Fundamental Algorithms

Lecture #5

@cs.utcluj

Agenda

- Hash Tables
- Trees
  - Binary and multiway
  - Representation
  - Basic operations

Hash Tables

- Stores a dynamic set of data for fast access = data whose content varies (e.g. symbol table in a compiler, routing table in a router)
- Frequent operation = search
- DS that maintains a set of items (identified by a key) subject to the following operations:
  - insert (item): add item to set
  - delete (item): remove item from set
  - search (key): return item with key if it exists
- goal: O(1) time per operation.

Hash Tables - direct access table

- Items stored in an array (hash table) indexed by key (identifier of item)

1	/
2	/
…	…
key1	item1
…	…
key2	item2
…	…

Limitations:

keys must be nonnegative integers

large key range = large storage space

Solution

Reduce universe *U* of all keys down to reasonable range for table

⇔ project *U* onto sa table of size *m*

Hash Tables

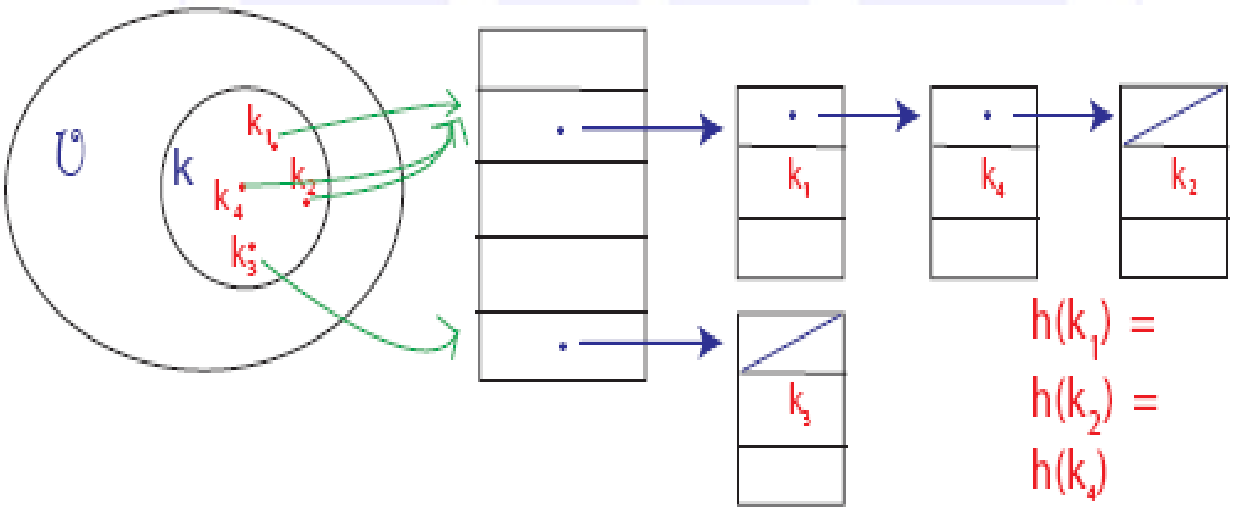
- m - size of the table
- |U| - size of the universe, number of possible keys
- n – number of keys stored in the hash table
- *h:U->{0,1,…m-1}* mapping. Properties?
- *h* computes the key location in the table
- h(key1)=h(key2)
  - Is it possible?
    - Collision
  - What happens?
    - avoidance
    - resolution

Hash Tables – collision resolution

- **Chaining** (linked lists) ~ find alternative solution (e.g. go outside table space)
  - Hash functions
  - Universal hash
- **Open addressing** (in table) ~ find alternative place
  - Linear probing
  - Quadratic probing
  - Double hashing

Chaining (linked lists)

- Linked list of colliding elements in each slot of table



Picture taken from MIT OpenCourseWare, Introduction to Algorithms http://ocw.mit.edu6.006Introduction

Chaining (linked lists)

- h(k)=Dispersion value
- Search must go through the whole list
- Load factor: α=n/m =average number of keys per slot (n=|K|, K= key set, m=|T|, T=table)
- *Expected* performance of chaining assuming *simple uniform hashing* O(1+ α)
- *Worst* case: all keys in K to the same slot => O(n)

