Laboratorio 4, Tópicos en análisis datos 1

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```
knitr::opts_chunk$set(warning = FALSE, message = FALSE)
library(tidyverse) # To manipulate data, and other things
```

```
-- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
v dplyr     1.1.2     v readr     2.1.4
v forcats   1.0.0     v stringr   1.5.0
v ggplot2   3.4.2     v tibble    3.2.1
v lubridate 1.9.2     v tidyr     1.3.0
v purrr     1.0.1
-- Conflicts ------------------------------------------ tidyverse_conflicts() --
x dplyr::filter() masks stats::filter()
x dplyr::lag()    masks stats::lag()
i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
```

```
library(ggforce) # To use ggcircle
library(openxlsx) # To manipulate Excel
library(kableExtra) # To make beatiful tables
```

```
Attaching package: 'kableExtra'

The following object is masked from 'package:dplyr':

    group_rows
```

```
tryCatch(
    {
        # Directorio donde se ubica el qmd
```

```
        directory <- dirname(rstudioapi::getSourceEditorContext()$path)
        setwd(directory) # Establecer el directorio del archivo como la raiz
    },
    error = function(e) {
        message("")
        print("")
    }
)
```

```
[1] ""
```

1

```
# We construct a function to make all the things that are wanted in the by the homework
fn_acp_homework <- function(df, scale = TRUE, name_acp) {
    require(tidyverse) # To manipulate data

    require(ggforce) # To make the unit circle

    require(kableExtra) # To make beatiful tables

    # We select the name of columns of individuals
    name_observations <- colnames(df)[1]

    # We delete the first columns that have names
    names_df <- df[, 1]

    # We select the variables
    df <- df[, -1]

    # We get the number of observations
    n <- nrow(df)

    # We get the center of gravity
    g <- apply(df, 2, mean)

    # We estimate the standard deviation
    sd_n <- sqrt((n - 1) / n) * apply(df, 2, sd)
```

```r
# We assign a name to the acp
df <- (df - matrix(rep(g, n), nrow = n, byrow = TRUE)) /
    matrix(rep(sd_n, n), nrow = n, byrow = TRUE)

# We assign the name to the ACP
assign(name_acp, prcomp(df), envir = .GlobalEnv)

# We get the acp to use it
acp <- get(name_acp)

# We make the main plain
df_pc <- as.data.frame(as.matrix(df) %*% as.matrix(acp$rotation))

# We add the name of the observations
df_pc <- cbind(names_df, df_pc)

# We add the name of the first column
colnames(df_pc)[1] <- name_observations

# We select the names of the variables
names_cols <- colnames(df)

# We print the ggplot that is the main plain
print(ggplot(, aes(x = df_pc$PC1, y = df_pc$PC2, label = df_pc[, 1])) +
    geom_label(vjust = 1) +
    geom_point() +
    geom_hline(yintercept = 0) +
    geom_vline(xintercept = 0) +
    theme_minimal() +
    # theme(text = element_text(size = 16)) +
    labs(x = "PC1", y = "PC2", caption = paste("Porcentaje de inercia:",
        sum((acp$sdev)[c(1, 2)]^(2)) / sum(acp$sdev^(2)),
        sep = " "
    )))

# We estimate the correlation of the principal components
df_correlations <- data.frame("Variable" = names_cols[1], (n - 1) /
    n * cor(df[, names_cols[1]], df_pc[, -1]))

for (i in colnames(df)[-1]) {
    df_aux <- data.frame("Variable" = i, (n - 1) / n * cor(df[, i], df_pc[, -1]))
```

```
    df_correlations <- rbind(df_correlations, df_aux)
}

# We show the correlation circle of variables
print(ggplot(, aes(
    x = df_correlations$PC1, y = df_correlations$PC2,
    label = df_correlations$Variable
)) +
    geom_label(vjust = 1) +
    geom_point() +
    geom_hline(yintercept = 0) +
    geom_vline(xintercept = 0) +
    geom_circle(aes(x0 = 0, y0 = 0, r = 1)) +
    coord_fixed() +
    theme_minimal() +
    # theme(text = element_text(size = 16)) +
    labs(x = "PC1", y = "PC2", caption = paste("Porcentaje de inercia:",
        sum((acp$sdev)[c(1, 2)]^(2)) / sum(acp$sdev^(2)),
        sep = " "
    )))

# We save the eigen values
eigen_values <- data.frame(
    "Valor propio" = 1:ncol(df),
    "Valor" = acp$sdev^(2), "Porcentaje de inercia" = acp$sdev^(2) /
        sum(acp$sdev^(2)) * 100,
    "Porcentaje de inercia acumulado" = cumsum(acp$sdev^(2)) /
        sum(acp$sdev^(2)) * 100
)

# We show the table with values
print(ggplot(, aes(x = 1:length(acp$sdev), y = acp$sdev^(2))) +
    geom_line() +
    geom_point() +
    theme_minimal() +
    # theme(text = element_text(size = 16)) +
    labs(x = "Número", y = "Valor propio"))

# We estimate the euclidean norm squared
df_quality_representation <- apply(df, 1, function(x) sum((x)^(2)))
```

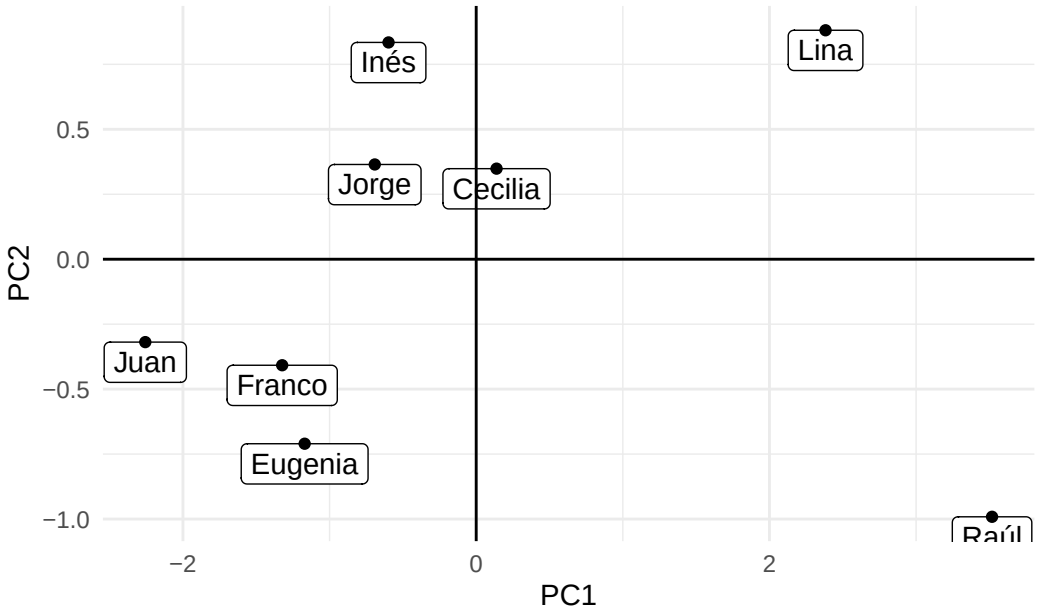
```
    # We estimate the quality of representation of each projection
    df_quality_representation <- df_pc[, -1]^2 /
        matrix(rep(df_quality_representation, ncol(df)), ncol = ncol(df))

    # We add the name of the columns
    df_quality_representation <- cbind(
        names_df,
        df_quality_representation
    )

    # We estimate the quality of the main plain
    quality_main_plain_id <- df_quality_representation[, 2] + df_quality_representation[, 3]

    df_quality_representation$Calidad.representacion.plano.principal <- quality_main_plain_id

    # We add the name of the column
    colnames(df_quality_representation)[1] <- name_observations

    #We estimate the communalities
    df_comunalities <- df_correlations %>%
        mutate("Comunalidad" = PC1^2 + PC2^2) %>%
        select(Variable, Comunalidad)

    #We return the list the list with the tables
    list(
        eigen_values = eigen_values,
        df_pc = df_pc,
        df_correlations = df_correlations,
        df_quality_representation = df_quality_representation,
        df_comunalities = df_comunalities
    )
}
```

1.1 Ejercicio 8 del libro

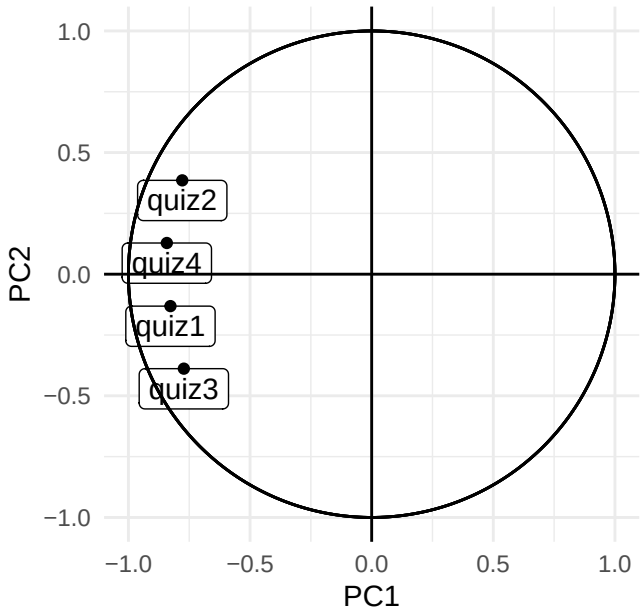
```
df_notas_quices <- data.frame(
    Estudiante = c(
        "Inés",
        "Jorge",
        "Lina",
```

```
        "Franco",
        "Cecilia",
        "Raúl",
        "Eugenia",
        "Juan"
    ),
    quiz1 = c(90, 80, 50, 85, 75, 56, 92, 100),
    quiz2 = c(88, 85, 65, 82, 78, 43, 75, 85),
    quiz3 = c(56, 67, 40, 80, 60, 48, 77, 80),
    quiz4 = c(77, 82, 63, 87, 75, 33, 86, 100)
)
```

```
tablas_notas <- fn_acp_homework(df_notas_quices, name_acp = "acp_notas_quices")
```



Porcentaje de inercia: 0.956391821364111



Porcentaje de inercia: 0.956391821364111

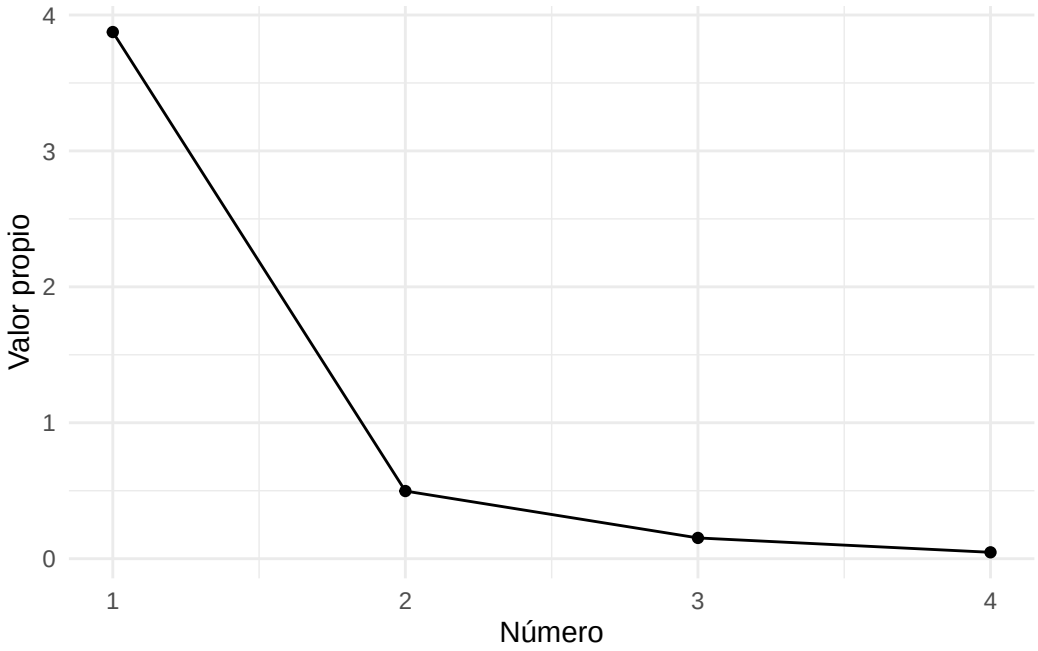


Table 1: Autovalores

Valor.propio	Valor	Porcentaje.de.inercia	Porcentaje.de.inercia.acumulado
1	3.8745495	84.755769	84.75577
2	0.4975274	10.883413	95.63918
3	0.1526199	3.338559	98.97774
4	0.0467318	1.022259	100.00000

Table 2: Componentes Principales

Estudiante	PC1	PC2	PC3	PC4
Inés	-0.5977494	0.8343720	0.7622808	-0.0287586
Jorge	-0.6917970	0.3646483	-0.1392705	-0.2547475
Lina	2.3825901	0.8811835	-0.4862643	0.2231486
Franco	-1.3231527	-0.4081149	-0.3801178	-0.2823218
Cecilia	0.1387472	0.3485541	-0.0516079	-0.0801053
Raúl	3.5181265	-0.9913841	0.2899247	-0.0570101
Eugenia	-1.1699366	-0.7101870	0.0124181	0.1801895
Juan	-2.2568282	-0.3190719	-0.0073632	0.2996051

```
knitr::kable(tablas_notas$eigen_values,
    booktabs = T,
    caption = "Autovalores"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_notas$df_pc,
    booktabs = T,
    caption = "Componentes Principales"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

Table 3: Correlaciones

Variable	PC1	PC2	PC3	PC4
quiz1	-0.8271359	-0.1312476	0.2483969	0.0504404
quiz2	-0.7786527	0.3857977	0.0260690	-0.0990233
quiz3	-0.7720692	-0.3883273	-0.1125139	-0.0779527
quiz4	-0.8420812	0.1282213	-0.1649344	0.1134910

Table 4: Calidad de proyección

Estudiante	PC1	PC2	PC3	PC4	Calidad.representacion.plano.principal
Inés	0.2184840	0.4256971	0.3553132	0.0005057	0.6441811
Jorge	0.6877735	0.1910893	0.0278745	0.0932627	0.8788628
Lina	0.8423121	0.1152145	0.0350848	0.0073886	0.9575266
Franco	0.8175319	0.0777767	0.0674716	0.0372198	0.8953086
Cecilia	0.1284919	0.8109009	0.0177771	0.0428302	0.9393928
Raúl	0.9204194	0.0730881	0.0062508	0.0002417	0.9935075
Eugenia	0.7182259	0.2646561	0.0000809	0.0170371	0.9828820
Juan	0.9637411	0.0192637	0.0000103	0.0169849	0.9830049

```
knitr::kable(tablas_notas$df_correlations,
    booktabs = T,
    caption = "Correlaciones"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_notas$df_quality_representation,
    booktabs = T,
    caption = "Calidad de proyección"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_notas$df_comunalities,
    booktabs = T,
    caption = "Comunalidad"
) %>%
```

Table 5: Comunalidad

Variable	Comunalidad
quiz1	0.7013797
quiz2	0.7551398
quiz3	0.7468890
quiz4	0.7255414

```
    kable_styling(font_size = 10)
```

De lo anterior se puede destacar que el mayor porcentaje de inercia está explicado por las primeras dos componentes principales, ya que con estas dos se obtiene un porcentaje cercano al 96%. Además, al observar el gráfico de los valores propios se obtiene que el codo se da en la segunda componente principal. Se encuentra que se tiene un efecto talla, según el círculo de correlaciones.

Se procede a usar la función PCA del paquete FactoMineR para comparar los resultados obtenidos

```
#We load the package FactoMineR
library(FactoMineR)
library(factoextra)

#We estimate the principal component, it normalized
pca_notas_facto <- PCA(df_notas_quices[-1],)
pca_notas_facto$eig
```

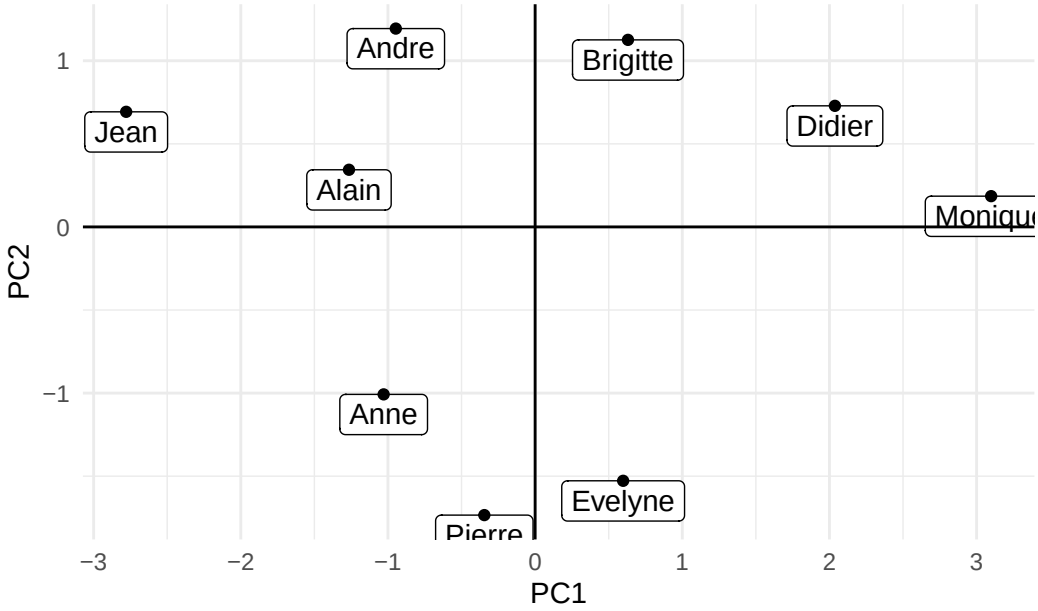
```
       eigenvalue percentage of variance cumulative percentage of variance
comp 1 3.39023078              84.755769                           84.75577
comp 2 0.43533651              10.883413                           95.63918
comp 3 0.13354237               3.338559                           98.97774
comp 4 0.04089034               1.022259                          100.00000
```

Se puede observar que los porcentaje de inercia son iguales, sin embargo los autovalores varían ligeramente, posible producto de que se emplee el estimador insesgado en el último caso y en la función programada por nosotros lo hace con el estimador no insesgado la varianza.

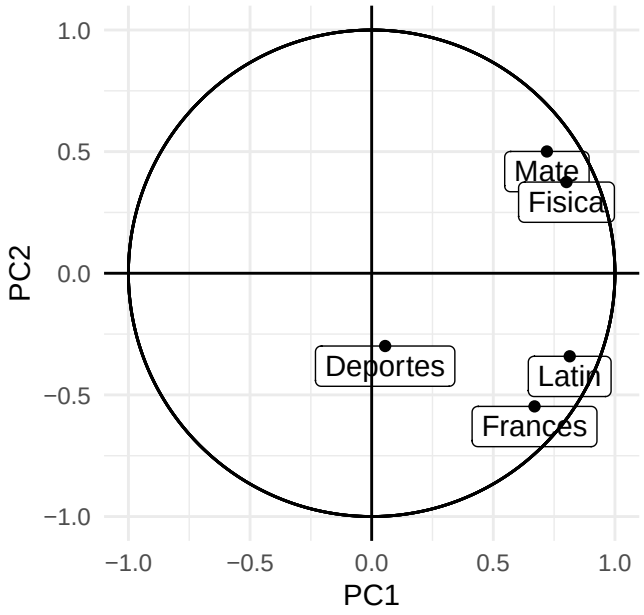
1.2 Ejercicio 9 del libro

```
df_notas_frances <- read.xlsx("./Ejercicios-Cap3.xlsx", sheet = "9.NotasFrancesas")
```

```
tablas_notas_francesas <- fn_acp_homework(df_notas_frances, name_acp = "acp_notas_quices")
```



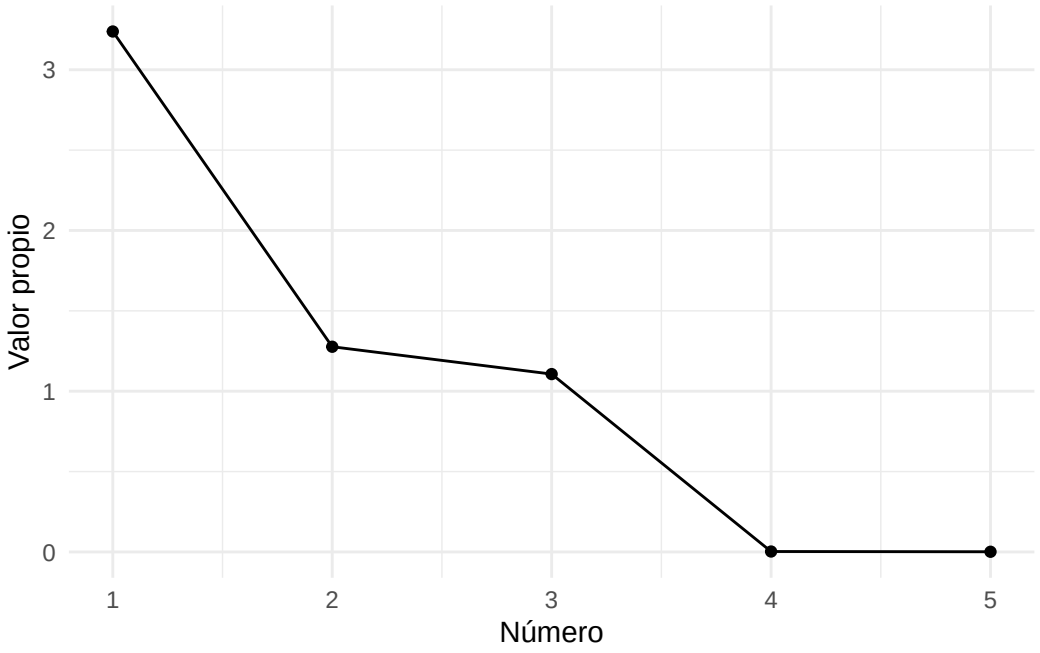
Porcentaje de inercia: 0.802608037670347



