Thermodynamic Processes of Vapour

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9.1. Introduction

We have already discussed the thermodynamic processes of perfect gases in Chapter 3. These processes are also applicable to vapours, but with different results. In each of these processes, some property of the expanding vapour will remain constant, which enables us to determine the final conditions of vapour after the process. The first and second laws of thermodynamics as applied to gases is also applicable to vapours, for non-flow and flow processes.

We know that for a unit mass,

$$\delta q = du + \delta w = du + p\,dv \qquad \text{...(For non-flow processes)}$$

$$= du - v\,dp \qquad \text{...(For flow processes)}$$

$$= T\,dS \qquad \text{...(For non-flow as well as flow processes)}$$

It may be noted that the general gas equation $p\,v = mRT$ and Joule's law etc. are not applicable to vapours.

9.2. Thermodynamic Processes of Vapour

The following thermodynamic processes for heating or *cooling of vapour are important from the subject point of view :

1. Constant volume process (or Isochoric process),

2. Constant pressure process (or Isobaric process),

3. Constant temperature process (or Isothermal process),

4. Hyperbolic process (or $p\,v = C$ process),

5. Reversible Adiabatic process (or Isentropic process or constant entropy process),

6. Polytropic process (or $p\,v^n = C$ process), and

7. Throttling process (or Constant enthalpy process or constant total heat process).

We shall now discuss these processes, in detail, in the following pages.

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* The cooling is regarded as negative heating

9.3. Constant Volume Process (or Isochoric Process)

The heating or cooling of the steam in a closed vessel is an example of constant volume process. In this process, the volume (or mass) of the steam before and after the process is constant. It may be noted that, in this process, no work is done.

Now consider 1 kg of the wet steam being heated at constant volume from an initial state 1 to the final state 2.

Let

p_1 = Initial pressure of the wet steam in bar,

v_{g1} = Specific volume of the dry saturated steam in m³/kg corresponding to initial pressure p_1 (from steam tables),

x_1 = Initial dryness fraction of the wet steam, and

p_2, v_{g2} and x_2 = Corresponding values for the final condition of the steam.

We know that initial volume of 1 kg of wet steam,

$$v_1 = x_1 v_{g1}$$

and final volume of 1 kg of wet steam,

$$v_2 = x_2 v_{g2}$$

Since the steam is being heated at a constant volume, therefore initial volume of steam is equal to final volume of steam, *i.e.*

$$v_1 = v_2 \quad \text{or} \quad x_1 v_{g1} = x_2 v_{g2}$$

$\therefore$
$$* \quad x_2 = \frac{x_1 v_{g1}}{v_{g2}}$$

The following points may be noted :

(a) If the value of x_2 is less than x_1, then the steam is being cooled at constant volume.

(b) If the final condition of the steam is dry saturated, then $x_2 = 1$. In such a case,

$$v_2 = v_{g2} = x_1 v_{g1}$$

(c) If the final condition of the steam is superheated, then

$$v_2 = v_{sup}, \quad \textit{i.e. Specific volume of superheated steam}$$

The final condition of superheated steam is specified by its temperature (*i.e.* T_{sup}). Since the superheated steam behaves like a perfect gas, therefore T_{sup} may be obtained by using Charles' law, *i.e.*

$$\frac{v_{sup}}{T_{sup}} = \frac{v_{g2}}{T_2}$$

The value of x_2 may also be obtained as discussed below :

We know that initial mass of the steam.

$$m_1 = \frac{\text{Total volume of steam in m}^3}{\text{Specific volume of wet steam in m}^3/\text{kg}} = \frac{v_1}{x_1 v_{g1}}$$

and final mass of the steam. $\quad m_2 = \frac{v_2}{x_2 v_{g2}}$

Since in a constant volume process, $m_1 = m_2$ and $v_1 = v_2$, therefore

$$x_1 v_{g1} = x_2 v_{g2} \quad \text{or} \quad x_2 = \frac{x_1 v_{g1}}{v_{g2}}$$

where T_{sup} = Absolute temperature of the superheated steam in K,

v_{g2} = Specific volume of dry steam in m^3/kg, corresponding to pressure p_2 (from steam tables), and

T_2 = Saturation temperature corresponding to pressure p_2 (from steam tables) in K.

Now let us derive the following important relations for the constant volume process.

1. *Workdone during the process*

We know that workdone during the process,

$$w_{1-2} = \text{Pressure} \times \text{Change in volume}$$

Since there is no change in volume, therefore

$$w_{1-2} = 0$$

2. *Change in internal energy of steam*

We know that initial internal energy of steam,

$$u_1 = h_1 - 100\, p_1\, v_1 = h_1 - 100\, p_1\, x_1\, v_{g1} \text{ kJ/kg}$$

and final internal energy of steam,

$$u_2 = h_2 - 100\, p_2\, v_2 \text{ kJ/kg}$$
$$= h_2 - 100\, p_2\, x_2\, v_{g2} \qquad \ldots \text{(For wet steam)}$$
$$= h_2 - 100\, p_2\, v_{g2} \qquad \ldots \text{(For dry steam)}$$
$$= h_2 - 100\, p_2\, v_{sup} \qquad \ldots \text{(For superheated steam)}$$

∴ Change in internal energy of steam,

$$du = u_2 - u_1$$

If du is $-ve$, then it indicates that the steam is being cooled and there is a decrease in internal energy.

Note : The initial and final enthalpy of steam, *i.e.* h_1 and h_2 may be obtained as discussed in Art. 7.4.

3. *Heat absorbed or heat transferred*

According to the first law of thermodynamics, we know that heat absorbed or heat transferred,

$$q_{1-2} = du + w_{1-2} = (u_2 - u_1) + w_{1-2}$$

Since $w_{1-2} = 0$, therefore

$$q_{1-2} = (u_2 - u_1) \text{ kJ/kg}$$

Thus we find that during a constant volume heating process, the heat absorbed or heat transferred by steam is equal to the change in internal energy.

Note : If q_{1-2} is $-ve$, then it indicates that the steam is being cooled and the heat is rejected from the steam.

Example 9.1. *Steam at a pressure of 4 bar and dryness 0.7 is allowed to expand at a constant volume, until the pressure rises to 5.5 bar. Find the final condition of steam and the heat absorbed by 1 kg of steam.*

Solution. Given : $p_1 = 4$ bar ; $x_1 = 0.7$; $p_2 = 5.5$ bar

The expansion of steam at constant volume is shown on p-v, T-s and h-s diagrams in Fig. 9.1.
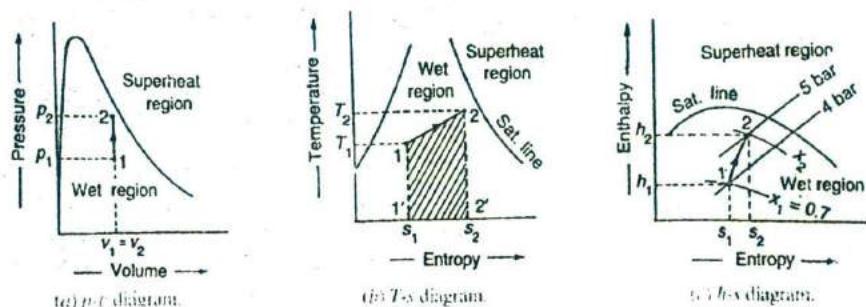
Fig. 9.1

Final condition of steam

Let x_2 = Final dryness fraction of steam.

From steam tables, corresponding to a pressure of 4 bar, we find that

$$h_{f1} = 604.7 \text{ kJ/kg} ; \ h_{fg1} = 2132.9 \text{ kJ/kg} ; \text{ and } v_{g1} = 0.462 \text{ m}^3/\text{kg}$$

and corresponding to a pressure of 5.5 bar , we find that

$$h_{f2} = 655.8 \text{ kJ/kg} ; h_{fg2} = 2095.9 \text{ kJ/kg} ; \text{ and } v_{g2} = 0.342 \text{ m}^3/\text{kg}$$

We know that $\quad x_2 = \dfrac{x_1 v_{g1}}{v_{g2}} = \dfrac{0.7 \times 0.462}{0.342} = 0.945$ Ans.

Heat absorbed by 1 kg of steam

We know that initial internal energy of steam,

$$
\begin{aligned}
u_1 &= h_1 - 100\, p_1 v_1 \\
&= h_{f1} + x_1 \, h_{fg1} - 100\, p_1 \, x_1 \, v_{g1} \\
&= 604.7 + 0.7 \times 2132.9 - 100 \times 4 \times 0.7 \times 0.462 \\
&= 2097.7 - 129.4 = 1968.3 \text{ kJ/kg}
\end{aligned}
$$

and final internal energy of steam,

$$
\begin{aligned}
u_2 &= h_2 - 100\, p_2 v_2 \\
&= h_{f2} + x_2 \, h_{fg2} - 100\, p_2 \, x_2 \, v_{g2} \\
&= 655.8 + 0.945 \times 2095.9 - 100 \times 5.5 \times 0.945 \times 0.342 \\
&= 2636.4 - 177.7 = 2458.7 \text{ kJ/kg}
\end{aligned}
$$

∴ Change in internal energy,

$$du = u_2 - u_1 = 2458.7 - 1968.3 = 490.4 \text{ kJ/kg}$$

We know that in constant volume process, heat absorbed by the steam,

$$q_{1-2} = u_2 - u_1 = 490.4 \text{ kJ/kg} \text{ Ans.}$$

Notes : 1. The area under the T-s diagram (*i.e.* 1-2-2'-1') represents the heat absorbed by the steam.

2. The values of h_1 and h_2 may be read directly from the h-s diagram (Mollier chart) corresponding to a pressure $p_1 = 4$ bar, $x_1 = 0.7$ and $p_2 = 5.5$ bar, $x_2 = 0.945$.

Example 9.2. *A pressure cooker contains 2 kg of steam at 5 bar and 0.9 dry. Calculate the quantity of heat which must be rejected so as the quality of steam becomes 0.5 dry.*

Solution. Given : $m = 2$ kg ; $p_1 = 5$ bar ; $x_1 = 0.9$; $x_2 = 0.5$
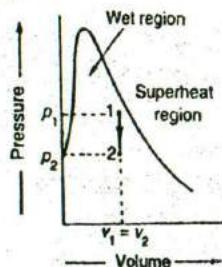
From steam tables, corresponding to a pressure of 5 bar, we find that

$$h_{f1} = 640.1 \text{ kJ/kg} ; \ h_{fg1} = 2107.4 \text{ kJ/kg} ; \text{ and } v_{g1} = 0.375 \text{ m}^3/\text{kg}$$
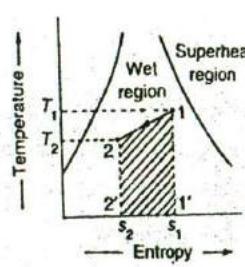
Let v_{g2} = Final specific volume of dry steam.
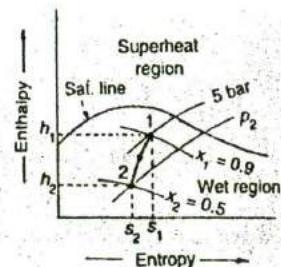
Since the final dryness fraction of steam ($x_2 = 0.5$) is less than the initial dryness friction of steam ($x_1 = 0.9$), therefore it is cooling process and the cooling in a pressure cooker is a constant volume process. The constant volume cooling process on the *p-v*, *T-s*, and *h-s* diagrams is shown in Fig. 9.2.

(a) *p-v* diagram.   (b) *T-s* diagram.   (c) *h-s* diagram.

Fig. 9.2

We know that for constant volume process,

$$x_1 v_{g1} = x_2 v_{g2} \text{ or } v_{g2} = \frac{x_1}{x_2} \times v_{g1} = \frac{0.9}{0.5} \times 0.375 = 0.675 \text{ m}^3/\text{kg}$$

From steam tables, corresponding to $v_{g2} = 0.675$ m³/kg, we find that

$$p_2 = 2.65 \text{ bar} ; \ h_{f2} = 543.4 \text{ kJ/kg} ; \text{ and } h_{fg2} = 2175.4 \text{ kJ/kg}$$

We know that initial internal energy of steam,

$$
\begin{aligned}
u_1 &= h_1 - 100 \, p_1 \, x_1 \, v_{g1} \\
&= h_{f1} + x_1 \, h_{fg1} - 100 \, p_1 \, x_1 \, v_{g1} \\
&= 640.1 + 0.9 \times 2107.4 - 100 \times 5 \times 0.9 \times 0.375 \\
&= 2536.8 - 168.8 = 2368 \text{ kJ/kg}
\end{aligned}
$$

and final internal energy of steam,

$$
\begin{aligned}
u_2 &= h_2 - 100 \, p_2 \, x_2 \, v_{g2} \\
&= h_{f2} + x_2 \, h_{fg2} - 100 \, p_2 \, x_2 \, v_{g2} \\
&= 543.4 + 0.5 \times 2175.4 - 100 \times 2.65 \times 0.5 \times 0.675 \\
&= 1631 - 89.4 = 1541.6 \text{ kJ/kg}
\end{aligned}
$$

∴ Change in internal energy,

$$du = u_2 - u_1 = 1541.6 - 2368 = -826.4 \text{ kJ/kg}$$

We know that in a constant volume process, heat transferred per kg of steam

$$q_{1-2} = u_2 - u_1 = -826.4 \text{ kJ/kg}$$

∴ Total heat transferred $= m \times q_{1-2} = 2 \times -826.4 = -1652.8 \text{ kJ}$ **Ans.**

The −ve sign indicates that the heat is rejected from the steam.

Note : The area under the *T-s* diagram (1-2-2′-1) represents the heat rejected from the steam.

Example 9.3. *A closed vessel of 0.2 m^3 contains steam at a pressure of 10 bar and a temperature of 250° C. The vessel is cooled till the pressure of steam in the vessel falls to 3.5 bar. Find the final temperature, final dryness fraction, change in internal energy, heat transferred and change in entropy.*

Solution. Given : $v_1 = 0.2 \text{ m}^3$; $p_1 = 10 \text{ bar}$; $T_{sup} = 250° C = 250 + 273 = 523 \text{ K}$; $p_2 = 3.5 \text{ bar}$

From steam tables, corresponding to a pressure of 10 bar, we find that

$$| \ T_1 = 179.9° C = 179.9 + 273 = 452.9 \text{ K} ; \ v_{g1} = 0.1943 \text{ m}^3/\text{kg}$$

and corresponding to a pressure of 3.5 bar, we find that

$$v_{g2} = 0.524 \text{ m}^3/\text{kg} ; h_{f2} = 584.3 \text{ kJ/kg} ; h_{fg2} = 2147.3 \text{ kJ/kg} ;$$
$$s_{f2} = 1.727 \text{ kJ/kg K} ; \ s_{fg2} = 5.212 \text{ kJ/kg K}$$

Final temperature of the steam

Let T_2 = Final temperature of the steam.

First of all, let us find the final volume (v_2) of steam at a pressure of 3.5 bar.

Since the initial condition of steam is superheated, therefore specific volume of superheated steam (v_{sup}) is given by

$$\frac{v_{sup}}{T_{sup}} = \frac{v_{g1}}{T_1}$$

∴
$$v_{sup}^* = v_{g1} \times \frac{T_{sup}}{T_1} = 0.1943 \times \frac{523}{452.9} = 0.2244 \text{ m}^3/\text{kg}$$

We know that mass of steam in the vessel,

$$m = \frac{\text{Total volume of steam in the vessel}}{\text{Specific volume of steam}} = \frac{v_1}{v_{sup}}$$

$$= \frac{0.2}{0.2244} = 0.89 \text{ kg}$$

∴ Final volume of the steam at a pressure of 3.5 bar,

$$v_2 = m \, v_{g2} = 0.89 \times 0.524 = 0.466 \text{ m}^3$$

Since the final volume of the steam (v_2) is more than the initial volume of the steam (v_1), therefore the final condition of steam at 3.5 bar is **wet. The final temperature must, therefore, be the saturation temperature corresponding to a pressure of 3.5 bar. From steam tables, we find that

$$T_2 = 138.9° C \text{ Ans.}$$

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* The value of v_{sup} may be read directly from steam tables for superheated steam corresponding to a pressure of 10 bar and 250° C.

** We may also express in the following way :
Since final specific volume of dry steam (v_{g2}) at a pressure of 3.5 bar is more than initial specific volume of superheated steam (v_{sup}), therefore the final condition of the steam is wet.

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Final dryness fraction of steam

Let x_2 = Final dryness fraction of steam.

Since the vessel is a closed one, therefore the cooling takes place at constant volume.

∴ Initial specific volume of superheated steam

= Final specific volume of wet steam

i.e. $v_{sup} = x_2 v_{g2}$ or $x_2 = \dfrac{v_{sup}}{v_{g2}} = \dfrac{0.2244}{0.524} = 0.43$ **Ans.**

The constant volume cooling process on the p-v, T-s and h-s diagrams is shown in Fig. 9.3.

(a) p-v diagram.   (b) T-s diagram.   (c) h-s diagram.
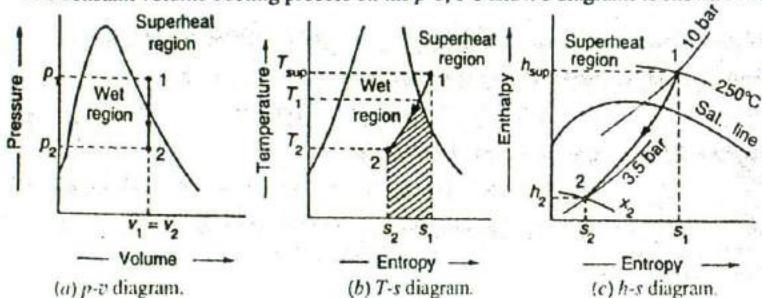
Fig. 9.3

Change in internal energy

From steam tables for superheated steam, corresponding to a pressure of 10 bar and 250° C, we find that

$$h_{sup} = 2943 \text{ kJ/kg}$$

We know that initial internal energy of superheated steam,

$$u_1 = h_{sup} - 100\, p_1\, v_{sup}$$
$$= 2943 - 100 \times 10 \times 0.2244 = 2718.6 \text{ kJ/kg}$$

and final internal energy of wet steam,

$$u_2 = h_2 - 100\, p_2\, v_2$$
$$= h_{f2} + x_2\, h_{fg2} - 100\, p_2\, x_2\, v_{g2}$$
$$= 584.3 + 0.43 \times 2147.3 - 100 \times 3.5 \times 0.43 \times 0.524$$
$$= 1507.6 - 79 = 1428.6 \text{ kJ/kg}$$

∴ Change in internal energy,

$$du = u_2 - u_1 = 1428.6 - 2718.6 = -1290 \text{ kJ/kg}$$

and total change in internal energy

$$= m(u_2 - u_1) = 0.89 \times -1290 = -1148 \text{ kJ}$$ **Ans.**

The −ve sign indicates that there is a decrease in internal energy.

Heat transferred

We know that in a constant volume process, total heat transferred,

$$q_{1-2} = m(u_2 - u_1) = -1148 \text{ kJ}$$ **Ans.**

The −ve sign indicates that the heat is rejected from the steam.

Change in entropy

From steam tables for superheated steam, corresponding to a pressure of 10 bar and 250° C, we find that

Initial entropy of superheated steam,

$$s_1 = 6.926 \text{ kJ/kg K}$$

We know that final entropy of wet steam,

$$s_2 = s_{f2} + x_2 \, s_{fg2} = 1.727 + 0.43 \times 5.212 = 3.97 \text{ kJ/kg K}$$

∴ Change in entropy, $ds = s_2 - s_1 = 3.97 - 6.926 = -2.956 \text{ kJ/kg K}$

and total change in entropy $= m(s_2 - s_1) = 0.89 \times -2.956 = -2.63 \text{ kJ/K}$ **Ans.**

The −ve sign indicates that there is a decrease in entropy.

