Formulas Statistics 2 - Pivotal Variables for Confidence Intervals and Test Statistics for Hypothesis Testing

	Parameter to estimate	Population(s) type	Sample size	Is σ^2 known?	Test statistic	Pivotal variable	Sampling distribution
1	μ	normal	any	yes	$\dfrac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$	$\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}}$	$\cap\, n(0,1)$
2	μ	any	n > 30	yes	$\dfrac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$	$\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}}$	$\dot{\cap}\, n(0,1)$
3	μ	normal	any	no	$\dfrac{\bar{X}-\mu_0}{S'/\sqrt{n}}$	$\dfrac{\bar{X}-\mu}{S'/\sqrt{n}}$	$\cap\, t_{(n-1)}$
4	μ	any	n > 30	no	$\dfrac{\bar{X}-\mu_0}{S'/\sqrt{n}}$	$\dfrac{\bar{X}-\mu}{S'/\sqrt{n}}$	$\dot{\cap}\, n(0,1)$
5	$\mu_1 - \mu_2$	normal	any	$(\sigma_1^2\,;\,\sigma_2^2)$ yes	$\dfrac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)_0}{\sqrt{\dfrac{\sigma_1^2}{n_1}+\dfrac{\sigma_2^2}{n_2}}}$	$\dfrac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1}+\dfrac{\sigma_2^2}{n_2}}}$	$\cap\, n(0,1)$
6	$\mu_1 - \mu_2$ (SPSS)	normal	any	$(\sigma_1^2\,;\,\sigma_2^2)$ No, but $\sigma_1^2 = \sigma_2^2$	$\dfrac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)_0}{\sqrt{\dfrac{(n_1-1)S'^2_1+(n_2-1)S'^2_2}{n_1+n_2-2}\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)}}$	$\dfrac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{\sqrt{\dfrac{(n_1-1)S'^2_1+(n_2-1)S'^2_2}{n_1+n_2-2}\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)}}$	$\cap\,\left(t_{n_1+n_2-2}\right)$
7	$\mu_1 - \mu_2$	any	$n_1 > 30$ and $n_2 > 30$	$(\sigma_1^2\,;\,\sigma_2^2)$ No, but $\sigma_1^2 = \sigma_2^2$	$\dfrac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)_0}{\sqrt{\dfrac{(n_1-1)S'^2_1+(n_2-1)S'^2_2}{n_1+n_2-2}\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)}}$	$\dfrac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{\sqrt{\dfrac{(n_1-1)S'^2_1+(n_2-1)S'^2_2}{n_1+n_2-2}\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)}}$	$\dot{\cap}\, n(0,1)$

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8	$\mu_1 - \mu_2$ (SPSS)	normal	any	$(\sigma_1^2\ ;\ \sigma_2^2)$ No, but $\sigma_1^2 \neq \sigma_2^2$	$\frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\frac{S'^2_1}{n_1} + \frac{S'^2_2}{n_2}}}$	$\frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S'^2_1}{n_1} + \frac{S'^2_2}{n_2}}}$	$\cap\ t_{(v)}$ with $v = \frac{\left(S_1^{2'}/n_1 + S_2^{'2}/n_2\right)^2}{\frac{\left(S_1^{'2}/n_1\right)^2}{(n_1-1)} + \frac{\left(S_2^{'2}/n_2\right)^2}{(n_2-1)}}$
9	$\mu_1 - \mu_2$	any	$n_1 > 30$ and $n_2 > 30$	$(\sigma_1^2\ ;\ \sigma_2^2)$ No, but $\sigma_1^2 \neq \sigma_2^2$	$\frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\frac{S'^2_1}{n_1} + \frac{S'^2_2}{n_2}}}$	$\frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S'^2_1}{n_1} + \frac{S'^2_2}{n_2}}}$	$\dot{\cap}\ n(0,1)$
10	σ^2	normal	any	-	$\frac{(n-1)S'^2}{\sigma_0^2}$	$\frac{(n-1)S'^2}{\sigma^2}$	$\cap\ \chi^2_{(n-1)}$
11	μ_d (paired samples)	normal	any	no	$\frac{\bar{D} - \mu_{d0}}{S'_d/\sqrt{n}}$	$\frac{\bar{D} - \mu_d}{S'_d/\sqrt{n}}$	$\cap\ t_{(n-1)}$
12	P (Wald)	Bernoulli	n > 30	-	$\frac{\bar{X}_b - p_0}{\sqrt{\frac{\bar{X}_b(1-\bar{X}_b)}{n}}}$ *Continuity corrected :* $\frac{\lvert n\bar{X}_b - np_0\rvert - 0{,}5}{\sqrt{n\bar{X}_b(1-\bar{X}_b)}}$	$\frac{\bar{X}_b - p}{\sqrt{\frac{\bar{X}_b(1-\bar{X}_b)}{n}}}$	$\dot{\cap}\ n(0,1)$
13	P (Scores)	Bernoulli	n > 30	-	$\frac{\bar{X}_b - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$ *Continuity corrected:* $\frac{\lvert n\bar{X}_b - np_0\rvert - 0{,}5}{\sqrt{np_0 \times q_0}}$	$\frac{\bar{X}_b - p}{\sqrt{\frac{pq}{n}}}$	$\dot{\cap}\ n(0,1)$