Porcentaje de inercia: 0.802608037670347

Table 6: Autovalores

Valor.propio	Valor	Porcentaje.de.inercia	Porcentaje.de.inercia.acumulado
1	3.2380037	57.5645100	57.56451
2	1.2766665	22.6962937	80.26080
3	1.1065557	19.6721007	99.93290
4	0.0026705	0.0474754	99.98038
5	0.0011036	0.0196201	100.00000




```
knitr::kable(tablas_notas_francesas$eigen_values,
    booktabs = T,
    caption = "Autovalores"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_notas_francesas$df_pc,
    booktabs = T,
    caption = "Componentes Principales"
) %>%
```

Table 7: Componentes Principales

Estudiante	PC1	PC2	PC3	PC4	PC5
Jean	-2.7788226	0.6927433	0.7345732	-0.0510649	0.0363393
Alain	-1.2649120	0.3439755	0.5524236	-0.0295369	-0.0141203
Anne	-1.0292266	-1.0077668	0.2982004	0.0575102	-0.0051318
Monique	3.0987283	0.1850850	1.1325100	-0.0558442	-0.0327358
Didier	2.0369954	0.7287226	0.1946418	0.0209966	0.0595180
Andre	-0.9467285	1.1936845	-1.1514813	-0.0064080	-0.0467194
Pierre	-0.3451875	-1.7342903	0.9249510	0.0392019	-0.0129008
Brigitte	0.6309889	1.1257092	-0.7065004	0.0801117	-0.0020132
Evelyne	0.5981647	-1.5278632	-1.9793185	-0.0549664	0.0177639

Table 8: Correlaciones

Variable	PC1	PC2	PC3	PC4	PC5
Mate	0.7203070	0.5006327	-0.1420863	-0.0129358	-0.0170974
Fisica	0.8007919	0.3748935	-0.0870517	0.0228062	0.0145789
Frances	0.6694954	-0.5472836	0.2044894	0.0203232	-0.0122891
Latin	0.8140929	-0.3412845	0.1000455	-0.0278015	0.0109147
Deportes	0.0554290	-0.2991169	-0.8352113	0.0014693	-0.0003123

```
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_notas_francesas$df_correlations,
    booktabs = T,
    caption = "Correlaciones"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_notas_francesas$df_quality_representation,
    booktabs = T,
    caption = "Calidad de proyección"
) %>%
```

Table 9: Calidad de proyección

Estudiante	PC1	PC2	PC3	PC4	PC5	Calidad.representacion.plano.principal
Jean	0.8829746	0.0548746	0.0617016	0.0002982	0.0001510	0.9378492
Alain	0.7902943	0.0584418	0.1507345	0.0004309	0.0000985	0.8487361
Anne	0.4888001	0.4686293	0.0410323	0.0015262	0.0000122	0.9574294
Monique	0.8790618	0.0031361	0.1174185	0.0002855	0.0000981	0.8821979
Didier	0.8786797	0.1124540	0.0080227	0.0000934	0.0007501	0.9911338
Andre	0.2456066	0.3904526	0.3633314	0.0000113	0.0005981	0.6360592
Pierre	0.0299071	0.7549312	0.2147342	0.0003857	0.0000418	0.7848383
Brigitte	0.1833990	0.5837219	0.2299209	0.0029563	0.0000019	0.7671210
Evelyne	0.0541040	0.3529855	0.5924059	0.0004569	0.0000477	0.4070896

Table 10: Comunalidad

Variable	Comunalidad
Mate	0.7694753
Fisica	0.7818128
Frances	0.7477435
Latin	0.7792223
Deportes	0.0925433

```
    kable_styling(latex_options = "scale_down", font_size = 10)
```

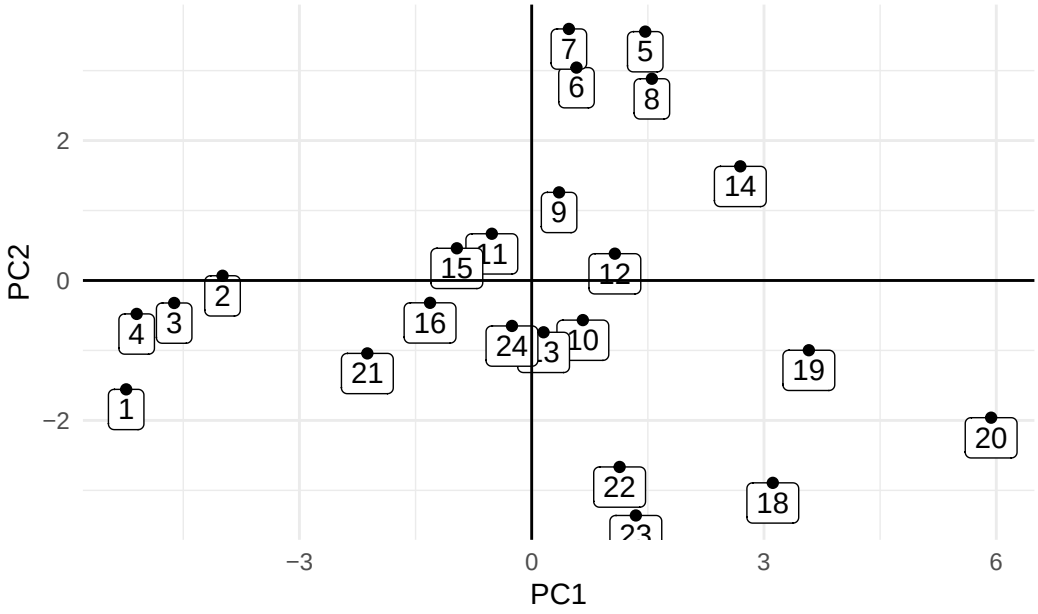
```
knitr::kable(tablas_notas_francesas$df_comunalities,
    booktabs = T,
    caption = "Comunalidad"
) %>%
    kable_styling(font_size = 10)
```

De lo observado en este caso se puede decir que 84% de la inercia está explicada por las primeras dos componentes principales. Un aspecto a tener en consideración es que se observan lo que parecen dos codos, sin embargo, el primero se encuentra en la primera componente principal. Por lo que se podría seleccionar estas dos componentes principales. Sin embargo, la variabled de educación física no se ve bien representada por estas dos al observar el círculo de correlaciones. La calidad de proyección es buena para la mayoría en la primera componente y en la segunda. Exceto evelyn que tiene una mejor calidad en la tercera componente principal.

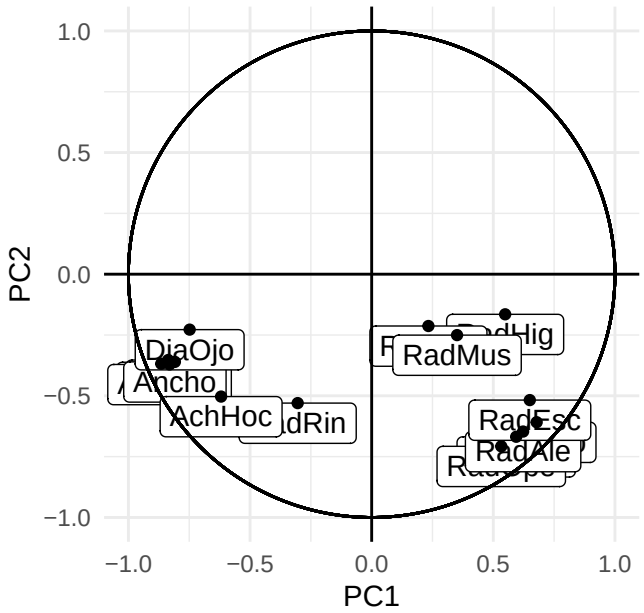
1.3 Ejercicio 10 del libro

```
df_amiard <- read.xlsx("./Ejercicios-Cap3.xlsx", sheet = "10.Amiard")
```

```
tablas_amiard <- fn_acp_homework(df_amiard, name_acp = "acp_notas_quices")
```



Porcentaje de inercia: 0.701631272044215



Porcentaje de inercia: 0.701631272044215

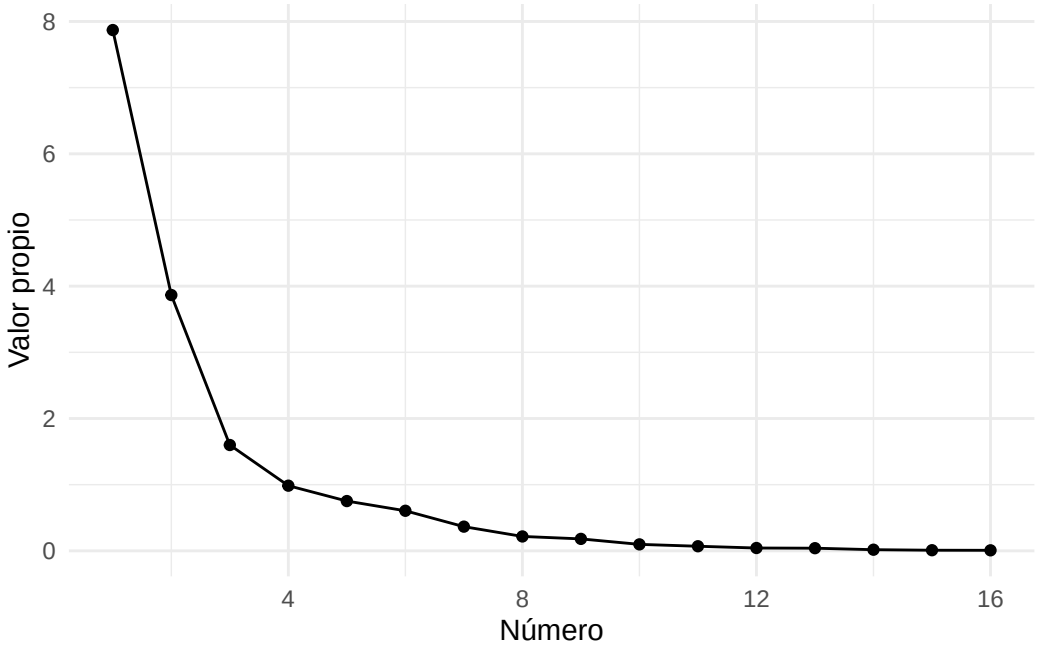


Table 11: Autovalores

Valor.propio	Valor	Porcentaje.de.inercia	Porcentaje.de.inercia.acumulado
1	7.8704164	47.0514022	47.05140
2	3.8659613	23.1117250	70.16313
3	1.5994246	9.5617775	79.72490
4	0.9847454	5.8870648	85.61197
5	0.7523633	4.4978239	90.10979
6	0.6054877	3.6197633	93.72956
7	0.3657228	2.1863865	95.91594
8	0.2175356	1.3004847	97.21643
9	0.1806235	1.0798144	98.29624
10	0.0978307	0.5848572	98.88110
11	0.0695315	0.4156775	99.29678
12	0.0429282	0.2566362	99.55341
13	0.0400988	0.2397212	99.79313
14	0.0172793	0.1033000	99.89643
15	0.0094216	0.0563248	99.95276
16	0.0079021	0.0472408	100.00000

```
knitr::kable(tablas_amiard$eigen_values,
    booktabs = T,
    caption = "Autovalores"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_amiard$df_pc,
    booktabs = T,
    caption = "Componentes Principales"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_amiard$df_correlations,
    booktabs = T,
    caption = "Correlaciones"
) %>%
```

Table 12: Componentes Principales

Pez	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10	PC11	PC12	PC13	PC14	PC15	PC16
1	-5.2372536	-1.5564601	-0.3323934	-0.3179194	-1.2155544	-0.1024358	-0.6992818	-0.1955694	-0.1040166	-0.0054939	0.3468870	-0.5937963	-0.1193976	-0.0364869	0.0382667	-0.0276538
2	-3.9927735	0.0677263	0.9285664	-0.4088455	-0.2393649	-0.3313678	0.7489649	-0.4703521	-0.6056871	-0.3176685	-0.1507686	0.0333647	0.1087371	-0.0528510	0.0083998	-0.1278729
3	-4.6173255	-0.3217696	0.1224370	-0.0893397	0.5163818	0.4373925	0.2957067	0.1083518	-0.4978421	0.2046696	-0.0052627	0.1093069	0.2055963	0.1707935	-0.1275611	-0.0768142
4	-5.1016465	-0.4777752	-0.1844123	0.0289435	0.3990341	0.2796090	0.7225024	-0.4130194	0.6666010	-0.1610426	-0.1006676	-0.0074127	-0.0444183	-0.0488419	-0.1542849	0.1374152
5	1.4665709	3.5575396	-1.1033454	-0.4303608	0.6396394	0.6467673	-0.0070807	-0.0488429	0.3070590	-0.1733654	-0.1778199	-0.0288550	-0.2245479	0.0450718	-0.2032282	-0.0773372
6	0.5783974	3.0430877	-0.6038556	-0.5002485	-0.6890424	0.1719122	-0.1278501	0.1126513	-0.3536192	0.3399207	-0.0312822	0.1490840	0.0025291	-0.0954101	-0.0037324	-0.0392600
7	0.4804574	3.5957372	-0.2316868	-0.6408542	0.2667918	0.5995246	0.0637408	0.2507223	-0.1085116	-0.0840738	0.3269115	-0.1192793	-0.2368157	-0.0081810	0.0765589	-0.0136768
8	1.5521614	2.8837410	0.2498233	-0.0774123	-2.0669387	0.3475083	0.5594655	-0.1940939	0.1291686	0.5395349	0.0052245	-0.0714591	0.1300446	0.1131221	0.0578813	0.1225987
9	0.3537393	1.2596924	0.3407250	-0.0889677	0.1493151	-1.0598508	-0.4408661	-0.1113806	0.1156687	0.3127086	-0.3484778	0.0620830	0.0677131	-0.3476393	0.0346939	-0.1295587
10	0.6605632	-0.5665006	1.3855670	-0.3058448	-0.4741584	-0.7519692	1.2442551	-0.2737576	0.6863467	-0.2375600	-0.0934702	0.1418554	-0.0006657	0.1241582	0.1643887	-0.0566191
11	-0.5148971	0.6678867	-1.2971026	0.8615936	0.9072267	-0.6309839	-0.3251704	-0.1541086	0.7590946	0.2327353	0.0597172	0.0011867	-0.0971929	0.0933254	0.0370725	0.0353978
12	1.0737763	0.3830753	-0.1457304	0.6608903	0.2511096	0.9326664	-1.2658294	-0.8131461	0.4477079	-0.3413688	0.2585888	0.1279971	0.4773805	0.0368514	0.0618795	-0.0611420
13	0.1532323	-0.7403641	1.9139142	2.8406649	0.0934549	0.5193534	-0.2987725	0.6115184	0.0639469	0.2239099	-0.4004547	-0.2065492	-0.1357872	0.1370587	0.0140995	-0.0899715
14	2.6943191	1.6315530	-0.2324715	0.5910551	-0.3071497	-1.7216972	-0.2804995	0.2001392	-0.3341520	-0.8559892	-0.1617297	-0.1891666	-0.0002402	0.0315705	-0.0339062	0.0956158
15	-0.9667424	0.4602281	0.2471948	0.8754006	-0.1258295	0.2364653	-0.1421807	0.5114140	-0.5273024	-0.0591658	-0.0734143	0.1602788	0.3469697	-0.0543641	-0.0251774	0.1874735
16	-1.3126686	-0.3192968	1.6679717	0.6432685	-0.1793423	0.2604957	-0.2738815	0.2497061	-0.1749027	-0.2265660	0.5379838	0.4927278	-0.4154812	-0.0599165	0.0351930	0.0176700
18	3.1138079	-2.8927078	-0.2488603	-1.2970084	-1.3330084	-0.4238839	-0.1673000	0.9527058	0.4397253	-0.0463520	0.2150463	0.0803472	0.1572143	0.0851857	-0.1606241	-0.1066537
19	3.5794128	-0.9971309	-1.9151727	0.5012267	1.3874249	0.6134744	1.2219300	0.2674677	-0.4292095	-0.0696996	0.2861973	-0.2068910	0.1352364	-0.0159248	0.1020629	-0.0563972
20	5.9337234	-1.9613316	2.9695640	-0.4858227	0.6655724	0.3138540	0.1186565	-0.7371611	-0.1773968	0.2178160	0.1459133	-0.2133854	-0.0448531	-0.1163794	-0.1318144	0.0706922
21	-2.1226130	-1.0424430	0.2547975	-0.9079201	1.1224402	-0.0461927	0.0292173	0.9397558	0.5286730	0.1271195	0.0021657	-0.0698300	0.1261436	-0.2005959	0.0823140	0.0843519
22	1.1355502	-2.6654758	-0.8302429	-1.5566248	-0.3793574	1.6511486	-0.5216230	-0.0513736	-0.1639215	-0.2767149	-0.6362508	0.0679774	-0.1996577	0.0270876	0.1178644	0.0489007
23	1.3450303	-3.3590558	-2.9773697	1.5069656	-0.7769344	-0.5125506	0.2355771	-0.5026803	-0.2064426	0.2435876	0.0168650	0.1923296	-0.1731983	-0.1021257	-0.0408881	0.0157000
24	-0.2548218	-0.6499558	0.0220828	-1.4028396	1.3882897	-1.4292398	-0.6896804	-0.2389471	-0.4609876	0.4130583	-0.0219019	0.0880860	-0.0653089	0.2744917	0.0505418	0.0471415

Table 13: Correlaciones

Variable	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10	PC11	PC12	PC13	PC14	PC15	PC16
RadOjo	0.6781867	-0.6084619	0.0565977	-0.1056566	-0.0566079	-0.2137055	0.0122365	0.0553840	0.0056768	-0.1164693	0.0190763	-0.0061280	0.0171301	0.0250748	-0.0147384	0.0563412
RadBra	0.5941287	-0.6683107	0.0750845	0.0534942	-0.0687210	0.0990901	0.0698650	0.1519824	0.2385636	-0.0537277	0.0287542	0.0141817	0.0419169	-0.0269475	0.0141129	-0.0278514
RadOpe	0.5327014	-0.7076030	-0.2254930	-0.2186188	0.0509600	0.0535342	0.0286780	0.0809749	-0.0505723	0.0532231	-0.0612205	0.0245109	-0.0553714	0.0743323	-0.0048270	-0.0249809
RadAle	0.6224750	-0.6467572	-0.1515350	-0.1669414	0.1343407	-0.0028805	0.0248477	0.0510220	-0.0851249	0.1498929	0.0108938	-0.0411839	-0.0165302	-0.0707740	-0.0075486	0.0136754
RadHig	0.5486968	-0.1648956	0.5545420	-0.0323249	0.1557506	-0.4860266	0.0365894	-0.1070428	-0.0542176	-0.0174687	-0.0210098	0.0057351	-0.0012041	-0.0095528	0.0133818	-0.0290774
RadDig	0.2332937	-0.2129625	-0.5472958	0.6435336	0.1888148	-0.0935057	0.2245889	-0.0696635	-0.0140920	-0.0293335	-0.0131912	-0.0207105	-0.0110781	0.0027389	0.0026139	-0.0003859
RadRin	-0.3040760	-0.5299079	-0.2724501	0.1385305	-0.6036233	-0.2015580	-0.1708169	-0.1064880	0.0138694	0.0400826	-0.0269038	0.0077741	-0.0028376	-0.0102861	-0.0018389	-0.0066464
RadEsc	0.6500026	-0.5175131	-0.0855494	-0.0811677	0.1344774	0.2900342	-0.1459470	-0.2735942	-0.0608175	-0.0522472	0.0580790	0.0284526	0.0391338	0.0013439	0.0137117	-0.0035764
RadMus	0.3509391	-0.2504071	0.6971948	0.4185337	-0.0159860	0.2099293	-0.0414559	-0.0614586	0.0728733	0.0889043	-0.0583763	-0.0079453	-0.0181793	0.0155919	-0.0225186	0.0152675
Peso	-0.8650169	-0.3681464	0.0012295	-0.0832691	0.0918297	-0.0100115	0.0515787	-0.0371916	-0.0174716	-0.0396351	0.0082842	-0.0308799	0.0572325	-0.0025914	-0.0701413	-0.0231412
Long	-0.8363185	-0.3527976	0.1346753	-0.0156194	-0.0212249	0.0229677	0.2126647	-0.0256182	-0.0260123	0.0026778	0.0311642	0.1465589	-0.0450565	-0.0222560	-0.0046229	0.0107579
LonEst	-0.8214582	-0.3619655	0.1100118	-0.1250255	-0.0615164	0.0524445	0.2234175	-0.0352463	-0.0371883	0.0570146	-0.0772040	-0.0299445	0.1023099	0.0178599	0.0357560	0.0118024
AncCab	-0.8308477	-0.3710574	0.1415134	-0.1002159	-0.0137846	-0.0164184	0.0916957	-0.0885954	0.0948842	0.0170390	0.1280932	-0.0870624	-0.0786646	0.0219536	0.0143078	0.0005540
Ancho	-0.8089344	-0.3604470	-0.0683712	-0.1145116	0.2197639	0.0417123	-0.1332450	-0.0405298	0.0832185	-0.1017226	-0.1447372	-0.0205444	-0.0605363	-0.0309323	0.0102053	0.0068603
AchHoc	-0.6189104	-0.5030252	0.2576046	0.3036205	-0.0259320	0.0938402	-0.1395520	0.1834764	-0.2250455	-0.0692346	0.0383582	-0.0192135	-0.0035181	-0.0040114	0.0169780	-0.0095357
DiaOjo	-0.7482163	-0.2278672	-0.1452990	0.1335477	0.3541557	-0.2087039	-0.2572501	0.0482904	0.1002313	0.1009880	0.0429522	0.0440166	0.0455046	0.0225279	0.0068930	0.0094821

```
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_amiard$df_quality_representation,
    booktabs = T,
    caption = "Calidad de proyección"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_amiard$df_comunalities,
    booktabs = T,
    caption = "Comunalidad"
) %>%
    kable_styling(font_size = 10)
```

En este caso se encuentra que las dos componentes principales ocupan un 96% de la inercia total. Además, al observar el gráfico de los autovalores se observa que estos tienen un claro codo en la