Example 9.4. *A closed vessel of 1.2 m³ capacity contains steam at 3 bar and 0.85 dryness. Steam at 10 bar and 0.96 dryness is supplied to the vessel until the pressure in the vessel becomes 5 bar. Calculate the mass of steam supplied to the closed vessel and the final dryness fraction in the vessel. Neglect the volume of the moistures and the thermal capacity of the vessel.*

Solution. Given : $v = 1.2 \text{ m}^3$; $p_1 = 3$ bar ; $x_1 = 0.85$; $p_2 = 10$ bar ; $x_2 = 0.96$; $p_3 = 5$ bar.

From steam tables, corresponding to a pressure of 3 bar, we find that

$$v_{g1} = 0.6055 \text{ m}^3/\text{kg} ; \ h_{f1} = 561.4 \text{ kJ/kg} ; \text{ and } h_{fg1} = 2163.2 \text{ kJ/kg}$$

Corresponding to a pressure of 10 bar, we find that

$$v_{g2} = 0.1943 \text{ m}^3/\text{kg} ; \ h_{f2} = 762.6 \text{ kJ/kg} ; \text{ and } h_{fg2} = 2013.6 \text{ kJ/kg}$$

and corresponding to a pressure of 5 bar, we find that

$$v_{g3} = 0.3746 \text{ m}^3/\text{kg} ; \ h_{f3} = 640.1 \text{ kJ/kg} ; \text{ and } h_{fg3} = 2107.4 \text{ kJ/kg}$$

Mass of steam supplied to the closed vessel

Let m_2 = Mass of steam supplied to the closed vessel at a pressure of 10 bar.

∴ Enthalpy of steam supplied.

$$h_2 = m_2 (h_{f2} + x_2 \, h_{fg2})$$
$$= m_2 (762.6 + 0.96 \times 2013.6) = 2696 \, m_2 \text{ kJ} \qquad \qquad \dots (i)$$

We know that initial mass of steam in the vessel,

$$m_1 = \frac{\text{Total volume of steam in the vessel}}{\text{Specific volume of steam}} = \frac{v}{x_1 \, v_{g1}}$$

$$= \frac{1.2}{0.85 \times 0.6055} = 2.33 \text{ kg}$$

∴ Total initial internal energy of the steam at a pressure of 3 bar,

$$u_1 = m_1 (h_1 - 100 \, p_1 \, v_1)$$
$$= m_1 (h_{f1} + x_1 \, h_{fg1} - 100 \, p_1 \, x_1 \, v_{g1})$$
$$= 2.33 (561.4 + 0.85 \times 2163.2 - 100 \times 3 \times 0.85 \times 0.6055)$$
$$= 2.33 (2400.1 - 154.4) = 5232.5 \text{ kJ} \qquad \qquad \dots (ii)$$

Let $\qquad m_3$ = Final mass of the steam after mixing = $m_1 + m_2 = (2.33 + m_2)$,

$\qquad\qquad x_3$ = Final dryness fraction of the steam.

Since it is a closed vessel, therefore volume of steam is constant. We know that mass of steam after mixing,

$$m_3 = \frac{\text{Total volume of steam}}{\text{Specific volume of steam after mixing}}$$

$$(2.33 + m_2) = \frac{1.2}{x_3 \, v_{g3}} = \frac{1.2}{x_3 \times 0.3746} = \frac{3.2}{x_3}$$

$\therefore \qquad\qquad x_3 = \dfrac{3.2}{2.33 + m_2}$ \hfill ... (*iii*)

We know that final internal energy of steam after mixing,

$$u_3 = m_3 (h_3 - 100 \, p_3 \, v_3)$$

$$= m_3 (h_{f3} + x_3 \, h_{fg3} - 100 \, p_3 \, x_3 \, v_{g3})$$

$$= (2.33 + m_2) [640.1 + x_3 \times 2107.4 - 100 \times 5 \times x_3 \times 0.3746]$$

$$= (2.33 + m_2) (640.1 + 1920 \, x_3)$$

$$= (2.33 + m_2) \left(640.1 + \frac{1920 \times 3.2}{2.33 + m_2} \right) \qquad ... \text{[From equation (\textit{iii})]}$$

$$= 640.1 \, m_2 + 7632.4$$

Since the enthalpy of steam supplied is equal to the change in internal energy, therefore

$$h_2 = u_3 - u_1$$

or $\qquad 2696 \, m_2 = 640.1 \, m_2 + 7632.4 - 5232.5$

$\qquad 2055.9 \, m_2 = 2399.9$ or $m_2 = 1.167$ kg **Ans.**

Final dryness fraction of steam in the vessel

Substituting the value of m_2 in equation (*iii*), we have

$$x_3 = \frac{3.2}{2.33 + 1.167} = 0.917 \text{ Ans.}$$

9.4. Constant Pressure Process (or Isobaric Process)

The generation of steam in steam boilers is an example of constant pressure process. In this process, the pressure of steam before and after the process is constant. We have already discussed that the heating of wet steam is done at constant pressure in order to convert it into dry saturated steam. Since the wet steam can only exist at a particular temperature corresponding to the saturation temperature (given in the steam tables) at a given pressure, therefore the heating of wet steam to convert it into dry saturated steam at constant pressure is also a constant temperature process (or isothermal process). We also know that the superheating is done at a constant pressure, but its temperature rises according to Charles' law. It, therefore, follows that once the steam is superheated, it will no longer be an isothermal process.

Consider 1 kg of wet steam being heated at constant pressure from an initial state 1 to a final state 2.

Let $\qquad\qquad p$ = Constant pressure,

x_1 = Initial dryness fraction of steam, and

x_2 = Final dryness fraction of steam.

Since the pressure (p) of steam is constant, therefore the specific volume of dry steam (v_g) corresponding to the pressure (from the steam tables) will also remain constant, *i.e.* $v_{g1} = v_{g2} = v_g$.

We know that initial volume of steam,

$$v_1 = x_1 v_g$$

and final volume of steam, $v_2 = x_2 v_g$

It may be noted that

(*a*) When the final condition of steam is dry saturated, then

$$x_2 = 1, \text{ and } v_2 = v_g$$

(*b*) When the final condition of steam is superheated, then

$$v_2 = v_{sup}$$

Now let us derive the following relations for the constant pressure process.

1. *Workdone during the process*

We know that workdone during the process,

$$w_{1-2} = \text{Pressure} \times \text{Change in volume}$$

$$= 100\, p\, (v_2 - v_1) \text{ kJ/kg} \qquad \dots \text{(when } p \text{ is in bar)}$$

2. *Change in internal energy*

We know that initial internal energy of steam,

$$u_1 = h_1 - 100\, p\, v_1 \text{ kJ/kg}$$

and final internal energy of steam,

$$u_2 = h_2 - 100\, p\, v_2 \text{ kJ/kg}$$

∴ Change in internal energy,

$$du = (u_2 - u_1) \text{ kJ/kg}$$

3. *Heat absorbed or heat transferred*

According to the first law of thermodynamics, we know that heat absorbed or heat transferred,

$$q_{1-2} = du + w_{1-2} = (u_2 - u_1) + w_{1-2}$$

$$= [(h_2 - 100\, p\, v_2) - (h_1 - 100\, p\, v_1)] + 100\, p\, (v_2 - v_1)$$

$$= (h_2 - h_1) \text{ kJ/kg}$$

Thus we find that during a constant pressure heating process, the heat absorbed or heat transferred by the steam is equal to the change in enthalpy (or total heat) of steam.

Example 9.5. *One kg of steam at a pressure of 17.5 bar and dryness 0.95 is heated at a constant pressure, until it is completely dry. Determine : 1. the increase in volume ; 2. the quantity of heat supplied ; and 3. the change in entropy.*

Solution. Given : p = 17.5 bar ; x_1 = 0.95 ; x_2 = 1

The heating of steam at constant pressure is shown on p-v, T-s and h-s diagrams in Fig. 9.4.

(a) p-v diagram.   (b) T-s diagram.   (c) h-s diagram
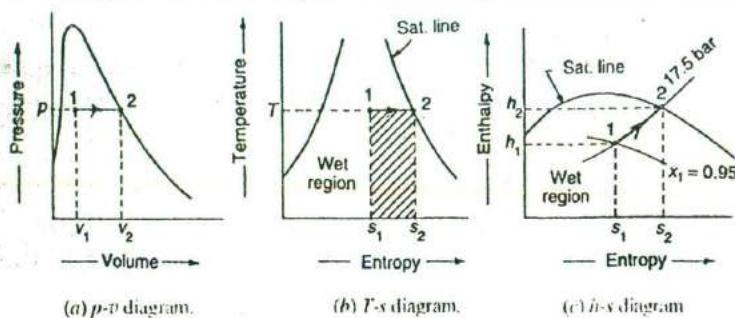
Fig. 9.4

From steam tables, corresponding to a pressure of 17.5 bar, we find that

$$h_{f1} = h_{f2} = 878.2 \text{ kJ/kg} \; ; \; h_{fg1} = h_{fg2} = 1915.9 \text{ kJ/kg} \; ; \; v_g = 0.1134 \text{ m}^3/\text{kg} \; ;$$

$$s_{f1} = s_{f2} = 2.384 \text{ kJ/kg K} \; ; \; \text{and } s_{fg1} = s_{fg2} = 4.001 \text{ kJ/kg K}$$

1. Increase in volume

We know that initial volume of 1 kg of wet steam,

$$v_1 = x_1 v_g = 0.95 \times 0.1134 = 0.1077 \text{ m}^3$$

Since the final condition of steam is completely dry, therefore final volume of 1 kg of dry steam,

$$v_2 = v_g = 0.1134 \text{ m}^3$$

∴ Increase in volume $= v_2 - v_1 = 0.1134 - 0.1077 = 0.0057 \text{ m}^3$ Ans.

2. Quantity of heat supplied

We know that initial enthalpy or total heat of wet steam,

$$h_1 = h_{f1} + x_1 h_{fg1} = 878.2 + 0.95 \times 1915.9 = 2698.3 \text{ kJ/kg}$$

and final enthalpy or total heat of dry steam,

$$h_2 = h_{f2} + h_{fg2} = 878.2 + 1915.9 = 2794.1 \text{ kJ/kg}$$

Since the steam is heated at constant pressure, therefore

Heat supplied   = Change in enthalpy of steam

          $= h_2 - h_1 = 2794.1 - 2698.3 = 95.8 \text{ kJ/kg}$ Ans.

3. Change in entropy

We know that initial entropy of wet steam,

$$s_1 = s_{f1} + x_1 s_{fg1} = 2.384 + 0.95 \times 4.001 = 6.185 \text{ kJ/kg K}$$

and final entropy of dry steam,

$$s_2 = s_{f2} + s_{fg2} = 2.384 + 4.001 = 6.385 \text{ kJ/kg K}$$

∴ Change in entropy,

$$ds = s_2 - s_1 = 6.385 - 6.185 = 0.2 \text{ kJ/kg K Ans.}$$

Example 9.6. *Two boilers discharge equal amounts of steam into the same main. The steam from one is at 18 bar and 380° C, and from the other at 18 bar and 0.95 quality. Determine : 1. the equilibrium condition after mixing, 2. the loss of entropy by the high temperature steam, 3. the gain in entropy by the low temperature steam, and 4. net increase or decrease of entropy.*

Take c_p for superheated steam = 2.3 kJ/kg K.

Solution. Given : $p_1 = 18$ bar ; $T_{sup} = 380°$ C = 380 + 273 = 653 K ; $p_2 = 18$ bar ; $x_2 = 0.95$; $c_p = 2.3$ kJ/kg K

From steam tables, corresponding to a pressure of 18 bar, we find that

$$h_{f1} = h_{f2} = 884.8 \text{ kJ/kg}$$

$$h_{fg1} = h_{fg2} = 1910.3 \text{ kJ/kg}$$

$$h_{g1} = 2795.1 \text{ kJ/kg}$$

$$T_1 = T_2 = 207° \text{ C} = 207 + 273 = 480 \text{ K}$$

$$s_{f2} = 2.398 \text{ kJ/kg K} ; \; s_{fg2} = 3.978 \text{ kJ/kg K}$$

From boiler 1
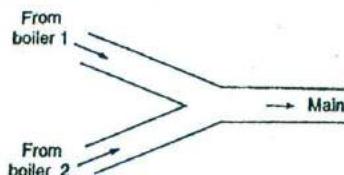
From boiler 2

→ Main

Fig. 9.5

1. *Equilibrium condition after mixing*

Consider that each boiler discharge 1 kg of steam. Therefore

Mass of steam in the main, $m = 2$ kg

We know that enthalpy of steam discharged from boiler 1,

$$h_1 = {}^* h_{sup} = h_{f1} + h_{fg1} + c_p(T_{sup} - T_1)$$

$$= 884.8 + 1910.3 + 2.3 (653 - 480) = 3193 \text{ kJ/kg}$$

and enthalpy of steam discharged from boiler 2,

$$h_2 = h_{f2} + x_2 \, h_{fg2} = 884.8 + 0.95 \times 1910.3 = 2700 \text{ kJ/kg}$$

Let h = Enthalpy of the mixture in kJ/kg.

For equilibrium, $h_1 + h_2 = m \, h$

∴ $3193 + 2700 = 2 \times h$ or $h = 2946.5$ kJ/kg

Since the enthalpy of the mixture ($h = 2946.5$ kJ/kg) is more than the enthalpy of the dry saturated steam at a pressure of 18 bar ($h_{g1} = 2795.1$ kJ/kg) ; therefore the condition of the mixture is superheated.

Let T_m = Temperature of the superheated steam after mixing.

∴ Enthalpy of the superheated mixture (h),

$$2946.5 = h_{g1} + c_p(T_m - T_1) = 2795.1 + 2.3(T_m - 480)$$

or $T_m = 545.8$ K = 272.8° C **Ans.**

2. *Loss of entropy by high temperature steam*

From steam tables for superheated steam, corresponding to a pressure of 18 bar and 380° C, we find that

$$s_1 = 7.113 \text{ kJ/kg K}$$

---

*   The value of h_{sup} may be directly taken from steam tables of superheated steam, corresponding to a pressure of 18 bar and 380° C.

and corresponding to a pressure of 18 bar and 272.8° C, we find that

$$s_m = 6.707 \text{ kJ/kg K}$$

∴ Loss of entropy by high temperature steam

$$= s_1 - s_m = 7.113 - 6.707 = 0.406 \text{ kJ/kg K Ans.}$$

3. *Gain in entropy by the low temperature steam*

We know that entropy of low temperature steam (*i.e.* wet steam from boiler 2),

$$s_2 = s_{f2} + x_2 \, s_{fg2} = 2.398 + 0.95 \times 3.978 = 6.18 \text{ kJ/kg K}$$

∴ Gain in entropy by the low temperature steam

$$= s_m - s_2 = 6.707 - 6.18 = 0.527 \text{ kJ/kg K}$$

4. *Net increase or decrease of entropy*

We know that net increase of entropy

$$= 0.527 - 0.406 = 0.121 \text{ kJ/kg Ans.}$$

9.5.   Constant Temperature Process (or Isothermal Process)

We have already discussed in the previous article that the wet steam has only one temperature (saturation temperature) corresponding to a given pressure. It is thus obvious, that the heating of wet steam at a constant temperature till it becomes dry saturated is similar to that of heating at a constant pressure. Thus for an isothermal process (which is limited to wet steam region only), the same expressions for the workdone, change in internal energy and heat absorbed may be used as for constant pressure process.

Once the steam is superheated, it behaves like a perfect gas. In other words, the superheated steam follows the law

$$\frac{pv}{T} = \text{Constant} \quad \text{or} \quad pv = \text{Constant} \qquad \dots (\because T \text{ is constant})$$

The superheating of steam at constant temperature may be regarded as hyperbolic. In other words, it follows Boyle's law.

Example 9.7. *Steam at a pressure of 5.4 bar and dryness fraction 0.8 expands in a cylinder reversibly and isothermally to a pressure of 1 bar. Find : 1. Final condition of steam ; 2. Change in internal energy ; 3. Change of entropy ; 4. Heat transferred ; and 5. Workdone per kg of steam.*

Solution. Given : $p_1 = 5.4$ bar ; $x_1 = 0.8$; $p_2 = 1$ bar

(a) p-v diagram.   (b) T-s diagram.   (c) h-s diagram.
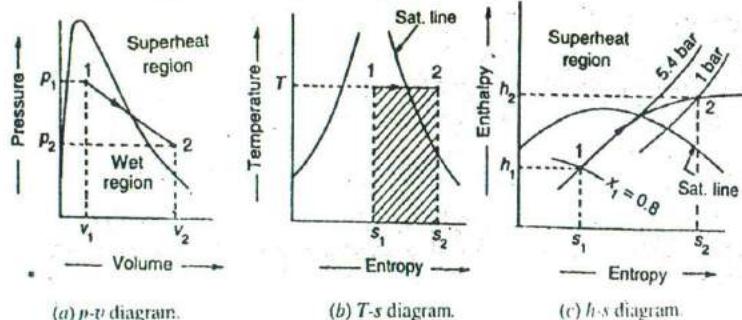
Fig. 9.6

The isothermal expansion of steam on p-v, T-s and h-s diagram is shown in Fig. 9.6.

From steam tables, corresponding to a pressure of 5.4 bar, we find that

$$T = 154.8° C = 154.8 + 273 = 427.8 \text{ K} ; \; h_{f1} = 652.8 \text{ kJ/kg} ; \; h_{fg1} = 2098 \text{ kJ/kg} ;$$

$$v_{g1} = 0.348 \text{ m}^3/\text{kg} ; \; s_{f1} = 1.89 \text{ kJ/kg K} ; \text{ and } s_{fg1} = 4.903 \text{ kJ/kg K}$$

and corresponding to a pressure of 1 bar, we find that

$$h_{f2} = 417.5 \text{ kJ/kg} ; \; h_{fg2} = 2258 \text{ kJ/kg}$$

1. *Final condition of steam*

From steam tables, corresponding to a pressure of 5.4 bar, we find that the saturation temperature,

$$T = 154.8° C$$

Thus, the final condition of steam at state 2 is superheated having $T_{sup} = T = 154.8° C$. **Ans.**

2. *Change in internal energy*

We know that initial internal energy,

$$u_1 = h_1 - 100 \, p_1 \, v_1 = h_{f1} + x_1 \, h_{fg1} - 100 \, p_1 \, x_1 \, v_{g1}$$

$$= 652.8 + 0.8 \times 2098 - 100 \times 5.4 \times 0.8 \times 0.348 \text{ kJ/kg}$$

$$= 2331.3 - 150.3 = 2181 \text{ kJ/kg}$$

and final internal energy, $\quad u_2 = h_2 - 100 \, p_2 \, v_2 = {}^* h_{sup} - 100 \, p_2 \, v_{sup}$

$$= 2785.8 - 100 \times 1 \times 1.96 = 2589.8 \text{ kJ/kg}$$

∴ Change in internal energy,

$$du = u_2 - u_1 = 2589.8 - 2181 = 408.8 \text{ kJ/kg} \text{ Ans.}$$

3. *Change in entropy*

We know that initial entropy of wet steam,

$$s_1 = s_{f1} + x_1 \, s_{fg1} = 1.89 + 0.8 \times 4.903 = 5.81 \text{ kJ/kg K}$$

and final entropy of superheated steam from steam tables for superheated steam corresponding to 1 bar and 154.8° C,

$$s_2 = 7.635 \text{ kJ/kg K}$$

---

* From steam tables for superheated steam. We shall find the values of h_{sup} and v_{sup} corresponding to a pressure of 1 bar and 154.8° C, as discussed below :

$$h_{sup} = h_{sup} \text{ at } 150° C + (h_{sup} \text{ at } 200° C - h_{sup} \text{ at } 150° C) \, 4.8 / 50$$

$$= 2776.3 + (2875.4 - 2776.3) \, 4.8 / 50 = 2776.3 + 9.5 = 2785.8 \text{ kJ/kg}$$

Similarly $\qquad v_{sup} = v_{sup} \text{ at } 150° C + (v_{sup} \text{ at } 200° C - h_{sup} \text{ at } 150° C) \, 4.8 / 50$

$$= 1.936 + (2.172 - 1.936) \, 4.8 / 50 = 1.936 + 0.024 = 1.96 \text{ m}^3/\text{kg}$$

The value of v_{sup} may also be obtained as follows :

We know that $\qquad \dfrac{v_{sup}}{T_{sup}} = \dfrac{v_{g2}}{T_2} \; \text{ or } \; v_{sup} = v_{g2} \times \dfrac{T_{sup}}{T_2}$

From steam tables, corresponding to a pressure of 1 bar, we find that

$$v_{g2} = 1.694 \text{ m}^3/\text{kg} ; \text{ and } T_2 = 99.63° C$$

∴ $\qquad v_{sup} = 1.694 \times \dfrac{154.8 + 273}{99.63 + 273} = 1.96 \text{ m}^3/\text{kg}$

$\therefore$ Change in entropy,

$$ds = s_2 - s_1 = 7.635 - 5.81 = 1.825 \text{ kJ/kg K} \quad \text{Ans.}$$

4. Heat transferred

We know that heat transferred,

$$q_{1-2} = T ds = 427.8 \times 1.825 = 781 \text{ kJ/kg} \quad \text{Ans.}$$

5. Workdone per kg of steam

We know that workdone,

$$w_{1-2} = q_{1-2} - du \qquad \qquad \dots (\because q_{1-2} = du + w_{1-2})$$
$$= 781 - 408.8 = 372.2 \text{ kJ/kg} \quad \text{Ans.}$$

9.6. Hyperbolic Process (or $p\,v = C$ Process)

We have discussed above that superheated steam behaves like a perfect gas and the process of superheating the steam at constant temperature (i.e. isothermal process in the superheat region) is regarded as a *hyperbolic process (or $p\,v = C$ process). Now consider 1 kg of wet steam being heated hyperbolically from an initial state 1 to the final state 2.

