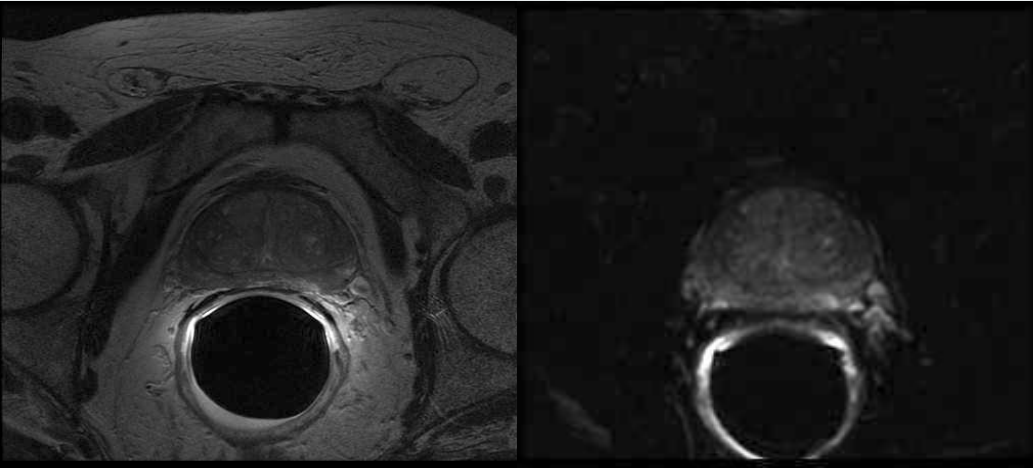




# Image Registration of DWI-T2w Images of the Prostate Gland

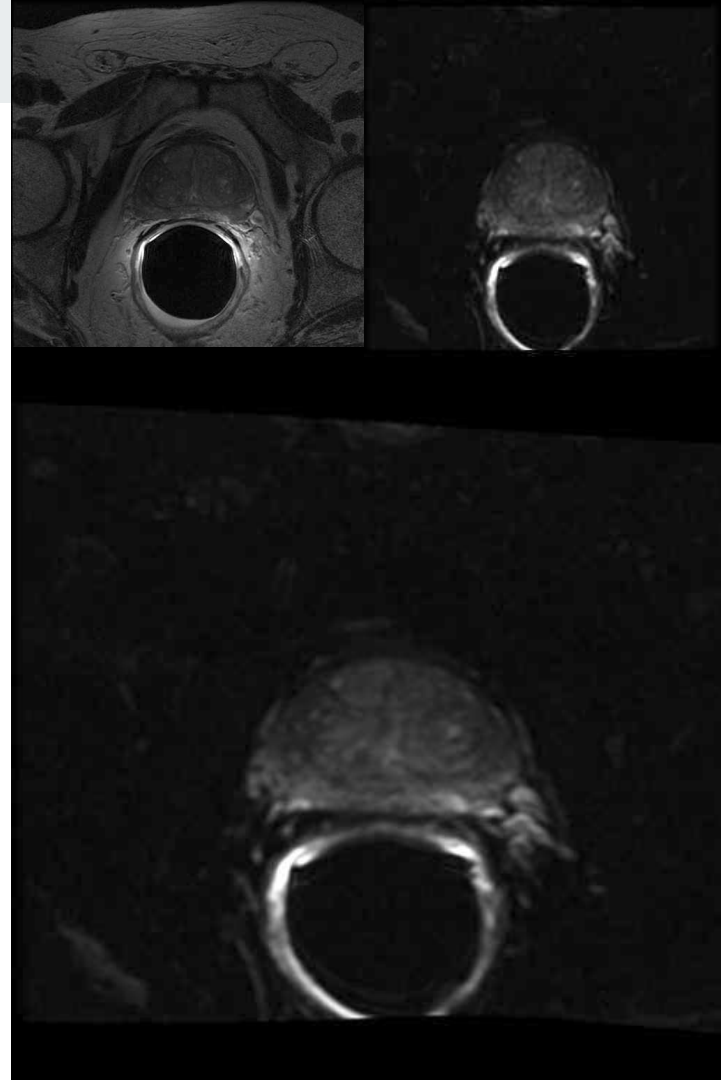


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Jurgena Gjuni, 265891

# Objectives:

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- Development of a semi-automatic system for the **DWI-T2w alignment** by selecting key points on the images;
- Implementation of: **Rigid**, **Affine** and **Thin-Plate Spline** transformations;
- **Evaluation** of the registration's performance for the dataset.