Hash functions

- **Division Method**: $h(k) = k \bmod m$
- k_1 and k_2 collide when $k_1 = k_2 (\bmod m)$ i.e. when m divides $|k_1 - k_2|$
- OK if keys are uniformly distributed
- Not OK if they are on a *pattern* distribution
- **Good Practice**: to *avoid* <u>common</u> regularities in keys make *m a prime number* that is NOT too close to a power of 2 or 10.
- Drawbacks:
  - find prime numbers, if they need to be large
  - consecutive keys map to consecutive hash values
  - division is slow

Hash functions - cont

- **Multiplication Method**:

  $h(k) = m\{kA\}$ fractional part from kA

  $= m(kA-[kA])$, [KA] – integer part of KA

  where $0<A<1$, A=ct.

- **Good practice**:
  - considering w= the length of the word of the machine,
  - $m = 2^p$ for some integer p, so that m fits a single word.
  - thus, h(k) easy to calculate (check the textbook for justification).
- Knuth shows that **A=(sqrt5-1)/2** is a good value
- malicious keys => all keys in the SAME location! => O(n) => any possible hash function is vulnerable

Universal hash

- **Randomly** select the hash function at the beginning of each execution, from a set of functions

Note: again, randomness helps efficiency

- **Theorem**: if $n \leq m$, the average number of collisions per key < 1, if *a class of Universal hash functions* is considered
- Homework: Check the textbook for **a class of Universal hash functions**

Open addressing

- All keys kept in the table (no linked lists),

1 key/slot => $m \geq n$, always

- The hash function specifies the order of slots to try for a key, not just one slot
- Sequence to try for a key k:

$\langle h(k,0), h(k,1), \ldots, h(k,m-1) \rangle$

- Ideally, the sequence should be a permutation

of $\langle 0,1,\ldots,m-1 \rangle$

/
$item_3$
$item_1$
…
$item_2$
…

Probing Strategies

- **Linear probing**

  $h(k, i)=(h'(k)+i) \bmod m$, where $h'(k)$ is ordinary hash function, $i=0,1,\ldots,m-1$

  - Drawback: *clustering =* consecutive group of filled slots grows=>average search time grows

- **Quadratic probing**

  $h(k, i)=(h'(k)+c_1*i+c_2*i^2) \bmod m$

  - Drawback: *secondary clustering*

- **Double hashing**

  $h(k, i)=(h'(k)+ih''(k)) \bmod m$, where $h'(k)$ and $h''(k)$ are ordinary hash functions

  $h''(k)$ should be relatively prime to m

Open addressing - evaluation

- $\alpha = n/m < 1$, α load factor
- **Theorem**

  average **un**successful search time is $1/(1- \alpha)$
- **Theorem**

  average successful search time is $1/\alpha * \ln(1/(1- \alpha))$

Trees

- Dynamic structures
- Target
  - Faster (than on linked lists) retrieval of elements
  - Maintain good running time for other operations
- Basic operations (n=#nodes in T; h=height of T)
  - Traversal O(n)
    - (pre, in, post) order
  - Search O(n) regular; O(h) BST
  - Insert O(h)
  - Remove O(n) regular; O(h) BST

  $h \in [\lg n, n]$ Why? Best? Worst?
  - Find: min, max, pred, succ in BST only O(h)

Trees (binary) - representation

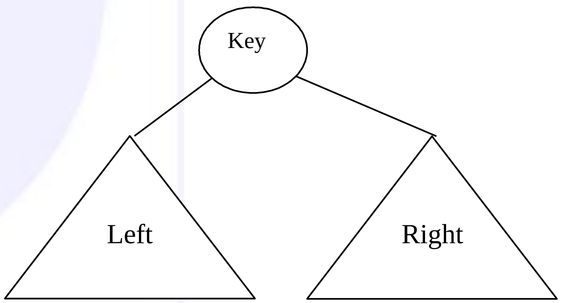
- Dynamic linked structures
- Minimal data representation:
  - key, left, right
  - parent, info
  - other (like balance, size, … depending on type)
- Empty tree = nil
- Types of nodes:
  - root (just one in a tree),
  - internal (non root, non leaf)
  - leaves (all nodes with both children nil)