Let p_1 = Initial pressure of the steam in bar,

v_{g1} = Specific volume of the dry steam in m³/kg corresponding to pressure, p_1 (from steam tables),

x_1 = Initial dryness fraction of the steam, and

p_2, v_{g2}, x_2 = Corresponding values for the final condition of the steam.

We know that initial volume of 1 kg of steam,

$$v_1 = x_1 v_{g1}$$

and final volume of 1 kg of steam,

$$v_2 = x_2 v_{g2}$$

Since the steam is heated hyperbolically (i.e. $pv = C$), therefore

$$p_1 v_1 = p_2 v_2 \quad \text{or} \quad p_1 (x_1 v_{g1}) = p_2 (x_2 v_{g2})$$

$\therefore$

$$x_2 = \frac{p_1 x_1 v_{g1}}{p_2 v_{g2}} \qquad \dots (i)$$

The value of x_2 gives us an important information, about the final condition of the steam, e.g.

1. If the value of x_2 is less than one, the final condition of steam is wet.

2. If the value of x_2 is equal to one, then the final condition of steam is dry saturated. In such a case, $v_2 = v_{g2}$.

3. If the value of x_2 is greater than one, then the final condition of steam is superheated. In such a case, $v_2 = v_{sup}$.

In case of superheated steam, we shall find out its volume and temperature.

Let v_{sup} = Volume of superheated steam, and

T_{sup} = Temperature of superheated steam in K.

---

*   The isothermal process in the wet region is not hyperbolic.

We know that $p_1 x_1 v_{g1} = p_2 v_{sup}$

$\therefore$ $v_{sup} = \dfrac{p_1 x_1 v_{g1}}{p_2}$

Now T_{sup} may be found out by Charles' law, *i.e.*

$$\dfrac{v_{sup}}{T_{sup}} = \dfrac{v_{g2}}{T_2}$$

where T_2 = Saturation temperature corresponding to the pressure p_2,
                         in K (from steam tables).

Now let us derive the following important relations for the hyperbolic process.

1. *Workdone during the process*

We know that workdone during the process,

$$w_{1-2} = 100\, p_1\, v_1\, \log_e\left(\dfrac{v_2}{v_1}\right) = 2.3 \times 100\, p_1\, v_1\, \log\left(\dfrac{v_2}{v_1}\right) \text{kJ/kg}$$

$$\dots \text{(when } p \text{ is in bar)}$$

$$= 230\, p_1\, v_1\, \log\left(\dfrac{p_1}{p_2}\right) \text{kJ/kg} \qquad \dots \left(\because p_1 v_1 = p_2 v_2 \text{ or } \dfrac{v_2}{v_1} = \dfrac{p_1}{p_2}\right)$$

2. *Change in internal energy of steam*

We know that initial internal energy of steam,

$$u_1 = h_1 - 100\, p_1\, v_1 \text{ kJ/kg}$$

and final internal energy of steam,

$$u_2 = h_2 - 100\, p_2\, v_2 \text{ kJ/kg}$$

Since in hyperbolic process, $p_1 v_1 = p_2 v_2$, therefore

$\therefore$ Change in internal energy

$$du = u_2 - u_1 = h_2 - h_1 \text{ kJ/kg}$$

Thus, we find that during hyperbolic process, the change in internal energy is equal to the change in enthalpy.

3. *Heat absorbed or heat transferred*

According to the first law of thermodynamics, we know that heat absorbed or heat transferred,

$$q_{1-2} = du + w_{1-2} = (u_2 - u_1) + w_{1-2} \text{ kJ/kg}$$

Example 9.8. *Steam at a pressure of 10 bar and 0.9 dry expands to atmospheric pressure hyperbolically. Find : 1. Work done, 2. Change in enthalpy, 3. Change in internal energy, and 4. Heat absorbed. Specific heat of steam at constant pressure = 2 kJ/kg K.*

Solution. Given : p_1 = 10 bar ; x_1 = 0.9 ; p_2 = 1.013 bar ; c_p = 2 kJ/kg K

The hyperbolic process on the *p-v*, *T-s* and *h-s* diagram is shown in Fig. 9.7.

From steam tables, corresponding to a pressure of 10 bar, we find that

$$h_{f1} = 762.6 \text{ kJ/kg} ; \quad h_{fg1} = 2013.6 \text{ kJ/kg} ; \quad v_{g1} = 0.1943 \text{ m}^3\text{/kg} ;$$

$$T_1 = 179.9^\circ \text{C} = 179.9 + 273 = 452.9 \text{ K}$$

and corresponding to atmospheric pressure of 1.013 bar, we find that

$$h_{f2} = 419 \text{ kJ/kg}; \; h_{fg2} = 2257 \text{ kJ/kg}; \; v_{g2} = 1.673 \text{ m}^3/\text{kg};$$

$$T_2 = 100^\circ C = 100 + 273 = 373 \text{ K}$$

(a) p-v diagram   (b) T-s diagram   (c) h-s diagram
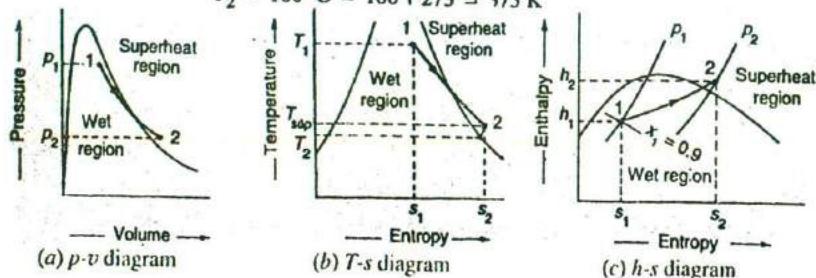
Fig. 9.7

1. Workdone

We know that workdone during hyperbolic expansion,

$$w_{1-2} = 230 \, p_1 \, v_1 \log\left(\frac{v_2}{v_1}\right) = 230 \, p_1 \, x_1 \, v_{g1} \log\left(\frac{p_1}{p_2}\right)$$

$$\dots (\because \; v_1 = x_1 v_{g1}, \text{ and } v_2/v_1 = p_1/p_2)$$

$$= 230 \times 10 \times 0.9 \times 0.1943 \log\left(\frac{10}{1\,013}\right) = 400 \text{ kJ/kg Ans.}$$

2. Change in enthalpy

First of all, let us find the final condition of steam. We know that for hyperbolic expansion,

$$p_1 v_1 = p_2 v_2 \quad \text{or} \quad p_1 x_1 v_{g1} = p_2 x_2 v_{g2}$$

$$\therefore \qquad x_2 = \frac{p_1 x_1 v_{g1}}{p_2 v_{g2}} = \frac{10 \times 0.9 \times 0.1943}{1.013 \times 1.673} = 1.032$$

Since the value of x_2 is more than one, therefore the steam after expansion is *superheated. First of all, let us find out the volume of the superheated steam (v_{sup}) after expansion. We know that

$$p_1 x_1 v_{g1} = p_2 v_{sup}$$

$$\therefore \qquad v_{sup} = \frac{p_1 x_1 v_{g1}}{p_2} = \frac{10 \times 0.9 \times 0.1943}{1.013} = 1.726 \text{ m}^3/\text{kg}$$

Now let us find out the temperature of the superheated steam (T_{sup}) by Charles' law. We know that

$$\frac{v_{sup}}{T_{sup}} = \frac{v_{g2}}{T_2} \quad \text{or} \quad T_{sup} = T_2 \times \frac{v_{sup}}{v_{g2}} = 373 \times \frac{1.726}{1.673} = 385 \text{ K}$$

---

* We may also say as follows :

We know that $p_1 v_1 = p_2 v_2$ or $v_2 = \dfrac{p_1 v_1}{p_2} = \dfrac{p_1 x_1 v_{g1}}{p_2} = \dfrac{10 \times 0.9 \times 0.1943}{1.013} = 1.726 \text{ m}^3/\text{kg}$

Since $v_2 = 1.726 \text{ m}^3/\text{kg}$ is more than $v_{g2} = 1.673 \text{ m}^3/\text{kg}$, therefore the steam is superheated.

We know that initial enthalpy of wet steam,

$$h_1 = h_{f1} + x_1 h_{fg1} = 762.6 + 0.9 \times 2013.6 = 2574.8 \, kJ/kg$$

and final enthalpy of superheated steam,

$$h_2 = h_{sup} = h_{f2} + h_{fg2} + c_p (T_{sup} - T_2)$$
$$= 419 + 2257 + 2 (385 - 373) = 2700 \, kJ/kg$$

∴ Change in enthalpy $dh = h_2 - h_1 = 2700 - 2574.8 = 125.2 \, kJ/kg$ **Ans.**

3. *Change in internal energy*

We know that in hyperbolic process, change in internal energy is equal to the change in enthalpy. Therefore change in internal energy,

$$du = 125.2 \, kJ/kg \text{ **Ans.**}$$

4. *Heat absorbed*

We know that heat absorbed,

$$q_{1-2} = du + w_{1-2} = 125.2 + 400 = 525.2 \, kJ/kg \text{ **Ans.**}$$

9.7. Reversible Adiabatic Process (or Isentropic Process or Constant Entropy Process)

The adiabatic process may be reversible or irreversible. The reversible adiabatic process (*i.e.* frictionless adiabatic process) is known as *isentropic process* or *constant entropy process*. But when there is some friction involved in the process, it is then known as *irreversible adiabatic process*. We have already discussed that irreversibility leads to increase in entropy. Thus in an irreversible adiabatic process, the entropy *increases.

We know that in a reversible adiabatic or isentropic process :

1. there is no transfer of heat through the cylinder walls.

2. there is no change in entropy, *i.e.* entropy before and after the process is constant.

3. the work is done on the piston by the expanding steam.

Now consider 1 kg of wet steam being heated by reversible adiabatic process *i.e.* isentropically from the initial state 1 to the final state 2.

Let p_1 = Initial pressure of the wet steam in bar,

         v_{g1} = Specific volume of the dry saturated steam in m^3/kg, corresponding to initial pressure p_1 (from steam tables),

         x_1 = Initial dryness fraction of the steam, and

         p_2, v_{g2}, x_2 = Corresponding values for the final condition of the steam.

We know that initial entropy of steam before expansion,

$$s_1 = s_{f1} + \frac{x_1 h_{fg1}}{T_1} = s_{f1} + x_1 s_{fg1}$$

and final entropy of steam after expansion,

$$s_2 = s_{f2} + \frac{x_2 h_{fg2}}{T_2} = s_{f2} + x_2 s_{fg2}$$

---

* See Example 9.11.

Since the entropy before expansion is equal to the entropy after expansion, therefore

$$s_{f1} + \frac{x_1 h_{fg1}}{T_1} = s_{f2} + \frac{x_2 h_{fg2}}{T_2}$$

or

$$s_{f1} + x_1 s_{fg1} = s_{f2} + x_2 s_{fg2}$$

where s_{f1}, h_{fg1} and T_1 represents entropy of water, latent heat or enthalpy of evaporation and temperature corresponding to pressure p_1,

s_{f2}, h_{fg2} and T_2 represents the corresponding values at pressure p_2, and

s_{fg1} and s_{fg2} represents entropy of evaporation.

From the above equation, the value of *x_2 is obtained. The value of x_2 gives us an important information about the final condition of the steam, e.g.

1. If the value of x_2 is less than one, then the final condition of the steam is *wet*.

2. If the value of x_2 is equal to one, then the final condition of the steam is *dry saturated*.

3. If the value of x_2 is more than one, then the final condition of the steam is *superheated*.

In case of superheated steam, the absolute temperature of superheated steam (i.e. T_{sup}) may be obtained by using the Charles' law, i.e.

$$\frac{v_{sup}}{T_{sup}} = \frac{v_{g2}}{T_2}$$

where

v_{sup} = Specific volume of superheated steam in m^3/kg, and

T_2 = Saturation temperature corresponding to pressure p_2 (from steam tables) in K.

Now let us derive the following important relations for a reversible adiabatic process or isentropic process.

1. *Change in internal energy*

We know that initial internal energy of the steam,

$$u_1 = h_1 - 100 p_1 v_1$$

where

v_1 = Initial volume of steam = $x_1 v_{g1}$ ...(For wet steam)

and final internal energy of the steam,

$$u_2 = h_2 - 100 p_2 v_2$$

where

v_2 = Final volume of steam

= $x_2 v_{g2}$ ...(For wet steam)

= v_{g2} ...(For dry saturated steam)

= v_{sup} ...(For superheated steam)

∴ Change in internal energy,

$$du = u_2 - u_1$$

---

* The final condition of steam x_2 may be directly obtained from the Mollier diagram by plotting the initial and final conditions of steam. This has been explained in the previous chapter.

2. Heat absorbed or heat transferred

Since no heat is added or subtracted during an adiabatic process (reversible or irreversible), therefore

$$q_{1-2} = 0$$

3. Workdone during the process

According to first law of thermodynamics, the heat absorbed or heat transferred,

$$q_{1-2} = du + w_{1-2}$$

Since $q_{1-2} = 0$, during a non-flow reversible adiabatic or isentropic process, therefore workdone during the process,

$$w_{1-2} = -du = u_1 - u_2$$

This shows that during a reversible adiabatic process or isentropic process, workdone is equal to the change in internal energy.

If the process is steady flow reversible adiabatic or isentropic process, then according to steady flow energy equation,

$$h_1 + q_{1-2} = h_2 + w_{1-2} \qquad \qquad \dots \text{(Neglecting kinetic energy)}$$

Since $q_{1-2} = 0$, therefore

$$w_{1-2} = h_1 - h_2$$

This shows that during a steady flow reversible adiabatic process or isentropic process, workdone is equal to the change in enthalpy.

Notes : 1. The adiabatic expansion of steam, as a rough approximation, is assumed to follow the general law

$$p\,v^n = \text{Constant}$$

where

$$n = 1.13 \qquad \qquad \dots \text{(For wet steam)}$$
$$= 1.135 \qquad \qquad \dots \text{(For dry steam)}$$
$$= 1.3 \qquad \qquad \dots \text{(For superheated steam)}$$

Then work done on the piston

$$w_{1-2} = \frac{100\,(p_1 v_1 - p_2 v_2)}{(n-1)}$$

This method should not be used unless the value of n for the steam during the expansion is given. The value for the work done on the piston is given by :

$$w_{1-2} = -du = u_1 - u_2$$

2. The dryness fraction of steam decreases during an adiabatic expansion. In other words, the steam becomes wetter during an adiabatic process. But, in some cases such as expansion of steam in the nozzle of a steam turbine, the steam remains in its state of dry saturated or superheated steam because it does not get enough time for expansion.

Example 9.9. *Steam at a pressure of 10 bar and 0.95 dry expands isentropically to a pressure of 4 bar. Determine the final dryness fraction of steam, by 1. Using steam tables, and 2. Using Mollier chart.*

Solution. Given : $p_1 = 10$ bar ; $x_1 = 0.95$; $p_2 = 4$ bar

Let $\qquad \qquad \qquad x_2 = $ Final dryness fraction of steam.

The isentropic process on *p-v*, *T-s* and *h-s* diagrams is shown in Fig 9.8.

1. *Using steam tables*

From steam tables, corresponding to a pressure of 10 bar, we find that

$$s_{f1} = 2.138 \text{ kJ/kg K} ; \quad s_{fg1} = 4.445 \text{ kJ/kg K}$$

and corresponding to a pressure of 4 bar, we find that

$$s_{f2} = 1.776 \text{ kJ/kg K} ; \quad s_{fg2} = 5.118 \text{ kJ/kg K}$$

We know that for an isentropic process,

$$s_{f1} + x_1 \, s_{fg1} = s_{f2} + x_2 \, s_{fg2}$$
$$2.138 + 0.95 \times 4.445 = 1.776 + x_2 \times 5.118$$
$$2.138 + 4.223 = 1.776 + 5.118 \, x_2 \quad \text{or} \quad x_2 = 0.896 \text{ Ans.}$$

2. *Using Mollier chart*

First of all, mark point 1 where the pressure line through p_1 (*i.e.* 10 bar) and dryness fraction x_1 (*i.e.* 0.95) meets, as shown in Fig. 9.8 (*c*). Since the steam expands isentropically, therefore draw a vertical line through 1 to meet the pressure line through p_2 (*i.e.* 4 bar) at point 2.

The final dryness fraction of the steam, as read from the chart at point 2, is $x_2 = 0.896$. Ans.

(a) p-v diagram.   (b) T-s diagram.   (c) h-s diagram.
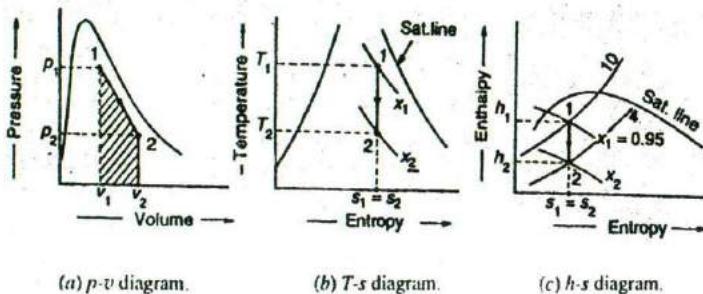
Fig. 9.8

Example 9.10. *Steam from an initial pressure of 7 bar and 200° C is expanded isentropically to a pressure of 1 bar. Calculate : 1. the final condition of the steam ; 2. change in internal energy ; 3. workdone during the process ; 4. the value of n, if the expansion follows the law p v^n = Constant.*

Solution.   Given : $p_1 = 7$ bar ; $T_{sup} = 200°$ C $= 200 + 273 = 473$ K ; $p_2 = 1$ bar

The isentropic process on the p-v, T-s and h-s diagrams is shown in Fig. 9.9.

From steam tables of superheated steam, corresponding to a pressure of 7 bar and 200° C, we find that

$$s_{sup} = 6.886 \text{ kJ/kg K} ; h_{sup} = 2844 \text{ kJ/kg} ; \text{ and } v_{sup} = 0.3 \text{ m}^3/\text{kg}$$

and from steam tables for saturated steam, corresponding to a pressure of 1 bar, we find that

$$s_{f2} = 1.303 \text{ kJ/kg K} ; s_{fg2} = 6.057 \text{ kJ/kg K} ; h_{f2} = 417.5 \text{ kJ/kg} ;$$
$$h_{fg2} = 2258 \text{ kJ/kg} ; \text{ and } v_{g2} = 1.694 \text{ m}^3/\text{kg}$$

1. *Final condition of steam*

Let x_2 = Final dryness fraction of steam.