Table 14: Calidad de proyección

Pez	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10	PC11	PC12	PC13	PC14	PC15	PC16	Calidad.representacion.plano.principal
1	0.8418954	0.0743579	0.0033912	0.0031023	0.0453523	0.0003221	0.0150091	0.0011740	0.0003321	0.0000009	0.0036934	0.0108225	0.0004376	0.0000409	0.0000449	0.0000235	0.9162532
2	0.8641647	0.0002486	0.0467383	0.0090608	0.0031058	0.0059521	0.0304068	0.0119920	0.0198858	0.0054701	0.0012322	0.0000603	0.0006409	0.0001514	0.0000038	0.0008863	0.8644133
3	0.9518285	0.0046224	0.0006693	0.0003563	0.0119047	0.0085412	0.0039039	0.0005241	0.0110652	0.0018702	0.0000012	0.0005334	0.0018872	0.0013023	0.0007265	0.0002634	0.9564509
4	0.9379891	0.0082267	0.0012256	0.0000302	0.0057385	0.0028176	0.0188129	0.0061478	0.0160143	0.0009347	0.0003652	0.0000020	0.0000711	0.0000860	0.0008579	0.0006805	0.9462157
5	0.1243548	0.7317387	0.0703849	0.0107083	0.0236552	0.0241854	0.0000029	0.0001379	0.0054513	0.0017377	0.0018282	0.0000481	0.0029152	0.0001175	0.0023879	0.0003458	0.8560936
6	0.0303643	0.8405045	0.0330961	0.0227134	0.0430926	0.0026824	0.0014836	0.0011518	0.0113497	0.0104874	0.0000888	0.0020173	0.0000006	0.0008262	0.0000013	0.0001399	0.8708689
7	0.0161153	0.9026206	0.0037474	0.0286713	0.0049691	0.0250925	0.0002836	0.0043885	0.0008220	0.0004935	0.0074609	0.0009933	0.0039152	0.0000047	0.0004092	0.0000131	0.9187359
8	0.1515390	0.5230741	0.0039257	0.0003769	0.2687237	0.0075959	0.0196878	0.0023696	0.0010495	0.0183101	0.0000017	0.0003212	0.0010637	0.0008049	0.0002107	0.0009454	0.6746131
9	0.0350686	0.4447134	0.0325356	0.0022183	0.0062483	0.3148043	0.0544709	0.0034767	0.0037496	0.0274051	0.0340331	0.0010802	0.0012850	0.0338695	0.0003373	0.0047042	0.4797820
10	0.0754136	0.0554653	0.3317995	0.0161668	0.0388568	0.0977284	0.2675713	0.0129525	0.0814156	0.0097537	0.0015100	0.0034779	0.0000001	0.0026642	0.0046705	0.0005540	0.1308788
11	0.0515652	0.0867603	0.3272383	0.1443844	0.1600837	0.0774376	0.0205654	0.0046192	0.1120746	0.0105351	0.0006936	0.0000003	0.0018373	0.0016940	0.0002673	0.0002437	0.1383254
12	0.2062142	0.0262458	0.0037983	0.0781177	0.0112776	0.1555763	0.2865769	0.1182573	0.0358493	0.0208420	0.0119594	0.0029302	0.0407587	0.0002429	0.0006848	0.0006686	0.2324599
13	0.0017590	0.0410637	0.2744176	0.6045150	0.0006543	0.0202066	0.0066873	0.0280147	0.0003063	0.0037559	0.0120136	0.0031961	0.0013813	0.0014073	0.0000149	0.0006064	0.0428227
14	0.5034356	0.1846070	0.0037479	0.0242271	0.0065425	0.2055698	0.0054564	0.0027779	0.0077434	0.0508139	0.0018140	0.0024816	0.0000000	0.0000691	0.0000797	0.0006340	0.6880426
15	0.3338907	0.0756709	0.0218304	0.2737767	0.0056565	0.0199764	0.0072221	0.0934390	0.0993350	0.0012506	0.0019255	0.0091777	0.0430097	0.0010559	0.0002265	0.0125563	0.4095617
16	0.2847977	0.0168506	0.4598368	0.0683929	0.0053161	0.0112157	0.0123980	0.0103059	0.0050561	0.0084843	0.0478371	0.0401273	0.0285318	0.0005934	0.0002047	0.0000516	0.3016483
18	0.4212396	0.3635421	0.0026906	0.0730855	0.0771989	0.0078062	0.0012160	0.0394333	0.0084006	0.0000933	0.0020091	0.0002805	0.0010738	0.0003153	0.0011209	0.0004942	0.7847817
19	0.5840327	0.0453229	0.1671974	0.0114520	0.0877471	0.0171556	0.0680622	0.0032610	0.0083975	0.0002214	0.0037337	0.0019512	0.0008337	0.0000116	0.0004748	0.0001450	0.6293556
20	0.7128371	0.0778821	0.1785341	0.0047785	0.0089686	0.0019943	0.0002850	0.0110017	0.0006371	0.0009605	0.0004310	0.0009219	0.0000407	0.0002742	0.0003518	0.0001012	0.7907193
21	0.5007213	0.1207700	0.0072151	0.0916114	0.1400170	0.0002371	0.0000949	0.0981487	0.0310619	0.0017959	0.0000005	0.0005419	0.0017684	0.0044720	0.0007530	0.0007908	0.6214913
22	0.0847151	0.4667641	0.0452854	0.1591900	0.0094546	0.1791102	0.0178756	0.0001734	0.0017653	0.0050305	0.0265953	0.0003036	0.0026189	0.0000482	0.0009127	0.0001571	0.5514792
23	0.0707121	0.4410256	0.3464933	0.0887638	0.0235938	0.0102684	0.0021692	0.0098767	0.0016658	0.0023192	0.0000111	0.0014458	0.0011725	0.0004077	0.0000653	0.0000096	0.5117376
24	0.0087343	0.0568226	0.0000656	0.2647096	0.2592471	0.2747666	0.0639808	0.0076799	0.0285846	0.0229497	0.0000645	0.0010437	0.0005737	0.0101347	0.0003436	0.0002989	0.0655569

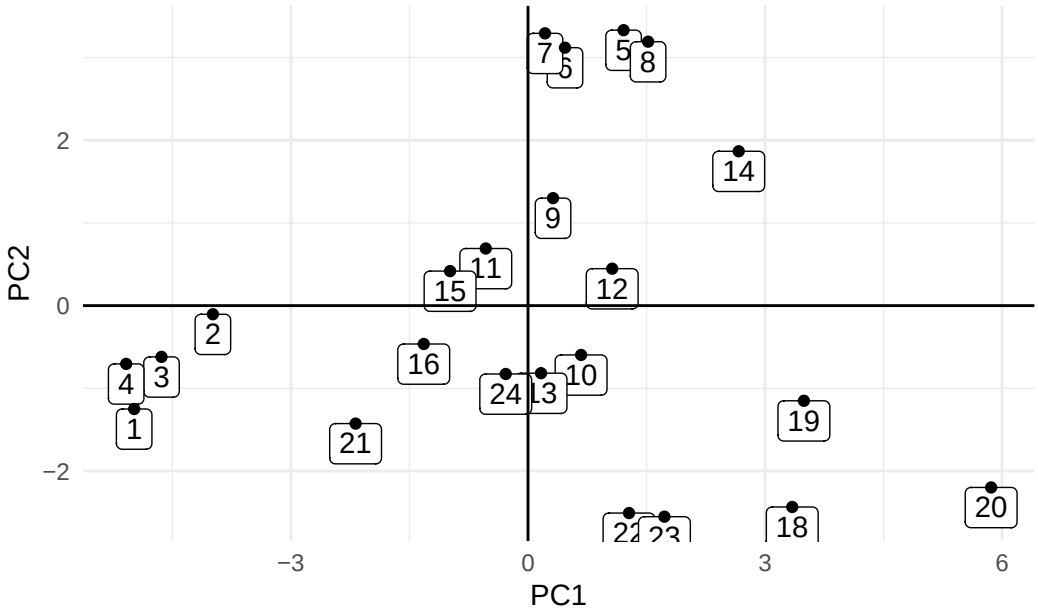
Table 15: Comunalidad

Variable	Comunalidad
RadOjo	0.8301631
RadBra	0.7996281
RadOpe	0.7844727
RadAle	0.8057700
RadHig	0.3282588
RadDig	0.0997790
RadRin	0.3732646
RadEsc	0.6903233
RadMus	0.1858619
Peso	0.8837861
Long	0.8238948
LonEst	0.8058127
AncCab	0.8279916
Ancho	0.7842969
AchHoc	0.6360845
DiaOjo	0.6117511

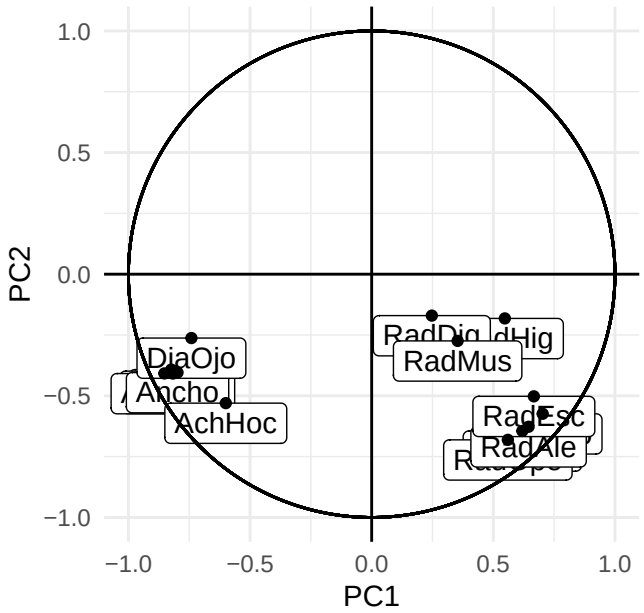
tercera componente principal. En esre caso sería mejor quedarse con las primeras dos. Se encuentra que la calidad de proyección en estad soc componentes principales es buena, superando el 80% en todos los casos.

Para realizar lo solicitado en el libro de proyectar como variable suplementaria

```
tablas_amiard_without_x_7 <- fn_acp_homework(df_amiard[, -8], name_acp = "acp_notas_quices")
```



Porcentaje de inercia: 0.725644388023151



Porcentaje de inercia: 0.725644388023151

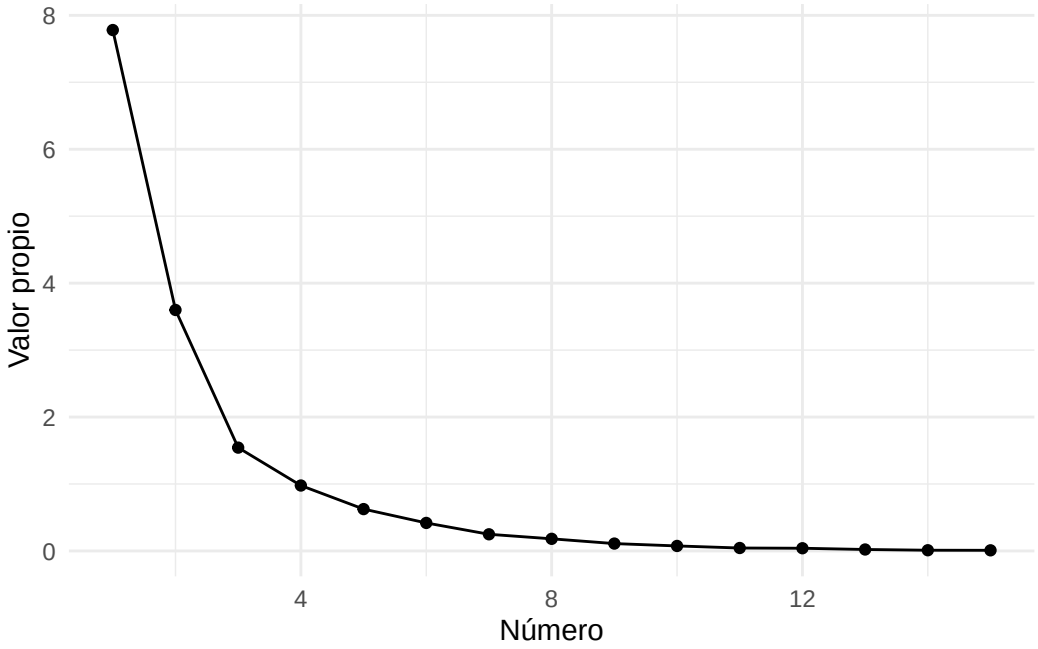


Table 16: Autovalores

Valor.propio	Valor	Porcentaje.de.inercia	Porcentaje.de.inercia.acumulado
1	7.7802116	49.6129433	49.61294
2	3.5992118	22.9514955	72.56444
3	1.5429127	9.8388635	82.40330
4	0.9768908	6.2294488	88.63275
5	0.6251004	3.9861477	92.61890
6	0.4180901	2.6660817	95.28498
7	0.2487069	1.5859570	96.87094
8	0.1810379	1.1544445	98.02538
9	0.1098397	0.7004272	98.72581
10	0.0746707	0.4761610	99.20197
11	0.0437638	0.2790735	99.48104
12	0.0401936	0.2563070	99.73735
13	0.0215814	0.1376207	99.87497
14	0.0103883	0.0662441	99.94122
15	0.0092185	0.0587845	100.00000

```
knitr::kable(tablas_amiard_without_x_7$eigen_values,
    booktabs = T,
    caption = "Autovalores"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_amiard_without_x_7$df_pc,
    booktabs = T,
    caption = "Componentes Principales"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_amiard_without_x_7$df_correlations,
    booktabs = T,
    caption = "Correlaciones"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

Table 17: Componentes Principales

Pez	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10	PC11	PC12	PC13	PC14	PC15
1	-4.9842152	-1.2500897	-0.0899996	-0.5239628	-0.3260811	0.1608543	0.3170805	0.1601555	-0.3422373	0.4739459	0.5921364	0.1672482	-0.0149629	0.0321649	0.0353278
2	-3.9880424	-0.1045719	0.9015586	-0.4571094	0.1917206	-0.7187794	-0.5567528	0.6338044	-0.2681508	-0.2329772	-0.0348660	-0.1073047	-0.0945352	-0.0862450	0.0702952
3	-4.6381836	-0.6199352	-0.0844338	0.0299637	-0.2341662	-0.0134446	-0.1026966	0.4672957	0.3227635	-0.0099895	-0.0990498	-0.2121472	0.0915088	-0.1980112	-0.0469020
4	-5.0852659	-0.7046033	-0.3453921	0.1291000	-0.1539843	-0.3797818	-0.7037375	-0.6497348	-0.1260092	-0.1488812	-0.0018421	0.0467075	-0.0390642	0.0333359	-0.2029940
5	1.2096217	3.3309610	-1.1653778	-0.2890233	-0.3302396	0.4218274	-0.2536775	-0.3184257	-0.0308441	-0.2690505	-0.0084517	0.2319130	-0.0341349	-0.2021878	-0.1212872
6	0.4680287	3.1192839	-0.3699021	-0.6127541	-0.3865776	-0.0392077	0.1905649	0.3439853	0.3273472	0.0552937	-0.1287377	-0.0208595	-0.0405730	0.0217866	-0.0002220
7	0.2150812	3.2925878	-0.2814444	-0.5672096	-0.4356442	0.1661514	0.1046056	0.0794275	-0.0134756	0.2242577	0.0365659	0.2638030	-0.1120287	-0.0187178	0.0885438
8	1.5199621	3.1915245	0.7481852	-0.4304927	-1.1103854	-1.0072560	-0.1097208	-0.1071809	0.3383243	0.2409090	0.1535377	-0.1446376	0.2484801	0.1452569	-0.0219731
9	0.3168432	1.2996486	0.4548297	-0.1363992	1.0994549	0.3078785	-0.0049947	-0.1071000	0.3176782	-0.2395748	-0.0205394	-0.0808943	-0.2554332	0.0660397	0.0499466
10	0.6717755	-0.5963792	1.4002628	-0.4081712	0.4547458	-1.3323460	-0.5298609	-0.6722523	-0.2220414	-0.1487678	-0.1360208	-0.0137892	0.1171182	0.0026793	0.1802234
11	-0.5343500	0.6907345	-1.3057207	0.9870548	0.9226341	0.4432359	-0.1631642	-0.7461523	0.1383823	0.1466554	0.0130191	0.0895459	0.1322798	0.0337355	0.0165629
12	1.0654997	0.4467989	-0.1015875	0.6865780	-0.6582564	1.5217120	-0.4504372	-0.3572785	-0.6130936	0.2553155	-0.0886696	-0.4867787	0.0343649	-0.0172404	0.0838644
13	0.1649989	-0.8169477	1.9532231	2.7960689	-0.4867167	0.2644449	0.5746685	-0.1166469	0.3820041	-0.3470512	0.2294864	0.1438367	0.1245239	-0.0974399	0.0680172
14	2.6666537	1.8664734	0.0488413	0.4596878	1.4568144	-0.3509963	0.5864982	0.3563790	-0.8668171	-0.3170463	0.1834363	0.0179639	0.0292253	0.0423481	-0.0734142
15	-0.9861755	0.4173428	0.2969190	0.8489295	-0.2964554	0.0011322	0.5635020	0.4897549	0.0492643	-0.1176115	-0.1334364	-0.3584624	-0.0220635	0.1418190	-0.1196433
16	-1.3198209	-0.4640274	1.6538101	0.5840206	-0.3001808	0.1774938	0.3638685	0.1674745	-0.2673646	0.4028296	-0.5866198	0.3862392	-0.0731323	0.0338281	0.0211938
18	3.3439644	-2.4350988	-0.0998325	-1.4918657	-0.1252368	-0.5679464	1.1828615	-0.4836329	-0.0892805	0.2054292	-0.0772245	-0.1611992	0.0402738	-0.1888653	-0.0756290
19	3.4913712	-1.1508616	-2.2812455	0.8328307	-0.1918826	-0.6314533	-0.3098119	0.3670264	0.1743085	0.1170621	0.1229934	-0.0886877	-0.2199868	-0.0788514	0.1459076
20	5.8612539	-2.1987829	2.6620946	-0.4131209	0.0389949	0.4317503	-0.9218108	0.2116902	0.1670499	0.1896323	0.1906677	0.0691247	-0.1384285	0.0056483	-0.1531273
21	-2.1819732	-1.4265481	-0.1346490	-0.6860017	0.4565016	0.2191272	0.6043790	-0.6223744	0.3893446	-0.0884493	0.0277470	-0.1030399	-0.2672846	0.1169196	0.0268075
22	1.2784723	-2.5083758	-0.9678184	-1.5192809	-1.6042442	0.7311412	0.0234501	0.1691313	-0.1361888	-0.6728714	-0.0343029	0.1789576	0.1156521	0.1184782	0.0683685
23	1.7262996	-2.5523534	-2.6505101	1.3977481	0.0775462	-0.6687880	-0.2498563	0.2626906	0.0060801	0.1766375	-0.1501063	0.1388876	0.0601109	0.0947016	-0.0768122
24	-0.2817993	-0.8267805	-0.2418106	-1.2165906	1.9416387	0.8632502	-0.1549576	0.4719633	0.3629559	0.1043026	-0.0497229	0.0435732	0.3180898	-0.0011830	0.0369454

Table 18: Correlaciones

Variable	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10	PC11	PC12	PC13	PC14	PC15
RadOjo	0.7030125	-0.5738248	0.0447695	-0.1151862	0.1655981	-0.1132228	0.1072794	-0.0031231	-0.1378312	0.0047632	0.0106114	-0.0178099	0.0440772	0.0356313	-0.0416794
RadBra	0.6193486	-0.6444110	0.0532334	0.0512683	-0.1331246	-0.1001363	0.1209999	-0.2488319	-0.0355132	0.0027728	-0.0213247	-0.0397444	-0.0460919	-0.0135161	0.0263694
RadOpe	0.5598466	-0.6809127	-0.2726864	-0.1942245	-0.0366116	-0.0164159	0.0548672	0.0415542	0.0823085	-0.0513745	-0.0201133	0.0502323	0.0721006	-0.0411461	0.0142132
RadAle	0.6449880	-0.6271993	-0.2032233	-0.1354322	0.0518144	0.0195701	0.0063836	0.0753329	0.1689027	0.0349566	0.0363252	0.0213984	-0.0678945	0.0251837	-0.0186780
RadHig	0.5474154	-0.1818078	0.5392392	-0.0367228	0.5196719	-0.0442209	-0.1096251	0.0609693	-0.0154380	-0.0265599	-0.0075244	0.0013677	-0.0191303	-0.0161789	0.0266330
RadDig	0.2472516	-0.1702096	-0.5444302	0.6792145	0.1143858	-0.1945037	-0.1198825	0.0166975	-0.0241749	-0.0198604	0.0192419	0.0129853	0.0006097	-0.0002973	0.0028259
RadEsc	0.6666720	-0.5020573	-0.1314800	-0.0494469	-0.1967608	0.2816223	-0.2416148	0.0826097	-0.0991432	0.0547363	-0.0276718	-0.0408472	0.0006080	0.0037740	0.0136552
RadMus	0.3533759	-0.2741978	0.6952070	0.4004292	-0.1912252	0.1062604	-0.0841119	-0.0713346	0.0903360	-0.0303659	0.0168439	0.0158515	0.0288990	0.0005243	-0.0272699
Peso	-0.8521779	-0.4078949	-0.0437161	-0.0621767	0.0392839	-0.0181863	-0.0549129	0.0192670	-0.0344353	-0.0049580	0.0289702	-0.0521767	-0.0283083	-0.0604122	-0.0456913
Long	-0.8239164	-0.3921867	0.1035363	-0.0124226	-0.0457032	-0.1897061	-0.0825711	0.0241258	0.0171257	0.0143425	-0.1544839	0.0332125	-0.0116530	0.0138667	-0.0117679
LonEst	-0.8081971	-0.3984343	0.0786542	-0.1250607	-0.0861426	-0.2004334	-0.0950146	0.0351548	0.0723101	-0.0553523	0.0492805	-0.1022094	0.0290884	0.0265170	0.0244891
AncCab	-0.8171883	-0.4084016	0.1100466	-0.0982954	0.0040161	-0.0844350	-0.0935816	-0.0865282	-0.0387439	0.1415454	0.0750691	0.0870085	0.0156989	0.0007227	0.0139224
Ancho	-0.7980370	-0.4042367	-0.1290577	-0.0738471	0.0569436	0.2096860	-0.0418686	-0.0804558	-0.0633714	-0.1674635	0.0197623	0.0611720	-0.0223187	0.0205284	0.0022741
AchHoc	-0.5996651	-0.5300576	0.2393489	0.2989109	-0.1066571	0.0684560	0.2472139	0.2155976	-0.0551935	0.0143959	0.0108042	0.0074169	-0.0156165	-0.0012526	0.0202297
DiaOjo	-0.7409575	-0.2625517	-0.1927077	0.1826902	0.3373089	0.2823616	0.0782780	-0.1027930	0.0805066	0.0697005	-0.0354481	-0.0506718	0.0322186	0.0081802	0.0016245

Table 19: Calidad de proyección

Pez	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10	PC11	PC12	PC13	PC14	PC15	Calidad.representacion.plano.principal
1	0.8978421	0.0564792	0.0002927	0.0099222	0.0038429	0.0009351	0.0036337	0.0009270	0.0042331	0.0081183	0.0126721	0.0010109	0.0000081	0.0000374	0.0000451	0.9543213
2	0.8661399	0.0005955	0.0442646	0.0113791	0.0020017	0.0281358	0.0168808	0.0218766	0.0039159	0.0029559	0.0000662	0.0006271	0.0004867	0.0004051	0.0002691	0.8667354
3	0.9604807	0.0171587	0.0003183	0.0000401	0.0024482	0.0000081	0.0004709	0.0097494	0.0046512	0.0000045	0.0004380	0.0020094	0.0003739	0.0017505	0.0000982	0.9776395
4	0.9348614	0.0179477	0.0043127	0.0006025	0.0008572	0.0052142	0.0179036	0.0152613	0.0005740	0.0008013	0.0000001	0.0000789	0.0000552	0.0000402	0.0014897	0.9528092
5	0.0999660	0.7580404	0.0927869	0.0057071	0.0074509	0.0121569	0.0043966	0.0069274	0.0000650	0.0049456	0.0000049	0.0036745	0.0000796	0.0027929	0.0010050	0.8580065
6	0.0201033	0.8929622	0.0125573	0.0344584	0.0137150	0.0001411	0.0033328	0.0108593	0.0098342	0.0002806	0.0015210	0.0000399	0.0001511	0.0000436	0.0000000	0.9130656
7	0.0039657	0.9293634	0.0067904	0.0275802	0.0162695	0.0023666	0.0009380	0.0005408	0.0000156	0.0043113	0.0001146	0.0059658	0.0010759	0.0000300	0.0006721	0.9333291
8	0.1461042	0.6441596	0.0354010	0.0117200	0.0779731	0.0641618	0.0007613	0.0007265	0.0072387	0.0036703	0.0014908	0.0013230	0.0039046	0.0013344	0.0000305	0.7902638
9	0.0281407	0.4734757	0.0579887	0.0052152	0.3388447	0.0265708	0.0000070	0.0032153	0.0282892	0.0160890	0.0001183	0.0018343	0.0182894	0.0012225	0.0006993	0.5016163
10	0.0780059	0.0614786	0.3389206	0.0287981	0.0357451	0.3068407	0.0485291	0.0781166	0.0085221	0.0038256	0.0031981	0.0000329	0.0023710	0.0000012	0.0056144	0.1394845
11	0.0555434	0.0928118	0.3316512	0.1895237	0.1655922	0.0382165	0.0051788	0.1083019	0.0037251	0.0041839	0.0000330	0.0015598	0.0034038	0.0002214	0.0000534	0.1483553
12	0.2030757	0.0357088	0.0018460	0.0843201	0.0775071	0.4142058	0.0362927	0.0228331	0.0672364	0.0116602	0.0014064	0.0423852	0.0002112	0.0000532	0.0012581	0.2387845
13	0.0020396	0.0500013	0.2858223	0.5857174	0.0177478	0.0052392	0.0247416	0.0010194	0.0109327	0.0090236	0.0039455	0.0015500	0.0011617	0.0007113	0.0003466	0.0520409
14	0.4931766	0.2416088	0.0001654	0.0146553	0.1471898	0.0085443	0.0238563	0.0088083	0.0521104	0.0069713	0.0023337	0.0000224	0.0000592	0.0001244	0.0003738	0.7347854
15	0.3475457	0.0622428	0.0315050	0.2575412	0.0314067	0.0000005	0.1134735	0.0857158	0.0008673	0.0049431	0.0063629	0.0459188	0.0001740	0.0071874	0.0051154	0.4097885
16	0.2879467	0.0355934	0.4521196	0.0563817	0.0148953	0.0052077	0.0218862	0.0046364	0.0118165	0.0268241	0.0568847	0.0246601	0.0008841	0.0001892	0.0000743	0.3235401
18	0.5214588	0.2765221	0.0004648	0.1037902	0.0007314	0.0150422	0.0652476	0.0109076	0.0003717	0.0019680	0.0002781	0.0012118	0.0000756	0.0016634	0.0002667	0.7979809
19	0.6028204	0.0655001	0.2573593	0.0343012	0.0018208	0.0197187	0.0047467	0.0066618	0.0015026	0.0006777	0.0007481	0.0003890	0.0023933	0.0003075	0.0010528	0.6683206
20	0.7205712	0.1014053	0.1486424	0.0035797	0.0000319	0.0039099	0.0178229	0.0009399	0.0005853	0.0007543	0.0007625	0.0001002	0.0004019	0.0000007	0.0004918	0.8219765
21	0.5568143	0.2380041	0.0021204	0.0550379	0.0243723	0.0056157	0.0427199	0.0453018	0.0177288	0.0009150	0.0000900	0.0012417	0.0083552	0.0015988	0.0000840	0.7948184
22	0.1101020	0.4238361	0.0630958	0.1554851	0.1733619	0.0360093	0.0000370	0.0019269	0.0012494	0.0304984	0.0000793	0.0021573	0.0009010	0.0009456	0.0003149	0.5339381
23	0.1556211	0.3401869	0.3668554	0.1020220	0.0003140	0.0233568	0.0032600	0.0036035	0.0000019	0.0016293	0.0011766	0.0010073	0.0001887	0.0004683	0.0003081	0.4958080
24	0.0108589	0.0934727	0.0079957	0.2023920	0.5155150	0.1019009	0.0032834	0.0304594	0.0180141	0.0014876	0.0003381	0.0002596	0.0138358	0.0000002	0.0001866	0.1043316

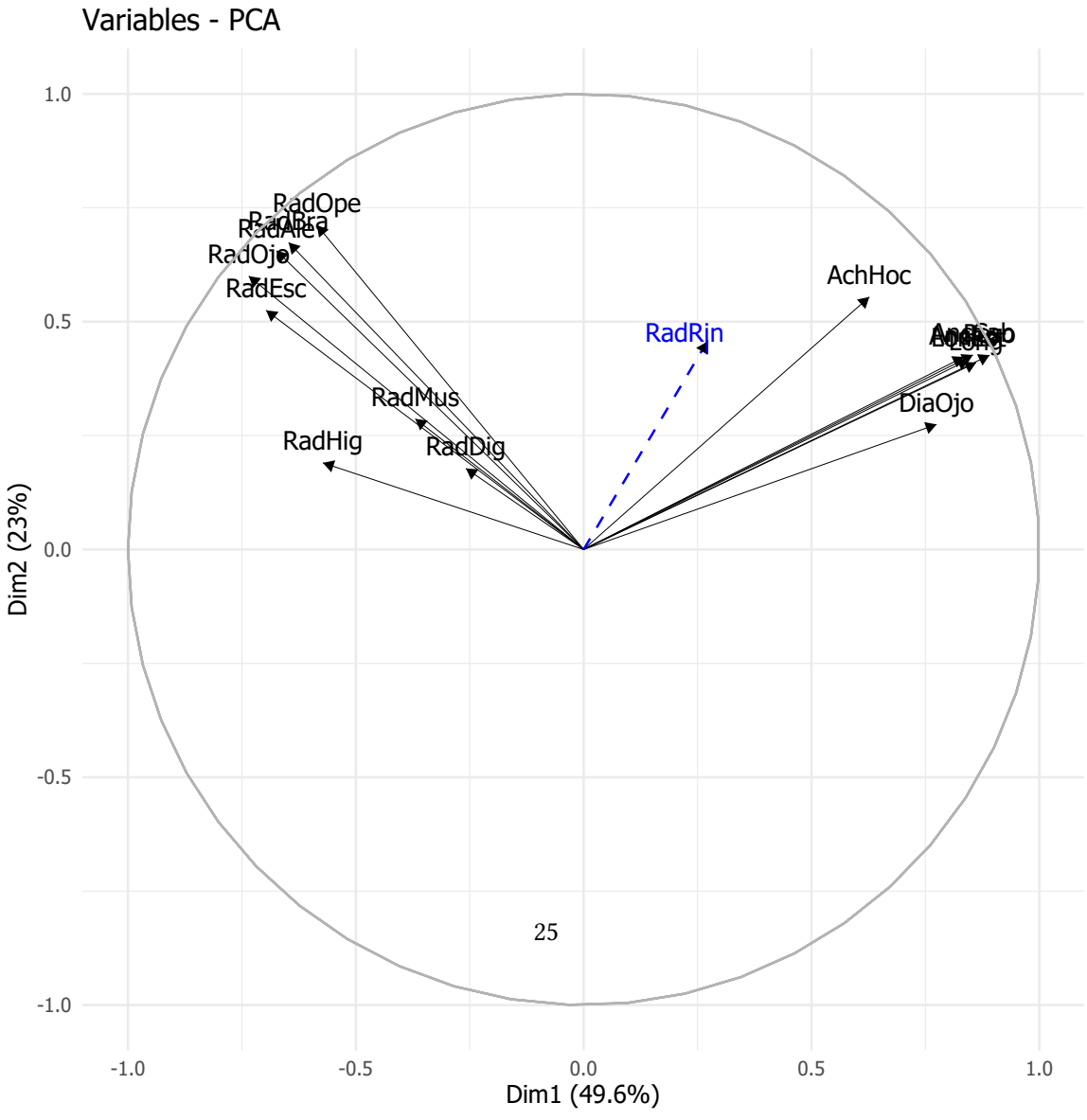
```
knitr::kable(tablas_amiard_without_x_7$df_quality_representation,
    booktabs = T,
    caption = "Calidad de proyección"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_amiard_without_x_7$df_comunalities,
    booktabs = T,
    caption = "Comunalidad"
) %>%
    kable_styling(font_size = 10)
```