Trees – representation - cont

- Multiway trees
  - A node has more than just 2 children (unspecified; unknown; variable)
  - Represented as:
    - a tree with exactly m children (array of m potential children)
    - a tree with just one child (linked list)
    - a binary tree:
      - left link= first child (proper tree link)
      - right link = brother (next child of the parent’s node; right links form a singly linked list of brothers)
- Transformation?

Binary Search Trees (BST)

- Binary Trees – if no order imposed on keys, NO improvement over lists! Why to have them?
- BST = BT with a **total order relation** defined on the set of keys.
- $\forall x \in Left, \forall y \in Right, x \leq Key \leq y$
- Any subtree of a BST is a BST
- In general, the **properties** of a structure with **recurrent definition** are **shared** by the component structures (subtrees in our case)
- So, how do you check a BT is a B**S**T?



BST traversal

- Preorder: **Key**, Left, Right
- Inorder: Left, **Key**, Right
- Postorder: Left, Right, **Key**

(pre)=> **7** 3 2 5 4 6 11

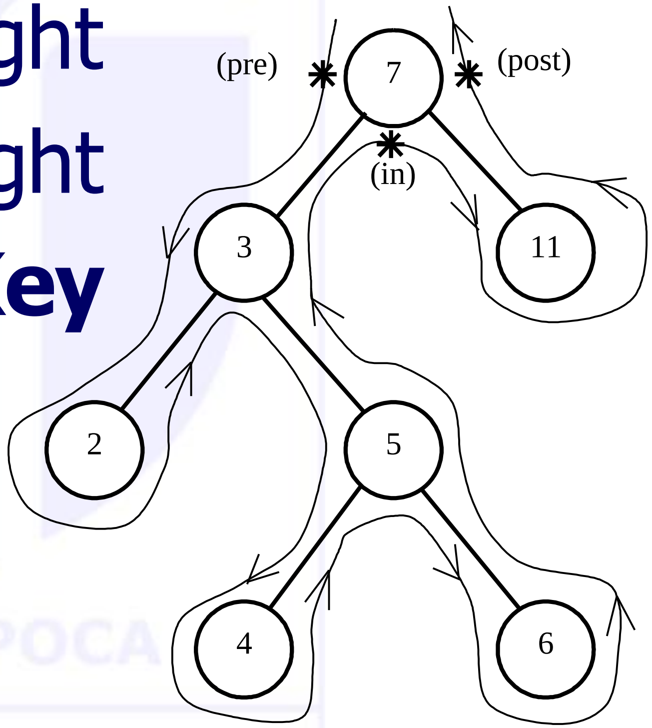
(in) => 2 3 4 5 6 **7** 11

(post)=> 2 4 6 5 3 11 **7**

Root boldface

Left underlined

Inorder: keys are in nondecreasing order



BST traversal - code

```
tree_walk(x, order)  //x=root; order=in, pre,post
if x<>nil
  then      if order= pre
                 then write key[x]
            tree_walk(left[x],order)
            if order= in
                 then write key[x]
            tree_walk(right[x],order)
            if order= post
                 then write key[x]
```

Note: Just ONE `write` statement is executed (one color)

Hw: Look for the non-recursive implementation!!!

BST traversal - eval

- order $\in$ {in, pre, post}
- Only one of the 3 statements **`write key[x]`** is executed
- O(n) (assuming constant time for the operation(s) performed at the level of each individual node – write in our case)

BST – search -recursive

r_tree_search(x,k) //x=root; k=searched

```
if x=nil or k=key[x]
      then return(x)
      else if k<key[x]
            then r_tree_search(left[x],k)
            else r_tree_search(right[x],k)
```

Running time: O(h)

In a BST, h∈[lgn,n]

Discussion on recursive vs iterative implementation

Recursive implementation: where to place the conditional statement (if) & why

BST – search -iterative

i_tree_search(x,k) //x=root; k=searched

```
while x<>nil and k<>key[x]
do
  if k<key[x]
     then x<-left[x]
     else x<-right[x]
return x
```

How does the time differ between iterative vs recursive implementation?