Since the initial condition of steam is superheated, therefore for isentropic process,

$$s_{sup} = s_2 = s_{f2} + x_2 \, s_{fg2}$$

$$6.886 = 1.303 + x_2 \times 6.057 \quad \text{or} \quad x_2 = 0.92 \text{ Ans.}$$

| (*a*) *p-v* diagram. | (*b*) *T-s* diagram. | (*c*) *h-s* diagram. |
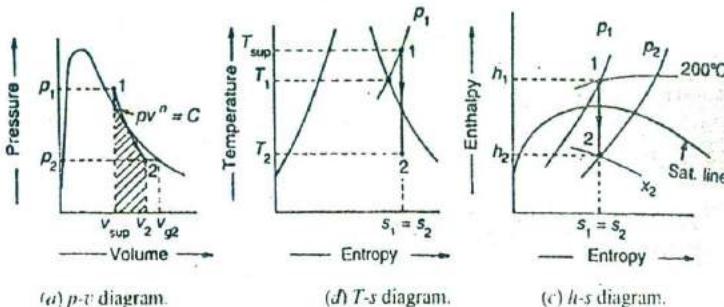
Fig. 9.9

Note : The final condition of steam (*i.e.* x_2) may also be obtained by using Mollier diagram as shown in Fig. 9.9 (*c*). The vertical line drawn from point 1 of an initial condition of steam, *i.e.* pressure 7 bar and temperature 200° C, upto the pressure line of 1 bar. The dryness fraction of steam is read at the point of intersection (*i.e.* point 2) from which $x_2 = 0.92$. **Ans.**

2. *Change in internal energy*

We know that initial internal energy of the steam,

$$u_1 = h_1 - 100 \, p_1 \, v_1 = h_{sup} - 100 \, p_1 \, v_{sup}$$

$$= 2844 - 100 \times 7 \times 0.3 = 2634 \text{ kJ/kg}$$

and final internal energy of the steam,

$$u_2 = h_2 - 100 \, p_2 \, v_2 = h_{f2} + x_2 \, h_{fg2} - 100 \, p_2 \, x_2 \, v_{g2}$$

$$= 417.5 + 0.92 \times 2258 - 100 \times 1 \times 0.92 \times 1.694$$

$$= 2495 - 156 = 2339 \text{ kJ/kg}$$

∴ Change in internal energy,

$$du = u_2 - u_1 = 2339 - 2634 = -295 \text{ kJ/kg} \text{ Ans.}$$

The −ve sign indicates that there is a decrease in internal energy.

3. *Workdone during the process*

Since the heat transferred (q_{1-2}) during an isentropic process is zero, therefore workdone during the process,

$$w_{1-2} = -du = -(-295) = 295 \text{ kJ/kg Ans.} \qquad \ldots (\because \, q_{1-2} = du + w_{1-2})$$

4. *Value of n*

Since the expansion follows the law $p \, v^n = \text{Constant}$, therefore

$$p_1 \, v_1^n = p_2 \, v_2^n \quad \text{or} \quad p_1 \, (v_{sup})^n = p_2 \, (x_2 \, v_{g2})^n$$

Taking logarithm on both sides,

$$\log p_1 + n \log (v_{sup}) = \log p_2 + n \log (x_2 \, v_{g2})$$

or $\quad n \, [\log (v_{sup}) - \log (x_2 \, v_{g2})] = \log p_2 - \log p_1$

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$\therefore$ $n = \dfrac{\log p_2 - \log p_1}{\log (v_{sup}) - \log (x_2 v_{g2})} = \dfrac{\log (1 \times 10^5) - \log (7 \times 10^5)}{\log (0.3) - \log (0.92 \times 1.694)}$

... (p_1 and p_2 are taken in N/m²)

$= \dfrac{5 - 5.845}{-0.523 - 0.1927} = \dfrac{-0.845}{-0.7157} = 1.18$ Ans.

Example 9.11. *Steam at a pressure of 20 bar and 350° C expands adiabatically to a pressure of 1 bar till the steam is dry saturated at the end of expansion. Find : 1. Change of internal energy, 2. Workdone ; and 3. Change of entropy during the process.*

Solution. Given : $p_1 = 20$ bar ; $T_{sup} = 350°$ C ; $p_2 = 1$ bar ; $x_2 = 1$

The adiabatic process (irreversible) on the p-v, T-s and h-s diagrams is shown in Fig. 9.10.

From steam tables for superheated steam, corresponding to a pressure of 20 bar and 350° C, we find that

$$h_{sup} = 3138.6 \text{ kJ/kg} ; v_{sup} = 0.1386 \text{ m}^3/\text{kg} ; s_{sup} = 6.96 \text{ kJ/kg K}$$

and from steam tables for saturated steam, corresponding to a pressure of 1 bar, we find that

$$h_{f2} = 417.5 \text{ kJ/kg} ; h_{fg2} = 2258 \text{ kJ/kg} ; v_{g2} = 1.694 \text{ m}^3/\text{kg} ;$$

$$s_{f2} = 1.303 \text{ kJ/kg K} ; s_{fg2} = 6.057 \text{ kJ/kg K}$$

(a) p-v diagram.   (b) T-s diagram.   (c) h-s diagram.
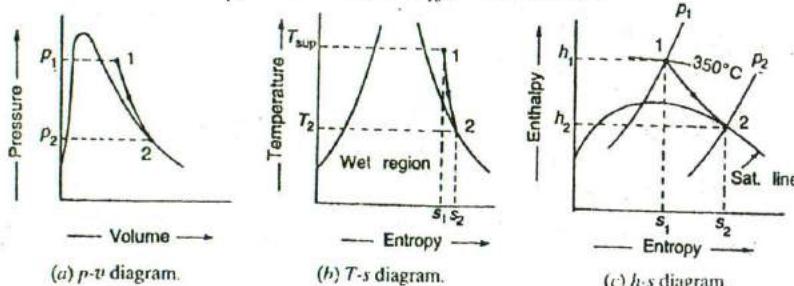
Fig. 9.10

1. *Change in internal energy*

We know that initial internal energy of superheated steam,

$$u_1 = h_1 - 100 p_1 v_1 = h_{sup} - 100 p_1 v_{sup}$$

$$= 3138.6 - 100 \times 20 \times 0.1386 = 2861.4 \text{ kJ/kg}$$

and final internal energy of dry saturated steam,

$$u_2 = h_2 - 100 p_2 v_2 = h_{f2} + h_{fg2} - 100 p_2 v_{g2}$$

$$= 417.5 + 2258 - 100 \times 1 \times 1.694 = 2506.1 \text{ kJ/kg}$$

$\therefore$ Change of internal energy,

$$du = u_2 - u_1 = 2506.1 - 2861.4 = -355.3 \text{ kJ/kg Ans.}$$

The –ve sign indicates that there is a decrease in internal energy.

2. *Workdone*

Since the heat transferred (q_{1-2}) is zero during an adiabatic process, therefore workdone

$$w_{1-2} = -du = -(-355.3) = 355.3 \text{ kJ/kg Ans.}$$

3. *Change of entropy*

We know that initial entropy of superheated steam,

$$s_1 = s_{sup} = 6.96 \text{ kJ/kg K}$$

and final entropy of dry saturated steam,

$$s_2 = s_{f2} + s_{fg2} = 1.303 + 6.057 = 7.36 \text{ kJ/kg K}$$

∴ Change in entropy $= s_2 - s_1 = 7.36 - 6.96 = 0.4 \text{ kJ/kg K}$ **Ans.**

Note : From above, we see that the entropy has increased. Thus the adiabatic process is irreversible.

9.8. Polytropic Process ($pv^n = C$ Process)

Consider 1 kg of wet steam being heated polytropically from an initial state 1 to a final state 2.

Let p_1 = Initial pressure of the steam in bar,

v_{g1} = Specific volume of dry steam, corresponding to a pressure p_1 (from steam tables),

x_1 = Initial dryness fraction of the steam,

p_2, v_{g2}, x_2 = Corresponding values for the final condition of the steam, and

n = Polytropic index.

We know that initial volume of 1 kg of steam,

$$v_1 = x_1 v_{g1}$$

and final volume of 1 kg of steam,

$$v_2 = x_2 v_{g2}$$

Since the steam is heated polytropically (*i.e.* $pv^n = C$), therefore

$$p_1 v_1^n = p_2 v_2^n \quad \text{or} \quad p_1 (x_1 v_{g1})^n = p_2 (x_2 v_{g2})^n$$

∴ $$x_2 = \frac{x_1 v_{g1}}{v_{g1}} \times \left(\frac{p_1}{p_2}\right)^{1/n}$$

The value of x_2 gives us an important information about the final condition of the steam, e.g.

1. If the value of x_2 is less than one, then the final condition of the steam is *wet*.

2. If the value of x_2 is equal to one, then the final condition of the steam is *dry saturated*. In such a case, $v_2 = v_{g2}$.

3. If the value of x_2 is more than one, then the final condition of the steam is *superheated*. In such a case, $v_2 = v_{sup}$.

In case of superheated steam, we shall find out its volume and temperature as follows :

Let v_{sup} = Volume of the superheated steam, and

T_{sup} = Absolute temperature of the superheated steam.

We know that $p_1 (x_1 v_{g1})^n = p_2 (v_{sup})^n$

∴ $$v_{sup} = x v_{g1} \left(\frac{p_1}{p_2}\right)^{1/n}$$

Now T_{sup} may be found out by Charles' law, i.e.

$$\frac{v_{sup}}{T_{sup}} = \frac{v_{g2}}{T_2} \quad \text{or} \quad T_{sup} = T_2 \times \frac{v_{sup}}{v_{g2}}$$

where $\quad\quad\quad\quad T_2$ = Saturation temperature corresponding to pressure p_2 (from steam tables) in K.

Now let us derive the following important relations for the polytropic process :

1. *Workdone during the process*

We know that workdone during the polytropic process,

$$w_{1-2} = \frac{100\,(p_1 v_1 - p_2 v_2)}{n-1} \text{ kJ/kg} \qquad \qquad \text{... (when } p \text{ is in bar)}$$

2. *Change in internal energy*

We know that initial internal energy of steam,

$$u_1 = h_1 - 100\,p_1\,v_1 \text{ kJ/kg}$$

and final internal energy of steam,

$$u_2 = h_2 - 100\,p_2\,v_2 \text{ kJ/kg}$$

3. *Heat absorbed or heat transferred*

According to the first law of thermodynamics, we know that heat absorbed or heat transferred,

$$q_{1-2} = du + w_{1-2} = (u_2 - u_1) + w_{1-2} \text{ kJ/kg}$$

Example 9.12. *One kg of steam at a pressure of 1 bar and 0.8 dry is compressed in a cylinder to a pressure of 2 bar. The law of compression is $pv^{1.2}$ = Constant. Find : 1. the final condition of the steam ; 2. change in internal energy ; and 3. the heat that passes through the cylinder walls.*

Solution. Given : p_1 = 1 bar ; x_1 = 0.8 ; p_2 = 2 bar ; n = 1.2

From steam tables, corresponding to a pressure of 1 bar, we find that

$$h_{f1} = 417.5 \text{ kJ/kg} ; \quad h_{fg1} = 2258 \text{ kJ/kg} ; \quad \text{and} \quad v_{g1} = 1.694 \text{ m}^3/\text{kg}$$

and corresponding to a pressure of 2 bar, we find that

$$h_{f2} = 504.7 \text{ kJ/kg} ; \quad h_{fg2} = 2201.6 \text{ kJ/kg} ; \quad \text{and} \quad v_{g2} = 0.885 \text{ m}^3/\text{kg}$$

1. *Final condition of the steam*

Let $\quad\quad\quad\quad x_2$ = Final dryness fraction of the steam.

We know that $\quad\quad p_1 v_1^n = p_2 v_2^n \quad \text{or} \quad p_1 (x_1 v_{g1})^n = p_2 (x_2 v_{g2})^n$

$$\therefore \quad\quad x_2 = \frac{x_1 v_{g1}}{v_{g2}} \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = \frac{0.8 \times 1.694}{0.885} \left(\frac{1}{2}\right)^{\frac{1}{1.2}} = 0.86$$

2. *Change in internal energy*

We know that initial internal energy of the steam,

$$u_1 = h_1 - 100\,p_1\,v_1 = h_{f1} + x_1\,h_{fg1} - 100\,p_1\,x_1\,v_{g1}$$

$$= 417.5 + 0.8 \times 2258 - 100 \times 1 \times 0.8 \times 1.694 \text{ kJ/kg}$$

$$= 2224 - 135.5 = 2088.5 \text{ kJ/kg}$$

and final internal energy of the steam,

$$u_2 = h_2 - 100\, p_2\, v_2 = h_{f2} + x_2\, h_{fg2} - 100\, p_2\, x_2\, v_{g2}$$
$$= 504.7 + 0.86 \times 2201.6 - 100 \times 2 \times 0.86 \times 0.885 \text{ kJ/kg}$$
$$= 2398 - 152.2 = 2245.8 \text{ kJ/kg}$$

∴ Change in internal energy,

$$du = u_2 - u_1 = 2245.8 - 2088.5 = 157.3 \text{ kJ/kg Ans.}$$

3. *Heat passing through the cylinder walls*

We know that workdone during the compression process,

$$w_{1-2} = \frac{100\,(p_1\, v_1 - p_2\, v_2)}{n - 1} = \frac{100\,(p_1\, x_1\, v_{g1} - p_2\, x_2\, v_{g2})}{n - 1}$$
$$= \frac{100\,(1 \times 0.8 \times 1.694 - 2 \times 0.86 \times 0.885)}{1.2 - 1} = -83.5 \text{ kJ/kg}$$

The −ve sign indicates that the work is done on the steam.

We know that heat passing through the cylinder walls or heat transferred,

$$q_{1-2} = du + w_{1-2} = 157.3 - 83.5 = 73.84 \text{ kJ/kg Ans.}$$

Example 9.13. *Steam of dryness fraction 0.96 is expanded in a cylinder according to* $pv^{1.02} = Constant$. *The pressure at the beginning of expansion is 20 bar, and is continued till the pressure reduces to 1 bar. Determine : 1. the final condition of the steam ; 2. the workdone during expansion ; 3. the change in internal energy, and 4. the heat exchange that occurs between the steam and cylinder walls per kg.*

Solution. Given : $x_1 = 0.96$; $n = 1.02$; $p_1 = 20$ bar ; $p_2 = 1$ bar

The polytropic process on p-v, T-s and h-s diagrams is shown in Fig. 9.11.

(a) p-v diagram.   (b) T-s diagram.   (c) h-s diagram.
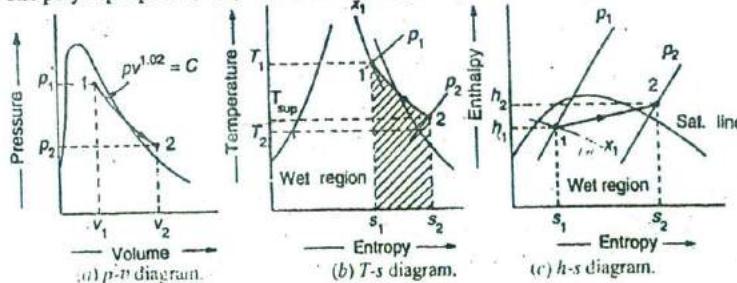
Fig. 9.11

From steam tables, corresponding to a pressure of 20 bar, we find that

$$h_{f1} = 908.6 \text{ kJ/kg} ; \quad h_{fg1} = 1888.7 \text{ kJ/kg} ; \quad \text{and } v_{g1} = 0.0995 \text{ m}^3\text{/kg}$$

and corresponding to a pressure of 1 bar, we find that

$$h_{f2} = 417.5 \text{ kJ/kg} ; \quad h_{fg2} = 2258 \text{ kJ/kg} ; \quad v_{g2} = 1.694 \text{ m}^3\text{/kg} ; \quad \text{and}$$
$$T_2 = 99.6° \text{ C} = 99.6 + 273 = 372.6 \text{ K}$$

1. *Final condition of the steam*

Let $\qquad x_2 = $ Final dryness fraction of the steam.

We know that $\quad p_1 v_1^n = p_2 v_2^n \quad$ or $\quad p_1 (x_1 v_{g1})^n = p_2 (x_2 v_{g2})^n$

$$\therefore \qquad x_2 = \frac{x_1 v_{g1}}{v_{g2}} \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = \frac{0.96 \times 0.0995}{1.694}\left(\frac{20}{1}\right)^{\frac{1}{1.02}} = 1.062$$

Since the final dryness fraction of the steam in greater than one, therefore the final condition of steam at the end of expansion is superheated.

Let $\qquad v_{sup}$ = Volume of the superheated steam in m^3/kg, and

$\qquad\qquad\qquad T_{sup}$ = Temperature of the superheated steam in K.

We know that $\quad p_1 v_1^n = p_2 v_2^n \quad$ or $\quad p_1 (x_1 v_{g1})^n = p_2 (v_{sup})^n$

$$\therefore \qquad v_{sup} = x_1 v_{g1}\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = 0.96 \times 0.0995 \left(\frac{20}{1}\right)^{\frac{1}{1.02}} = 1.8 \ m^3/kg$$

Now let us find out the final condition of the superheated steam (*i.e.* T_{sup}) by Charles' law. We know that

$$\frac{v_{sup}}{T_{sup}} = \frac{v_{g2}}{T_2} \quad \text{or} \quad T_{sup} = T_2 \times \frac{v_{sup}}{v_{g2}} = 372.6 \times \frac{1.8}{1.694} = 396 \ K \ \text{Ans.}$$

2. Workdone during expansion

We know that workdone during expansion,

$$w_{1-2} = \frac{100 \, (p_1 v_1 - p_2 v_2)}{n-1} = \frac{100 \, (p_1 x_1 v_{g1} - p_2 v_{sup})}{n-1}$$

$$= \frac{100 \, (20 \times 0.96 \times 0.0995 - 1 \times 1.8)}{1.02-1} = 552 \ kJ/kg \ \text{Ans.}$$

3. Change in internal energy

We know that initial internal energy of the wet steam,

$$u_1 = h_1 - 100 \, p_1 \, v_1 = h_{f1} + x_1 \, h_{fg1} - 100 \, p_1 \, x_1 \, v_{g1}$$

$$= 908.6 + 0.96 \times 1888.7 - 100 \times 20 \times 0.96 \times 0.0995$$

$$= 2721.8 - 191 = 2530.8 \ kJ/kg$$

and final internal energy of the superheated steam,

$$u_2 = h_2 - 100 \, p_2 \, v_2 = {}^* h_{sup} - 100 \, p_2 \, v_{sup}$$

$$= [h_{f2} + h_{fg2} + c_p \, (T_{sup} - T_2)] - 100 \, p_2 \, v_{sup}$$

$$= [417.5 + 2258 + 2 \, (396 - 372.6)] - 100 \times 1 \times 1.8$$

$$= 2722.3 - 180 = 2542.3 \ kJ/kg$$

$\qquad\qquad\qquad\qquad\qquad\qquad$ ... (Taking c_p for superheated steam = 2 kJ/kg K)

$\therefore$ Change in internal energy,

$$du = u_2 - u_1 = 2542.3 - 2530.8 = 11.5 \ kJ/kg \ \text{Ans.}$$

*   The value of h_{sup} may also be read directly from steam tables for superheated steam, corresponding to a pressure of 1 bar and 123° C.

4. Heat exchange between the steam and cylinder walls

We know that heat exchange between the steam and cylinder walls or heat transferred,

$$q_{1-2} = du + w_{1-2} = 11.5 + 552 = 563.5 \text{ kJ/kg Ans.}$$

9.9. Throttling Process (Constant Enthalpy or Constant Total Heat Process)

It is also known as wire drawing when the working substance is vapour. Steam is said to be throttled when it passes through a restricted opening such as a narrow aperture or a slightly opened valve. The leakage of a fluid through a crack in the vessel is an example of throttling.