```
pca_peces <- PCA(df_amiard[, -c(1)], quanti.sup = 7)


ggsave("./pca_peces.svg", fviz_pca_var(pca_peces))
```

Table 20: Comunalidad

Variable	Comunalidad
RadOjo	0.8235014
RadBra	0.7988581
RadOpe	0.7770704
RadAle	0.8093884
RadHig	0.3327177
RadDig	0.0901047
RadEsc	0.6965131
RadMus	0.2000589
Peso	0.8925855
Long	0.8326486
LonEst	0.8119324
AncCab	0.8345887
Ancho	0.8002704
AchHoc	0.6405592
DiaOjo	0.6179514

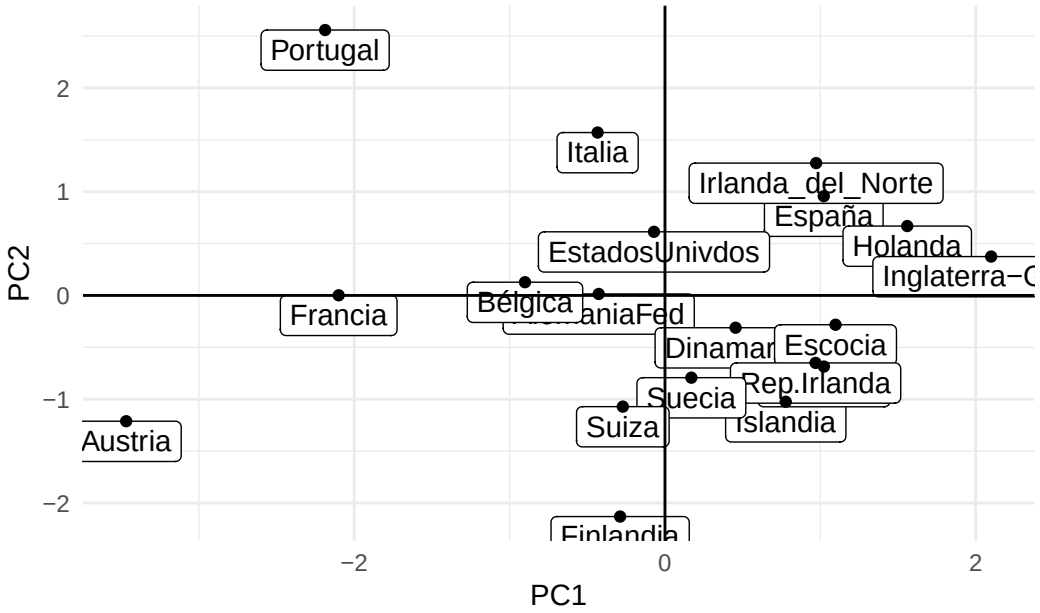


Se puede observar que el comportamiento en los dos ACP es similar sin la variable en cuestión. Salvo que la calidad de proyección mejora en las primeras componentes principales y la comunalidad sigue siendo muy similar.

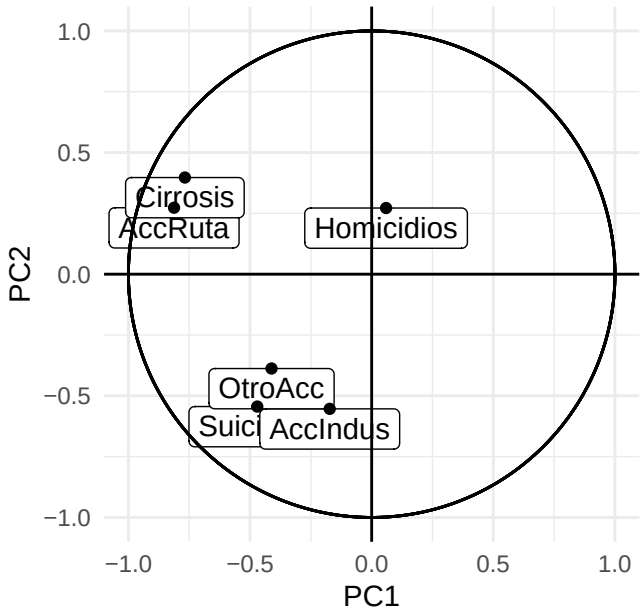
1.4 Ejercicio 11 del libro

```
df_suicidios <- read.xlsx("./Ejercicios-Cap3.xlsx", sheet = "11.Suicidios")
```

```
tablas_suicidios <- fn_acp_homework(df_suicidios, name_acp = "acp_notas_quices")
```



Porcentaje de inercia: 0.507483874942314



Porcentaje de inercia: 0.507483874942314

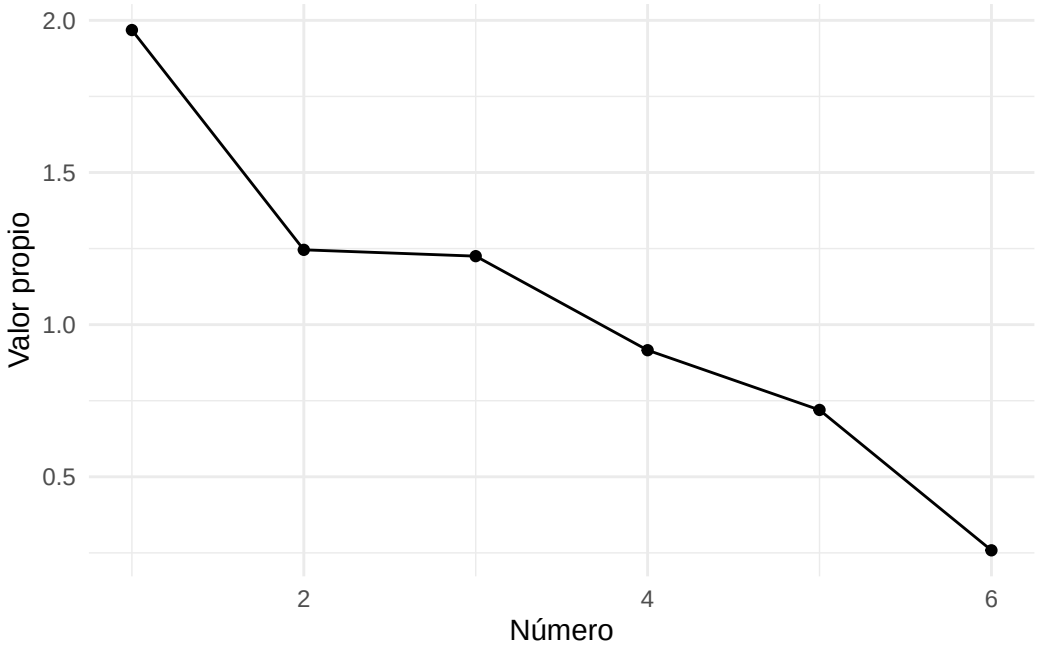


Table 21: Autovalores

Valor.propio	Valor	Porcentaje.de.inercia	Porcentaje.de.inercia.acumulado
1	1.9681680	31.076337	31.07634
2	1.2458965	19.672051	50.74839
3	1.2251310	19.344174	70.09256
4	0.9160556	14.464036	84.55660
5	0.7196778	11.363334	95.91993
6	0.2584043	4.080068	100.00000

```
knitr::kable(tablas_suicidios$eigen_values,
    booktabs = T,
    caption = "Autovalores"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_suicidios$df_pc,
    booktabs = T,
    caption = "Componentes Principales"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_suicidios$df_correlations,
    booktabs = T,
    caption = "Correlaciones"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_suicidios$df_quality_representation,
    booktabs = T,
    caption = "Calidad de proyección"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_suicidios$df_comunalities,
    booktabs = T,
    caption = "Comunalidad"
```

Table 22: Componentes Principales

País	PC1	PC2	PC3	PC4	PC5	PC6
AlemaniaFed	-0.4263880	0.0144191	-0.5089637	-1.3894771	-0.9541737	-0.1431914
Austria	-3.4678533	-1.2119566	0.9887456	-1.3854849	0.4686380	0.0822603
Bélgica	-0.9009976	0.1275439	-0.4837521	0.7236669	-0.1771543	-0.8569980
Dinamarca	0.4546154	-0.3110710	-1.3593127	-0.3848613	-1.5893768	0.0284894
Escocia	1.0977323	-0.2822200	-0.1398691	0.6662465	0.4506068	-0.0362594
España	1.0222325	0.9568950	-0.7982641	-0.4108638	0.7975313	1.0422848
EstadosUnivdos	-0.0707360	0.6130628	2.1772600	-0.2297338	-0.8456354	0.5557428
Finlandia	-0.2886004	-2.1304610	0.3772323	0.2826297	-0.6814870	0.0844775
Francia	-2.0999897	0.0016273	-0.8918319	2.4845971	0.5342944	0.6025602
Holanda	1.5594452	0.6684562	-0.8637664	-0.1963577	-0.3084496	-0.4925402
Inglaterra-Gales	2.0994769	0.3745679	-0.6841417	-0.4257898	-0.1590795	0.0000428
Irlanda_del_Norte	0.9729925	1.2749864	2.9912583	1.0971453	-0.8574852	-0.0613361
Islandia	0.7775545	-1.0242814	0.0607711	0.6560139	0.6957939	-0.5754188
Italia	-0.4334712	1.5696774	-0.4477220	-0.9163658	0.7570791	0.7020554
Noruega	1.0231734	-0.6850448	0.6639792	-0.4309550	1.4656643	-0.3179390
Rep.Irlanda	0.9694392	-0.6500950	0.6473598	-0.4390444	1.4788264	-0.2581765
Portugal	-2.1873391	2.5575186	-0.5242618	-0.0356717	0.1077829	-0.8551336
Suecia	0.1699913	-0.7923031	-0.8961674	1.1467201	-0.5556294	0.3681398
Suiza	-0.2712779	-1.0713214	-0.3085535	-0.8124142	-0.6277462	0.1309400

Table 23: Correlaciones

Variable	PC1	PC2	PC3	PC4	PC5	PC6
Suicidios	-0.4703241	-0.5440539	-0.2224989	-0.0874461	-0.5675731	0.0318713
Homicidios	0.0592404	0.2720680	0.8037618	0.1824316	-0.3585770	0.1099302
AccRuta	-0.8123853	0.2732462	0.2046983	-0.1299413	0.0130355	-0.3223623
AccIndus	-0.1724769	-0.5535392	0.5085991	-0.4476254	0.3088373	0.0832526
OtroAcc	-0.4116345	-0.3877757	0.1023246	0.7172821	0.2296034	0.0035040
Cirrosis	-0.7678530	0.3973579	-0.1873719	-0.0788631	0.1137343	0.3094386

Table 24: Calidad de proyección

País	PC1	PC2	PC3	PC4	PC5	PC6	Calidad.representacion.plano.principal
AlemaniaFed	0.0550486	0.0000630	0.0784350	0.5845739	0.2756713	0.0062083	0.0551116
Austria	0.7236552	0.0883862	0.0588274	0.1155085	0.0132156	0.0004072	0.8120414
Bélgica	0.3452099	0.0069176	0.0995134	0.2226965	0.0133456	0.3123170	0.3521275
Dinamarca	0.0428234	0.0200499	0.3828526	0.0306903	0.5234156	0.0001682	0.0628733
Escocia	0.6171742	0.0407935	0.0100198	0.2273447	0.1039945	0.0006734	0.6579676
España	0.2327793	0.2039734	0.1419509	0.0376046	0.1416904	0.2420014	0.4367527
EstadosUnivdos	0.0008073	0.0606395	0.7648326	0.0085152	0.1153751	0.0498303	0.0614468
Finlandia	0.0156681	0.8538281	0.0267695	0.0150265	0.0873652	0.0013425	0.8694962
Francia	0.3666685	0.0000002	0.0661310	0.5132763	0.0237356	0.0301884	0.3666687
Holanda	0.6078019	0.1116780	0.1864723	0.0096365	0.0237788	0.0606325	0.7194799
Inglaterra-Gales	0.8439609	0.0268634	0.0896174	0.0347129	0.0048454	0.0000000	0.8708243
Irlanda_del_Norte	0.0703213	0.1207477	0.6646234	0.0894121	0.0546161	0.0002794	0.1910690
Islandia	0.2082623	0.3613995	0.0012722	0.1482433	0.1667670	0.1140557	0.5696618
Italia	0.0394907	0.5178392	0.0421299	0.1764866	0.1204638	0.1035898	0.5573298
Noruega	0.2383606	0.1068498	0.1003795	0.0422863	0.4891082	0.0230157	0.3452104
Rep.Irlanda	0.2222907	0.0999617	0.0991223	0.0455929	0.5172667	0.0157657	0.3222524
Portugal	0.3875825	0.5298702	0.0222653	0.0001031	0.0009411	0.0592379	0.9174526
Suecia	0.0089771	0.1950137	0.2494943	0.4085048	0.0959076	0.0421025	0.2039908
Suiza	0.0308205	0.4806733	0.0398723	0.2764174	0.1650360	0.0071805	0.5114938

Table 25: Comunalidad

Variable	Comunalidad
Suicidios	0.5171994
Homicidios	0.0775304
AccRuta	0.7346334
AccIndus	0.3361540
OtroAcc	0.3198130
Cirrosis	0.7474916

```
) %>%
    kable_styling(font_size = 10)
```

En este caso se encuentra que el mayor porcentaje de inercia cercano al 90% se alcanza en la tercera componente principal. Sin embargo, el codo se presenta en la segunda componente principal. Por lo que se pueden seleccionar las primeras dos covariables. Además, en cuanto a la calidad de proyección se encuentra que AlemaniaFed, Estados Unidos, Suiza y Dinamarca no tienen la mejor calidad de proyección. Sin embargo, con la tercera componente principal mejoraría la calidad de manera significativa.

Se verifican los valores obtenidos con `dudi.pca`

```r
library(ade4)
dudi_pca_suicidios <- dudi.pca(df_suicidios[,-1], scannf = FALSE, scale = TRUE, center = TRUE)
```

Se obtienen valores similares, más no exactamente iguales, posiblemente motivado por la forma en que se estiman estos con los pesos, podemos verlo acontinuación donde se estima la desviación estándar sin el estimador insesgado y coincide con lo empleado por la función `dudi.pca`.