Same efficiency (big Oh), smaller machine time (if LCO does not apply) for iterative version (reason: overhead with stack)

BST – insert

- **Always as a leaf, regardless of the particularity of the BT!!!!**
- **NEVER EVER internal node. There is NO exception!**
- Running time: O(h).
- Range of h LARGE for regular trees
- Rooted tree = tree as a DS, root[T] its root

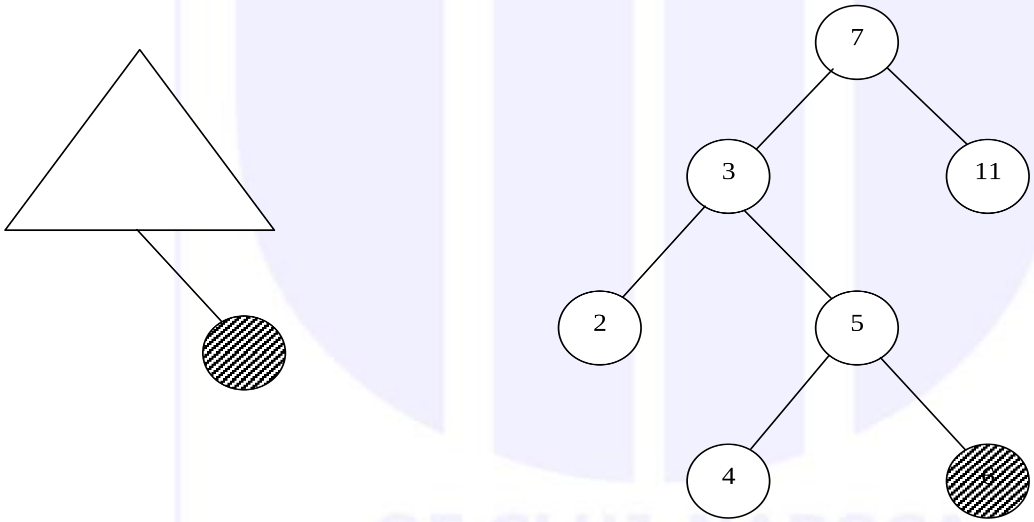
BST – insert - code

```
tree_insert(T,z)    //x=root; z=new node, already allocated
y<-nil              //y=x's parent; stays 1 step behind x;
x<-root[T]
while x<>nil        //search loop to find the position to insert
   do  y<-x         // y=x at the prev step
   if key[z]<key[x]
       then x<-left[x]
       else x<-right[x]
p[z]<-y       //position found; x=nil; y=new node (z)'s parent
if y=nil      //in case the tree was empty before this insertion
       then root[T]<-z
       else   if key[z]<key[y]
                   then left[y]<-z
                   else right[y]<-z
```

