the reliability of this statement and decided to ask a market research team to carry out a study to decide if the company is right. The following results were obtained from the data collected from 20 bus trips from Lisbon to Faro:

$$\sum_{i=1}^{20} X_i = 2360 \text{ min.} \quad \sum_{i=1}^{20} (X_i - \bar{X})^2 = 60.192 \text{ min.}^2$$

Assuming that the bus trip duration is normally distributed:

- Calculate the unbiased estimates for the mean and variance of the bus trip duration from Lisbon to Faro.
- Construct a 99% confidence interval for the true mean of the trip duration. Based on this result, who do you think is right, the company or the costumers committee?
- The market research team presented the following confidence interval for σ^2 :

$$]IC_{\lambda}[\sigma^2 =]1,999 ; 5,959[$$

Which was the confidence level λ used by the research team to construct the previous interval? Justify your answer.

If your answer to the previous question was negative, which changes would you propose to (1st Exam 4th June 2012)

24. 350 electors were asked about their voting intentions in the next elections for President. The following results were obtained for candidate A:

Number of votes in favor of candidate A	185
Number of votes in favor of other candidates	130
Do not know / Have not decided yet	35

Is it possible for candidate A to get absolute majority in the next elections? Answer the question with:

- A point estimate.
- A confidence interval with $\lambda = 0,95$. Interpret the result.
- The results from the previous question were communicated to candidate A campaign manager who made the following comment: "The results from this poll are highly reliable since the error margin is only $\pm 2,5\%$ ". Show if this comment is correct.
- the fieldwork manager to make the comment correct? Justify and present all the necessary calculations.

(2nd Exam 16th June 2008)

25. A hotel unit wants to estimate the probability of room reservation being canceled by the clients. A random sample of 300 reservations has been collected, and for each one, a registration has been made whether it was canceled or not.

- From the 300 reservations analyzed, 12 of them have been canceled.
 - Find an estimate for the population parameter and justify your choice.
 - Construct a 95% confidence interval for the population parameter and interpret the result.
 - The hotel manager stated recently in a public conference of the Portuguese Hotel Association that the percentage of canceled reservations is around 5%. Do you agree with his statement? Why?

c₄) If you wanted to reduce to half the error margin of the confidence interval obtained in question c₂), keeping everything else constant, how many reservations should be analyzed?
(1st Test 31th March 2014)

26. If you wanted to reduce the range of a 99% confidence interval for the parameter mean of a normal population with known variance σ^2 , without any change in the quality of the final results, which of the following options would you choose?

- ☐ a) increase the sample dimension and decrease the confidence level;
- ☐ b) increase the sample dimension and keep the confidence level;
- ☐ c) both answers a) and b) are correct;
- ☐ d) None of the previous answers.

(1st Test 31th March 2014)

27. The following confidence interval for the true mean age (in years) of the Portuguese electors was constructed from the results of a random sample of dimension 1010:

$$]IC_{\lambda}[\mu^* =]48,0683; 49,9317[$$

The mean age of the 1010 interviewed electors is:

- ☐ a) 48 years;
- ☐ b) 48,45 years;
- ☐ c) 49 years;
- ☐ d) None of the previous answers id correct.

(1st Test 31th March 2014)

28. The time spent to repair a machine, in minutes, is a random variable with normal distribution with parameters μ and σ^2 . A random sample of dimension $n=25$ was collected and the following results were obtained:

$$\sum_{i=1}^{25} x_i = 1500$$

$$\sum_{i=1}^{25} x_i^2 = 92160$$

- a) Is the sample mean an unbiased estimator for the mean time spent to repair that type of machine? Justify your answer.
- b) Calculate an unbiased estimate for the standard deviation of the time spent to repair that type of machine. Justify your answer.
- c) Construct a 99% confidence interval for the real mean time spent to repair that type of machine and interpret the result.
- d) Based on the collected sample, the following confidence interval was constructed for the population variance:

$$]IC_{\lambda}[\sigma^2^* =]54,822; 174,19[$$

Which is the confidence level associated to this interval?

(1st Exam 7th June 2014)

29. For quality control purposes, the manager of the Company Beer++ wants to investigate if the true quantity of beer contained in the cans is 33 centilitres as specified in the label. Consider that the quantity of beer in each can follows a normal distribution with mean μ and standard deviation $\sigma=2$.

a) Estimate the true mean of the quantity of beer in the cans with a 95% confidence interval. The results of a random sample of 25 cans show a mean of 32.5 centilitres. Interpret the result.

b) If you intended to reduce to half the range of the confidence interval, keeping the same confidence level, what would be the dimension of the sample to be collected? Justify your answer.

c) Suppose now the standard deviation σ is unknown.

c₁) Which estimator would you propose to estimate σ ? Why?.

c₂) Construct now a 99% confidence interval for μ using the same results from the sample of 25 cans. Compare the range of this interval with the one obtained in question a) and explain all possible differences between the two intervals.