# Introduction

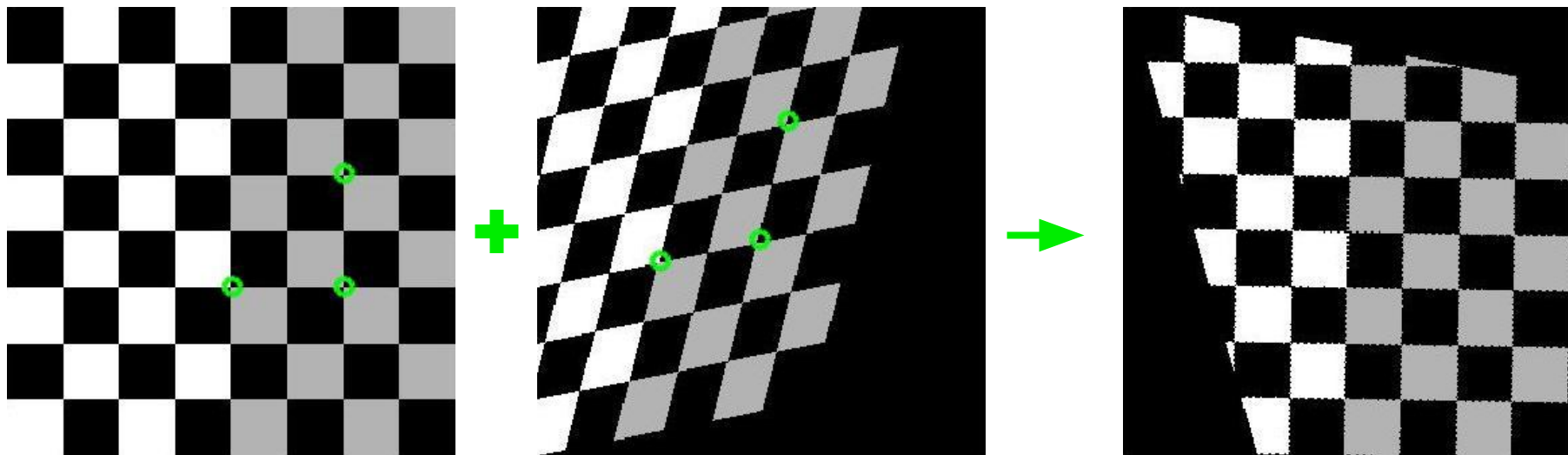


- T2 weighted imaging → **morphological** images, low contrast
- Diffusion weighted imaging → **differentiation** of cancer tissue and non-cancerous tissue
- DWI-T2w alignment → 3 different models of geometric transformation:
  - Rigid transformation
  - Affine transformation
  - Thin Plate spline transformation
- Evaluation of performance of each model
  - Normalized Mutual Information
  - Normalized Cross-Correlation
  - Target Registration Error

# Methods

Image Alignment → fix displacement of points in the DWI image compared to the T2w.

Example of the desired effect:



# Rigid Transformation

Preserves the Euclidean distance between every pair of points.

- Includes translation, rotation and reflection (not considered)
- $T_{\text{rigid}} = T_{\text{translation}} \cdot T_{\text{rotation}}$

$$\begin{bmatrix} x_m \\ y_m \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & x_{\text{trans}} \\ -\sin \theta & \cos \theta & y_{\text{trans}} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_f \\ y_f \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \phantom{x} \\ \phantom{y} \end{bmatrix}$$

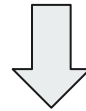
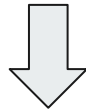
$$\begin{bmatrix} x_m \\ y_m \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \cdot \begin{bmatrix} x_f \\ y_f \end{bmatrix} + \begin{bmatrix} x_{\text{trans}} \\ y_{\text{trans}} \end{bmatrix}$$

$$Q = R \cdot P + t$$

# Rigid Transformation

$$\begin{bmatrix} x_m \\ y_m \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \cdot \begin{bmatrix} x_f \\ y_f \end{bmatrix} + \begin{bmatrix} x_{\text{trans}} \\ y_{\text{trans}} \end{bmatrix}$$

$$Q = R \cdot P + t$$



Obtained using **Singular Value Decomposition**

# Affine transformation

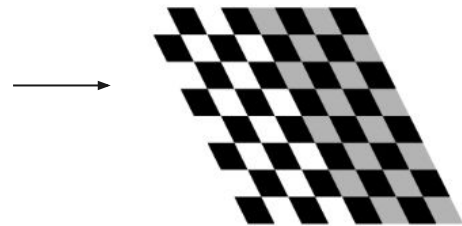
- Geometric transformation that preserves points, straight lines and planes
- $T_{\text{affine}} = T_{\text{translation}} \cdot T_{\text{rotation}} \cdot T_{\text{scale}} \cdot T_{\text{shear}}$

$$T_{\text{translation}} = \begin{bmatrix} 1 & 0 & x_{\text{trans}} \\ 0 & 1 & y_{\text{trans}} \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_{\text{scale}} = \begin{bmatrix} sc_x & 0 & 0 \\ 0 & sc_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_{\text{rotation}} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_{\text{shear}} = \begin{bmatrix} 1 & 0 & sh_x \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