During the throttling process, the fluid is forced out by its pressure through the restricted opening or aperture. The aperture is so narrow that due to frictional resistance between the fluid and the sides of the aperture, the velocity of the outcoming fluid is almost reduced to zero. The kinetic energy is converted into heat by friction. Hence, if the steam is initially wet, it will start drying up as it expands. If throttled, the dry steam will become super-heated.


Fig. 9.12. Throttling process.

It may be noted that during a throttling process :

1. No heat is supplied or rejected (*i.e.* $q_{1-2} = 0$).

2. No work is done by the expanding fluid (*i.e.* $w_{1-2} = 0$).

3. No change in the internal energy of fluid (*i.e.* $du = 0$).

4. The enthalpy or total heat of the fluid remains constant (*i.e.* $h_1 = h_2$).

Consider 1 kg of wet steam being throttled through a narrow aperture, from an initial state 1 to the final state 2, as shown in Fig. 9.12. The throttling process is irreversible adiabatic process.

Applying *steady flow equation between sections 1-1 and 2-2, we have for a unit mass of fluid,

$$h_1 + \frac{V_1^2}{2} + g z_1 + q_{1-2} = h_2 + \frac{V_2^2}{2} + g z_2 + w_{1-2}$$

Since the datum level is same, therefore $z_1 = z_2$. Also the velocities V_1 and V_2 are small, therefore $V_1^2/2$ and $V_2^2/2$ are negligible. Hence the above equation reduces to

$$h_1 = h_2 \qquad \qquad \dots (\because q_{1-2} = 0 \text{ and } w_{1-2} = 0)$$

This shows that during throttling process, enthalpy remains constant.

Let $\qquad\qquad\qquad p_1$ = Initial pressure of the steam in bar,

$\qquad\qquad\qquad v_{g1}$ = Specific volume of dry steam in m³/kg corresponding to an initial pressure p_1 (from steam tables).

$\qquad\qquad\qquad x_1$ = Initial dryness fraction of the steam, and

$\qquad\qquad p_1, v_{g2}, x_2$ = Corresponding values for the final condition of the steam.

We know that initial enthalpy or total heat of steam before expansion,

$$h_1 = h_{f1} + x_1 h_{fg1}$$

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* For details, please refer chapter 3. Art 3.18.

and final enthalpy or total heat of steam after expansion,

$$h_2 = h_{f2} + x_2 \, h_{fg2}$$

Since enthalpy remains constant during throttling process, therefore

$$h_1 = h_2 \quad \text{or} \quad h_{f1} + x_1 \, h_{fg1} = h_{f2} + x_2 \, h_{fg2}$$

$$\therefore \qquad x_2 = \frac{h_{f1} + x_1 \, h_{fg1} - h_{f2}}{h_{fg2}}$$

The value of x_2 gives us an important information, *e.g.*

1. If the value of x_2 is less than one, then the final condition of the steam is *wet.*

2. If the value of x_2 is equal to one, then the final condition of the steam is *dry saturated.*

3. If the value of x_2 is greater than one, then the final condition of the steam is *superheated.*

Note : The throttling process may be represented on Mollier chart by a horizontal line as discussed in the previous chapter. The final condition of steam may also be read directly from the chart.

Example 9.14. *One kg of steam at 14 bar pressure and dryness fraction 0.9 is throttled to 1.4 bar. Determine the final condition of the steam by using : 1. Steam tables, and 2. Mollier chart.*

Solution. Given : $p_1 = 14$ bar ; $x_1 = 0.9$; $p_2 = 1.4$ bar

Let $\qquad\qquad x_2$ = Final dryness fraction of the steam.

1. Using steam tables

From steam tables, corresponding to a pressure of 14 bar, we find that

$$h_{f1} = 830.1 \text{ kJ/kg} \; ; \; h_{fg1} = 1957.7 \text{ kJ/kg}$$

and corresponding to a pressure of 1.4 bar, we find that

$$h_{f2} = 458.4 \text{ kJ/kg} \; ; \; h_{fg2} = 2232 \text{ kJ/kg}$$

We know that for a throttling process,

$$h_{f1} + x_1 \, h_{fg1} = h_{f2} + x_2 \, h_{fg2}$$

$$830.1 + 0.9 \times 1957.7 = 458.4 + x_2 \times 2232$$

$$\therefore \qquad\qquad x_2 = 0.96 \text{ Ans.}$$

2. Using Mollier chart

First of all, mark point 1 where the pressure p_1 (*i.e.* 14 bar) and dryness fraction x_1 (*i.e.* 0.9) meet as shown in Fig. 9.13. Since the steam is throttled, therefore draw a horizontal line through 1 to meet the pressure line p_2 (*i.e.* 1.4 bar) at point 2.
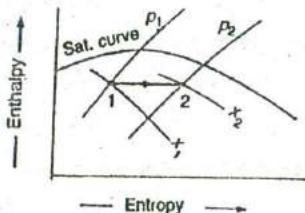

Fig. 9.13

The final dryness fraction of the steam, as read from the point 2 is $x_2 = 0.96$. Ans.

Example 9.15. *Steam at 10 bar and 0.9 dryness is throttled to a pressure of 2 bar. Using steam tables only, evaluate the final dryness fraction or degree of superheat. Estimate the change of entropy during this process.*

Solution. Given : $p_1 = 10$ bar ; $x_1 = 0.9$; $p_2 = 2$ bar

From steam tables, corresponding to a pressure of 10 bar, we find that

$$h_{f1} = 762.6 \text{ kJ/kg} \; ; \; h_{fg1} = 2013.6 \text{ kJ/kg} \; ; \; s_{f1} = 2.138 \text{ kJ/kg K} \; ; \text{ and}$$

$$s_{fg1} = 4.445 \text{ kJ/kg K}$$

and corresponding to a pressure of 2 bar, we find that

$$h_{f2} = 504.7 \text{ kJ/kg} ; \ h_{fg2} = 2201.6 \text{ kJ/kg} ; \ s_{f2} = 1.53 \text{ kJ/kg K} ; \text{ and}$$

$$s_{fg2} = 5.597 \text{ kJ/kg K}$$

Let $\qquad x_2$ = Final dryness fraction of steam.

We know that for a throttling process,

$$h_{f1} + x_1 \, h_{fg1} = h_{f2} + x_2 \, h_{fg2}$$

$$762.6 + 0.9 \times 2013.6 = 504.7 + x_2 \times 2201.6$$

∴ $\qquad x_2 = 0.94$ Ans.

Change of entropy

We know that entropy before expansion,

$$s_1 = s_{f1} + x_1 \, s_{fg1} = 2.138 + 0.9 \times 4.445 = 6.14 \text{ kJ/kg K}$$

and entropy after expansion, $s_2 = s_{f2} + x_2 \, s_{fg2} = 1.53 + 0.94 \times 5.597 = 6.79 \text{ kJ/kg K}$

∴ Change of entropy, $s = s_2 - s_1 = 6.79 - 6.14 = 0.65 \text{ kJ/kg K}$ Ans.

Note : From above we see that the entropy increases during the throttling process. Hence, the throttling process is irreversible.

Example 9.16. *Steam at a pressure of 14 bar with 80 K of superheat is throttled to a pressure of 8.5 bar. Determine the temperature of steam after throttling. The specific heat of superheated steam may be assumed as 2.2 kJ/kg K.*

Solution. Given : $p_1 = 14$ bar ; $T_{sup} - T_1 = 80$ K ; $p_2 = 8.5$ bar ; $c_p = 2.2$ kJ/kg K

Let $\qquad T$ = Temperature of steam after throttling.

From steam tables, corresponding to a pressure of 14 bar, we find that

$$h_{g1} = 2787.8 \text{ kJ/kg}$$

and corresponding to a pressure of 8.5 bar, we find that

$$h_{g2} = 2769 \text{ kJ/kg} \text{ and } T_2 = 172.9° C = 445.9 \text{ K}$$

We know that initial enthalpy of superheated steam,

$$h_1 = h_{g1} + c_p \, (T_{sup} - T_1) = 2787.8 + 2.2 \times 80 = 2963.8 \text{ kJ/kg} \quad \dots (i)$$

and final enthalpy of superheated steam,

$$h_2 = h_{g2} + c_p \, (T - T_2)$$

$$= 2769 + 2.2 \, (T - 445.9) = 1788 + 2.2 \, T \qquad \dots (ii)$$

Equating equations (i) and (ii),

$$2963.8 = 1788 + 2.2 \, T \text{ or } T = 534.4 \text{ K} = 261.4° C \text{ Ans.}$$

Example 9.17. *Steam initially at a pressure of 15 bar and 0.95 dryness expands isentropically to 7.5 bar and then throttled until it is dry. Using steam tables only, Calculate : 1. change in entropy, and 2. change in internal energy per kg of steam during the entire process. Show the process in a h-s plane. Is the entire process reversible. Justify your statement.*

Solution. Given : $p_1 = 15$ bar ; $x_1 = 0.95$; $p_2 = 7.5$; $x_3 = 1$

First of all, let us find the dryness fraction of steam (x_2) after isentropic expansion and pressure (p_3) at the end of throttling process.

From steam tables, corresponding to a pressure of 15 bar, we find that

$$h_{f1} = 844.6 \text{ kJ/kg}; \quad h_{fg1} = 1945.3 \text{ kJ/kg};$$
$$v_{g1} = 0.1317 \text{ m}^3/\text{kg}; \quad s_{f1} = 2.314 \text{ kJ/kg K};$$
$$s_{fg1} = 4.126 \text{ kJ/kg K}$$

Fig. 9.14

and corresponding to a pressure of 7.5 bar, we find that

$$h_{f2} = 709.3 \text{ kJ/kg}; \quad h_{fg2} = 2055.5 \text{ kJ/kg};$$
$$v_{g2} = 0.2553 \text{ m}^3/\text{kg}; \quad s_{f2} = 2.019 \text{ kJ/kg K};$$
$$s_{fg2} = 4.662 \text{ kJ/kg K}$$
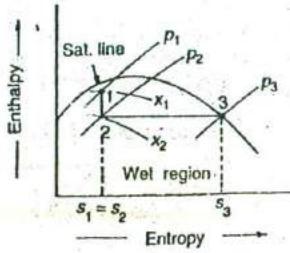
We know that entropy of wet steam before isentropic expansion,

$$s_1 = s_{f1} + x_1 s_{fg1} = 2.314 + 0.95 \times 4.126 = 6.324 \text{ kJ/kg K}$$

and entropy of wet steam after isentropic expansion,

$$s_2 = s_{f2} + x_2 s_{fg2} = 2.019 + x_2 \times 4.662 = 2.019 + 4.662 \, x_2$$

Since the entropy remains constant during isentropic process, therefore

$$s_1 = s_2$$
$$6.324 = 2.019 + 4.662 \, x_2 \quad \text{or} \quad x_2 = 0.923$$

We know that enthalpy of wet steam before isentropic expansion,

$$h_1 = h_{f1} + x_1 \, h_{fg1} = 844.6 + 0.95 \times 1945.3 = 2693 \text{ kJ/kg}$$

and enthalpy of wet steam after isentropic expansion or before throttling,

$$h_2 = h_{f2} + x_2 \, h_{fg2} = 709.3 + 0.923 \times 2055.5 = 2606 \text{ kJ/kg}$$

We know that enthalpy of steam after throttling,

$$h_3 = \text{Enthalpy of steam before throttling } (h_2) = 2606 \text{ kJ/kg}$$

From steam tables, corresponding to the enthalpy of dry steam equal to 2606 kJ/kg, we find that pressure after throttling,

$$p_3 = 0.18 \text{ bar}; \quad v_{g3} = 8.445 \text{ m}^3/\text{kg}; \quad \text{and} \quad s_3 = 7.946 \text{ kJ/kg K}$$

1. *Change of entropy*

We know that change of entropy during the entire process,

$$= s_3 - s_1 = 7.946 - 6.324 = 1.622 \text{ kJ/kg K} \quad \textbf{Ans.}$$

2. *Change in internal energy*

We know that initial internal energy of the steam,

$$u_1 = h_1 - 100 \, p_1 \, v_1 = h_1 - 100 \, p_1 \, x_1 \, v_{g1}$$
$$= 2693 - 100 \times 15 \times 0.95 \times 0.1317 = 2505 \text{ kJ/kg}$$

and final internal energy of the steam,

$$u_3 = h_3 - 100 \, p_3 \, v_3 = h_3 - 100 \, p_3 \, x_3 \, v_{g3}$$
$$= 2606 - 100 \times 0.18 \times 1 \times 8.445 = 2454 \text{ kJ/kg}$$

$$\dots (\because h_3 = h_2 \text{ and } x_3 = 1)$$

∴ Change in internal energy during the entire process

$$= u_3 - u_1 = 2454 - 2505 = -51 \text{ kJ/kg Ans.}$$

The $-$ve sign indicates that the internal energy has decreased.

The process on the $h\text{-}s$ diagram is shown in Fig. 9.14. We know that for a reversible process, the change in entropy is zero whereas for an irreversible process, the entropy increases. Thus from above, we see that the isentropic process 1-2 is reversible because $s_1 = s_2$ or $(s_2 - s_1) = 0$ and throttling process 2-3 is irreversible because s_3 is greater than s_2 or entropy increases.

EXERCISES

1. Steam at a pressure of 6 bar and dryness 0.8 is heated at a constant volume to a pressure of 7 bar. Determine the final dryness fraction and heat absorbed by 1 kg of steam. [Ans. 0.92 ; 234 kJ/kg]

2. A pressure cooker contains 1.5 kg of saturated steam at a pressure of 5 bar. Find the quantity of heat which must be rejected so as to reduce the quality of steam to 60% dry. Determine the pressure and temperature of the steam at the new state. [Ans. 1218.5 kJ ; 2.9 bar ; 132.4° C]

3. A closed vessel of 0.255 m³ contains steam at a pressure of 18 bar and 0.75 dry. The vessel is cooled till the pressure in the vessel falls to 12 bar. Find 1. the final condition of steam ; 2. the heat transferred, and 3. the change in entropy. [Ans. 0.507 ; 1426 kJ (rejected); 3 kJ/K (decrease)]

4. A quantity of steam at 10 bar and 0.85 dryness occupies 0.15 m³. The steam is heated at constant pressure to raise its temperature upto 300° C. Determine : 1. the workdone, 2. the change in internal energy, and 3. the heat supplied. [Ans. 84.4 kJ ; 440.4 kJ ; 524.8 kJ]

5. One kg of steam at 18 bar and 280° C undergoes a constant pressure process until the quality of steam becomes 0.5 dry. Find the workdone, the heat transferred and the change in entropy.

[Ans. $-$ 142 kJ/kg ; $-$ 1142.8 kJ/kg ; $-$ 2.351 kJ/kg K]

6. A cylinder contains steam at a pressure of 7 bar and 0.95 dry. The steam is expanded isothermally and reversibly to a pressure of 1.5 bar. Find : 1. the final condition of steam ; 2. the change in enthalpy ; 3. the change in internal energy ; 4. the change in entropy ; 5. the heat transferred ; and 6. the workdone per kg of steam. [Ans. 165° C ; 143.5 kJ/kg ; 127 kJ/kg ; 1.015 kJ/kg K ; 445 kJ/kg ; 318 kJ/kg]

7. One kg of dry saturated steam at 230° C expands isothermally to a pressure of 4 bar. Find : 1. change in enthalpy ; 2. change in internal energy ; 3. change in entropy ; 4. heat transferred, and 5. workdone during the process. [Ans. 121 kJ/kg ; 93 kJ/kg ; 1.0854 kJ/kg K ; 546 kJ/kg ; 453 kJ/kg]

8. One kg of steam at a pressure of 8 bar and dryness 0.8 is expanded hyperbolically in a cylinder to a pressure of 0.5 bar. Determine the final condition of steam and the heat that passes through the cylinder walls into the steam. [Ans. 0.95 ; 598.6 kJ/kg]

9. Dry saturated steam at a pressure of 7 bar is expanded hyperbolically to 0.36 bar. Find: 1. the final condition of steam ; 2. change of entropy ; and 3. heat exchange through the cylinder walls per kg of steam. [Ans. Superheated (143.5° C) ; 1.354 kJ/kg K ; 564 kJ/kg]

10. 1 m³ of steam at a pressure of 25 bar and dryness fraction 0.9 expands to 2.5 bar. If the steam expands hyperbolically, find : 1. final condition of the steam ; 2. workdone ; and 3. heat transferred.

[Ans. Dry ; 5755 kJ ; 7142.2 kJ]

11. 0.05 m³ of steam at a pressure of 1.5 bar and dryness 0.8 is contained in a cylinder. The steam is compressed till its volume becomes one-fifth of the original volume. If the compression follows the law $pv = $ constant, find : 1. the final pressure of steam ; 2. the final dryness fraction of steam ; 3. the change in internal energy ; 4. the change in entropy ; and 5. the heat flow to or from the steam during the compression process. [Ans. 7.5 bar ; 0.726 ; $-$ 0.44 kJ ; $-$ 0.0354 kJ/K ; $-$ 20.57 kJ]

12. Steam at a pressure of 5 bar and a temperature of 200° C expands isentropically to a pressure 0.7 bar. Find the final dryness of steam by using 1. Steam tables, and 2. Mollier chart. [Ans. 0.93]

13. Steam at a pressure of 22 bar and 380° C expands isentropically in a perfectly insulated cylinder till the temperature falls to 90° C. Determine the quality of steam at the final state and the workdone if the process is 1. non-flow, and 2. steady flow. [Ans. 0.925 ; 574 kJ/kg ; 712 kJ/kg]

14. Steam at a pressure of 120 bar and 400° C expands adiabatically to a pressure of 38 bar. If the final condition of steam is dry saturated, find the workdone by the steam. State whether the process is reversible or irreversible.   [Ans. 200 kJ/kg : Reversible]

15. Dry saturated steam at 7 bar is expanded isentropically to 4 bar after which it is condensed at a constant volume to 1 bar. Determine the final dryness fraction of the steam.   [Ans. 0.26]

16. Steam having dryress fraction 0.9 expands isentropically in a cylinder from a pressure of 4 bar to 1 bar. Determine the final condition of the steam and the work done on the piston.

Assuming the expansion to be according to $pv^n = $ constant, find the value of n.
                                                                [Ans. 0.84 : 184 kJ/kg : 1.127]

17. Dry saturated steam at a pressure of 11 bar expands polytropically according to $pv^{1.13} = $ constant, to a pressure of 1 bar. Determine 1. the final condition of steam ; 2. the amount of heat transferred per kg of steam ; and 3. the change of entropy.   [Ans. 0.874 ; 325.7 kJ/kg ; 0.075 kJ/kg K]

18. Steam at a pressure of 15 bar and 250° C expands according to $p v^{1.25} = $ constant, to a pressure of 1.5 bar. Find 1. the final condition of steam ; 2. the workdone ; 3. the heat transferred ; and 4. change in entropy.
                                                                [Ans. 0.827 ; 336 kJ/kg ; − 195 kJ/kg ; −0.487 kJ/kg K]

19. Steam is throttled from a pressure of 11.5 bar to a pressure 1.4 bar. If the steam is dry saturated at the end of expansion, what is its dryness fraction at the beginning ? By how much is the entropy of the steam increased by throttling ?   [Ans. 0.954 ; 0.912 kJ/kg K]

20. Steam at a pressure of 10 bar and 200° C is throttled to a pressure of 3 bar and then expanded isentropically to a pressure of 0.5 bar. By using Mollier chart, find out the change in entropy and enthalpy during these two processes. Also find the quality of steam at the end of each process.
                                                                [Ans. 0.54 kJ/kg K. 0 ; 0. − 300 kJ/kg : 184° C. 0.94]

QUESTIONS

1. Show that the heat supplied is equal to the change of internal energy, when steam is expanded at a constant volume.

2. Is the expansion of superheated steam isothermal ? If not, state the reason.

3. Prove that during expansion of steam according to $pv = C$, the change in internal energy is equal to the change in total heat of steam.