```r
# SD using weigth equal to n
apply(df_suicidios[, -1], 2, sd) * sqrt((nrow(df_suicidios) - 1) / nrow(df_suicidios))
```

```
Suicidios Homicidios    AccRuta   AccIndus    OtroAcc   Cirrosis
75.584995  34.041119  57.048893   9.124258  82.395102 110.463265
```

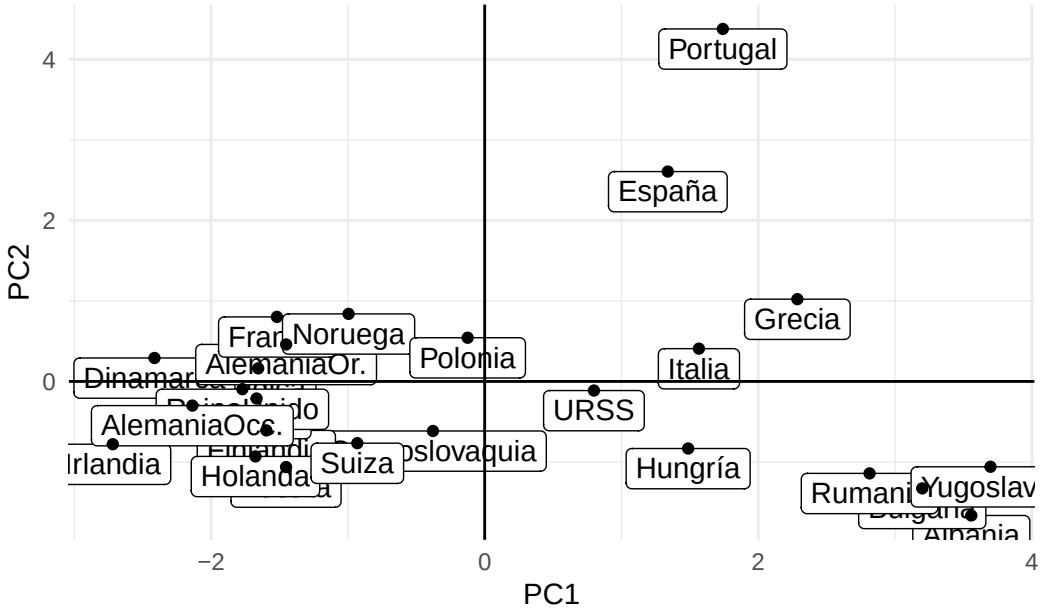
```r
# SD used by dudi.pca
dudi_pca_suicidios$norm
```

```
Suicidios Homicidios    AccRuta   AccIndus    OtroAcc   Cirrosis
75.584995  34.041119  57.048893   9.124258  82.395102 110.463265
```

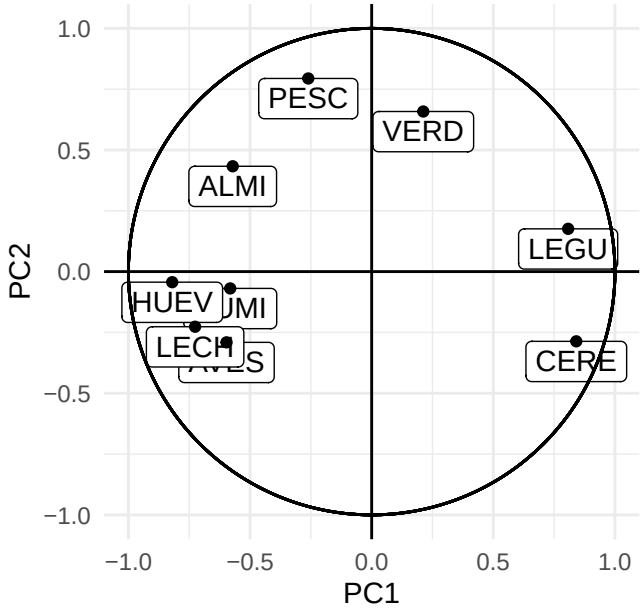
1.5 Ejercicio 12 del libro

```r
df_proteinas <- read.xlsx("./Ejercicios-Cap3.xlsx", sheet = "12.Proteinas")
```

```r
tablas_proteinas <- fn_acp_homework(df_proteinas, name_acp = "acp_notas_quices")
```



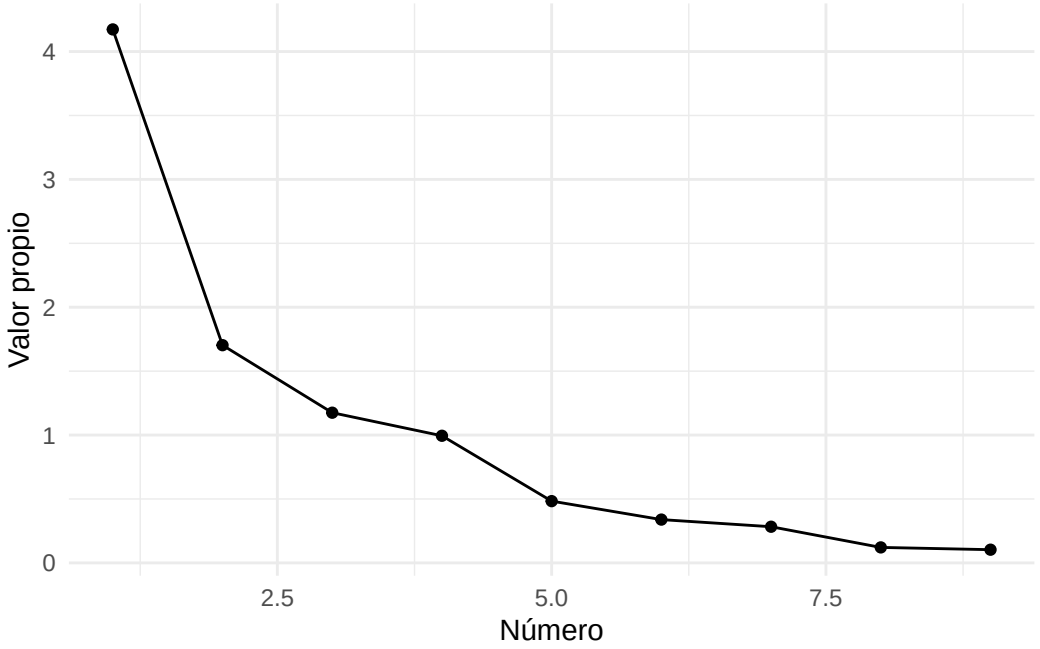
Porcentaje de inercia: 0.626826335654979



Porcentaje de inercia: 0.626826335654979

Table 26: Autovalores

Valor.propio	Valor	Porcentaje.de.inercia	Porcentaje.de.inercia.acumulado
1	4.1733725	44.515973	44.51597
2	1.7031244	18.166660	62.68263
3	1.1749162	12.532439	75.21507
4	0.9944416	10.607377	85.82245
5	0.4831650	5.153760	90.97621
6	0.3386781	3.612566	94.58878
7	0.2829233	3.017848	97.60662
8	0.1211374	1.292132	98.89876
9	0.1032416	1.101243	100.00000




```
knitr::kable(tablas_proteinas$eigen_values,
    booktabs = T,
    caption = "Autovalores"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

Table 27: Componentes Principales

País	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9
Albania	3.5572381	-1.6641015	-1.7975512	-0.2343937	0.0237335	1.0555921	0.4814696	0.7772550	-0.1053824
Austria	-1.4520059	-1.0627022	1.3653904	-0.1715636	-0.9526949	-0.2229322	0.1848897	-0.2561783	-0.2219302
Bélgica	-1.6554798	0.1627845	0.2209995	-0.5314638	0.7706609	0.2957811	0.1996301	-0.2075042	-0.0338555
Bulgaria	3.1987084	-1.3278945	0.1544093	-0.2186061	-0.4947412	0.7099212	-0.4743666	-0.8249113	-0.3060463
Checoslovaquia	-0.3781039	-0.6150959	1.2206030	0.4735497	0.2621197	0.8400632	-0.3214429	0.0125517	-0.1525300
Dinamarca	-2.4140424	0.2913319	-0.7677756	0.9872885	-0.7679488	0.1738420	0.2304724	-0.6338267	0.4901831
AlemaniaOr.	-1.4515379	0.4595864	1.3293995	1.1593857	0.4316642	0.6616811	0.5662228	-0.1665420	-0.2652606
Finlandia	-1.5961042	-0.6082926	-2.0917697	1.4444931	0.0379703	-0.8514022	-0.7412059	0.2305761	-0.1358342
Francia	-1.5186657	0.8015600	0.0019214	-1.9978218	0.2556233	0.9174853	-0.9659921	-0.0226782	0.5548173
Grecia	2.2858842	1.0217014	-0.9008033	-1.8313202	-0.4133282	-1.1680767	0.1504300	-0.3121371	0.3959653
Hungría	1.4874933	-0.8327767	1.9536390	0.2218710	-0.0422514	-0.5502252	0.7839406	0.1486213	0.5480166
Irlandia	-2.7184004	-0.7794547	-0.0202906	-0.4436925	1.0353047	-0.4922718	0.0292598	0.0234733	-0.0811045
Italia	1.5662092	0.4072145	0.1286897	-1.2476685	-0.8201099	-0.2185002	-0.1530848	-0.0820641	-0.7474531
Holanda	-1.6752932	-0.9307968	0.7822937	0.1287530	-0.7769756	-0.3036571	0.0633762	0.4694096	0.2662514
Noruega	-0.9948020	0.8389795	-1.7392158	1.1610802	-0.4234287	0.0576157	0.0436700	-0.1095594	-0.1503678
Polonia	-0.1243826	0.5427068	1.5052005	0.4676712	-0.0236983	-0.6004312	-1.2867195	0.1955472	-0.2255570
Portugal	1.7410299	4.3773749	0.0445325	0.9119854	-0.3932337	0.7114712	-0.0474585	0.2092500	0.2689214
Rumania	2.8136598	-1.1418555	0.0715260	0.6276933	0.3236348	-0.1332081	0.1358233	-0.0274482	0.3450022
España	1.3388579	2.6061797	0.5259092	-0.3666114	0.5265405	-0.6830894	0.6095257	0.2401807	-0.4865664
Suecia	-1.6674187	-0.2116609	-1.3067741	0.7492390	-0.8367302	-0.0449855	0.5523214	-0.0737070	-0.1106274
Suiza	-0.9311308	-0.7665460	-0.1574349	-1.1945801	-0.8480945	0.0921032	-0.5228547	0.5402120	0.0679377
ReinoUnido	-1.7711527	-0.0959174	-1.1764506	-1.7694421	1.1062911	0.0985562	0.6643929	-0.2441417	-0.1346565
URSS	0.7987342	-0.1130543	-0.3773022	0.9467001	1.7039849	-0.1892536	-0.5859401	-0.0530999	0.0935758
AlemaniaOcc.	-2.1370117	-0.2998369	0.8205581	-0.1110385	-0.0697704	0.2051376	0.4661958	0.3639830	-0.0258005
Yugoslavia	3.6977167	-1.0594337	0.2102961	0.8384921	0.3854781	-0.3612167	-0.0625551	-0.1972615	0.1523015

```
knitr::kable(tablas_proteinas$df_pc,
    booktabs = T,
    caption = "Componentes Principales"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_proteinas$df_correlations,
    booktabs = T,
    caption = "Correlaciones"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_proteinas$df_quality_representation,
    booktabs = T,
    caption = "Calidad de proyección"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

Table 28: Correlaciones

Variable	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9
RUMI	-0.5814774	-0.0690503	-0.3033985	-0.6063861	0.2106329	0.2517302	-0.0752139	-0.0065009	0.0743479
AVES	-0.5967476	-0.2907432	0.6360970	0.0346982	-0.1962522	0.0662386	0.0098379	-0.0091261	0.1790386
HUEV	-0.8198817	-0.0433755	0.1850779	-0.2937434	0.0517236	-0.1977455	0.2217742	-0.1608068	-0.1007585
LECH	-0.7258197	-0.2265872	-0.3931989	0.0031125	-0.1310333	-0.3385294	-0.2311918	0.0266555	0.0538219
PESC	-0.2606573	0.7939868	-0.3275565	0.2025628	-0.1896263	0.0748784	0.0532300	-0.1469038	0.0945447
CERE	0.8411434	-0.2866086	0.0977931	0.0058194	0.1557176	-0.0442067	-0.2026085	-0.2301432	0.0460170
ALMI	-0.5711746	0.4331020	0.2477262	0.3158057	0.4811901	-0.0808319	-0.0764243	0.0374974	0.0368289
LEGU	0.8076912	0.1759172	-0.0554513	-0.3098052	0.0984232	-0.2446908	0.2037584	0.0601716	0.1566370
VERD	0.2121767	0.6581862	0.4155255	-0.4334020	-0.1526766	-0.0648935	-0.2251294	0.0301065	-0.0613372

Table 29: Calidad de proyección

País	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	Calidad.representacion.plano.principal
Albania	0.6121536	0.1339659	0.1563137	0.0026578	0.0000272	0.0539047	0.0112143	0.0292255	0.0005372	0.7461195
Austria	0.3379926	0.1810480	0.2988712	0.0047187	0.1455051	0.0079674	0.0054802	0.0105210	0.0078959	0.5190405
Bélgica	0.7092928	0.0068581	0.0126404	0.0731013	0.1537108	0.0226422	0.0103141	0.0111438	0.0002966	0.7161509
Bulgaria	0.7406468	0.1276407	0.0017259	0.0034593	0.0177181	0.0364823	0.0162888	0.0492580	0.0067801	0.8682875
Checoslovaquia	0.0455790	0.1206225	0.4749976	0.0714947	0.0219049	0.2249917	0.0329420	0.0000502	0.0074174	0.1662015
Dinamarca	0.6628456	0.0096538	0.0670488	0.1108691	0.0670791	0.0034374	0.0060417	0.0456945	0.0273300	0.6724994
AlemaniaOr.	0.3255241	0.0326333	0.2730470	0.2076740	0.0287885	0.0676432	0.0495337	0.0042852	0.0108711	0.3581573
Finlandia	0.2374903	0.0344944	0.4078976	0.1945156	0.0001344	0.0675761	0.0512155	0.0049562	0.0017201	0.2719847
Francia	0.2537586	0.0706917	0.0000004	0.4391469	0.0071895	0.0926178	0.1026700	0.0000566	0.0338685	0.3244503
Grecia	0.4266774	0.0852392	0.0662600	0.2738544	0.0139502	0.1114125	0.0018478	0.0079558	0.0128028	0.5119165
Hungría	0.2761104	0.0865426	0.4762790	0.0061429	0.0002228	0.0377793	0.0766901	0.0027564	0.0374766	0.3626530
Irlandia	0.7764984	0.0638404	0.0000433	0.0206861	0.1126290	0.0254638	0.0000900	0.0000579	0.0006912	0.8403388
Italia	0.4459006	0.0301429	0.0030104	0.2829678	0.1222595	0.0086784	0.0042599	0.0012242	0.1015562	0.4760435
Holanda	0.5302788	0.1636939	0.1156278	0.0031321	0.1140611	0.0174217	0.0007589	0.0416319	0.0133939	0.6939727
Noruega	0.1574434	0.1119835	0.4812361	0.2144745	0.0285241	0.0005281	0.0003034	0.0019096	0.0035972	0.2694269
Polonia	0.0031572	0.0601060	0.4623555	0.0446343	0.0001146	0.0735722	0.3378742	0.0078035	0.0103825	0.0632632
Portugal	0.1273314	0.8049161	0.0000833	0.0349381	0.0064957	0.0212636	0.0000946	0.0018393	0.0030379	0.9322475
Rumania	0.8012558	0.1319624	0.0005178	0.0398770	0.0106008	0.0017959	0.0018671	0.0000763	0.0120468	0.9332182
España	0.1722684	0.6527478	0.0265802	0.0129166	0.0266440	0.0448427	0.0357043	0.0055439	0.0227521	0.8250162
Suecia	0.4543711	0.0073215	0.2790760	0.0917407	0.1144175	0.0003307	0.0498546	0.0008878	0.0020001	0.4616926
Suiza	0.2062345	0.1397708	0.0058958	0.3394459	0.1710915	0.0020179	0.0650282	0.0694174	0.0010979	0.3460054
ReinoUnido	0.3332290	0.0009773	0.1470206	0.3325856	0.1300079	0.0010318	0.0468901	0.0063316	0.0019261	0.3342063
URSS	0.1280142	0.0025646	0.0285649	0.1798367	0.5826194	0.0071869	0.0688906	0.0005658	0.0017570	0.1305788
AlemaniaOcc.	0.7956403	0.0156629	0.1173064	0.0021481	0.0008481	0.0073315	0.0378652	0.0230816	0.0001160	0.8113032
Yugoslavia	0.8605992	0.0706450	0.0027835	0.0442519	0.0093526	0.0082124	0.0002463	0.0024492	0.0014600	0.9312442

Table 30: Comunalidad

Variable	Comunalidad
RUMI	0.3428839
AVES	0.4406393
HUEV	0.6740874
LECH	0.5781560
PESC	0.6983573
CERE	0.7896668
ALMI	0.5138177
LEGU	0.6833119
VERD	0.4782281

```
knitr::kable(tablas_proteinas$df_comunalities,
    booktabs = T,
    caption = "Comunalidad"
) %>%
    kable_styling(font_size = 10)
```

En este caso se encuentra que para con la segunda componente principal se obtiene un porcentaje de inercia del 86%, además se encuentra el codo en el segundo autovalor. En cuanto a la calidad de la proyección se ubica Checoslovaquía la presenta la peor calidad de representación, sin embargo, con la tercera componente esto mejora de manera significativa. En cuanto a las covariables al que tiene mayor comunalidad es la de CERE.

Se comparan los autovalores obtenidos por `acp` de `amap`. Donde se puede notar donde los autovalores son similare, más no iguales motivado por la forma en que son escaladas las variables, diferencia con el estimador insesgado.