# Affine transformation

$$T_{\text{affine}} = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

Obtained using  
**Least Squares Method**



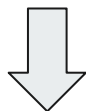
$$\begin{bmatrix} x_m \\ y_m \\ 1 \end{bmatrix} = T_{\text{affine}} \cdot \begin{bmatrix} x_f \\ y_f \\ 1 \end{bmatrix} \quad \longrightarrow \quad \begin{bmatrix} x_f & y_f & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_f & y_f & 1 \end{bmatrix} \cdot \begin{bmatrix} T_{11} \\ T_{12} \\ T_{13} \\ T_{21} \\ T_{22} \\ T_{23} \end{bmatrix} = \begin{bmatrix} x_m \\ y_m \end{bmatrix}$$



# Thin Plate spline transformation

- Data interpolation tool and the algebra behind it mimics the physical bending energy of a thin metal plate on point constraints.
- Allows the estimation of the parameters through the resolution of a linear system.

$$f(x, y) = a_1 + a_2x + a_3y + \sum_{i=1}^n w_i U(|P_i - (x, y)|)$$

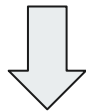


$$L^{-1}Y = (w_1 \ w_2 \ \dots \ w_n \mid a_1 \ a_2 \ a_3)$$

# Thin Plate spline transformation

- Data interpolation tool and the algebra of a thin metal plate on point constraints
- Allows the estimation of the parameters

$$f(x, y) = a_1 + a_2x + a_3y + \sum_{i=1}^n w_i U(|P_i|)$$



$$L^{-1}Y = (w_1 \ w_2 \ \dots \ w_n \mid a_1 \ a_2 \ a_3)$$

$$-U(r) = r_{ij}^2 \log r_{ij}^2 \quad L = \begin{bmatrix} K & P \\ P^\top & 0 \end{bmatrix}$$

$$Y = (V \mid 0 \ 0 \ 0)^\top$$

$$K = \begin{bmatrix} 0 & U(r_{12}) & \dots & U(r_{1n}) \\ U(r_{21}) & 0 & \dots & U(r_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ U(r_{n1}) & U(r_{n2}) & \dots & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} x_{f1} & x_{f2} & \dots & x_{fn} \\ y_{f1} & y_{f2} & \dots & y_{fn} \end{bmatrix}$$

# Evaluation methods



- Normalized Mutual Information
- Normalized Cross-Correlation
- Target Registration Error

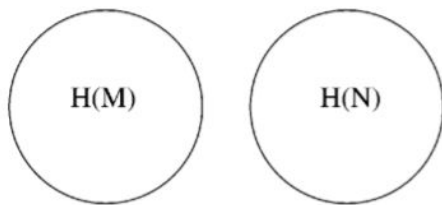
# Normalized Mutual Information

- Information theory measure of the statistical dependence between two random variables of the amount of information that one variable contains about the other → how well one image explains the other

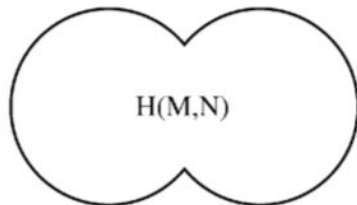
- Value 1 = images are equal
- Value 0 = images are not equal

$$MI(I_1, I_2) = H(I_1) + H(I_2) - H(I_1, I_2)$$

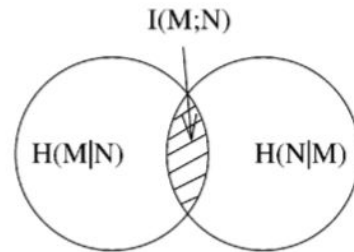
$$NMI(I_1, I_2) = \frac{MI(I_1, I_2)}{\sqrt{H(I_1)H(I_2)}}$$