4. What are the characteristics of adiabatic expansion ?

5. Find the amount of heat absorbed or rejected through the cylinder walls when the steam expands polytropically.

6. Explain throttling process of steam.

7. Show the throttling expansion of steam on a h-s plane.

OBJECTIVE TYPE QUESTIONS

1. In a constant volume process, heat transferred is equal to
   (a) workdone   (b) change in internal energy
   (c) change in enthalpy   (d) none of these

2. The process in which heat transferred is equal to the change of enthalpy, is known as
   (a) constant pressure process   (b) constant volume process
   (c) constant temperature process   (d) constant entropy process

3. The heating of wet steam at a constant temperature till it becomes dry saturated is similar to that of heating at a
   (a) constant volume   (b) constant pressure
   (c) constant entropy   (d) none of these

4. The superheating of steam at .:..... may be regarded as hyperbolic.
   (a) constant volume   (b) constant pressure
   (c) constant entropy   (d) constant temperature

5. In an isentropic process,
   (a) workdone is zero   (b) change in internal energy is zero
   (c) change in entropy is zero   (d) change in enthalpy is zero

6. The non-flow process, in which the workdone is equal to the change in internal energy, is known as
   (a) isentropic process   (b) isothermal process
   (c) isobaric process   (d) isochroic process

7. In a steady flow reversible adiabatic process, workdone is equal to
   (a) change in internal energy   (b) change in entropy
   (c) change in enthalpy   (d) heat transferred

8. The isentropic process on *h-s* diagram will be a
   (a) vertical line   (b) horizontal line   (c) curve

9. In a throttling process .... remains constant.
   (a) enthalpy   (b) temperature   (c) entropy

10. The throttling process on *h-s* diagram will be a
    (a) vertical line   (b) horizontal line   (c) curve

ANSWERS

1. (b)	2. (a)	3. (b)	4. (d)	5. (c)
6. (a)	7. (c)	8. (a)	9. (a)	10. (b)

Thermodynamic Vapour Cycles

10.1. Introduction

We have already discussed the thermodynamic cycles of air in chapter 6. But in this chapter, we shall discuss the thermodynamic cycles of vapour. In a vapour cycle, all the theory remains the same, except the working substance, which is steam. The steam may be in any form *i.e.* wet, dry saturated or superheated. Though there are many vapour cycles, yet the following are important from the subject point of view :

10.2. Carnot Cycle with Steam as Working Substance

The schematic diagram of a Carnot engine is shown in Fig. 10.1 and the Carnot cycle using steam as the working substance is represented on p-v and T-s diagrams as shown in Fig. 10.2 (*a*) and (*b*) respectively.
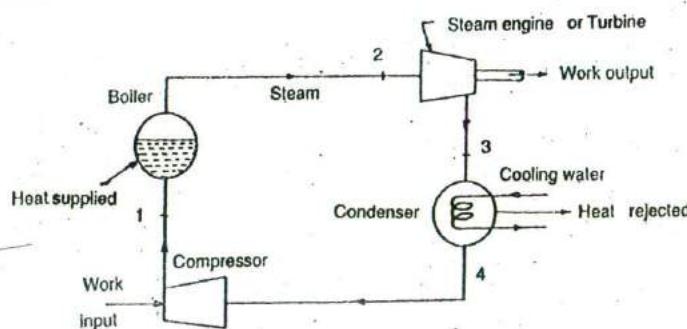

Fig. 10.1. Schematic diagram of a Carnot engine.

Consider 1 kg of saturated water at pressure p_1 and absolute temperature T_1, as represented by point 1 in Fig. 10.2 (*a*) and (*b*). The cycle is completed by the following four processes :

1. Process 1-2. The saturated water at point 1 is isothermally converted into dry saturated steam, in a boiler, and the heat is absorbed at a constant temperature T_1 and pressure p_1. The dry state of steam is represented by point 2. It means that the temperature T_2 (*i.e.* at point 2) and pressure p_2 (*i.e.* at point 2) is equal to temperature T_1 and pressure p_1 respectively. This isothermal process is represented by curve 1-2 on p-v and T-s diagrams in Fig. 10.2 (*a*) and (*b*).

We know that the heat absorbed by the saturated water during its conversion into dry steam is its latent heat of evaporation (*i.e.* $h_{fg1} = h_{fg2}$) corresponding to a pressure p_1 or p_2 ($\because p_1 = p_2$).
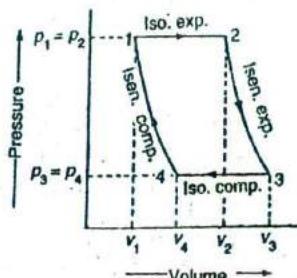
We also know that the area 1-2-*b*-*a* in the *T-s* diagram represents the heat absorbed to some scale, during the isothermal process.

∴ Heat absorbed during isothermal process (area 1-2-*b*-*a*),
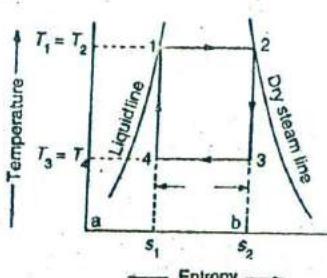
$$q_{1-2} = \text{Change in entropy} \times \text{Absolute temperature}$$

$$= (s_2 - s_1) T_1 = (s_2 - s_1) T_2 \qquad \ldots (\because T_1 = T_2) \quad \ldots (i)$$

2. *Process 2-3.* The dry steam at point 2 now expands isentropically in a steam engine or turbine. The pressure and temperature falls from p_2 to p_3 and T_2 to T_3 respectively. Since no heat is supplied or rejected during this process, therefore there is no change of entropy. The isentropic expansion is represented by the curve 2-3 as shown in Fig. 10.2 (*a*) and (*b*).


(*a*) *p-v* diagram.   (*b*) *T-s* diagram.

Fig. 10.2 Carnot cycle.

3. *Process 3-4* The wet steam at point 3 is now isothermally condensed in a condenser and the heat is rejected at a constant temperature T_3 and pressure p_3. It means that the temperature T_4 (*i.e.* at point 4) and pressure p_4 (*i.e.* at point 4) is equal to the temperature T_3 and pressure p_3 respectively. This isothermal process is represented by the curve 3-4 on *p-v* and *T-s* diagrams as shown in Fig. 10.2 (*a*) and (*b*).

We know that area 3-4-*b*-*a* in the *T-s* diagram represents the heat rejected to some scale during the isothermal process.

∴ Heat rejected during isothermal compression (area 3-4-*b*-*a*), .

$$q_{3-4} = (s_2 - s_1) T_3 = (s_2 - s_1) T_4 \qquad \ldots (\because T_3 = T_4) \quad \ldots (ii)$$

4. *Process 4-1.* The wet steam at point 4 is finally compressed isentropically in a compressor, till it returns back to its original state (point 1). The pressure and temperature rises from p_4 to p_1 and T_4 to T_1 respectively. The isentropic compression is represented by the curve 4-1 as shown in Fig. 10.2 (*a*) and (*b*). Since no heat is absorbed or rejected during this process, therefore entropy remains constant. This completes the cycle.

We know that work done during the cycle

$$= \text{Heat absorbed} - \text{Heat rejected}$$

$$= (s_2 - s_1) T_1 - (s_2 - s_1) T_3 = (s_2 - s_1) (T_1 - T_3)$$

and efficiency of the Carnot cycle,

$$\eta = \frac{\text{Work done}}{\text{Heat absorbed}}$$

$$= \frac{(s_2 - s_1)(T_1 - T_3)}{(s_2 - s_1) T_1} = \frac{T_1 - T_3}{T_1} = 1 - \frac{T_3}{T_1}$$

where
$T_1 = $ Highest temperature corresponding to the boiler pressure $p_1 = p_2$, and

$T_3 = $ Lowest temperature corresponding to the condenser pressure $p_3 = p_4$.

Notes : 1. Since the heat absorbed is at the highest temperature and rejected at the lowest temperature, the Carnot cycle would give a maximum possible efficiency.

2. In the above theory, we have taken temperature at points 1, 2, 3 and 4 as T_1, T_2, T_3 and T_4 respectively in order to keep similarity between Carnot cycle and other cycles. But some authors take it T_1 (for points 1 and 2) and T_2 (for points 3 and 4). In that case, they obtain the relation for efficiency as,

$$\eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$$

3. It may be noted that it is impossible to make a steam engine working on Carnot cycle. The simple reason for the same is that the isothermal expansion 1-2 will have to be carried out extremely slow to ensure that the steam is always at temperature T_1. Similarly, the isothermal compression 3-4 will have to be carried out extremely slow. But isentropic expansion 2-3 and isentropic compression 4-1 should be carried out as quickly as possible in order to approach ideal isentropic conditions. We know that sudden changes in the speed of an engine are not possible in actual practice. Moreover, it is impossible to completely eliminate friction between the various moving parts of the engine, and also heat losses due to conduction, radiation etc. It is thus obvious, that it is impossible to realise Carnot's engine in actual practice. However, such an imaginary engine is used as the ultimate standard of comparison of all steam engines.

Example 10.1. *A power plant is supplied with dry saturated steam at a pressure of 16 bar and exhausts at 0.2 bar. Using steam tables, find the efficiency of the Carnot cycle.*

Solution. Given : $p_1 = 16$ bar ; $p_2 = 0.2$ bar

From steam tables, corresponding to a pressure of 16 bar, we find that

$$T_1 = 201.4° C = 201.4 + 273 = 474.4 \text{ K}$$

and corresponding to a pressure of 0.2 bar, we find that

$$T_3 = 60.1° C = 60.1 + 273 = 333.1 \text{ K}$$

We know that efficiency of the Carnot cycle,

$$\eta = \frac{T_1 - T_3}{T_1} = \frac{474.4 - 333.1}{474.4} = 0.298 \text{ or } 29.8\% \text{ Ans.}$$

Example 10.2. *In a Carnot cycle, heat is supplied at 350° C and is rejected at 25° C. The working fluid is water, which while receiving heat, evaporates from liquid at 350° C to steam at 350° C. From the steam tables, the entropy change for this process is 1.438 kJ/kg K.*

If the cycle operates on a stationary mass of 1 kg of water, find the heat supplied, work done and heat rejected per cycle. What is the pressure of water during heat reception ?

Solution. Given : $T_1 = 350° C = 350 + 273 = 623$ K ; $T_3 = 25° C = 25 + 273 = 298$ K ; $s_2 - s_1 = 1.438$ kJ/kg K

Heat supplied per cycle

We know that the heat supplied per cycle,

$$= (s_2 - s_1) T_1 = 1.438 \times 623 = 895.87 \text{ kJ/kg Ans.}$$

Work done per cycle

We know that the work done per cycle,

$$= (s_2 - s_1)(T_1 - T_3) = 1.438 (623 - 298) = 467.35 \text{ kJ/kg} \text{ Ans.}$$

Heat rejected per cycle

We know that the heat rejected per cycle

$$= (s_2 - s_1) T_3 = 1.438 \times 298 = 428.52 \text{ kJ/kg} \text{ Ans.}$$

Pressure of water during heat reception

The pressure of water during heat reception is the formation pressure of steam corresponding to 350° C. From steam tables, corresponding to 350° C, the pressure is 165.35 bar. Ans.

10.3. Performance Criteria for Thermodynamic Vapour Cycles

Though, theoretically, the Carnot cycle is the most efficient cycle, yet it is not considered as a standard of reference for the comparison of performance of thermodynamic vapour cycles.

The following terms, in addition to the efficiency, are commonly used for the comparison of performance of thermodynamic vapour cycles.

1. *Efficiency ratio.* It is also known as relative efficiency. It is defined as the ratio of thermal efficiency (or actual cycle efficiency) to Rankine efficiency (or ideal cycle efficiency). Mathematically,

$$\text{Efficiency ratio} = \frac{\text{Thermal efficiency}}{{}^* \text{Rankine efficiency}}$$

Note : Thermal efficiency $= \dfrac{\text{Heat equivalent to one kilowatt hour (kWh)}}{\text{Total heat supplied to the steam per kWh}} = \dfrac{3600 \times P}{m_s (h_2 - h_{f3})}$

where m_s = Mass of steam supplied in kg/h, and

P = Power developed in kW.

2. *Work ratio.* It is defined as the ratio of net work output to the gross (engine or turbine) output. Mathematically,

$$\text{Work ratio} = \frac{\text{Net work output}}{\text{Gross output}} = \frac{\text{Turbine work} - \text{Compressor work}}{\text{Turbine work}}$$

It may be noted that the Carnot cycle, despite of its high ideal thermal efficiency, has low work ratio. It is one of the reason that Carnot cycle is not attempted. In order to have better performance of the plant, both efficiency ratio and work ratio are the important criteria. It is desirable to have the value of work ratio almost unity. The higher value of work ratio also means a smaller size of the plant.

3. *Specific steam consumption.* It is also known as steam rate or specific rate of flow of steam. It is defined as the mass of steam that must be supplied to a steam engine or turbine in order to develop a unit amount of work or power output. The amount of work or power output is usually expressed in kilowatt hour (kWh). Mathematically

$$= \frac{1 \text{ kWh}}{w'} = \frac{3600}{w} = \frac{3600}{h_2 - h_3} \text{ kg / kWh} \qquad \dots (\because 1 \text{ kWh} = 3600 \text{ kJ})$$

where w = Net workdone or power output = $(h_2 - h_3)$ kJ/kg

Note : In case of steam plants, the specific steam consumption is an indicator of the relative size of the plant.

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* Refer Art. 10.4.

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10.4. Rankine Cycle

The Rankine cycle is an ideal cycle for comparing the performance of steam plants. It is modified form of Carnot cycle, in which the condensation process (3-4) is continued until the steam is condensed into water. The schematic diagram of a steam engine or a turbine plant is shown in Fig. 10.3.
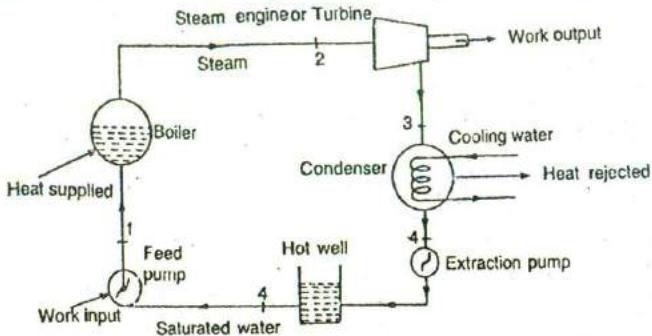

Fig. 10.3. Schematic diagram of a steam engine or turbine plant.

A Carnot cycle, using steam as a working substance, is represented on p-v and T-s diagrams as shown in Fig. 10.4 (a) and (b).

(a) p-v diagram.   (b) T-s diagram
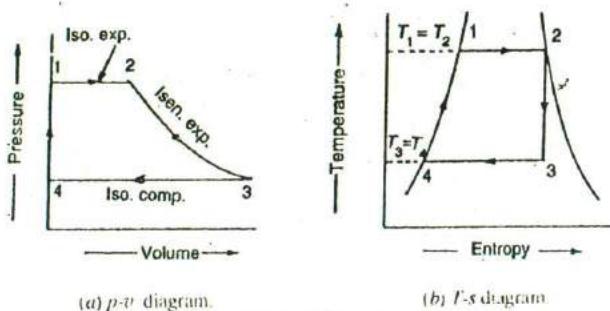
Fig. 10.4 Rankine cycle.

Consider 1 kg of saturated water at pressure p_1 and absolute temperature T_1 as represented by point 1 in Fig. 10.4 (a) and (b). The cycle is completed by the following four processes :

1. *Process 1-2* The saturated water at point 1 is isothermally converted into dry saturated steam in a boiler, and the heat is absorbed at a constant temperature T_1 and pressure p_1. The dry state of steam is represented by point 2. It means that the temperature T_2 (i.e. at point 2) and pressure p_2 (i.e. at point 2) is equal to temperature T_1 and pressure p_1 respectively. This isothermal process is represented by curve 1-2 on p-v and T-s diagrams in Fig. 10.4 (a) and (b).

We know that the heat absorbed during isothermal process by water during its conversion into dry steam is its latent heat of vaporisation (i.e. $h_{fg1} = h_{fg2}$), corresponding to a pressure p_1 or p_2 ($\because p_1 = p_2$).

2. *Process 2-3.* The dry saturated steam at point 2, now expands isentropically in an engine or turbine. The pressure and temperature falls from p_2 to p_3 and T_2 to T_3 respectively with a dryness fraction x_3. Since no heat is supplied or rejected during this process, therefore there is no change of entropy. The isentropic expansion is represented by the curve 2-3 as shown in Fig. 10.4 (a) and (b).

3. *Process 3-4.* The wet steam at point 3 is now isothermally condensed in a condenser and the heat is rejected at constant temperature T_3 and pressure p_3 until the whole steam is condensed into water. It means that the temperature T_4 and pressure p_4 (*i.e.* at point 4) is equal to the temperature T_3 and pressure p_3 respectively. The isothermal compression is represented by curve 3-4 on p-v and T-s diagrams in Fig. 10.4 (a) and (b). The heat rejected by steam is its latent heat (equal to $x_3\,h_{fg3}$).

4. *Process 4-1.* The water at point 4 is now warmed in a boiler at constant volume from temperature T_4 to T_1. Its pressure also rises from p_4 to p_1. This warming operation is represented by the curve 4-1 on p-v and T-s diagrams in Fig. 10.4 (a) and (b). The heat absorbed by water during this operation is equal to the sensible heat or liquid heat corresponding to the pressure p_1 *i.e.* equal to sensible heat at point 1 *minus* sensible heat at point 4.

Let $\quad h_{f1} = h_{f2} =$ Sensible heat or enthalpy of water at point 1 corresponding to a pressure of p_1 or p_2 ($\because p_1 = p_2$), and

$\qquad h_{f4} = h_{f3} =$ Sensible heat or enthalpy of water at point 4 orresponding to a pressure of p_4 or p_3 ($\because p_4 = p_3$).

∴ Heat absorbed during warming operation 4-1

$$= h_{f1} - h_{f4} = h_{f2} - h_{f3}$$

and heat absorbed during the complete cycle

$$= \text{Heat absorbed during isothermal operation 1-2}$$
$$+ \text{Heat absorbed during warming operation 4-1}$$

$$= h_{fg2} + (h_{f2} - h_{f3}) = h_{f2} + h_{fg2} - h_{f3} = h_2 - h_{f3}$$

$$\ldots (\because \text{For dry steam, } h_2 = h_{f2} + h_{fg2}) \quad \ldots (i)$$

We know that heat rejected during the cycle

$$= h_3 - h_{f4} = h_{f3} + x_3\,h_{fg3} - h_{f4} = x_3\,h_{fg3} \qquad \ldots(\because h_{f3} = h_{f4})$$

∴ *Workdone during the cycle

$$= \text{Heat absorbed} - \text{Heat rejected}$$

$$= (h_2 - h_{f3}) - x_3\,h_{fg3}$$

$$= h_2 - (h_{f3} + x_3\,h_{fg3}) = h_2 - h_3 \qquad \ldots(\because h_3 = h_{f3} + h_{fg3}) \quad \ldots (ii)$$

and efficiency (also called Rankine efficiency),

$$\eta_R = \frac{\text{Work done}}{\text{Heat absorbed}} = \frac{h_2 - h_3}{h_2 - h_{f3}} \qquad \ldots (iii)$$

Notes : 1. The difference of enthalpies $(h_2 - h_3)$ is known as isentropic heat drop.