```
library(amap)
(amap_pca_proteinas <- amap::acp(df_proteinas[, -1]))
```

```
Standard deviations:
   Comp 1    Comp 2    Comp 3    Comp 4    Comp 5    Comp 6    Comp 7    Comp 8
2.0016087 1.2786710 1.0620355 0.9770691 0.6810568 0.5702026 0.5211586 0.3410160
   Comp 9
0.3148204

Eigen values:
[1] 9.805840 6.264183 5.202890 4.786641 3.336483 2.793411 2.553146 1.670630
[9] 1.542299
```

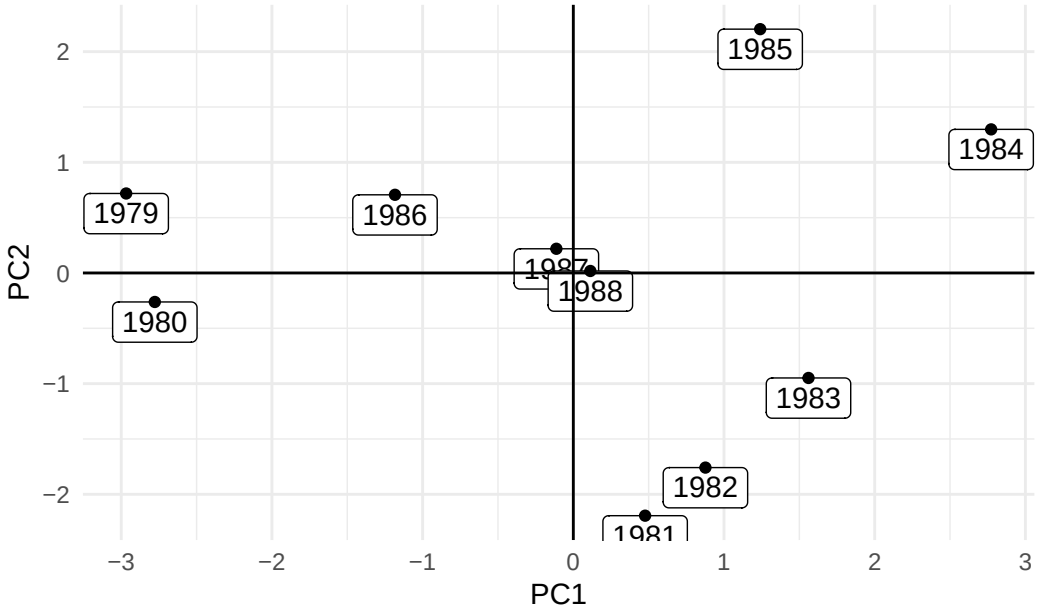
```
(amap_pca_proteinas$sdev)^2
```

```
   Comp 1    Comp 2    Comp 3    Comp 4    Comp 5    Comp 6    Comp 7    Comp 8
4.0064376 1.6349994 1.1279195 0.9546640 0.4638384 0.3251310 0.2716063 0.1162919
   Comp 9
0.0991119
```

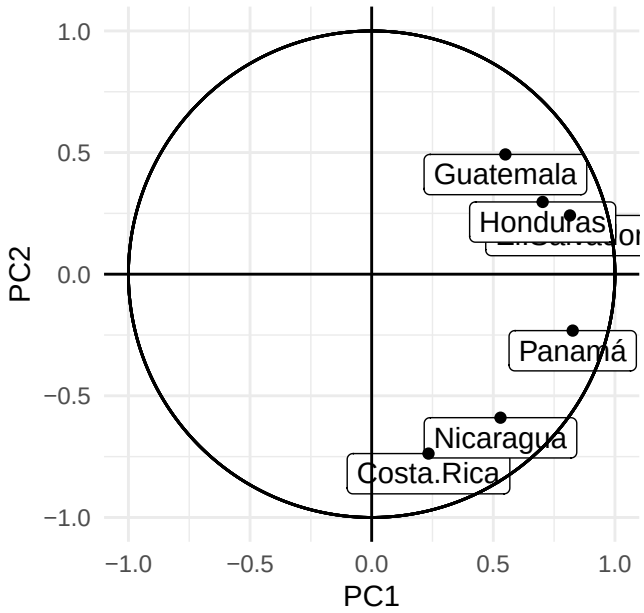
1.6 Ejercicio 13 del libro

```
df_importmex <- read.xlsx("./Ejercicios-Cap3.xlsx", sheet = "13.ImportMex")
```

```
tablas_importmex <- fn_acp_homework(df_importmex, name_acp = "acp_notas_quices")
```



Porcentaje de inercia: 0.784608554142161



Porcentaje de inercia: 0.784608554142161

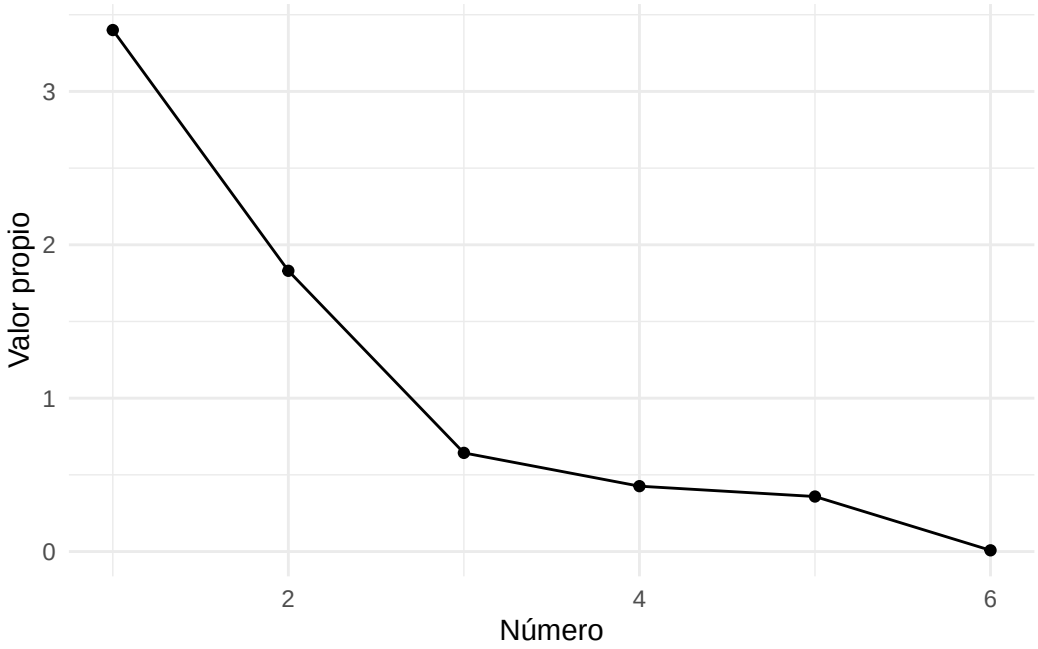


Table 31: Autovalores

Valor.propio	Valor	Porcentaje.de.inercia	Porcentaje.de.inercia.acumulado
1	3.4001930	51.0028951	51.00290
2	1.8305307	27.4579603	78.46086
3	0.6431720	9.6475806	88.10844
4	0.4262044	6.3930667	94.50150
5	0.3586333	5.3794990	99.88100
6	0.0079332	0.1189983	100.00000

Table 32: Componentes Principales

Año	PC1	PC2	PC3	PC4	PC5	PC6
1979	-2.9672548	0.7183595	-0.3488030	0.1357637	-0.4880140	0.0006900
1980	-2.7771519	-0.2619704	0.6256753	0.8753063	-0.0449730	-0.0754310
1981	0.4766095	-2.1932610	-0.0458554	0.7300205	0.8154158	0.0922968
1982	0.8768022	-1.7587370	-1.2182684	-0.2908561	-0.1386310	-0.0947363
1983	1.5607861	-0.9487933	0.2308284	-0.1674952	-1.2908399	0.0396005
1984	2.7728560	1.2977541	1.0519601	0.6445836	-0.1240581	-0.0238495
1985	1.2401064	2.2033840	-1.3556914	0.3926874	0.3455696	0.0012511
1986	-1.1834462	0.7064935	-0.1318465	-0.6099133	-0.0614058	0.1572698
1987	-0.1124194	0.2186811	0.4177914	-0.8847467	0.4175440	-0.1397303
1988	0.1131121	0.0180895	0.7742096	-0.8253502	0.5693925	0.0426390

```
knitr::kable(tablas_importmex$eigen_values,
    booktabs = T,
    caption = "Autovalores"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_importmex$df_pc,
    booktabs = T,
    caption = "Componentes Principales"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

Table 33: Correlaciones

Variable	PC1	PC2	PC3	PC4	PC5	PC6
Costa.Rica	0.2336431	-0.7374027	0.4123822	0.0575557	0.1948753	0.0173220
El.Salvador	0.8151204	0.2414903	-0.0427549	-0.2881325	0.0006389	0.0491173
Honduras	0.7029214	0.2970594	-0.1788121	0.3824151	0.2222687	0.0062261
Guatemala	0.5497487	0.4923393	0.4650566	0.0846228	-0.2040440	-0.0174848
Nicaragua	0.5295525	-0.5898742	-0.1922573	0.1577125	-0.3460243	0.0073210
Panamá	0.8264275	-0.2319109	-0.1084947	-0.2146963	0.1126796	-0.0516980

Table 34: Calidad de proyección

Año	PC1	PC2	PC3	PC4	PC5	PC6	Calidad.representacion.plano.principal
1979	0.9077942	0.0532061	0.0125441	0.0019004	0.0245551	0.0000000	0.9610004
1980	0.8620728	0.0076709	0.0437565	0.0856377	0.0002261	0.0006360	0.8697438
1981	0.0363683	0.7701554	0.0003366	0.0853233	0.1064525	0.0013639	0.8065237
1982	0.1408308	0.5666255	0.2718819	0.0154971	0.0035206	0.0016441	0.7074563
1983	0.4790256	0.1770171	0.0104773	0.0055167	0.3276549	0.0003084	0.6560427
1984	0.7046793	0.1543554	0.1014228	0.0380799	0.0014105	0.0000521	0.8590347
1985	0.1808340	0.5708769	0.2161144	0.0181324	0.0140421	0.0000002	0.7517108
1986	0.6043187	0.2153701	0.0075008	0.1605111	0.0016270	0.0106723	0.8196888
1987	0.0104305	0.0394679	0.1440591	0.6460401	0.1438885	0.0161140	0.0498984
1988	0.0078990	0.0002020	0.3700572	0.4205603	0.2001590	0.0011224	0.0081010

```
knitr::kable(tablas_importmex$df_correlations,
    booktabs = T,
    caption = "Correlaciones"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_importmex$df_quality_representation,
    booktabs = T,
    caption = "Calidad de proyección"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_importmex$df_comunalities,
    booktabs = T,
    caption = "Comunalidad"
) %>%
```

Table 35: Comunalidad

Variable	Comunalidad
Costa.Rica	0.5983519
El.Salvador	0.7227388
Honduras	0.5823428
Guatemala	0.5446216
Nicaragua	0.6283775
Panamá	0.7367650

```
    kable_styling(font_size = 10)
```

En este penúltimo intento se encuentra que las mayoría que la el codo se encuentra en la segunda componente principal, además el porcentaje de inercia explicado por las dos primeras componentes es cercano al 88%. Además, se tiene que encuanto a la calidad de proyección es buena en la mayoría de los años exceptuando los casos 1988 y 1987 que muestran una calidad inferior.

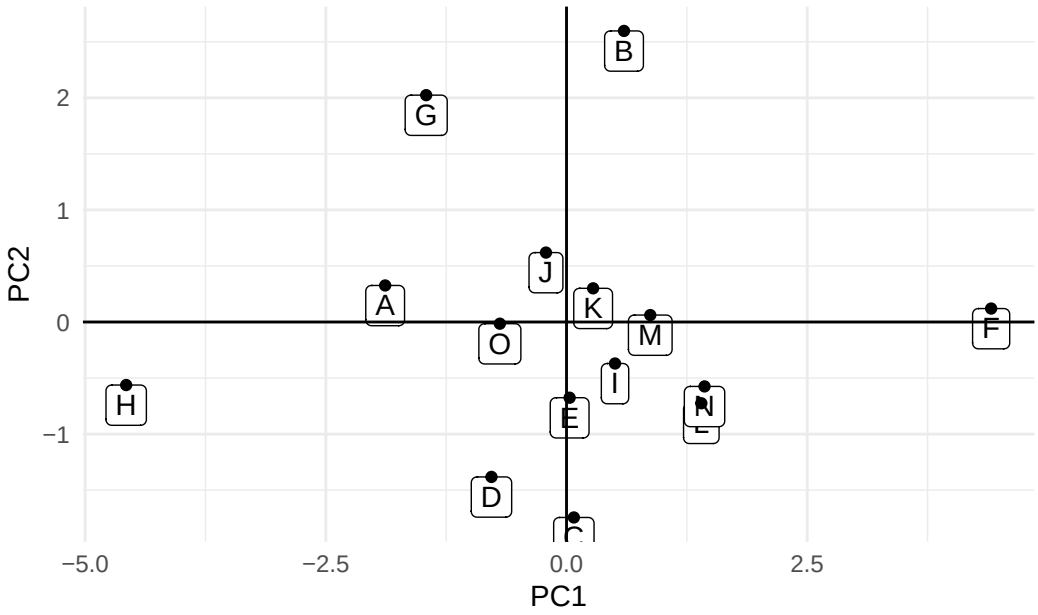
En cuanto a las trayectorías se puede observar que los años consecutivos en algunos casos se encuentran cercano, sin embargo, este no siempre es el caso teniéndose que por ejemplo el 1981 se aleja bastante de 1980, 1983 de 1984 y 1985 de 1986, de tal forma de que se describe un patrón el que cada dos años un año queda en el lado opuesto del plano. Podría considerarse por cada dos años consecutivos que quedan en el mismo lado del plano el año siguiente se contraponga.

Los países que tienen una mayor comunalidad son Nicaragua y Honduras.

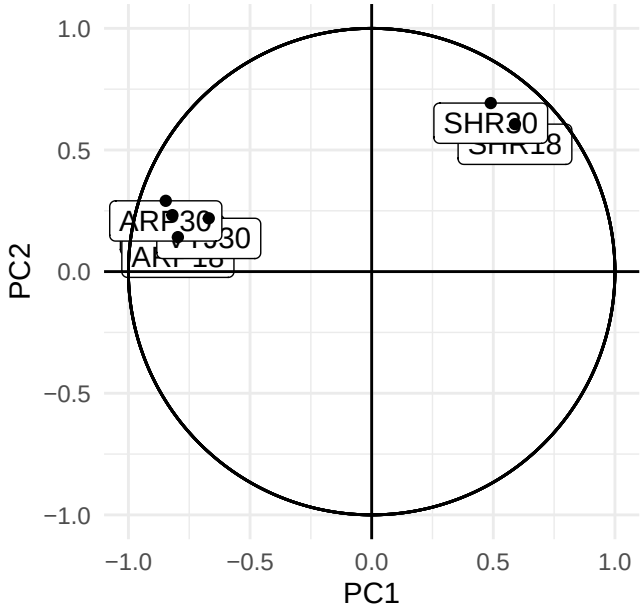
1.7 Ejercicio 14 del libro

```
df_atletas <- read.xlsx("./Ejercicios-Cap3.xlsx", sheet = "14.Atletas")
```

```
tablas_atletas <- fn_acp_homework(df_atletas, name_acp = "acp_notas_quices")
```



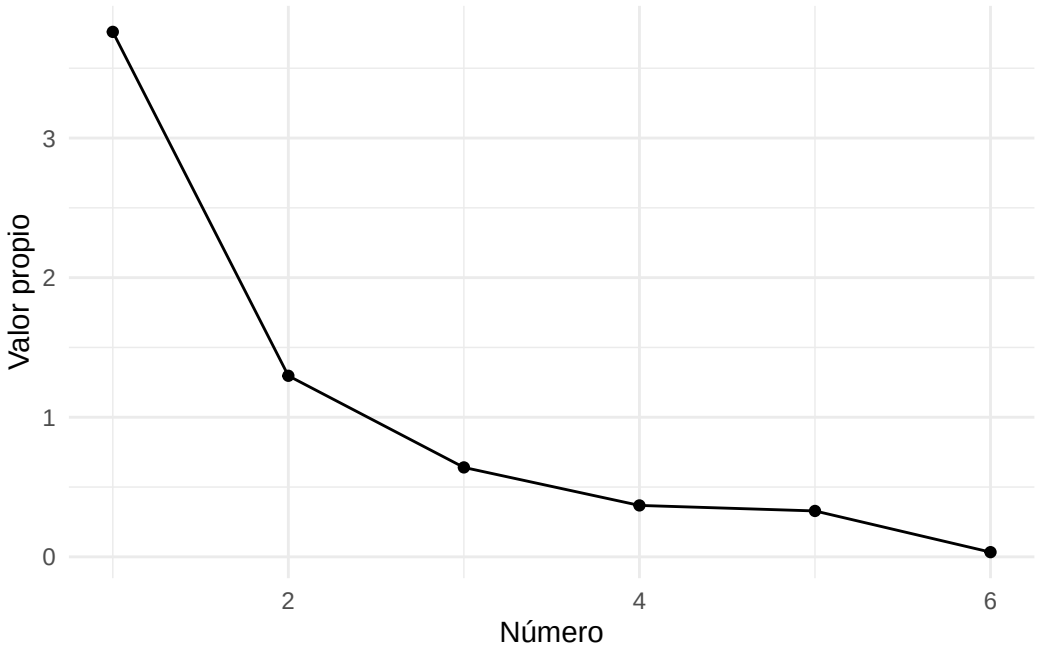
Porcentaje de inercia: 0.786671118889984



Porcentaje de inercia: 0.786671118889984

Table 36: Autovalores

Valor.propio	Valor	Porcentaje.de.inercia	Porcentaje.de.inercia.acumulado
1	3.7602622	58.4929674	58.49297
2	1.2969093	20.1741445	78.66711
3	0.6406396	9.9655050	88.63262
4	0.3683649	5.7301204	94.36274
5	0.3287639	5.1141053	99.47684
6	0.0336315	0.5231574	100.00000




```
knitr::kable(tablas_atletas$eigen_values,
    booktabs = T,
    caption = "Autovalores"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_atletas$df_pc,
    booktabs = T,
    caption = "Componentes Principales"
```

Table 37: Componentes Principales

Atleta	PC1	PC2	PC3	PC4	PC5	PC6
A	-1.8837319	0.3261354	1.5367830	0.2254737	0.3574525	-0.0163570
B	0.5971449	2.5969870	0.3580031	-0.4256968	0.4675157	-0.0118453
C	0.0762734	-1.7438160	0.0593636	-0.9027172	-0.5248150	0.3118900
D	-0.7795216	-1.3821643	0.2984881	0.6954546	-0.2941110	-0.1447266
E	0.0323358	-0.6754829	0.5645492	-0.3778249	-0.0285443	-0.1139363
F	4.4105926	0.1200233	-0.3062428	0.0564737	-0.6885021	-0.0701537
G	-1.4575360	2.0240042	-0.8595229	-0.2008677	-0.7379540	-0.1362108
H	-4.5737134	-0.5623959	-0.9254251	-0.1575316	0.2031243	-0.0715938
I	0.5038526	-0.3702258	-0.6567393	0.9606954	-0.0319918	-0.0252560
J	-0.2134435	0.6194915	0.2080364	0.9414966	0.3572075	0.4042477
K	0.2764408	0.3004657	-0.7431297	0.5548636	-0.3606290	0.1690071
L	1.4002613	-0.7250872	-1.4377604	-0.5030792	1.2421704	-0.0925911
M	0.8698717	0.0618640	0.5373761	-0.7662972	0.4303326	0.1614540
N	1.4335096	-0.5746856	0.8602273	0.4256053	0.4249951	-0.2745674
O	-0.6923362	-0.0151133	0.5059933	-0.5260481	-0.8162510	-0.0893609

```
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_atletas$df_correlations,
    booktabs = T,
    caption = "Correlaciones"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_atletas$df_quality_representation,
    booktabs = T,
    caption = "Calidad de proyección"
) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
knitr::kable(tablas_atletas$df_comunalities,
    booktabs = T,
    caption = "Comunalidad"
```

Table 38: Correlaciones

Variable	PC1	PC2	PC3	PC4	PC5	PC6
VTJ18	-0.8192073	0.2322053	-0.0418262	0.2328785	0.2894475	0.0795582
ARP18	-0.7966078	0.1417261	-0.3007462	-0.2189176	-0.2712385	0.0670622
SHR18	0.5890541	0.6062782	0.1947984	-0.3096748	0.1445122	0.0427135
VTJ30	-0.6697554	0.2192329	0.5705822	0.0834085	-0.2046457	-0.0086535
ARP30	-0.8463834	0.2910596	-0.1220536	-0.1437445	0.1431661	-0.1182136
SHR30	0.4895162	0.6928764	-0.2241838	0.2716966	-0.1633611	-0.0253584

Table 39: Calidad de proyección

Atleta	PC1	PC2	PC3	PC4	PC5	PC6	Calidad.representacion.plano.principal
A	0.5727558	0.0171683	0.3812031	0.0082058	0.0206238	0.0000432	0.5899241
B	0.0467402	0.8840379	0.0167998	0.0237537	0.0286499	0.0000184	0.9307781
C	0.0013728	0.7175575	0.0008316	0.1922909	0.0649932	0.0229540	0.7189303
D	0.1899969	0.5973233	0.0278576	0.1512264	0.0270466	0.0065492	0.7873202
E	0.0011212	0.4892599	0.3417547	0.1530707	0.0008737	0.0139199	0.4903811
F	0.9705475	0.0007187	0.0046790	0.0001591	0.0236501	0.0002455	0.9712662
G	0.2808856	0.5416437	0.0976800	0.0053347	0.0720028	0.0024531	0.8225293
H	0.9438742	0.0142712	0.0386419	0.0011197	0.0018617	0.0002313	0.9581454
I	0.1453297	0.0784659	0.2469069	0.5283465	0.0005859	0.0003652	0.2237955
J	0.0276104	0.2325826	0.0262292	0.5372099	0.0773299	0.0990379	0.2601930
K	0.0644656	0.0761576	0.4658573	0.2597144	0.1097097	0.0240954	0.1406232
L	0.3083742	0.0826876	0.3251119	0.0398045	0.2426734	0.0013483	0.3910618
M	0.4095143	0.0020713	0.1562843	0.3177995	0.1002230	0.0141077	0.4115856
N	0.5768519	0.0927094	0.2077254	0.0508484	0.0507027	0.0211622	0.6695613
O	0.2842046	0.0001354	0.1518052	0.1640771	0.3950430	0.0047347	0.2843400

Table 40: Comunalidad

Variable	Comunalidad
VTJ18	0.7250199
ARP18	0.6546702
SHR18	0.7145580
VTJ30	0.4966353
ARP30	0.8010806
SHR30	0.7197038

```
) %>%
    kable_styling(font_size = 10)
```

En este caso se encuentra que la mayoría de la inercia se encuentra en las primeras dos componentes principales alcanzando cerca del 92% de la inercia total. Adicional el codo se presenta en el segundo autovalor. En cuanto a la calidad de representaci'on se encuentra que salvo D, F y M la mayor'ia presentan una calidad aceptable.