(1st Test 13th March 2015)

30. The following confidence interval was presented for the mean weekly sales (m.u.) of a supermarket chain: $]IC_{0,95}[\mu^* =]148,5; 325,1[$. It is possible to conclude:

- ☐ a) It is expected that 95 out of 100 weeks will register mean weekly sales within that interval.
- ☐ b) The probability one week will show mean sales lower than 148,5 m.u. and higher than 325,1 m.u. is only 0.05.
- ☐ c) During the next year, exactly 95% of the weeks will register mean sales within that interval.
- ☐ d) None of the previous answers is correct.

(1st Test 13th March 2015)

31. A sports newspaper has carried out a survey to study the behaviour of its readers. One of the questions included in the questionnaire was the time (in minutes) spent to read the newspaper, which is assumed to follow a normal distribution with variance 225 minutes².

The following result was obtained from the collected sample:

$$\sum_{i=1}^{16} X_i = 1440$$

- a) Find an estimate for the time (in minutes) spent to read the newspaper. Justify your answer.
- b) Construct a confidence interval for the real mean time spent to read that newspaper with an error margin equal to 6.16875. Interpret the result.
- c) What is the confidence level associated with the previous interval? Show all the necessary calculations to answer this question.

(1st Exam 6th June 2015)

32. A poll was carried out by EuroSurvey, about the expected votes in the next parliamentary elections. The following 95% confidence interval was constructed for the population proportion of voters in the Socialist Party (PS):

$$]IC_{0.95}[p^* =]0.3385; 0.3995[$$

Technical file of the poll

Opinion survey carried out by EuroSurvey, for newspaper Expresso and SIC television, between the 4th and the 9th June 2015. Fixed phone interviews made by selected and supervised interviewers. The population includes all individuals aged 18 or more years, resident in the Portuguese mainland and belonging to households with fixed phone. The sample was random and stratified by region (.....), with a total of n valid interviews. (...) The error margin is 3.05%, for a confidence level of 95%.

- Comment the veracity of the following statement: 'The true population percentage of voters in PS, by the time of the poll, is between 33,85% and 39,95%, with a probability of 0.95'.
- Which estimate would you propose for the population percentage of voters in PS, by the time of the poll? Justify your answer.
- Calculate the sample dimension referred in the technical file (**n**) identifying all the assumptions needed to find that value.
- Suppose that EuroSurvey wanted to reduce the error margin to 2% and keep the same confidence level. What would be the change in the sample dimension? Answer the question without any additional calculations.

(2nd Exam 20th June 2015)

Solutions:

- $]I_{0.95}[p^* =]0.0; 10.2[$.
- The analyst should report an interval of 23.0 to 25.4 minutes at 90% confidence level.
 - Now the analyst could report an interval of 22.1 to 26.3 minutes at 99.7% confidence level.
 - For a fixed sample size, a higher confidence requires a wider confidence interval (higher error margin), so a higher confidence means lower precision.
- $]I_{0.90}[p^* =]4,35; 5,65[$; b) $n \geq 166$.
- The true mean increase in lifetime affected by this new liquid is estimated to be between 6.58 and 15.42 months at the 99% confidence. The company can advertise a 6-month extension of lifetime with the use of their new liquid battery solution.
- 101 students should be surveyed.
 - 403 students should be surveyed.
 - 175 students should be surveyed.
- a.i) $n \geq 355$; a.ii) $n \geq 200$; b.i) $n \geq 614$; b.ii) $n \geq 346$.