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14	$p_1 - p_2$ (Wald H_0) (SPSS H_0: $p_1 = p_2$)	Bernoulli	$n_1 > 30$ and $n_2 > 30$	-	$\frac{(\bar{X}_1 - \bar{X}_2) - (p_1 - p_2)_0}{\sqrt{\frac{[p_1(1-p_1)]_0}{n_1} + \frac{[p_2(1-p_2)]_0}{n_2}}}$	$\frac{(\bar{X}_1 - \bar{X}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{X}_1(1-\bar{X}_1)}{n_1} + \frac{\bar{X}_2(1-\bar{X}_2)}{n_2}}}$	$\dot{\cap}\, n(0,1)$
15	$p_1 - p_2$ (Wald)	Bernoulli	$\begin{cases} \bar{X}_1 \times n_1 > 10 \\ (1-\bar{X}_1) \times n_1 > 10 \\ \bar{X}_2 \times n_2 > 10 \\ (1-\bar{X}_2) \times n_2 > 10 \end{cases}$	-	$\frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\bar{X}(1-\bar{X})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \dot{\cap}\, n(0,1)$	$\frac{(\bar{X}_1 - \bar{X}_2) - (p_1 - p_2)}{\sqrt{\bar{X}(1-\bar{X})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$	$\dot{\cap}\, n(0,1)$
16	σ_1^2/σ_2^2	normal	any	-	$\frac{S'^2_1}{S'^2_2} \times \left(\frac{\sigma_2^2}{\sigma_1^2}\right)_0$	$\frac{S'^2_1}{S'^2_2} \times \frac{\sigma_2^2}{\sigma_1^2}$	$\cap\, F_{(n_1-1.n_2-1)}$
17	$\mu_1 = \mu_2 = \ldots = \mu_k$ Oneway ANOVA	normal	any	$(\sigma_1^2\,;\,\sigma_2^2;\ldots;\sigma_k^2)$ No, but $\sigma_1^2 = \sigma_2^2 = \cdots = \sigma_k^2$	$\frac{SSB/(K-1)}{SSW/(n-k)}$	-	$\cap\, F_{(k-1.n-k)}$
18	$\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$ (Levene)	normal	any	-	$\frac{SSB/(K-1)}{SSW/(n-k)}$ for the transformed variable $\lvert X_i - \bar{X} \rvert$	-	$\cap\, F_{(k-1.n-k)}$
19	Independence of 2 qualitative variables Chi-square test	any	any	-	$\sum_{i=1}^{c_1}\sum_{j=1}^{c_2}\frac{(O_{ij}-E_{ij})^2}{E_{ij}}$	-	$\cap\, \mathrm{X}^2_{(c_1-1)\times(c_2-1)}$
20	Goodness of fit Chi-square test	any	any	-	$\sum_{i=1}^{c}\frac{(O_i-E_i)^2}{E_i}$	-	$\cap\, \mathrm{X}^2_{(c-k-1)}$
21	Normality of one population K-S test	any	$n > 50$	-	$d_n = \max_{i=1,2,\ldots,n}\left\{\lvert F(x_i) - S(x_{i-1})\rvert ; \lvert F(x_i) - S(x_i)\rvert\right\}$	-	
22	Normality of one population Shapiro-Wilk test	any	$n \leq 50$	-	$W = \left(\sum_{i=1}^{n} a_i X_i\right)^2 \Big/ \sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$	-	$\dot{\cap}\, n(0,1)$