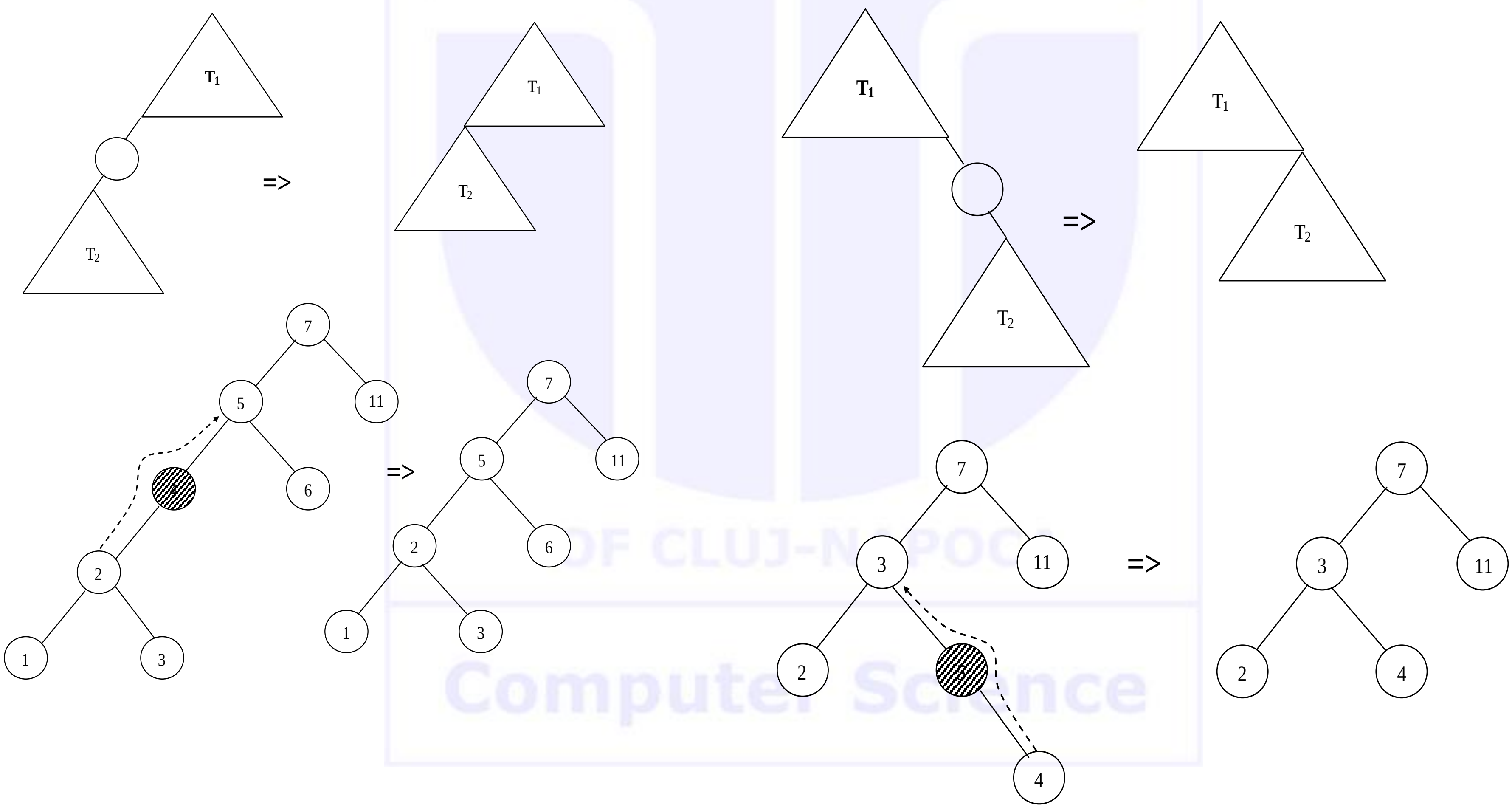
BST - delete

- Find the node
- Remove the node
- Cases:
  - Case 1: Leaf – remove it
  - Case 2: 1-child node – skip it (its only child will become its parent's child)
  - Case 3: 2-children nodes!
    - Chain the tree (fast, unbalances the tree)
    - Replace the operation with an easier one:
      - Keep the structure= keep the node,
      - place a different (*appropriate*) content,
      - remove of the node with the given content