Marginal Entropies



Joint Entropy

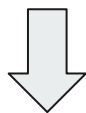


Mutual Information

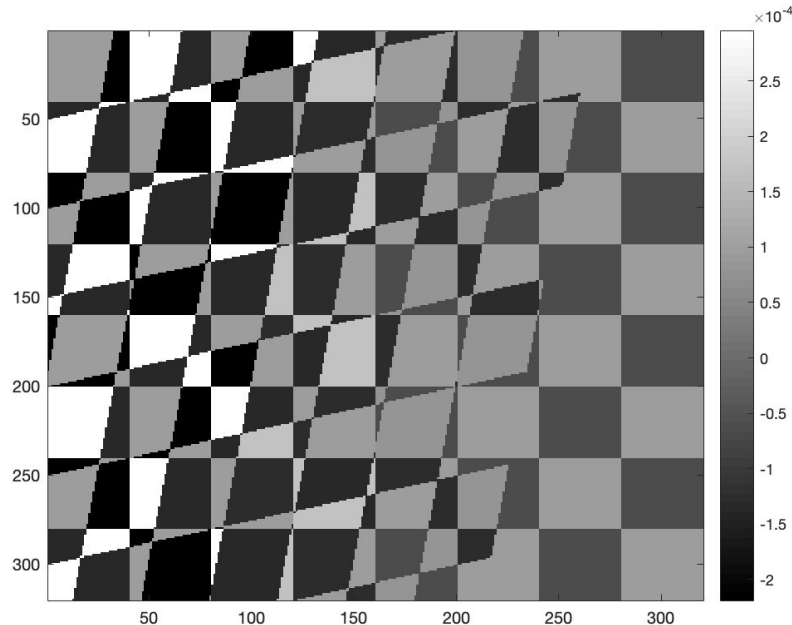
# Normalized Cross-Correlation

- Measures the degree of similarity between two images. It's a standard approach for feature detection.

$$I_{\text{NCC}}(x, y) = \frac{(I_1(x, y) - \mu_1)(I_2(x, y) - \mu_2)}{N\sigma_1\sigma_2} \longrightarrow$$

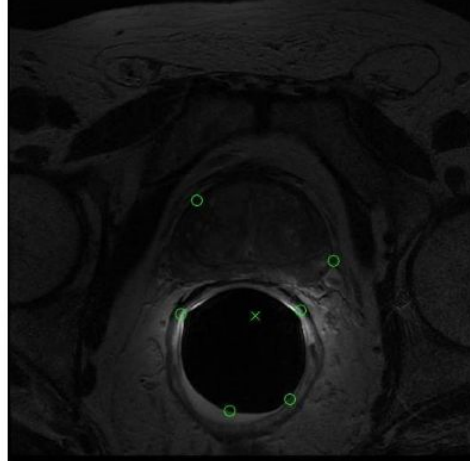


$$\text{NCC}(I_1, I_2) = \sum_{x, y} I_{\text{NCC}}(x, y)$$

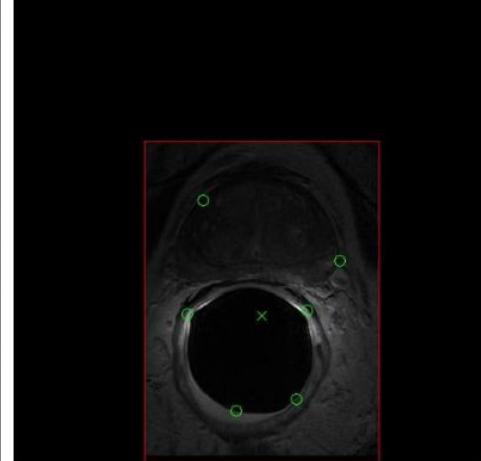


# Volume of Interest

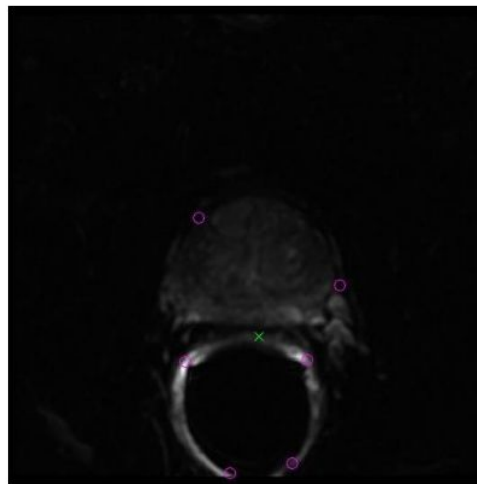
In order to minimize the background bias, a volume of interest region was selected.