7. At the time of this polling, between 51% and 61% of the voters are expected to favor this candidate.
8. a) The personnel manager can use 4% to 8% as a gauge of the usual percentage of absences. Then, for example, an absence rate of 10% on a given day would indicate excessive absenteeism.
b) The manager needs 1527 randomly chosen employee-day records to estimate p to within $\pm 1\%$ at 90% confidence.
9. The desired precision can just be met. The consumer advertising section can support its survey.
10. $\lambda \approx 0.77$.
11. The personnel manager can be 90% confident that the procedure reasonably describes the difference in the mean food cost for these selected items. Something between \$403 and \$635 would be a reasonable cost-of-living adjustment for the average food costs in Honolulu (in excess) over Dallas, so the manager can reasonably negotiate a value within this range.
12. a) The limits are 300.19 gm and 303.41 gm.
b) The interval is $]-1.74, 1.14[$, which includes the zero.
13. The 95% confidence interval to estimate the difference in the mean gas mileage is $]-4.42, -1.88[$
14. If Sue is willing to risk a 20% chance of being wrong, she could say the defective rate on the day shift is 1.5% to 8.7% higher than on the swing shift. If Sue is willing to risk only a 5% chance of being wrong, she could say the defective rate on the day shift is from 0.4% lower to 10.6% higher than on the swing shift.
15. a) $I_{0,95}^*[\sigma^2] =]0.041; 0.330[$; c) $\lambda = 0.90$.
16. $I_{0,95}^*[\sigma] =]7.64; 20.28[$. This interval does include the standard deviation of net weights of the old machine, which was 15.2 gm, so we should not report that the new machine is better.
17. **a** = 0,476; **b** = 20,42; **c** = 22,38.
18. a) $I_{0,95}^*[\sigma_1^2] =]208.20; 1466.67[$.
b) $I_{0,95}^*[\sigma_2^2] =]438.81; 2017.29[$. There is a considerable amount of overlap between the intervals for σ_1^2 and σ_2^2 , so it seems plausible that the variances for the two farms could be the same.
c) The 95% confidence interval for σ_1^2 / σ_2^2 is $]0.17, 2.07[$, which includes the value $\sigma_1^2 / \sigma_2^2 = 1$, meaning that the two population variances might be equal.
19. $I_{0,90}^*[\mu_D] =]3.52; 5.98[$.
20. $I_{0,95}^*[\mu_1 - \mu_2] =]5.45; 10.55[$.
21. a) $I_{0,90}^*[\sigma] =]0.2030; 0.3297[$ b) $\bar{x}^* = 75cl$ c) 0.1032 d) $\lambda = 0.95$.

22. b) $n=2402$ c) $]I_{0,99}[\mu^* =]0.57425; 0.62575[$ d) The error margin has increased to 0.02575; since the confidence level is now higher, the confidence interval has a higher range and the precision is lower e) $\hat{\sigma}^{2*} = 0.24$.
23. a) $s'^{2*} = 3.168 \text{ min.}^2$ b) $]I_{0,99}[\mu^* =]16.861; 119.139[$ c) $\lambda=0.90$.
24. a) $\bar{x}^* = 0.52857$ b) $]I_{0,95}[p^* =]0.47627; 0.58087[$ c) The comment is not correct since the error margin is 0.0523 d) Minimum sample dimension of 1531 electors (keeping $\lambda=0.95$) would make the comment correct.
25. c₁) $\bar{x}^* = 0.04$ c₂) $]I_{0,95}[p^* =]0.017825; 0.062175[$ c₃) Yes, he might be right c₄) $n=1200$.
26. b)
27. c)
28. a) Yes b) $s'^{*} = 9.49 \text{ min}$ c) $]I_{0,95}[\mu^* =]54.69; 65.31[$ d) $\lambda = 0.95$
29. a) $]IC_{0,95}[\mu^* =]31,72; 33,28[$ – It is expected that the true mean quantity of beer in the cans is between 31,72cl and 22,28cl, since this specific confidence interval has been based on a random interval with 0.95 probability of including the true population mean, i.e., if we construct 100 intervals, 95 of them will include the population mean.
b) $n=100$.
- c₁) S' because S'^2 is an unbiased estimator for small samples, and is also sufficient.
c₂) If $S' = 1,402$ the two intervals will have the same range; if $S' > 1,402$ the new interval will have higher range; if $S' < 1,402$ the new interval will have lower range.
30. a)
31. a) X: time readers take to read a sports newspaper per day (in minutes); $X \cap n (\mu; \sigma=15)$; the best estimate for μ is $\bar{x}^* = 90 \text{ minutos}$, since the sample mean is the best estimator for the population mean μ (is sufficient, unbiased, and the most efficient when compared to other unbiased estimators).
b) $]IC_{\lambda}[\mu^* =]83,83125; 96,16875[$ – It is expected that the true mean time readers take to read the sports newspaper is between 83.83 minutes and 96.17 minutes, since this specific confidence interval has been based on a random interval with λ probability of including the true population mean, i.e., if we construct 100 intervals, $100 \cdot \lambda$ of them will include the population mean.
c) 10%
32. a) The statement is false; the probability is related to the random CI and not to the population parameter.
b) X: Number of voters, in 1, that intend to vote PS; $X \cap B(p)$; the best estimate for p is $\bar{x}^* = 0,369$ since the sample mean is the best estimator for the parameter p of a Bernoulli population.
c) $n = 1033$ using the most pessimistic estimate for the population variance ($\hat{p} = 0,5$).
d) n would increase.