2

a-

```
princomp_notas_escolares <- princomp(df_notas_frances[,-1], cor = TRUE )
```

Se encuentra que esta función contiene la raíz cuadrada de los autovalores

```
princomp_notas_escolares$sdev
```

```
    Comp.1     Comp.2     Comp.3     Comp.4     Comp.5
1.69653338 1.06527681 0.99176864 0.04872137 0.03132099
```

Contiene las correlaciones de las variables respecto a las

```
princomp_notas_escolares$loadings
```

```
Loadings:
        Comp.1 Comp.2 Comp.3 Comp.4 Comp.5
Mate     0.478  0.529  0.161  0.299  0.614
Fisica   0.531  0.396        -0.527 -0.524
```

```
Frances   0.444 -0.578 -0.232 -0.469  0.441
Latin     0.540 -0.360 -0.113  0.642 -0.392
Deportes        -0.316  0.947

               Comp.1 Comp.2 Comp.3 Comp.4 Comp.5
SS loadings       1.0    1.0    1.0    1.0    1.0
Proportion Var    0.2    0.2    0.2    0.2    0.2
Cumulative Var    0.2    0.4    0.6    0.8    1.0
```

Contiene el centro de gravedad

```
princomp_notas_escolares$center
```

```
     Mate    Fisica   Frances     Latin  Deportes
 9.666667  9.833333 10.222222 10.055556 11.000000
```

Contiene la desviación estándar

```
princomp_notas_escolares$scale
```

```
    Mate   Fisica  Frances    Latin Deportes
3.374743 2.990726 3.473000 2.813109 2.943920
```

Contiene el número de observaciones

```
princomp_notas_escolares$n.obs
```

```
[1] 9
```

Contiene las componentes principales

```
princomp_notas_escolares$scores
```

```
      Comp.1     Comp.2     Comp.3       Comp.4       Comp.5
1 -2.7788226  0.6927433 -0.7345732  0.051064911 -0.036339304
2 -1.2649120  0.3439755 -0.5524236  0.029536890  0.014120268
3 -1.0292266 -1.0077668 -0.2982004 -0.057510172  0.005131786
4  3.0987283  0.1850850 -1.1325100  0.055844214  0.032735824
```

```
5  2.0369954  0.7287226 -0.1946418 -0.020996558 -0.059518017
6 -0.9467285  1.1936845  1.1514813  0.006407985  0.046719439
7 -0.3451875 -1.7342903 -0.9249510 -0.039201939  0.012900796
8  0.6309889  1.1257092  0.7065004 -0.080111713  0.002013151
9  0.5981647 -1.5278632  1.9793185  0.054966381 -0.017763943
```

Contiene la forma en que se empleó la función

```
princomp_notas_escolares$call
```

```
princomp(x = df_notas_frances[, -1], cor = TRUE)
```

b-

```
princomp_general_notas_escolares <- princomp(df_notas_frances[, -1])
princomp_notas_escolares
```

```
Call:
princomp(x = df_notas_frances[, -1], cor = TRUE)

Standard deviations:
    Comp.1     Comp.2     Comp.3     Comp.4     Comp.5
1.69653338 1.06527681 0.99176864 0.04872137 0.03132099

 5  variables and  9 observations.
```

Se puede observar que cambian los autovalores principalmente, además se obtienen las siguientes componentes principales, que son diferentes de las obtenidas al escalar las variables.

```
princomp_general_notas_escolares$scores
```

```
     Comp.1     Comp.2     Comp.3      Comp.4      Comp.5
1 -8.700907  1.7027046 -2.5539182  0.14945398 -0.11731596
2 -3.938596  0.7085441 -1.8104644  0.09068389  0.04349922
3 -3.209392 -3.4590552 -0.3006617 -0.17254286  0.01928215
4  9.755741 -0.2157421 -3.3436726  0.17347137  0.10041455
5  6.371422  2.1733326 -0.9570588 -0.07066256 -0.18799232
6 -2.974017  4.6509322  2.6349457  0.02321315  0.14809545
7 -1.050967 -6.2271742 -1.6880636 -0.11529582  0.04281219
8  1.980533  4.0685562  1.4007122 -0.24321198  0.01039742
9  1.766183 -3.4020982  6.6181814  0.16489082 -0.05919270
```

Table 41: Datos reconstruídos con ACP centrados

RadOjo	RadBra	RadOpe	RadAle	RadHig	RadDig	RadRin	RadEsc	RadMus	Peso	Long	LonEst	AncCab	Ancho	AchHoc	DiaOjo
-0.7240084	-0.7754864	-0.5976669	-0.6502150	-1.2479619	-0.7935851	2.1673312	-0.7370135	-0.8220535	1.8940638	1.3139966	1.6766595	2.3345787	1.6771133	1.7477208	1.3082673
-0.8572259	-1.2024680	-0.9497898	-1.0996498	0.1100354	-0.6507767	0.2862513	-0.9405049	-0.4653133	1.5145911	1.6491750	1.7404003	1.2939719	1.0120511	0.9595330	0.2706760
-1.2568786	-1.1248349	-0.5164077	-0.7850454	-1.0010533	-0.3458615	-0.0272620	-0.8363930	-0.4653133	1.7802220	1.6491750	1.7404003	1.2939719	1.2337385	1.3536269	1.3082673
-1.1236610	-0.6784451	-0.9362466	-1.1108857	-1.1862348	0.1095815	0.2862513	-0.9830961	-0.4653133	1.9320111	1.9284904	1.8041411	1.9183360	1.8988007	0.5654391	1.3082673
-0.9904435	-0.8919359	-0.5705805	-0.7288661	-0.8158719	0.0285281	-1.5948286	-0.2543129	-0.8220535	-0.9519816	-1.2557047	-1.3828977	-1.2034844	-0.5397606	-1.7991243	-0.7669153
-0.9904435	-1.1442432	-0.7330988	-0.5940356	-0.6306904	-0.6430574	-0.3407753	-0.6849575	-0.8220535	-0.8760870	-0.6970740	-0.6817492	-0.9953630	-0.9831354	-1.0109365	-0.7669153
-1.1236610	-1.1248349	-0.9904194	-0.8749324	-0.6924176	-0.7009527	-1.5948286	-0.6423663	-0.8220535	-0.8760870	-0.8088001	-0.9367123	-0.5791203	-0.7614480	-1.0109365	-0.7669153
-0.5907908	-0.5037708	-0.8549875	-0.7850454	-0.4455090	-0.6777945	0.2862513	-0.7417458	0.2481671	-1.3314543	-0.8088001	-0.3630453	-0.7872417	-1.8698849	-1.4050304	-1.8045066
-0.3243558	-0.4843625	-0.6112101	-0.1108932	0.9124883	-0.3458615	-0.0272620	-0.6092398	-0.1085731	-0.3827725	-0.4736217	-0.4267861	-0.5791203	-0.0963858	-0.6168426	0.2706760
0.7413846	0.8742153	0.0794926	-0.2120160	1.3445783	-0.2030531	-0.0272620	-0.3063688	0.6049073	-0.1171416	0.5319136	0.5293256	0.4614865	-0.3180732	-0.6168426	-0.7669153
-0.4575733	-0.2708717	-0.3403463	-0.3019030	-0.3220547	1.1902938	-0.0272620	-0.3678895	-0.4653133	-0.0791943	-0.3060325	-0.4905268	0.0452438	0.3469890	-0.6168426	1.3082673
-0.1911382	0.2919677	-0.4486918	-0.4479693	-0.3837818	0.1057219	-0.0272620	1.5108570	0.6049073	-0.3827725	-0.8646632	-0.8092307	-0.5791203	-0.0963858	-0.2227487	0.2706760
-0.1911382	0.7189493	-0.3132599	-0.3356106	0.4186711	0.9316948	-0.0272620	-0.4104807	2.7453486	-0.2689307	-0.0825802	-0.1080822	-0.1628776	-0.0963858	1.7477208	0.2706760
1.0078197	-0.2514635	-0.3945190	-0.3693182	1.3445783	0.6846748	-0.0272620	-0.4767337	-0.4653133	-1.1417179	-1.4791570	-1.5103792	-1.4116058	-0.9831354	-0.6168426	-0.7669153
-0.3243558	-0.3873213	-0.6112101	-0.4592052	-0.4455090	0.1404590	-0.0272620	-0.5051279	0.2481671	0.1484893	0.2525982	0.2743625	-0.3709989	-0.0963858	0.9595330	0.2706760
-0.1911382	0.0202521	-0.5705805	-0.6165074	0.2334897	-0.6430574	-0.0272620	-0.2353834	0.9616475	0.1864366	1.0905443	-0.0443414	0.6696079	0.1253016	1.3536269	0.2706760
2.2067775	2.3104262	2.0432552	1.6756103	0.5421254	-0.6739348	1.2267912	0.7726090	-0.1085731	-0.3827725	-0.5294848	-0.4267861	-0.3709989	-0.7614480	-0.2227487	-0.7669153
0.8746021	1.1071144	1.4744412	1.7205538	-0.1368732	2.3250417	-1.2813153	1.3830833	-0.4653133	-0.7242979	-0.8646632	-0.6817492	-0.9953630	-0.9831354	-0.6168426	-0.7669153
2.0735600	1.7475868	1.3390093	2.0801017	2.8260299	-0.6662155	-1.2813153	2.4194699	2.7453486	-1.2555597	-1.1439786	-1.0641938	-0.7872417	-1.4265102	-0.6168426	-1.8045066
-0.0579207	0.4278254	0.1336654	0.3610134	-0.2603275	-0.7048123	-0.6542887	-0.6660281	-0.4653133	0.9453820	0.7553659	0.9117702	0.8777292	1.2337385	0.5654391	1.3082673
0.8746021	1.0682979	1.9890824	1.3160625	-0.9393262	-0.6932333	0.5997646	1.8563192	-0.1085731	0.0346475	-0.0267171	0.3381032	-0.1628776	0.1120511	0.1713452	-0.7669153
1.1410372	1.0682979	1.6505027	1.6081951	-0.5072361	2.8924156	2.4808445	1.2316479	-0.4653133	-0.0032998	0.1967352	-0.1718230	-0.1628776	-0.0963858	0.1713452	0.2706760
0.4749495	-0.7948946	0.7295658	0.7205613	1.2828512	-0.6662155	-0.3407753	0.2236554	-0.8220535	0.3382257	-0.0267171	0.0831401	0.2533651	0.5686763	-0.2227487	1.3082673

3

a-

```
acp_peces <- prcomp(df_amiard[, -1], scale. = TRUE)

C <- acp_peces$x
acp_peces$sdev^2
```

```
 [1] 7.528224349 3.697876004 1.529884395 0.941930369 0.719651825 0.579162136
 [7] 0.349821836 0.208077555 0.172770306 0.093577150 0.066508397 0.041061787
[13] 0.038355396 0.016528001 0.009011967 0.007558528
```

```
u <- acp_peces$rotation
```

Se busca recontruir los datos entonces se obtiene

```
X_tilde <- matrix(rep(0, ncol(C) * nrow(C)), nrow = nrow(C))
for (i in 1:ncol(C)) {
    X_tilde <- X_tilde + C[, i] %*% t(u[, i])
}
X_tilde %>%
    kable(booktabs = T, caption = "Datos reconstruídos con ACP centrados") %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

Se encuentra que los datos originales centrados tienen lo siguiente

Table 42: Datos originales centrados

RadOjo	RadBra	RadOpe	RadAle	RadHig	RadDig	RadRin	RadEsc	RadMus	Peso	Long	LonEst	AncCab	Ancho	AchHoc	DiaOjo
-0.7240084	-0.7754864	-0.5976669	-0.6502150	-1.2479619	-0.7935851	2.1673312	-0.7370135	-0.8220535	1.8940638	1.3139966	1.6766595	2.3345787	1.6771133	1.7477208	1.3082673
-0.8572259	-1.2024680	-0.9497898	-1.0996498	0.1100354	-0.6507767	0.2862513	-0.9405049	-0.4653133	1.5145911	1.6491750	1.7404003	1.2939719	1.0120511	0.9595330	0.2706760
-1.2568786	-1.1248349	-0.5164077	-0.7850454	-1.0010533	-0.3458615	-0.0272620	-0.8363930	-0.4653133	1.7802220	1.6491750	1.7404003	1.2939719	1.2337385	1.3536269	1.3082673
-1.1236610	-0.6784451	-0.9362466	-1.1108857	-1.1862348	0.1095815	0.2862513	-0.9830961	-0.4653133	1.9320111	1.9284904	1.8041411	1.9183360	1.8988007	0.5654391	1.3082673
-0.9904435	-0.8919359	-0.5705805	-0.7288661	-0.8158719	0.0285281	-1.5948286	-0.2543129	-0.8220535	-0.9519816	-1.2557047	-1.3828977	-1.2034844	-0.5397606	-1.7991243	-0.7669153
-0.9904435	-1.1442432	-0.7330988	-0.5940356	-0.6306904	-0.6430574	-0.3407753	-0.6849575	-0.8220535	-0.8760870	-0.6970740	-0.6817492	-0.9953630	-0.9831354	-1.0109365	-0.7669153
-1.1236610	-1.1248349	-0.9904194	-0.8749324	-0.6924176	-0.7009527	-1.5948286	-0.6423663	-0.8220535	-0.8760870	-0.8088001	-0.9367123	-0.5791203	-0.7614480	-1.0109365	-0.7669153
-0.5907908	-0.5037708	-0.8549875	-0.7850454	-0.4455090	-0.6777945	0.2862513	-0.7417458	0.2481671	-1.3314543	-0.8088001	-0.3630453	-0.7872417	-1.8698849	-1.4050304	-1.8045066
-0.3243558	-0.4843625	-0.6112101	-0.1108932	0.9124883	-0.3458615	-0.0272620	-0.6092398	-0.1085731	-0.3827725	-0.4736217	-0.4267861	-0.5791203	-0.0963858	-0.6168426	0.2706760
0.7413846	0.8742153	0.0794926	-0.2120160	1.3445783	-0.2030531	-0.0272620	-0.3063688	0.6049073	-0.1171416	0.5319136	0.5293256	0.4614865	-0.3180732	-0.6168426	-0.7669153
-0.4575733	-0.2708717	-0.3403463	-0.3019030	-0.3220547	1.1902938	-0.0272620	-0.3678895	-0.4653133	-0.0791943	-0.3060325	-0.4905268	0.0452438	0.3469890	-0.6168426	1.3082673
-0.1911382	0.2919677	-0.4486918	-0.4479693	-0.3837818	0.1057219	-0.0272620	1.5108570	0.6049073	-0.3827725	-0.8646632	-0.8092307	-0.5791203	-0.0963858	-0.2227487	0.2706760
-0.1911382	0.7189493	-0.3132599	-0.3356106	0.4186711	0.9316948	-0.0272620	-0.4104807	2.7453486	-0.2689307	-0.0825802	-0.1080822	-0.1628776	-0.0963858	1.7477208	0.2706760
1.0078197	-0.2514635	-0.3945190	-0.3693182	1.3445783	0.6846748	-0.0272620	-0.4767337	-0.4653133	-1.1417179	-1.4791570	-1.5103792	-1.4116058	-0.9831354	-0.6168426	-0.7669153
-0.3243558	-0.3873213	-0.6112101	-0.4592052	-0.4455090	0.1404590	-0.0272620	-0.5051279	0.2481671	0.1484893	0.2525982	0.2743625	-0.3709989	-0.0963858	0.9595330	0.2706760
-0.1911382	0.0202521	-0.5705805	-0.6165074	0.2334897	-0.6430574	-0.0272620	-0.2353834	0.9616475	0.1864366	1.0905443	-0.0443414	0.6696079	0.1253016	1.3536269	0.2706760
2.2067775	2.3104262	2.0432552	1.6756103	0.5421254	-0.6739348	1.2267912	0.7726090	-0.1085731	-0.3827725	-0.5294848	-0.4267861	-0.3709989	-0.7614480	-0.2227487	-0.7669153
0.8746021	1.1071144	1.4744412	1.7205538	-0.1368732	2.3250417	-1.2813153	1.3830833	-0.4653133	-0.7242979	-0.8646632	-0.6817492	-0.9953630	-0.9831354	-0.6168426	-0.7669153
2.0735600	1.7475868	1.3390093	2.0801017	2.8260299	-0.6662155	-1.2813153	2.4194699	2.7453486	-1.2555597	-1.1439786	-1.0641938	-0.7872417	-1.4265102	-0.6168426	-1.8045066
-0.0579207	0.4278254	0.1336654	0.3610134	-0.2603275	-0.7048123	-0.6542887	-0.6660281	-0.4653133	0.9453820	0.7553659	0.9117702	0.8777292	1.2337385	0.5654391	1.3082673
0.8746021	1.0682979	1.9890824	1.3160625	-0.9393262	-0.6932333	0.5997646	1.8563192	-0.1085731	0.0346475	-0.0267171	0.3381032	-0.1628776	0.1120511	0.1713452	-0.7669153
1.1410372	1.0682979	1.6505027	1.6081951	-0.5072361	2.8924156	2.4808445	1.2316479	-0.4653133	-0.0032998	0.1967352	-0.1718230	-0.1628776	-0.0963858	0.1713452	0.2706760
0.4749495	-0.7948946	0.7295658	0.7205613	1.2828512	-0.6662155	-0.3407753	0.2236554	-0.8220535	0.3382257	-0.0267171	0.0831401	0.2533651	0.5686763	-0.2227487	1.3082673

```
df_original_amiard <- (df_amiard[, -1] - matrix(rep(
    acp_peces$center,
    nrow(df_amiard)
), nrow = nrow(df_amiard), byrow = TRUE)) /
    matrix(rep(apply(df_amiard[, -1], 2, sd), nrow(df_amiard)),
        nrow = nrow(df_amiard), byrow = TRUE
    )
df_original_amiard %>%
    kable(booktabs = T, caption = "Datos originales centrados") %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
autovalores_mayores_1 <- acp_peces$sdev^2
autovalores_mayores_1 <- autovalores_mayores_1[autovalores_mayores_1 >= 1]

C1 <- C[, 1:length(autovalores_mayores_1)]
U1 <- u[, 1:length(autovalores_mayores_1)]

X_1 <- matrix(rep(0, ncol(C) * nrow(C)), nrow = nrow(C))
for (i in 1:ncol(C1)) {
    X_1 <- X_1 + C1[, i] %*% t(U1[, i])
}
X_1 %>%
    kable(
        booktabs = T,
        caption = "Datos centrados reconstruídos con
         componentes principales asociadas a autovalores
```

Table 43: Datos centrados reconstruídos con componentes principales asociadas a autovalores mayores 1

RadOjo	RadBra	RadOpe	RadAle	RadHig	RadDig	RadRin	RadEsc	RadMus	Peso	Long	LonEst	AncCab	Ancho	AchHoc	DiaOjo
-0.8356030	-0.6270975	-0.3921020	-0.6379890	-1.0867918	-0.1286880	1.1068687	-0.8168063	-0.6692598	1.9925772	1.8871969	1.8725584	1.8897524	1.8958742	1.5534335	1.6887878
-0.9875575	-0.8504501	-0.9911899	-1.0658049	-0.3966899	-0.7748979	0.2242265	-1.0514590	0.0039800	1.2747667	1.3350516	1.2936787	1.3315028	1.1381717	1.1005157	0.9935520
-1.0571040	-0.9003575	-0.8183615	-0.9757566	-0.8597858	-0.4203786	0.5862993	-1.0385560	-0.4904423	1.5515161	1.5130161	1.4865188	1.5074189	1.4466550	1.1770735	1.3117103
-1.1433906	-0.9711978	-0.7986120	-0.9960170	-1.0858029	-0.3059933	0.7542453	-1.0912433	-0.7098590	1.7378640	1.6590616	1.6368988	1.6522571	1.6398970	1.2651597	1.5025095
-0.8319346	-1.0079451	-0.8416894	-0.7449868	-0.5178270	0.2238434	-0.9200540	-0.5456504	-0.9177700	-1.1702547	-1.2472505	-1.2339758	-1.2850376	-1.0615584	-1.5247220	-0.7074239
-0.8665938	-0.9907783	-0.9175531	-0.8366701	-0.4253570	-0.0210998	-0.7869553	-0.6545525	-0.6775530	-0.7827394	-0.8183320	-0.8176516	-0.8501115	-0.7234500	-1.0759101	-0.4574418
-1.0527303	-1.1857459	-1.2142988	-1.0960593	-0.3232305	-0.2605716	-1.0153913	-0.8566672	-0.5494485	-0.8589699	-0.8500444	-0.8601882	-0.8852855	-0.8208778	-1.1218822	-0.5417943
-0.5290014	-0.6655718	-0.8234246	-0.6629281	0.1790620	-0.3046238	-1.0446674	-0.4352060	-0.0369812	-1.0645751	-0.9968819	-1.0074372	-1.0203035	-1.0347039	-1.0760886	-0.8121832
-0.3022021	-0.3481622	-0.4672385	-0.3938191	0.1180776	-0.2660396	-0.4717516	-0.2850379	0.0749135	-0.3602645	-0.3086143	-0.3197428	-0.3181974	-0.3673183	-0.3459521	-0.2921809
0.4150477	0.4335576	0.0859955	0.1744770	0.8199003	-0.5052879	-0.2272954	0.2179018	0.9603694	-0.1006346	0.0546532	0.0328235	0.0693322	-0.1688679	0.2942226	-0.2819683
-0.4068957	-0.4318430	-0.1117146	-0.1866343	-0.7584513	0.4664480	0.1622988	-0.2167720	-0.9038329	0.0339226	-0.1092211	-0.0888827	-0.1240874	0.1005255	-0.3360977	0.2184432
0.1406215	0.0925685	0.0961953	0.1356018	0.1191680	0.1159064	-0.1967883	0.1649929	0.0054331	-0.4212687	-0.4227347	-0.4156846	-0.4250887	-0.3888744	-0.3811477	-0.3283062
0.3677994	0.4158083	-0.0477882	0.0503977	0.9736097	-0.7687436	-0.2398149	0.1054900	1.2216768	0.0974751	0.3042021	0.2696385	0.3225214	-0.0124717	0.5702469	-0.1829067
0.1422063	0.0023352	-0.0356645	0.0930447	0.3013013	0.1546657	-0.7126536	0.2201244	0.0011460	-1.1881295	-1.1716442	-1.1599374	-1.1833061	-1.1117650	-1.1073042	-0.9210024
-0.3816546	-0.3622397	-0.4111463	-0.4134858	-0.1247079	-0.2479971	-0.0758001	-0.3782920	-0.0452386	0.2217943	0.2424803	0.2298434	0.2374383	0.1892516	0.1525146	0.1841009
-0.1504100	-0.0736401	-0.4513674	-0.4036363	0.5242093	-0.8325952	-0.1369535	-0.3480611	0.8321589	0.4873388	0.6546942	0.6149745	0.6645466	0.3626298	0.7433480	0.2043490
1.7111746	1.7018854	1.7528768	1.7482470	0.7762365	0.7108531	0.5182513	1.5678261	0.6489409	-0.4377551	-0.4555146	-0.4190962	-0.4224880	-0.3702025	0.0025341	-0.4878402
1.1376161	1.0279555	1.4427202	1.4131191	-0.0586183	1.2905661	0.3066864	1.2768469	-0.5029128	-0.9605926	-1.1417192	-1.0779897	-1.1355666	-0.7796765	-0.9666930	-0.6471866
2.2731045	2.1950284	1.3623176	1.6789260	2.7465468	-0.6055448	-0.7885703	1.7669933	2.7486200	-1.5258054	-1.1507706	-1.1688942	-1.1028449	-1.5806794	-0.2115983	-1.7735226
-0.1872675	-0.0837106	-0.0766531	-0.1658120	-0.2258189	-0.1817713	0.4768556	-0.2453252	0.0080518	0.8885444	0.8854439	0.8735761	0.8926779	0.8252554	0.8226344	0.6875379
1.1104944	1.1470575	1.3830442	1.2840398	0.0852968	0.7761683	0.8093337	1.0672281	0.0248986	0.1548700	0.0536740	0.0898829	0.0771749	0.2154587	0.2742172	0.1060493
1.2873706	1.3066305	2.0858162	1.8401106	-0.7953316	1.8443285	1.4645980	1.4606647	-1.0928307	0.2209281	-0.1205449	-0.0360190	-0.1020200	0.4065917	-0.0458191	0.3895702
0.1469101	0.1759129	0.1898381	0.1616356	0.0150047	0.0414531	0.2070322	0.1155607	0.0659398	0.2093923	0.2037987	0.2051057	0.2097145	0.2001353	0.2373147	0.1471470

```
        mayores 1"
    ) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

Para comparar el error se estima el erro cuadrático medio

```
MSE_X1 <- mean(as.matrix((df_original_amiard - X_1)^2))
```

Se encuentra que error cuadrático medio de todas las covariables de 0.1939357.