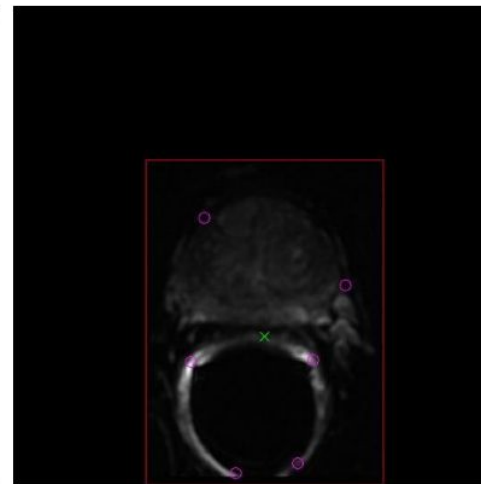
(a) T2 image



(b) VOI of T2 image



(c) DWI image



(d) VOI of DWI image

# Target Registration Error

- Compares the distance between partner points before and after registration
  - TRE= 0 → points are in the same position
  - TRE=1 → points didn't alter during transformation
  - TRE>1 → distance between points increased during the registration

$$d_b(i) = |(x_{fi}, y_{fi}) - (x_{mi}, y_{mi})| \quad \delta_{TR} = 1 - \frac{1}{n} \sum_{i=1}^n \frac{d_b(i) - d_a(i)}{d_b(i)}$$

# Results and Discussion

Examples of each transformation and measures of the registration's performance.

		Rigid		Affine		TPS	
		before	after	before	after	before	after
Total image	NMI	0.0650	0.4283	0.0650	0.5267	0.0646	0.3596
	NCC	0.0252	0.4110	0.0256	0.3857	0.0211	0.3073
VOI	NMI	0.1580	0.8548	0.0408	0.7544	0.2889	0.6904
	NCC	0.3168	0.9642	0.1659	0.8912	0.5426	0.8235
TRE		0.0118		$\approx 0$		0.0037	

The results of the table confirm that the registration algorithms are well built.