2. If the expansion of steam (2-3) is not isentropic and follows the general law $pv^n = $ constant, then work done during the process will not be $(h_2 - h_3)$. The work done in this case will be given by the relation :

$$W = p_2 v_2 + \frac{p_2 v_2 - p_3 v_3}{(n-1)} - p_3 v_3 = \frac{n\,(p_2 v_2 - p_3 v_3)}{n-1}$$

---

* Since the workdone by the extraction and boiler feed pumps in increasing the pressure of water from the condenser pressure $(p_3 = p_4)$ to the boiler pressure ($p_1 = p_2$) is very small, therefore it is neglected.

Example 10.3. *A steam power plant is supplied with dry saturated steam at a pressure of 12 bar and exhausts into a condenser at 0.1 bar. Calculate the Rankine efficiency by using 1. Steam tables, and 2. Mollier chart.*

Solution. Given : $x_2 = 1$; $p_2 = 12$ bar ; $p_3 = 0.1$ bar

1. *Using steam tables*

From steam tables, corresponding to a pressure of 12 bar, we find that

$$h_{f2} = 798.4 \text{ kJ/kg} ; \ h_{fg2} = 1984.3 \text{ kJ/kg} ; \ s_2 = s_{g2} = 6.519 \text{ kJ/kg K}$$

and corresponding to a pressure of 0.1 bar, we find that

$$h_{f3} = 191.8 \text{ kJ/kg} ; \ h_{fg3} = 2393 \text{ kJ/kg} ; \ s_{f3} = 0.649 \text{ kJ/kg K} ; \ s_{fg3} = 7.502 \text{ kJ/kg K}$$

First of all, let us find the dryness fraction (x_3) of steam after isentropic expansion. We know that in isentropic expansion,

Entropy before expansion (s_2) = Entropy after expansion (s_3)

$$6.519 = s_{f3} + x_3 \, s_{fg3} = 0.649 + x_3 \times 7.502$$

∴ $x_3 = 0.783$

Now enthalpy or total heat of dry saturated steam before expansion,

$$h_2 = h_{g2} = h_{f2} + x_2 \, h_{fg2} = 798.4 + 1 \times 1984.3 = 2782.7 \text{ kJ/kg}$$

and enthalpy or total heat of wet steam after expansion,

$$h_3 = h_{f3} + x_3 \, h_{fg3} = 191.8 + 0.783 \times 2393 = 2065.5 \text{ kJ/kg}$$

We know that Rankine efficiency,

$$\eta_R = \frac{h_2 - h_3}{h_2 - h_{f3}} = \frac{2782.7 - 2065.5}{2782.7 - 191.8} = 0.277 \quad \text{or} \quad 27.7 \% \text{ Ans.}$$

2. *Using Mollier chart*

In Fig. 10.5, the initial condition of steam is marked at point 2 and final condition at point 3. From Mollier chart, we find that

$$h_2 = 2780 \text{ kJ/kg} ; \ h_3 = 2060 \text{ kJ/kg} ; \text{ and}$$

$$h_{f3} = 191.8 \text{ kJ/kg} \qquad \text{...(From steam tables)}$$

We know that the Rankine efficiency,

$$\eta_R = \frac{h_2 - h_3}{h_2 - h_{f3}} = \frac{2780 - 2060}{2780 - 191.8}$$

$$= 0.278 \text{ or } 27.8\% \text{ Ans.}$$

Fig. 10.5

Example 10.4. *A simple Rankine cycle steam power plant operates between the temperatures of 260° C and 95° C. The steam is supplied to the turbine at a dry saturated condition. In the turbine, it expands in an isentropic manner. Determine the efficiency of the Rankine cycle followed by the turbine, and the efficiency of the Carnot cycle, operating between these two temperature limits. Draw the turbine cycle on a h-s or on a T-s diagram.*

Solution. Given : $T_1 = T_2 = 260° \text{ C} = 260 + 273 = 533 \text{ K}$; $T_3 = T_4 = 95° \text{ C} = 95 + 273 = 368 \text{ K}$
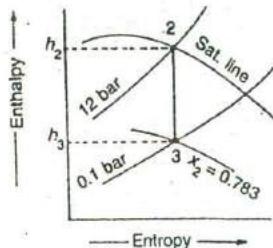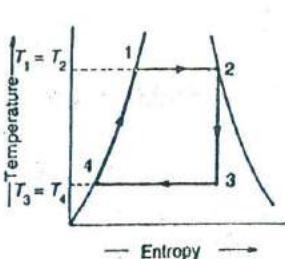
From steam tables, we find that the pressure corresponding to the highest temperature of 260° C,

$$p_2 = 46.94 \text{ bar}$$

and the pressure corresponding to the lowest temperature of 95° C,

$$p_3 = 0.845 \text{ bar}$$

(a) T-s diagram   (b) h-s diagram.
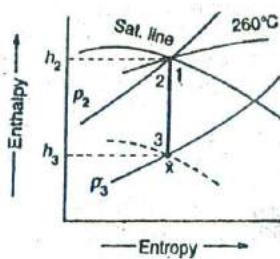
Fig. 10.6

The initial and final conditions of steam are shown in Fig. 10.6 (a) and (b). From Mollier chart we find that

$$h_2 = 2800 \text{ kJ/kg and } h_3 = 2150 \text{ kJ/kg}$$

From steam tables, we find that the enthalpy of water at 95° C,

$$h_{f3} = 398 \text{ kJ/kg}$$

Efficiency of the Rankine cycle

We know that efficiency of the Rankine cycle,

$$\eta_R = \frac{h_2 - h_3}{h_2 - h_{f3}} = \frac{2800 - 2150}{2800 - 398} = 0.27 \text{ or } 27\% \text{ Ans.}$$

Efficiency of the Carnot cycle

We know that efficiency of the Carnot cycle,

$$\eta_c = \frac{T_1 - T_3}{T_1} = \frac{533 - 368}{533} = 0.3096 \text{ or } 30.96\% \text{ Ans.}$$

Note : The value of h_2 and h_3 may be determined by using steam tables, as discussed below :

From steam tables, corresponding to a temperature of 260° C, we find that

$$h_2 = h_{g2} = 2796.4 \text{ kJ/kg} ; s_2 = s_{g2} = 6.001 \text{ kJ/kg K}$$

and corresponding to a temperature of 95° C, we find that

$$h_{f3} = 398 \text{ kJ/kg} ; h_{fg3} = 2270.2 \text{ kJ/kg} ; s_{f3} = 1.25 \text{ kJ/kg} ; s_{fg3} = 6.167 \text{ kJ/kg K}$$

First of all, let us find the dryness fraction of steam at point 3 (*i.e.* x_3). We know that for isentropic expansion 2-3,

$$s_2 = s_3$$

or

$$6.001 = s_{f3} + x_3 \, s_{fg3} = 1.25 + x_3 \times 6.167$$

∴

$$x_3 = 0.77$$

We know that

$$h_3 = h_{f3} + x_3 \, h_{fg3} = 398 + 0.77 \times 2270.2 = 2146 \text{ kJ/kg}$$

Example 10.5. *The steam consumption of a steam engine is 20 tonnes per shift of 8 hours when developing 220 kW. Dry and saturated steam enters the engine at 10 bar pressure and leaves it at 0.1 bar pressure. Estimate the Rankine efficiency and the thermal efficiency of the engine.*

Solution. Given : $m_s = 20/8 = 2.5$ t/h $= 2500$ kg/h ; $P = 220$ kW ; $x_2 = 1$; $p_1 = p_2 = 10$ bar ; $p_3 = p_4 = 0.1$ bar

From steam tables, corresponding to a pressure of 10 bar, we find that

$$h_2 = h_{g2} = 2776.2 \text{ kJ/kg} ; s_2 = s_{g2} = 6.583 \text{ kJ/kg K}$$

and corresponding to a pressure of 0.1 bar, we find that

$$h_{f3} = 191.8 \text{ kJ/kg} ; h_{fg3} = 2393 \text{ kJ/kg} ; s_{f3} = 0.649 \text{ kJ/kg K} ; s_{fg3} = 7.502 \text{ kJ/kg K}$$

Rankine efficiency

First of all, let us find the dryness fraction of steam at point 3 (*i.e.* x_3). We know that for isentropic expansion 2-3 (Refer Fig. 10.4),

Entropy before expansion (s_2) = Entropy after expansion (s_3)

$$6.583 = s_{f3} + x_3 \, s_{fg3} = 0.649 + x_3 \times 7.502$$

∴　　　　　　　　　　$$x_3 = 0.791$$

We know that enthalpy or total heat of steam at point 3,

$$h_3 = h_{f3} + x_3 \, h_{fg3} = 191.8 + 0.791 \times 2393 = 2084.7 \text{ kJ/kg}$$
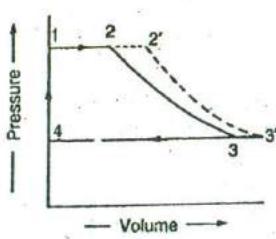
∴　Rankine efficiency, 　$$\eta_R = \frac{h_2 - h_3}{h_2 - h_{f3}} = \frac{2776.2 - 2084.7}{2776.2 - 191.8} = 0.267 \text{ or } 26.7\% \text{ Ans}$$

Thermal efficiency of the engine
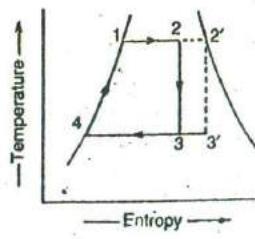
We know that thermal efficiency of the engine

$$\eta_{th} = \frac{3600 \times P}{m_s (h_2 - h_{f3})} = \frac{3600 \times 220}{2500 (2776.2 - 191.8)}$$

$$= 0.1226 \text{ or } 12.26\% \text{ Ans.}$$

## 10.5. Rankine Cycle with Incomplete Evaporation

Fig. 10.7. Rankine cycle with incomplete evaporation.

We have already discussed in the last article that in isothermal expansion of a Rankine cycle, the water is converted into dry saturated steam at a constant temperature T_1 and pressure p_1. Sometimes, the steam produced is not completely dry, but it is wet with dryness fraction equal to x_2. In such a case, the Rankine cycle may be represented on p-v and T-s diagrams as shown in Fig. 10.7 (a) and (b).

It may be noted from the above figure, that 1-2-3-4 represents the Rankine cycle with incomplete evaporation, whereas 1-2'-3'-4 represents the cycle with complete evaporation. In such a case, heat absorbed during the isothermal expansion 1-2 is equal to $x_2 h_{fg2}$.

∴ Heat absorbed during the complete cycle

$$= x_2 h_{fg2} + h_{f2} - h_{f3} = h_2 - h_{f3}$$

$$\ldots (\because \text{ For wet steam, } h_2 = h_{f2} + x_2 h_{fg2})$$

Example 10.6. *Dry saturated steam at 10 bar is supplied to a prime mover and the exhaust takes place at 0.2 bar. Determine the Rankine efficiency, efficiency ratio and specific steam consumption of the prime mover, if the indicated thermal efficiency is 20%.*

Also find the percentage change in the Rankine efficiency, if the steam is initially 90% dry.

Solution. Given : $p_2 = 10$ bar ; $p_3 = 0.2$ bar ; $n_{th} = 20\% = 0.2$

Rankine efficiency

The initial and final conditions of steam are shown in Fig. 10.8 (a) and (b). From Mollier chart, we find that
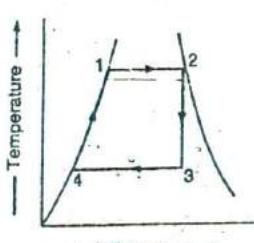
$$h_2 = 2775 \text{ kJ/kg}; \text{ and } h_3 = 2150 \text{ kJ/kg}$$

From steam tables, we find that enthalpy of water at 0.2 bar,
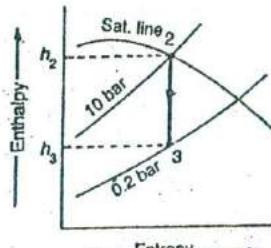
$$h_{f3} = 251.5 \text{ kJ/kg}$$

We know that Rankine efficiency,

$$\eta_R = \frac{h_2 - h_3}{h_2 - h_{f3}} = \frac{2775 - 2150}{2775 - 251.5} = 0.247 \text{ or } 24.7\% \text{ Ans}$$


(a) *T-s* diagram.   (b) *h-s* diagram.

Fig. 10.8

Efficiency ratio

We know that efficiency ratio

$$= \frac{\text{Indicated thermal efficiency}}{\text{Rankine efficiency}} = \frac{0.2}{0.247} = 0.81 \text{ or } 81 \% \text{ Ans.}$$

Specific steam consumption
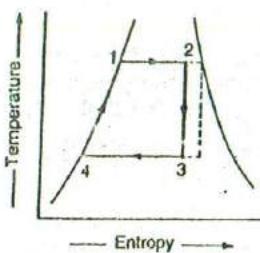
We know that specific steam consumption

$$= \frac{3600}{h_2 - h_3} = \frac{3600}{2775 - 2150} = 576 \text{ kg/kWh Ans.}$$
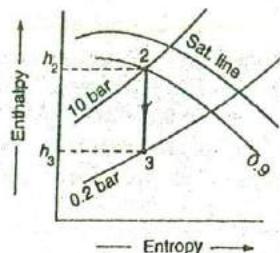
Percentage change in the Rankine efficiency if the steam is initially 90% dry

The initial and final conditions of steam are shown in Fig. 10.9 (a) and (b). From Mollier chart, we find that

$$h_2 = 2580 \text{ kJ/kg} ; \; h_3 = 2030 \text{ kJ/kg}$$

(a) T-s diagram.   (b) h-s diagram.

Fig. 10.9

We know that Rankine efficiency,
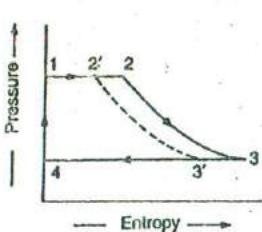
$$\eta_R = \frac{h_2 - h_3}{h_2 - h_{f3}} = \frac{2580 - 2030}{2580 - 251.5} = 0.236 \text{ or } 23.6\%$$

∴ Percentage change in Rankine efficiency
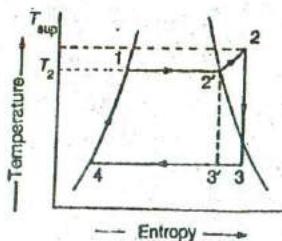
$$= \frac{24.7 - 23.6}{24.7} \times 100 = 4.45 \% \text{ Ans.}$$

10.6. Rankine Cycle with Superheated Steam

We have already discussed, in the last article, the case of a Rankine cycle where the steam produced is wet with dryness fraction x_2. But sometimes, the steam produced is superheated. In such a case, the Rankine cycle may be shown on p-v and T-s diagrams as shown in Fig. 10.10 (a) and (b).

(a) p-v diagram.   (b) T-s diagram.

Fig. 10.10. Rankine cycle with superheated steam.

In may be noted from the above figure, that 1-2-3-4 represents the Ranking cycle with superheated steam, where as 1-2'-3'-4 represents the cycle with complete evaporation. In such a case, heat absorbed during isothermal expansion,

$$h_2 = h_{sup} = h_{g2} + c_p (T_{sup} - T_2)$$

where h_{g2} = Enthalpy of dry saturated steam corresponding to a pressure of $p_1 = p_2$ (from steam tables),

T_{sup} = Temperature of superheated steam in K,

T_2 = Saturation temperature in K. corresponding to a pressure of $p_1 = p_2$ (from steam tables), and

c_p = Specific heat of superheated steam.

Note : The value of h_{sup} can be read directly from steam tables for superheated steam or Mollier chart corresponding to a pressure of p_2 and temperature T_{sup} in ° C.

The superheating of steam before entering for isentropic expansion has the following advantages :

1. The work done (equal to the area 2'-2-3-3' as shown in Fig. 10.10) increases.

2. The dryness fraction of steam at the end of isentropic expansion (*i.e.* leaving the steam engine or turbine) increases.

3. The specific steam consumption decreases.

4. The net efficiency of the cycle increases with the increase in degree of superheat (*i.e* $T_{sup} - T_2$). In other words, as the degree of superheat increases, the average temperature of heat addition of the cycle also increases and thus the efficiency increases.

Example 10.7. *A steam turbine receives steam at 15 bar and 350° C, and exhausts to the condenser at 0.06 bar. Determine the thermal efficiency of the ideal Rankine cycle operating between these two limits.*

Solution. Given : $p_1 = p_2 = 15$ bar ; $T_{sup} = 350°$ C ; $p_3 = p_4 = 0.06$ bar

From steam tables of superheated steam, corresponding to a pressure of 15 bar and 350° C, we find that

$$h_2 = h_{sup} = 3148.7 \text{ kJ/kg} ; \text{ and } s_2 = s_{sup} = 7.104 \text{ kJ/kg K}$$

and corresponding to a pressure of 0.06 bar, we find that

$$h_{f3} = 151.5 \text{ kJ/kg} ; h_{fg3} = 2416 \text{ kJ/kg} ; s_{f3} = 0.521 \text{ kJ/kg K} ; \text{ and } s_{fg3} = 7.81 \text{ kJ/kg K}$$

First of all, let us find out the dryness fraction of steam (x_3) after expansion. We know that

Entropy before expansion (s_2) = Entropy after expansion (s_3)

or $$s_{sup} = s_{f3} + x_3 \, s_{fg3}$$

$$7.104 = 0.521 + x_3 \times 7.81 \text{ or } x_3 = 0.843$$

∴ Enthalpy of steam at point 3,

$$h_3 = h_{f3} + x_3 \, h_{fg3} = 151.5 + 0.843 \times 2416 = 2188 \text{ kJ/kg}$$

We know that thermal efficiency of the ideal Rankine cycle,

$$\eta_R = \frac{h_2 - h_3}{h_2 - h_{f3}} = \frac{3148.7 - 2188}{3148.7 - 151.5} = 0.32 \text{ or } 32\% \text{ Ans.}$$

Note : The value of isentropic heat drop $(h_2 - h_3)$ may be obtained directly from the Mollier chart.

Example 10.8. *A steam turbine receives superheated steam at a pressure of 17 bar and having a degree of superheat of 110° C. The exhaust pressure is 0.07 bar and the expansion of steam takes place isentropically. Calculate 1. the heat supplied, 2. the heat rejected, 3. net workdone, and 4. the thermal efficiency.*

Solution. Given : $p_1 = p_2 = 17$ bar ; $T_{sup} - T_2 = 100°$ C ; $p_3 = p_4 = 0.07$ bar

The Rankine cycle with superheated steam is shown in Fig. 10.11.