Ahora se realiza lo mismo, pero únicamente con los primero autovalores

```
primeros_autovalores <- acp_peces$sdev[c(1, 2)]^2


C2 <- C[, 1:length(primeros_autovalores)]
U2 <- u[, 1:length(primeros_autovalores)]

X_2 <- matrix(rep(0, ncol(C) * nrow(C)), nrow = nrow(C))
for (i in 1:ncol(C2)) {
    X_2 <- X_2 + C2[, i] %*% t(U2[, i])
}

X_2 %>%
    kable(
        booktabs = T,
```

Table 44: Datos centrados reconstruídos con dos componentes principales

RadOjo	RadBra	RadOpe	RadAle	RadHig	RadDig	RadRin	RadEsc	RadMus	Peso	Long	LonEst	AncCab	Ancho	AchHoc	DiaOjo
-0.8200514	-0.6064662	-0.4540616	-0.6796269	-0.9344181	-0.2790707	1.0320065	-0.8403130	-0.4776888	1.9929150	1.9242022	1.9027868	1.9286366	1.8770876	1.6242165	1.6488634
-1.0310021	-0.9080852	-0.8181010	-0.9494863	-0.8223576	-0.3547924	0.4333598	-0.9857911	-0.5311883	1.2738229	1.2316745	1.2092334	1.2228768	1.1906536	0.9027778	1.1050838
-1.0628324	-0.9079570	-0.7955387	-0.9604193	-0.9159126	-0.3649852	0.6138748	-1.0298973	-0.5610074	1.5513917	1.4993852	1.4753841	1.4930959	1.4535750	1.1510006	1.3264164
-1.1347626	-0.9597516	-0.8329873	-1.0191178	-1.0012658	-0.3894258	0.7127116	-1.1042849	-0.6035751	1.7380515	1.6795922	1.6536696	1.6738302	1.6294742	1.3044302	1.4803594
-0.7803127	-0.9394617	-1.0473579	-0.8831994	-0.0120382	-0.2753363	-1.1685514	-0.6236786	-0.2818699	-1.1691333	-1.1244153	-1.1336358	-1.1559655	-1.1239187	-1.2897650	-0.8399488
-0.8383414	-0.9532977	-1.0301145	-0.9123132	-0.1485412	-0.2942984	-0.9229567	-0.6972570	-0.3295279	-0.7821257	-0.7511049	-0.7627360	-0.7794709	-0.7575795	-0.9473193	-0.5299720
-1.0418904	-1.1713654	-1.2574863	-1.1250819	-0.2170221	-0.3653922	-1.0675722	-0.8730520	-0.4159186	-0.8587344	-0.8242508	-0.8391182	-0.8581822	-0.8339726	-1.0725445	-0.5696226
-0.5406898	-0.6810781	-0.7768564	-0.6316335	0.0645395	-0.1915978	-0.9884017	-0.4175386	-0.1809639	-1.0648290	-1.0246947	-1.0301565	-1.0495284	-1.0205841	-1.1292884	-0.7821764
-0.3181435	-0.3693106	-0.4037258	-0.3511375	-0.0381154	-0.1118875	-0.3950129	-0.2609419	-0.1214593	-0.3606108	-0.3465471	-0.3507289	-0.3580563	-0.3480608	-0.4185093	-0.2512557
0.3502216	0.3475571	0.3442715	0.3480426	0.1847373	0.1215755	0.0847644	0.3158886	0.1618142	-0.1020429	-0.0996017	-0.0931822	-0.0927549	-0.0905567	-0.0008333	-0.1155452
-0.3462086	-0.3513334	-0.3535004	-0.3491183	-0.1638415	-0.1203920	-0.1298369	-0.3085026	-0.1562631	0.0352410	0.0351851	0.0290779	0.0276508	0.0272142	-0.0598802	0.0626458
0.1474398	0.1016138	0.0690305	0.1173467	0.1859728	0.0499745	-0.2296099	0.1546869	0.0894231	-0.4211205	-0.4065106	-0.4024317	-0.4080408	-0.3971110	-0.3501145	-0.3458102
0.2782536	0.2970138	0.3089740	0.2901477	0.0962449	0.0971566	0.1912402	0.2408413	0.1186149	0.0955299	0.0911264	0.0955840	0.0986270	0.0957014	0.1626797	0.0469772
0.1530828	0.0167645	-0.0789983	0.0639237	0.4078694	0.0494901	-0.7650112	0.2036841	0.1351282	-1.1878932	-1.1457632	-1.1387961	-1.1561110	-1.1249042	-1.0577995	-0.9489250
-0.3932201	-0.3775828	-0.3650681	-0.3825205	-0.2380254	-0.1361604	-0.0201264	-0.3608105	-0.1877064	0.2215431	0.2149601	0.2073631	0.2085209	0.2032229	0.0998745	0.2137919
-0.2284488	-0.1771692	-0.1404499	-0.1946947	-0.2404120	-0.0779652	0.2387099	-0.2301027	-0.1291569	0.4856435	0.4689992	0.4632865	0.4694231	0.4569024	0.3881541	0.4046922
1.7228179	1.7173319	1.7064882	1.7170731	0.8903175	0.5982629	0.4622026	1.5502267	0.7923686	-0.4375022	-0.4278091	-0.3964645	-0.3933757	-0.3842679	0.0555288	-0.5177313
1.2272207	1.1468280	1.0857234	1.1732114	0.8193233	0.4240965	-0.1246522	1.1414066	0.6008745	-0.9586461	-0.9285034	-0.9038208	-0.9115249	-0.8879206	-0.5588577	-0.8772216
2.1341684	2.0107110	1.9158577	2.0509139	1.3852576	0.7379564	-0.1197599	1.9769998	1.0371468	-1.5288236	-1.4813715	-1.4389511	-1.4502319	-1.4128418	-0.8439658	-1.4168427
-0.1991886	-0.0995256	-0.0291577	-0.1338943	-0.3426216	-0.0664949	0.5342415	-0.2273060	-0.1387978	0.8882854	0.8570774	0.8504045	0.8628710	0.8396564	0.7683754	0.7181421
1.1493387	1.1985897	1.2282832	1.1800379	0.4658916	0.4005467	0.6223449	1.0085136	0.5033993	0.1557138	0.1461047	0.1653865	0.1742988	0.1685340	0.4510171	0.0063273
1.4266719	1.4914324	1.5308211	1.4671448	0.5695359	0.4972958	0.7940296	1.2501062	0.6231412	0.2239542	0.2109250	0.2347478	0.2462801	0.2383129	0.5882107	0.0319527
0.1458770	0.1745423	0.1939544	0.1644018	0.0048816	0.0514439	0.2120057	0.1171224	0.0532126	0.2093699	0.2013403	0.2030975	0.2071312	0.2013834	0.2326121	0.1497994

```
        caption = "Datos centrados reconstruídos con dos componentes principales"
    ) %>%
    kable_styling(latex_options = "scale_down", font_size = 10)
```

```
MSE_X2 <- mean(as.matrix((df_original_amiard - X_2)^2))
```

El error cuadrático medio es de 0.2853962, por lo que se encuentra que el error es menor con las 3 componentes principales, como es de esperarse.

4

```
# To apply Gram - Schmidt algorithm
fn_gram_schmidt <- function(matrix_eigenvectors, M) {
    ortogonal_matrix <- diag(0, nrow = nrow(matrix_eigenvectors))
    ortogonal_matrix[, 1] <- matrix_eigenvectors[, 1]
    ortonormed_matrix <- diag(0, nrow = nrow(matrix_eigenvectors))
    ortonormed_matrix[, 1] <- (ortogonal_matrix[, 1]) *
        1 / sqrt(matrix(ortogonal_matrix[, 1], nrow = 1) %*%
            M %*%
            matrix(ortogonal_matrix[, 1], ncol = 1))
    for (i in 2:ncol(matrix_eigenvectors)) {
        v <- matrix_eigenvectors[, i]
        u <- 0
```

```
        for (j in 1:(i - 1)) {
            u_j <- ortogonal_matrix[, j]
            u <- u + (matrix(v, nrow = 1) %*% M %*% matrix(u_j, ncol = 1)) /
                (matrix(u_j, nrow = 1) %*% M %*% matrix(u_j, ncol = 1)) * u_j
        }
        ortogonal_matrix[, i] <- v - u
        # print(ortogonal_matrix, nrow = 1)
        ortonormed_matrix[, i] <- (ortogonal_matrix[, i]) *
            1 / sqrt(matrix(ortogonal_matrix[, i], nrow = 1) %*%
                M %*%
                matrix(ortogonal_matrix[, i], ncol = 1))
    }
    return(ortonormed_matrix)
}

g_quices <- apply(df_notas_quices[, -1], 2, mean)
df_centered_quices <- df_notas_quices[, -1] - matrix(rep(
    g_quices,
    nrow(df_notas_quices)
), nrow = nrow(df_notas_quices), byrow = TRUE)

V <- 1 / nrow(df_centered_quices) *
    t(df_centered_quices) %*%
        diag(1, nrow = nrow(df_centered_quices)) %*%
        as.matrix(df_centered_quices)

M <- diag(c(0.1, 0.2, 0.2, 0.5))

VM <- V %*% M


eigen_values <- eigen(VM)

eigen_vectors <- fn_gram_schmidt(eigen_values$vectors, M)

colnames(eigen_vectors) <- c("PC1", "PC2", "PC3", "PC4")
rownames(eigen_vectors) <- colnames(df_centered_quices)
eigen_vectors <- -eigen_vectors
# t(eigen_vectors) %*% M %*% (eigen_vectors) #Check that is ortonormed
eigen_values$values
```

```
[1] 257.232934  18.145209   6.408975   2.001944
```

```
eigen_vectors
```

```
             PC1        PC2        PC3        PC4
quiz1 -0.9107054 -1.0004551  2.1900749  1.8366484
quiz2 -0.7869173  1.0836439  1.2461767 -1.2858929
quiz3 -0.7408597 -1.7566908 -0.1375137 -1.1602820
quiz4 -1.1691357  0.3093852 -0.6418472  0.3541675
```

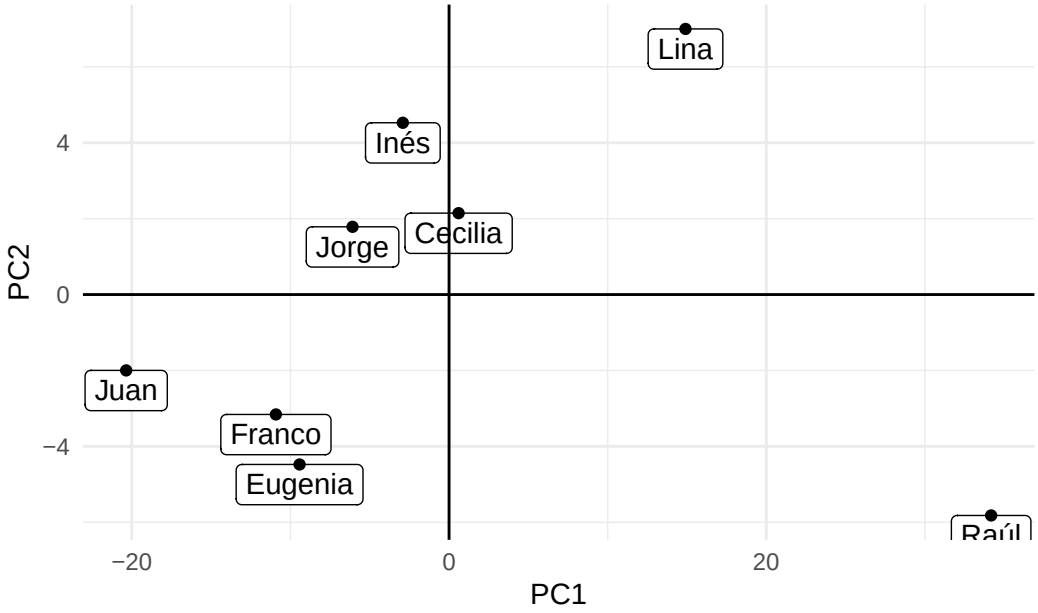
Ahora se procede a estimar el ACP, pero normado

```
acp_quices <- prcomp(df_notas_quices[, -1], scale. = TRUE)
acp_quices$sdev^2
```

```
[1] 3.39023078 0.43533651 0.13354237 0.04089034
```

Se encuentra que la diferencia relativa entre los valores obtenidos es

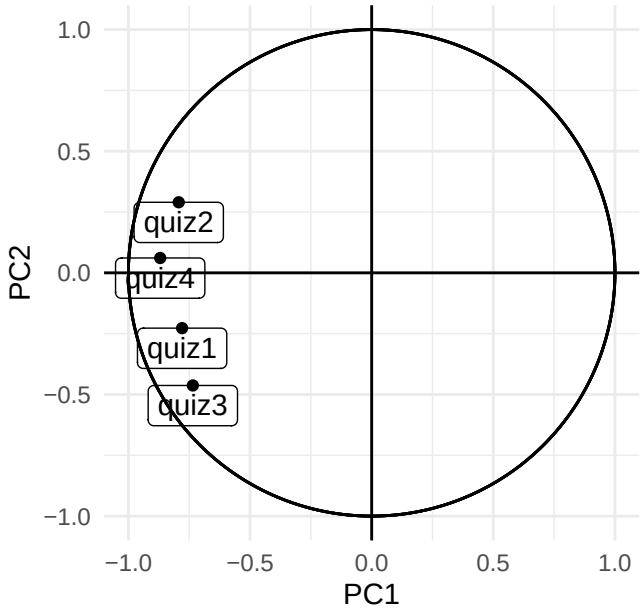
```
# We make the main plain
df_pc <- as.data.frame(as.matrix(df_centered_quices) %*% M %*% eigen_vectors)
df_pc <- cbind(df_notas_quices[, 1], df_pc)
colnames(df_pc)[1] <- "Nombres"
colnames(df_pc)[c(2, 3, 4)] <- c("PC1", "PC2", "PC3", "PC4")

names_cols <- colnames(df_centered_quices)
n <- nrow(df_centered_quices)

ggplot(, aes(x = df_pc$PC1, y = df_pc$PC2, label = df_pc[, 1])) +
    geom_label(vjust = 1) +
    geom_point() +
    geom_hline(yintercept = 0) +
    geom_vline(xintercept = 0) +
    theme_minimal() +
    # theme(text = element_text(size = 16)) +
    labs(x = "PC1", y = "PC2", caption = paste("Porcentaje de inercia:",
        sum((eigen_values$values)[c(1, 2)]) / sum(eigen_values$values),
        sep = " "
    ))
```



Porcentaje de inercia: 0.970362073910818

```
ggsave("ACP_general.svg")
df_correlations <- data.frame("Variable" = names_cols[1], (n - 1) /
    n * cor(df_centered_quices[, names_cols[1]], df_pc[, -1]))


# We make the correlation plot
for (i in colnames(df_centered_quices)[-1]) {
    df_aux <- data.frame("Variable" = i, (n - 1) /
        n *
        cor(df_centered_quices[, i], df_pc[, -1]))
    df_correlations <- rbind(df_correlations, df_aux)
}

ggplot(, aes(
    x = df_correlations$PC1, y = df_correlations$PC2,
    label = df_correlations$Variable
)) +
    geom_label(vjust = 1) +
    geom_point() +
    geom_hline(yintercept = 0) +
    geom_vline(xintercept = 0) +
    geom_circle(aes(x0 = 0, y0 = 0, r = 1)) +
```

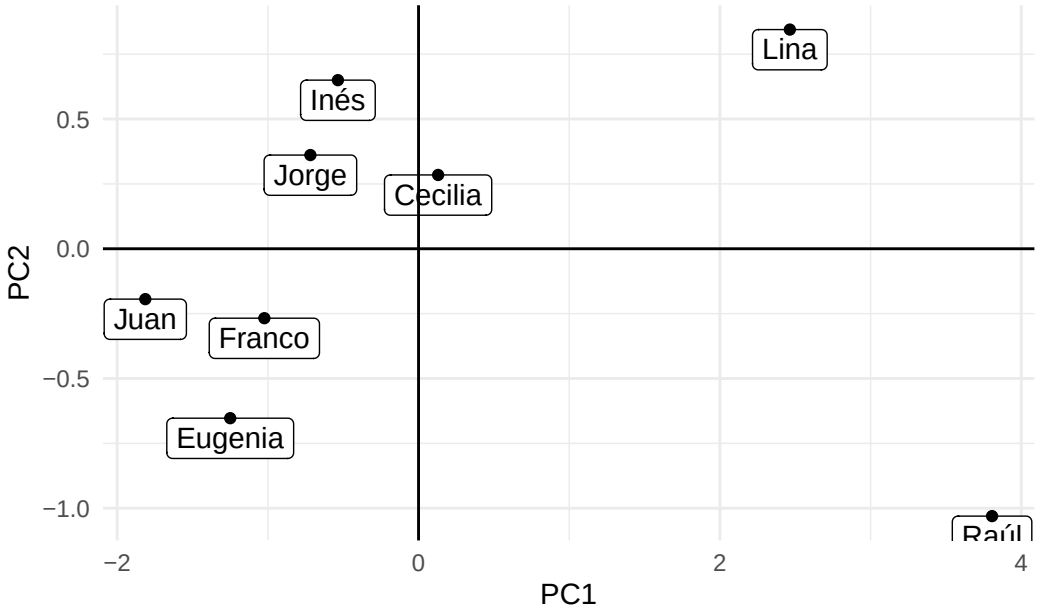
```
    coord_fixed() +
    theme_minimal() +
    # theme(text = element_text(size = 16)) +
    labs(x = "PC1", y = "PC2", caption = paste("Porcentaje de inercia:",
        sum((eigen_values$values)[c(1, 2)]) / sum(eigen_values$values),
        sep = " "
    ))
```



Porcentaje de inercia: 0.970362073910818

```
# We make the main plain
df_centered_quices <- as.matrix(df_centered_quices) / matrix(rep(apply(df_centered_quices, 2, s
df_pc <- as.data.frame(df_centered_quices %*% as.matrix(acp_quices$rotation))
df_pc <- cbind(df_notas_quices[, 1], df_pc)
colnames(df_pc)[1] <- "Nombres"

names_cols <- colnames(df_centered_quices)
n <- nrow(df_centered_quices)

ggplot(, aes(x = df_pc$PC1, y = df_pc$PC2, label = df_pc[, 1])) +
    geom_label(vjust = 1) +
    geom_point() +
    geom_hline(yintercept = 0) +
```

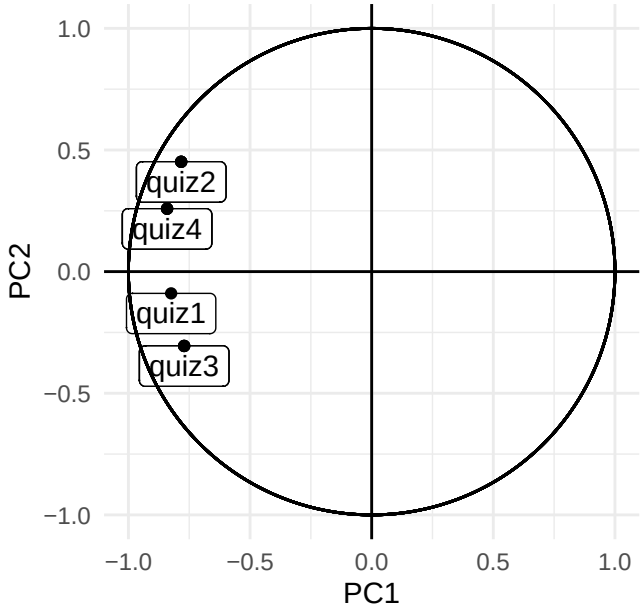
```
    geom_vline(xintercept = 0) +
    theme_minimal() +
    # theme(text = element_text(size = 16)) +
    labs(x = "PC1", y = "PC2", caption = paste("Porcentaje de inercia:",
        sum((eigen_values$values)[c(1, 2)]) / sum(eigen_values$values),
        sep = " "
    ))
```



Porcentaje de inercia: 0.970362073910818

```
df_correlations <- data.frame("Variable" = names_cols[1], (n - 1) /
    n * cor(df_centered_quices[, names_cols[1]], df_pc[, -1]))


# We make the correlation plot
for (i in colnames(df_centered_quices)[-1]) {
    df_aux <- data.frame("Variable" = i, (n - 1) / n * cor(df_centered_quices[, i] / matrix(rep
    df_correlations <- rbind(df_correlations, df_aux)
}

ggplot(, aes(
    x = df_correlations$PC1, y = df_correlations$PC2,
    label = df_correlations$Variable
```

```
)) +
    geom_label(vjust = 1) +
    geom_point() +
    geom_hline(yintercept = 0) +
    geom_vline(xintercept = 0) +
    geom_circle(aes(x0 = 0, y0 = 0, r = 1)) +
    coord_fixed() +
    theme_minimal() +
    # theme(text = element_text(size = 16)) +
    labs(x = "PC1", y = "PC2", caption = paste("Porcentaje de inercia:",
        sum((eigen_values$values)[c(1, 2)]) / sum(eigen_values$values),
        sep = " "
    ))
```



Se puede observar que en este caso se mantiene un comportamiento similar, teniéndose únicamente desplazamiento tanto en el círculo de correlaciones como en el plano principal.