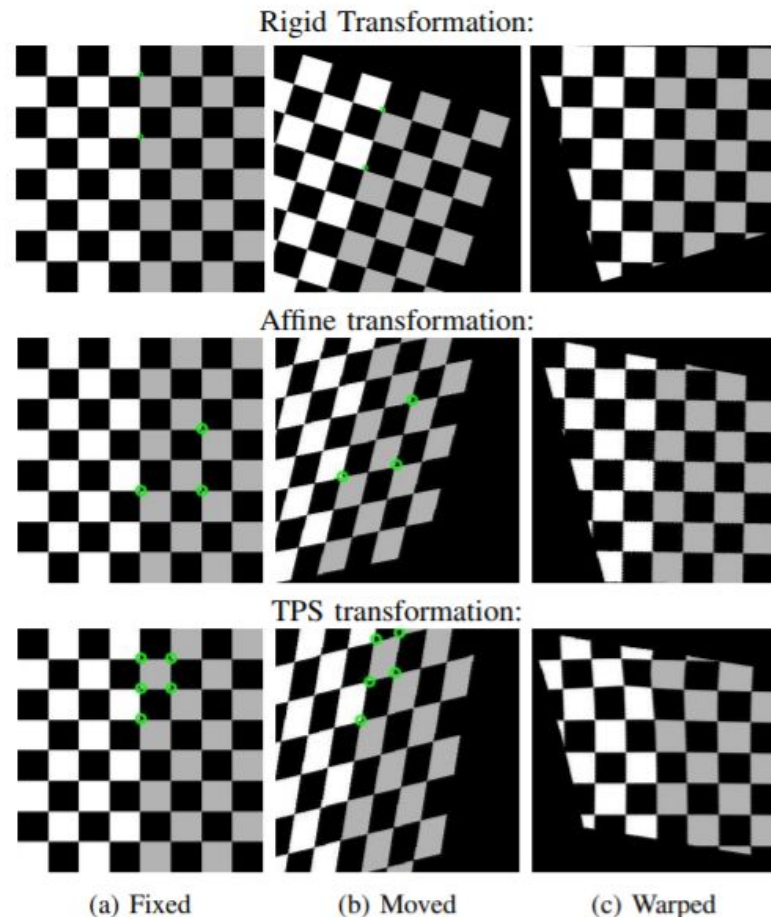


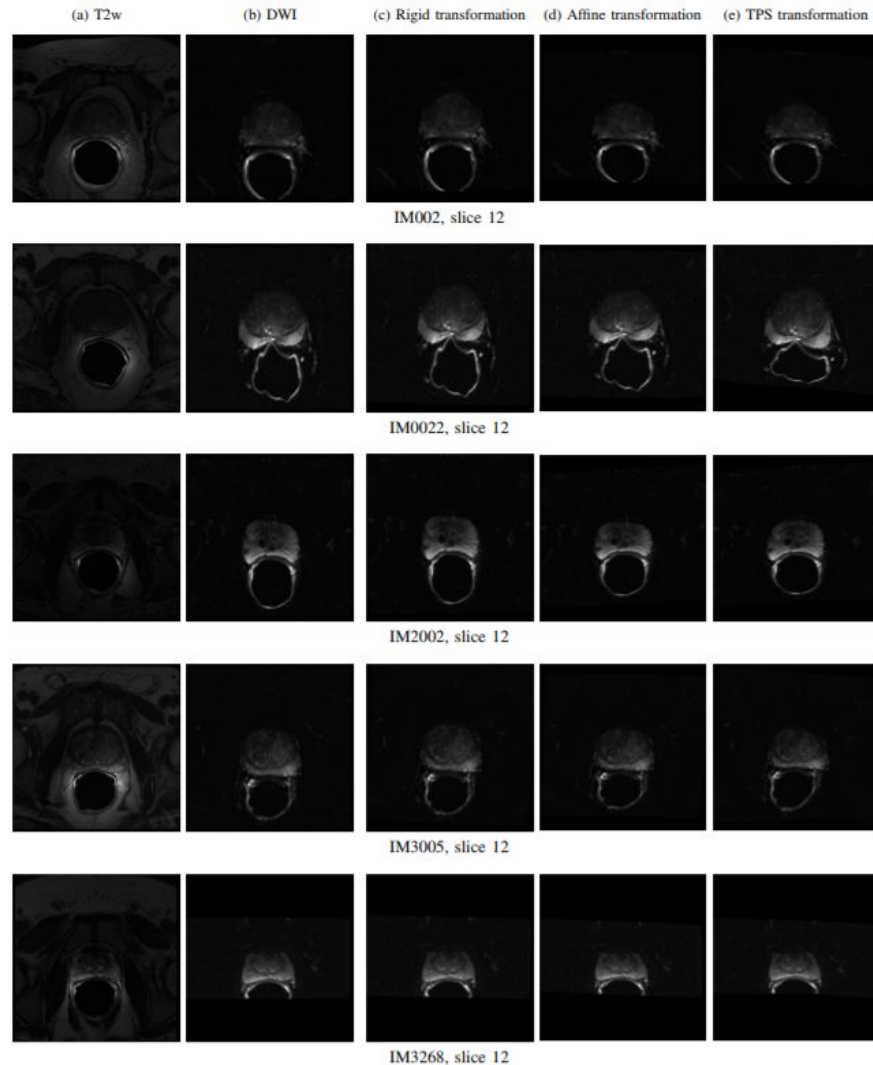
Fig: Three image alignment methods applied to checkerboard images with different deformations.



# Workflow

Registration of one slice of each volume/patient and the evaluation of this process:

1. **Slice 12** from 25 volumes (patients)
2. **Selection** of 6 pairs of **points** in each pair of T2w-DWI images;
3. Application of the 3 **registration** methods;
4. Application of the **evaluation** metrics (NMI, NCC and TRE) to the VOI of every image obtained



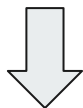
# TRE results

- **Rigid transformation:** distance between points **increased**.
- **Affine transformation:** distance between points **decreased**.
- **TPS transformation:** distance between points **decreased**.

Image	Slice	TRE		
		Rigid	Affine	TPS
IM0002	12	3.4791	0.4102	0.0757
IM0013	12	0.8691	0.1892	0.0270
IM0016	12	5.3973	2.6800	0.1395
IM0022	12	0.8803	0.8071	0.0275
IM0025	12	1.9888	0.5097	0.0444
IM0044	12	1.3700	0.2003	0.0305
IM2001	12	0.8222	0.4533	0.0247
IM2002	12	1.8800	0.6013	0.0406
IM2004	12	1.0895	0.2802	0.0187
IM2010	12	1.7462	0.1619	0.0438
IM2022	12	1.1142	0.5543	0.0320
IM3005	12	1.1288	0.5224	0.0373
IM3007	12	3.8645	0.5283	0.0856
IM3009	12	0.8600	0.5710	0.0688
IM3013	12	0.8204	0.1525	0.0337
IM3032	12	0.8620	0.3177	0.0279
IM3033	12	0.6025	0.3319	0.0146
IM3036	12	0.5714	0.3643	0.0393
IM3059	12	1.8549	1.2544	0.0539
IM3068	12	1.6344	1.2091	0.0807
IM3113	12	1.2575	0.3077	0.0471
IM3154	12	0.8469	0.5478	0.0707
IM3160	12	0.9403	0.9213	0.0888
IM3211	12	1.8030	1.9258	0.2917
IM3268	12	1.3060	0.7011	0.1167
Mean		1.5596	0.6601	0.0624