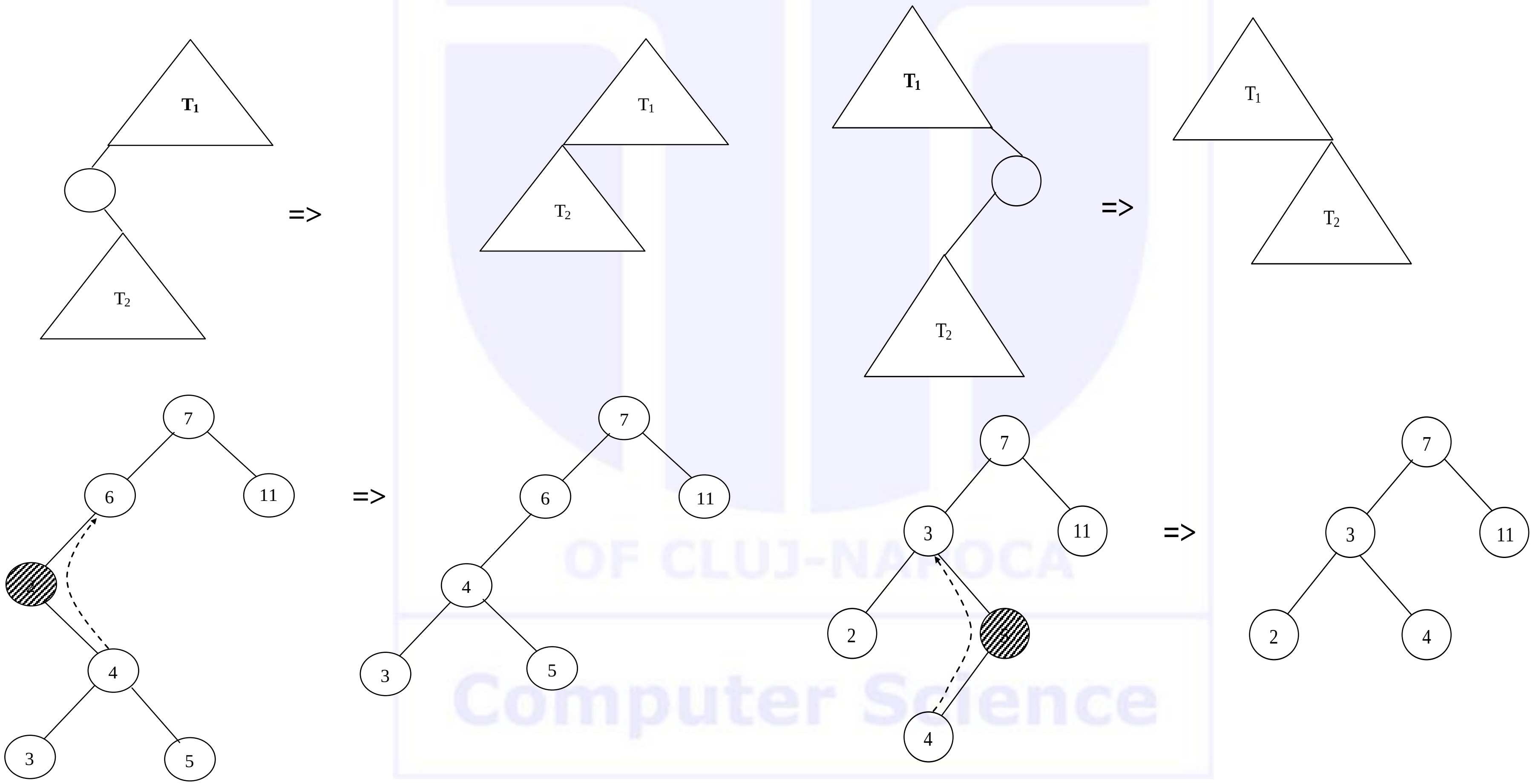
BST – delete – ex: leaf



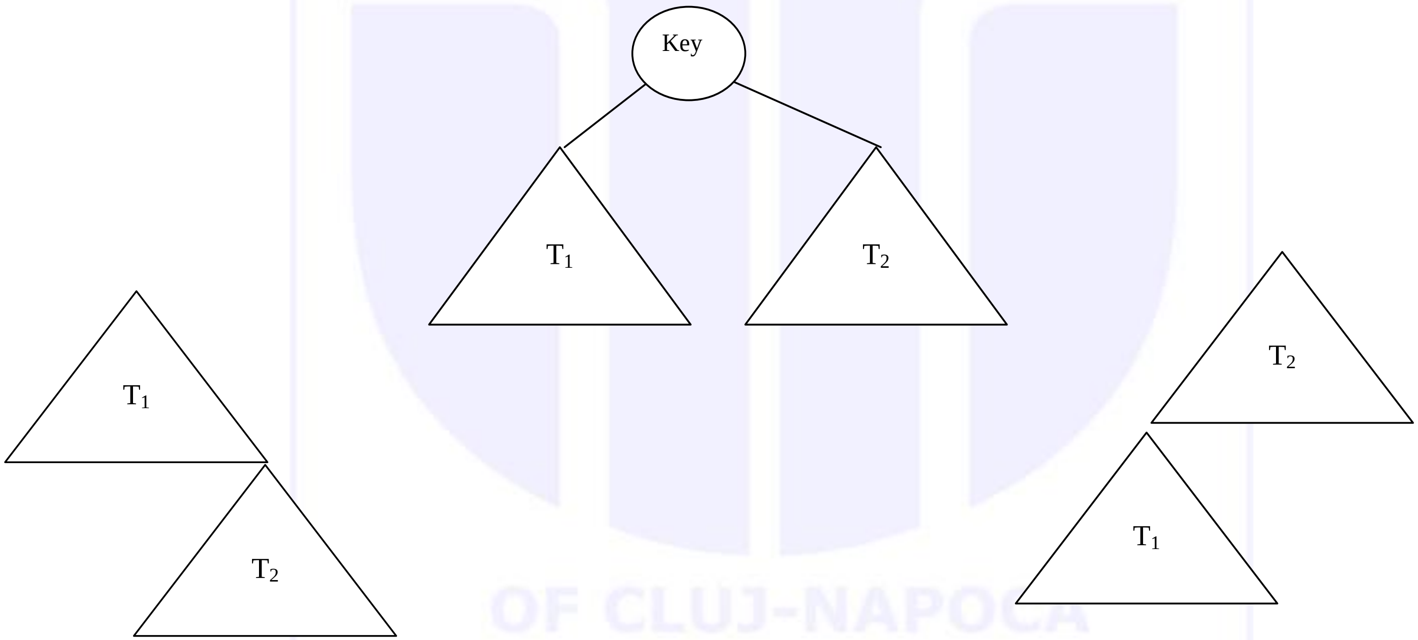
BST – delete – ex: a single successor node



BST – delete – ex: a single successor node



BST – delete – ex: root



Advantage: fast. How fast?

Drawback: increases the height

BST – delete - root

- Replace it with an easy-to-delete node
- Node to replace: *previous* or *next*. Why?
- Left<Key<Right
- Left'<*prev*(Key)<Key<*next*(Key)<Right
- Is it *prev*/*next* easier to del than the root node? Why?
- *prev*=max in Left=>no successor to the right
- *next*=min in Right=>no successor to the left
- =>both are nodes with at most 1 child (easy to del)

BST – delete - code

```
tree_delete(T,z)                //z=node to delete; y physically deleted
if left[z]=nil or right[z]=nil
      then y<-z                 //Case 1 OR 2; z has at most 1 child => del z
      else y<-tree_successor(z) //find replacement=min(right)
if left[y]<>nil                 //we are in Case 2; y is a single child node
      then x<-left[y]           //y has no child to the right; x=y's child
      else  x<-right[y]               //case 2 or 3. Why?
if x<>nil                       //y is not a leaf;
      then p[x]<-p[y]           // y's child redirected to y's parent = x's parent
   //becomes the former single (why?) grandparent
if p[y]=nil                     //means y were the root
      then root[T]<-x           //y's child becomes the new root
      else  if y=left[p[y]] //link y's parent to x which becomes its child
                  then left[p[y]]<-x
                  else right[p[y]]<-x
return[y]      //outside the procedure: copy y's info into z; dealloc y
```

BST – delete - eval

- Find node to delete O(h)
- Find successor O(h)
- BUT:
  - if finding node to delete takes O(h) =>case 1 => leaf (no succ. needed)
  - if node to delete not a leaf=> case 2 or 3 => succ. searched from that place down => find node + find succ.=O(h)
- Delete takes O(h) overall

Required Bibliography

- From the Bible – Chapter 11 (Hash Tables), Chapter 12 (Binary Search Trees)