# NMI results

NMI analyses the probability of finding the same value in the same regions in both images

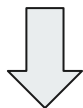


If the images have a different colors, the values don't reflect the correct alignment of shape

Image	Slice	NMI						
		DWI	Rigid	Affine	TPS	$\delta$ Rigid (%)	$\delta$ Affine (%)	$\delta$ TPS (%)
IM0002	12	0.3823	0.3553	0.3325	0.3413	-7.08	-13.05	-10.73
IM0013	12	0.3543	0.3310	0.3089	0.3366	-6.58	-12.83	-5.01
IM0016	12	0.3450	0.3216	0.2992	0.2996	-6.77	-13.26	-13.15
IM0022	12	0.2885	0.2601	0.2482	0.2302	-9.82	-13.96	-20.21
IM0025	12	0.3905	0.3831	0.3518	0.3572	-1.91	-9.91	-8.54
IM0044	12	0.4007	0.3634	0.3542	0.3575	-9.31	-11.60	-10.77
IM2001	12	0.4002	0.3789	0.3664	0.3797	-5.31	-8.44	-5.12
IM2002	12	0.3963	0.3955	0.3656	0.3573	-0.21	-7.76	-9.85
IM2004	12	0.3692	0.3310	0.3040	0.3307	-10.36	-17.67	-10.43
IM2010	12	0.4137	0.3855	0.3706	0.3860	-6.81	-10.43	-6.69
IM2022	12	0.4270	0.3740	0.3545	0.3448	-12.40	-16.98	-19.24
IM3005	12	0.3738	0.3452	0.3252	0.3366	-7.66	-12.99	-9.96
IM3007	12	0.4057	0.4008	0.3908	0.3866	-1.22	-3.69	-4.72
IM3009	12	0.4316	0.4266	0.4241	0.4261	-1.17	-1.76	-1.29
IM3013	12	0.3954	0.3579	0.3387	0.3333	-9.50	-14.35	-15.71
IM3032	12	0.3938	0.3846	0.3555	0.3630	-2.34	-9.72	-7.82
IM3033	12	0.4146	0.3639	0.3534	0.3526	-12.22	-14.78	-14.96
IM3036	12	0.4415	0.4298	0.4335	0.4334	-2.66	-1.81	-1.84
IM3059	12	0.3862	0.3789	0.3787	0.3891	-1.88	-1.93	0.77
IM3068	12	0.4350	0.4232	0.4162	0.4316	-2.71	-4.32	-0.79
IM3113	12	0.4134	0.4145	0.3957	0.3910	0.29	-4.26	-5.40
IM3154	12	0.3892	0.3815	0.3784	0.3837	-1.97	-2.76	-1.42
IM3160	12	0.2817	0.2703	0.2698	0.2734	-4.05	-4.20	-2.93
IM3211	12	0.4354	0.4279	0.4205	0.4264	-1.72	-3.42	-2.07
IM3268	12	0.3958	0.3958	0.3868	0.3847	0.01	-2.28	-2.79
Mean		0.3904	0.3712	0.3569	0.3613	-5.02	-8.73	-7.63

# NCC results

NCC: compares values in the same position, analyzing the similarity of values



More accurate results

Image	Slice	NCC						
		DWI	Rigid	Affine	TPS	$\delta$ Rigid (%)	$\delta$ Affine (%)	$\delta$ TPS (%)
IM0002	12	0.3876	0.4581	0.4283	0.4714	18.19	10.50	21.62
IM0013	12	0.3291	0.3411	0.4145	0.4582	3.64	25.92	39.20
IM0016	12	0.4681	0.4988	0.4895	0.4849	6.55	4.57	3.60
IM0022	12	0.3455	0.3152	0.3264	0.3553	-8.76	-5.52	2.85
IM0025	12	0.3593	0.3865	0.3995	0.4085	7.56	11.20	13.69
IM0044	12	0.4852	0.4914	0.5275	0.5317	1.28	8.70	9.57
IM2001	12	0.4720	0.4159	0.4050	0.3971	-11.88	-14.19	-15.88
IM2002	12	0.4764	0.5218	0.5415	0.5134	9.54	13.66	7.77
IM2004	12	0.3562	0.3364	0.3383	0.3857	-5.57	-5.02	8.27
IM2010	12	0.4812	0.4726	0.4802	0.4982	-1.77	-0.19	3.54
IM2022	12	0.5719	0.5079	0.5321	0.5913	-11.19	-6.95	3.40
IM3005	12	0.5172	0.4854	0.4612	0.4541	-6.15	-10.84	-12.20
IM3007	12	0.5221	0.5490	0.5441	0.5684	5.15	4.22	8.88
IM3009	12	0.5648	0.4944	0.5280	0.5781	-12.48	-6.51	2.35
IM3013	12	0.5392	0.5397	0.5089	0.5232	0.08	-5.62	-2.97
IM3032	12	0.5583	0.5651	0.5410	0.5733	1.21	-3.09	2.68
IM3033	12	0.4818	0.4942	0.4827	0.5018	2.58	0.18	4.16
IM3036	12	0.6303	0.6830	0.6886	0.6770	8.36	9.25	7.41
IM3059	12	0.5713	0.5521	0.5678	0.5781	-3.37	-0.62	1.19
IM3068	12	0.7360	0.7403	0.7133	0.7247	0.59	-3.08	-1.53
IM3113	12	0.5456	0.6109	0.5905	0.6145	11.97	8.23	12.63
IM3154	12	0.7131	0.6626	0.6708	0.6694	-7.08	-5.93	-6.12
IM3160	12	0.5110	0.4771	0.4875	0.4818	-6.64	-4.61	-5.72
IM3211	12	0.6505	0.6849	0.6832	0.6880	5.29	5.03	5.77
IM3268	12	0.6971	0.7453	0.7620	0.7685	6.92	9.32	10.24
Mean		0.5188	0.5212	0.5245	0.5399	0.56	1.54	4.98

# Discussion

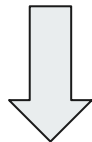


- **TRE:** presents low values (except for rigid transformation), but it doesn't necessarily mean that the obtained image is the most similar to the original one.
- **NMI:** it's not conclusive in our case.
- **NCC:** indicates that the images obtained after the registration are more similar to the original ones.

# Discussion

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- The registration methods are able to align the DWI images to the T2w images,
- The TPS gives the biggest improvement of similarity between the T2w image and the aligned DWI image,



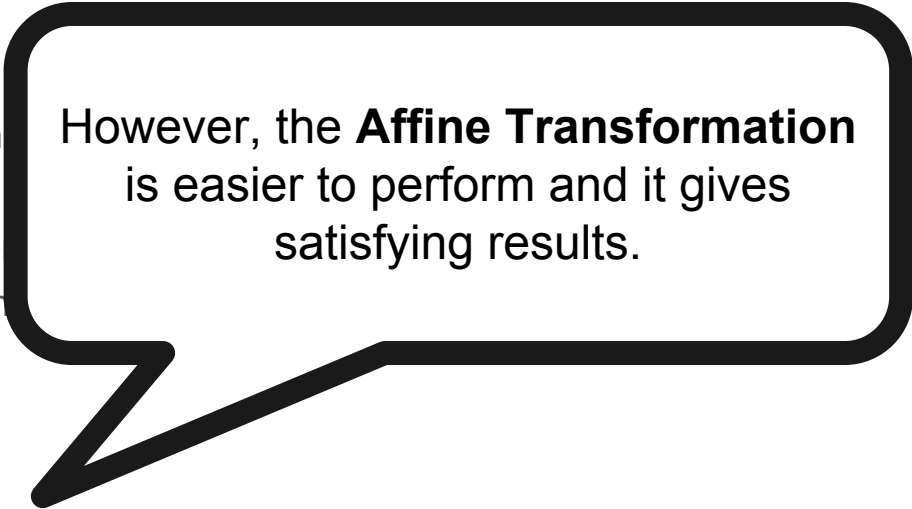
**TPS is the best method, between the three, to perform the registration.**



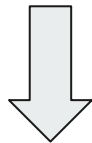
# Discussion



- They registration methods are a T2w images,
- The TPS gives the biggest improvement in the T2w image and the aligned DWI in



However, the **Affine Transformation** is easier to perform and it gives satisfying results.



TPS is the best method, between the three, to perform the registration.

# For the future...



- Implementation of an **automatic points selection** algorithm, to cancel the user's bias.
- Segmentation of the prostate gland's shape in each image in order to compare the different masks and **cancel the bias derived from the different colors** in each magnetic resonance methods.





Thanks for your attention

**And now the GUI**