From steam tables, corresponding to a pressure of 17 bar, we find that saturation temperature,

$$T_2 = 212.4° \text{ C}$$

∴ Temperature of superheated steam,

$$T_{sup} = 110 + 212.4 = 322.4° \text{ C}$$

Now from steam tables for superheated steam, corresponding to a pressure of 17 bar and 322.4° C, we find that

$$h_2 = h_{sup} = 3083.4 \text{ kJ/kg ; and}$$

$$s_2 = s_{sup} = 6.939 \text{ kJ/kg K}$$
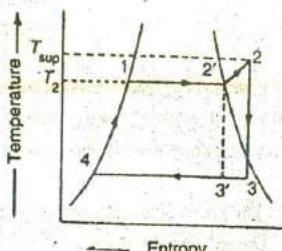
Fig. 10.11

and from steam tables for dry saturated steam, corresponding to a pressure of 0.07 bar, we find that

$$h_{f3} = h_{f4} = 163.4 \text{ kJ/kg ; } h_{fg3} = 2409.2 \text{ kJ/kg ; } s_{f3} = 0.559 \text{ kJ/kg K ; and } s_{fg3} = 7.718 \text{ kJ/kg K}$$

First of all, let us find the dryness fraction of steam at point 3 (*i.e.* x_3). We know, that for isentropic expansion 2-3,

Entropy before expansion (s_2) = Entropy after expansion (s_3)

or
$$s_{sup} = s_{f3} + x_3 \, s_{fg3}$$

$$6.939 = 0.559 + x_3 \times 7.718 \quad \text{or} \quad x_3 = 0.827$$

We know that enthalpy of steam at point 3,

$$h_3 = h_{f3} + x_3 \, h_{fg3} = 163.4 + 0.827 \times 2409.2 = 2155.8 \text{ kJ/kg}$$

1. *Heat supplied*

We know that heat supplied

$$= h_2 - h_{f3} = 3083.4 - 163.4 = 2920 \text{ kJ/kg Ans.}$$

2. *Heat rejected*

We know that heat rejected

$$= h_3 - h_{f4} = h_{f3} + x_3 \, h_{fg3} - h_{f3} = x_3 \, h_{fg3} \qquad \dots (\because h_{f4} = h_{f3})$$

$$= 0.827 \times 2409.2 = 1992.4 \text{ kJ/kg Ans.}$$

3. *Net workdone*

We know that net workdone = Heat supplied − Heat rejected

$$= 2920 - 1992.4 = 927.6 \text{ kJ/kg Ans.}$$

4. *Thermal efficiency*

We know that thermal efficiency

$$= \frac{\text{Net workdone}}{\text{Heat supplied}} = \frac{927.6}{2920} = 0.317 \text{ or } 31.7\% \text{ Ans.}$$

Example 10.9. *Steam at 50 bar, 400° C expands in a Rankine cycle to 0.34 bar. For a mass flow rate of 150 kg/s of steam, determine I. the power developed, 2. the thermal efficiency, and 3. specific steam consumption*

Solution. Given ; $p_1 = p_2 = 50$ bar ; $T_{sup} = 400°$ C ; $p_3 = p_4 = 0.34$ bar ; $m_s = 150$ kg/s

From steam tables for superheated steam, corresponding to a pressure of 50 bar and 400° C, we find that

$$h_2 = h_{sup} = 3198.3 \text{kJ/kg} ; \text{ and } s_2 = s_{sup} = 6.651 \text{ kJ/kg K}$$

and from steam tables for dry saturated steam, corresponding to a pressure of 0.34 bar, we find that

$$h_{f3} = 301.5 \text{ kJ/kg} ; h_{fg3} = 2329 \text{ kJ/kg} ; s_{f3} = 0.98 \text{ kJ/kg K} ; \text{ and } s_{fg3} = 6.747 \text{ kJ/kg K}$$

First of all, let us find the dryness fraction of steam at point 3 (*i.e.* x_3). We know that for isentropic expansion 2-3 (Refer Fig. 10.10),

Entropy before expansion (s_2)

$$= \text{Entropy after expansion } (s_3)$$

or

$$6.651 = s_{f3} + x_3 s_{fg3} = 0.98 + x_3 \times 6.747 \text{ or } x_3 = 0.84$$

We know that enthalpy of steam at point 3,

$$h_3 = h_{f3} + x_3 h_{fg3} = 301.5 + 0.84 \times 2329 = 2258 \text{ kJ/kg}$$

1. *Power developed*

We know that workdone,

$$w = h_2 - h_3 = 3198.3 - 2258 = 940.3 \text{ kJ/kg}$$

and total workdone

$$= m_s \times w = 150 \times 940.3 = 141\,045 \text{ kJ/s}$$

Power developed $= 141\,045$ kW Ans.   ...($\because$ 1 kJ/s = 1 kW)

2. *Thermal efficiency*

We know that thermal efficiency,

$$\eta_R = \frac{h_2 - h_3}{h_2 - h_{f3}} = \frac{3198.3 - 2258}{3198.3 - 301.5} = 0.3246 \text{ or } 32.46\% \text{ Ans.}$$

3. *Specific steam consumption*

We know that specific steam consumption

$$= \frac{3600}{h_2 - h_3} = \frac{3600}{w} = \frac{3600}{940.3} = 3.83 \text{ kg/kWh Ans.}$$

10.7. Modified Rankine Cycle

We have seen in the Rankine cycle, that the steam is expanded to the extreme toe of the *p-v* diagram (at point 3) as shown in Fig. 10.12 (*a*). But in actual reciprocating steam engines, it is found to be too uneconomical (due to larger size of the cylinder) to expand steam to the full limit (*i.e.* upto the point 3).

It may be noted that the diagram is very narrow at the toe, and the amount of work done (represented by area 5-3-6) during this final portion of the expansion stroke is extremely small. In fact, it is too small to overcome even the friction of the moving parts in the steam engine. The expansion of steam, therefore, is carried on in the engine cylinder at a pressure higher than that of the condenser pressure or exhaust pressure or back pressure. This higher pressure is known as release pressure (p_5).

In order to overcome the above mentioned difficulty, the Rankine cycle is slightly modified. In a modified Rankine cycle, the expansion stroke of the piston is stopped at point 5 by cutting the

toe of Rankine cycle, and the steam is exhausted from the cylinder at a constant volume. This causes a sudden drop of pressure from p_5 to p_6. The expansion of steam is, therefore, completed by a constant volume line 5-6 as shown on p-v and T-s diagram in Fig. 10.12 (a) and (b). By doing so, the size of the cylinder and stroke length is considerably reduced.

(a) p-v diagram. (b) T-s diagram.
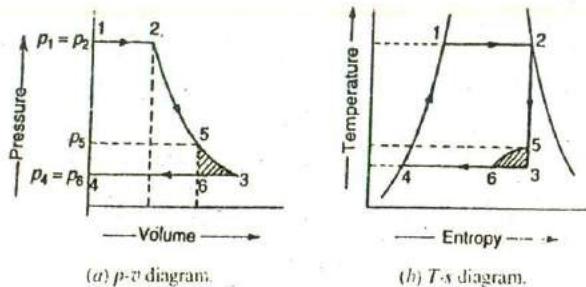
Fig. 10.12. Modified Rankine cycle.

10.8. Efficiency of Modified Rankine Cycle

Consider a modified Rankine cycle whose processes are shown in Fig. 10.13 (a) and (b).

(a) p-v diagram (b) T-s diagram.
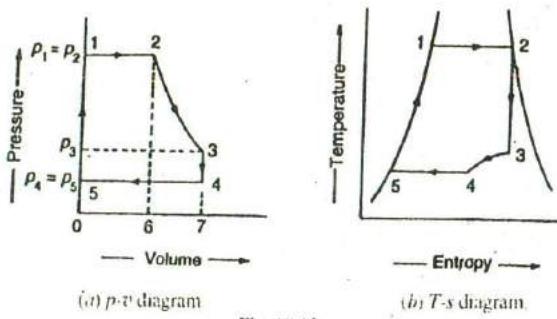
Fig. 10.13

Let $p_1 = p_2 = $ Pressure of steam at point 2,

$v_2 = $ Volume of steam at point 2,

$h_2 = $ Enthalpy or total heat of steam at point 2,

$u_2 = $ Internal energy of steam at point 2,

$p_3, v_3, h_3, u_3 = $ Corresponding values of steam at point 3.

$p_4 = $ Back pressure of steam at point 4, and

$h_{f4} = $ Sensible heat or enthalpy of water at point 4.

We know that work done during constant pressure process 1-2,

$$= \text{Area } 1\text{-}2\text{-}6\text{-}0 = 100\, p_2\, v_2 \qquad \ldots (i)$$

We also know that work done during isentropic expansion 2-3

$$= \text{Area } 2\text{-}3\text{-}7\text{-}6 = \text{Change in internal energy}$$

$$= u_2 - u_3 \qquad \qquad \ldots (ii)$$

and work done during constant pressure process 4-5

$$= \text{Area } 0\text{-}5\text{-}4\text{-}7 = 100 \, p_4 \, v_4 \qquad \qquad \ldots (iii)$$

∴ Work done during the cycle per kg of steam,

$$W = \text{Area } 1\text{-}2\text{-}3\text{-}4\text{-}5$$

$$= \text{Area } 1\text{-}2\text{-}6\text{-}0 + \text{Area } 2\text{-}3\text{-}7\text{-}6 - \text{Area } 0\text{-}5\text{-}4\text{-}7$$

$$= 100 \, p_2 \, v_2 + (u_2 - u_3) - 100 \, p_4 \, v_4$$

$$= 100 \, p_2 \, v_2 + [(h_2 - 100 \, p_2 \, v_2) - (h_3 - 100 \, p_3 \, v_3)] - 100 \, p_4 \, v_4$$

$$= h_2 - h_3 + 100 \, (p_3 - p_4) \, v_3 \qquad \qquad \ldots (\because \, v_3 = v_4)$$

We know that heat supplied per cycle

$$= h_2 - h_{f5} = h_2 - h_{f4} \qquad \qquad \ldots (\because \, h_{f5} = h_{f4})$$

∴ Efficiency of the modified Rankine cycle,

$$\eta_{\text{MR}} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{(h_2 - h_3) + 100 \, (p_3 - p_4) \, v_3}{h_2 - h_{f4}}$$

Example 10.10. *A steam engine admits steam at a pressure of 5.6 bar and 0.9 dry. The cut off occurs at one-half stroke, and pressure at release is 2.6 bar. The back pressure is 1 bar. Find the modified Rankine efficiency of the engine, assuming the pressure drop at release to take place at a constant volume. Neglect clearance volume.*

Solution. Given : $p_2 = 5.6$ bar ; $x_2 = 0.9$; $v_2 = v_3 / 2$; $p_3 = 2.6$ bar ; $p_4 = 1$ bar

From steam tables, corresponding to a pressure of 5.6 bar, we find that

$$h_{f2} = 658.8 \, \text{kJ/kg} \, ; \, h_{fg2} = 2093.7 \, \text{kJ/kg} \, ; \, v_{g2} = 0.3367 \, \text{m}^3/\text{kg} \, ;$$

$$s_{f2} = 1.904 \, \text{kJ/kg K} \, ; \, s_{fg2} = 4.877 \, \text{kJ/kg K}$$

Similarly, corresponding to a pressure of 2.6 bar, we find that

$$h_{f3} = 540.9 \, \text{kJ/kg} \, ; \, h_{fg3} = 2177.3 \, \text{kJ/kg} \, ; \, v_{g3} = 0.6925 \, \text{m}^3/\text{kg} \, ;$$

$$s_{f3} = 1.621 \, \text{kJ/kg K} \, ; \, s_{fg3} = 5.418 \, \text{kJ/kg K}$$

and corresponding to a pressure of 1 bar, we find that

$$h_{f4} = 417.5 \, \text{kJ/kg}$$

We know that enthalpy or total heat of steam at point 2,

$$h_2 = h_{f2} + x_2 \, h_{fg2} = 658.8 + 0.9 \times 2093.7 = 2543 \, \text{kJ/kg}$$

and volume at point 2, $\quad v_2 = x_2 \, v_{g2} = 0.9 \times 0.3367 = 0.303 \, \text{m}^3/\text{kg}$

∴ Volume of stroke at point 3 (*i.e.* stroke volume),

$$v_3 = 2 \times v_2 = 2 \times 0.303 = 0.606 \, \text{m}^3/\text{kg} \qquad \qquad \ldots (\because \, v_2 = v_3/2)$$

Now first of all, let us find out the dryness fraction of steam at point 3 (*i.e.* x_3). We know that for isentropic expansion 2-3 (Refer Fig. 10.13),

Entropy before expansion (s_2)

$$= \text{Entropy after expansion } (s_3)$$

or

$$s_{f2} + x_2\, s_{fg2} = s_{f3} + x_3\, s_{fg3}$$

$$1.904 + 0.9 \times 4.877 = 1.621 + x_3 \times 5.418 \quad \text{or} \quad x_3 = 0.86$$

Since the dryness fraction of steam at point 3 (*i.e.* x_3) is less than one, therefore the condition of steam at point 3 is wet.

∴ Enthalpy or total heat of steam at point 3,

$$h_3 = h_{f3} + x_3\, h_{fg3} = 540.9 + 0.86 \times 2177.3 = 2413.4 \text{ kJ/kg}$$

We know that modified Rankine efficiency,

$$\eta_{MR} = \frac{(h_2 - h_3) + 100\,(p_3 - p_4)\, v_3}{h_2 - h_{f4}}$$

$$= \frac{(2543 - 2413.4) + 100\,(2.6 - 1)\,0.606}{2543 - 417.5}$$

$$= 0.1066 \text{ or } 10.66\% \text{ Ans.}$$

Example 10.11. *A steam engine is supplied dry saturated steam at 15 bar. The pressure at release is 3 bar and the back pressure is 1 bar. Using steam tables or Mollier chart, determine the efficiency of the modified Rankine cycle.*

Solution. Given : $p_2 = 15$ bar ; $p_3 = 3$ bar ; $p_4 = 1$ bar

First of all, mark a point 2, where pressure line through p_2 (*i.e.* 15 bar) meets the saturation line. Since the steam expands isentropically, therefore draw a vertical line through the point 2 to meet the pressure line through p_3 (*i.e.* 3 bar) at point 3, as shown in Fig. 10.14.

Now from the Mollier diagram, we find that the enthalpy of steam at point 2,

$$h_2 = 2790 \text{ kJ/kg}$$

Similarly, enthalpy of steam at point 3,

$$h_3 = 2510 \text{ kJ/kg}$$

and dryness fraction of steam at point 3,

$$x_3 = 0.9$$

From steam tables, corresponding to a pressure of 3 bar, we find that specific volume of steam,

$$v_{g3} = 0.6055 \text{ m}^3/\text{kg}$$

∴ Volume of steam at point 3,

$$v_3 = x_3\, v_{g3} = 0.9 \times 0.6055 = 0.545 \text{ m}^3/\text{kg}$$

Fig. 10.14

Similarly, from steam tables, corresponding to a pressure of 1 bar, we find that sensible heat of water,

$$h_{f4} = 417.5 \text{ kJ/kg}$$
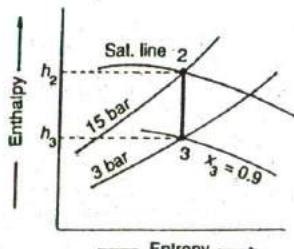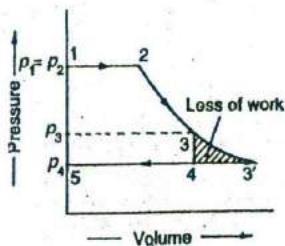
We know that modified Rankine efficiency,

$$\eta_{MR} = \frac{(h_2 - h_3) + 100\,(p_3 - p_4)\,v_3}{h_2 - h_{f4}}$$

$$= \frac{(2790 - 2510) + 100\,(3 - 1)\,0.545}{2790 - 417.5}$$

$$= 0.164 \text{ or } 16.4\% \text{ Ans.}$$
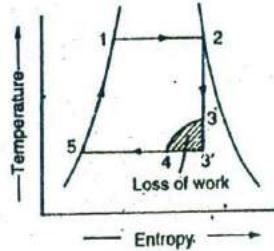
Example 10.12. *A steam engine takes dry steam at 20 bar and exhausts at 1.2 bar. The pressure at the release is 3 bar. Find ; 1. the theoretical loss of work per kg of steam due to incomplete expansion ; and 2. the loss in Rankine efficiency due to restricted expansion of steam.*

Solution. Given : $x_2 = 1$; $p_1 = p_2 = 20$ bar ; $p_4 = p_5 = 1.2$ bar ; $p_3 = 3$ bar

In the p-v and T-s diagram, as shown in Fig. 10.15 (*a*) and (*b*), 1-2-3-4-5-1 represents the modified Rankine cycle and 1-2-3′-5-1 represents the Rankine cycle.

(*a*) *p-v* diagram.   (*b*) *T-s* diagram.

Fig. 10.15

From steam tables, corresponding to a pressure of 20 bar, we find that

$$h_2 = h_{g2} = 2797.2 \text{ kJ/kg} ; \ v_2 = v_{g2} = 0.0995 \text{ m}^3\text{/kg} ;$$

$$s_2 = s_{g2} = 6.337 \text{ kJ/kg K}$$

Similarly, corresponding to a pressure of 3 bar, we find that

$$h_{f3} = 561.4 \text{ kJ/kg} ; \ h_{fg3} = 2163.2 \text{ kJ/kg} ; \ v_{g3} = 0.6055 \text{ m}^3\text{/kg} ;$$

$$s_{f3} = 1.672 \text{ kJ/kg K} ; \ s_{fg3} = 5.319 \text{ kJ/kg K}$$

and corresponding to a pressure of 1.2 bar, we find that

$$h_{f4} = h_{f3'} = 439.4 \text{ kJ/kg} ; \ h_{fg3'} = 2244.1 \text{ kJ/kg} ;$$

$$s_{f3'} = 1.361 \text{ kJ/kg K} ; \ s_{fg3'} = 5.937 \text{ kJ/kg K}$$

1. *Theoretical loss of work per kg of steam due to incomplete expansion*

First of all, let us find the dryness fraction of steam at points 3 and 3′ (*i.e.* x_3 and $x_{3'}$). We know that for isentropic expansion 2-3 (for modified Rankine cycle),

Entropy before expansion (s_2) = Entropy after expansion (s_3)

$$6.337 = s_{f3} + x_3\, s_{fg3} = 1.672 + x_3 \times 5.319$$

$\therefore$ $x_3 = 0.877$

Similarly, for isentropic expansion 2-3' (for Rankine cycle),

$$s_2 = s_{3'}$$

$$6.337 = s_{f3'} + x_{3'}\, s_{fg3'} = 1.361 + x_{3'} \times 5.937$$

$\therefore$ $x_{3'} = 0.838$

We know that enthalpy or total heat of steam at point 3,

$$h_3 = h_{f3} + x_3\, h_{fg3} = 561.4 + 0.877 \times 2163.2 = 2458.5 \text{ kJ/kg}$$

and enthalpy or total heat of steam at point 3',

$$h_{3'} = h_{f3'} + x_{3'}\, h_{fg3'}$$

$$= 439.4 + 0.838 \times 2244.1 = 2320 \text{ kJ/kg}$$

We also know that volume of steam at point 3,

$$v_3 = x_3\, v_{g3} = 0.877 \times 0.6055 = 0.531 \text{ m}^3\text{/kg}$$

We know that workdone during modified Rankine cycle,

$$w_1 = (h_2 - h_3) + 100\,(p_3 - p_4)\, v_3$$

$$= (2797.2 - 2458.5) + 100\,(3 - 1.2)\,0.531 = 434.3 \text{ kJ/kg}$$

and workdone during Rankine cycle

$$w_2 = h_2 - h_{3'} = 2797.2 - 2320 = 477.2 \text{ kJ/kg}$$

$\therefore$ Theoretical loss of work due to incomplete expansion,

$$w = w_2 - w_1 = 477.2 - 434.3 = 42.9 \text{ kJ/kg Ans.}$$

2. *Loss in Rankine efficiency due to restricted expansion of steam*

Since the heat supplied in modified Rankine cycle (*i.e.* $h_2 - h_{f4}$) is equal to the heat supplied in Rankine cycle (*i.e.* $h_2 - h_{f3'}$) because $h_{f4} = h_{f3'}$, therefore

Percentage loss in Rankine efficiency

$$= \text{Percentage loss of work}$$

$$= \frac{w}{w_2} \times 100 = \frac{42.9}{477.2} \times 100 = 9\% \text{ Ans.}$$

Example 10.13. *A steam engine receives steam at 8 bar superheated at 200° C, pressure at release being 2.8 bar and at exhaust 1 bar. Assuming isentropic expansion and constant volume conditions between release and commencement of exhaust, determine :*

1. Workdone in N-m per kg of steam, and 2. Efficiency of the unit.

Also compare these values with those for Rankine cycle working between the same pressure and temperature limits.

Solution. Given : $p_2 = 8$ bar ; $T_{sup} = 200°$ C ; $p_3 = 2.8$ bar ; $p_4 = 1$ bar

The modified Rankine cycle with superheated steam is shown in Fig. 10.16.

From steam tables of superheated steam, corresponding to a pressure of 8 bar and 200° C, we find that

$$h_2 = h_{sup} = 2838.6 \text{ kJ/kg} ; v_2 = v_{sup} = 0.3 \text{ m}^3\text{/kg} ; \text{ and } s_2 = s_{sup} = 6.886 \text{ kJ/kg K}$$

(a) *p-v* diagram.   (b) *T-s* diagram.
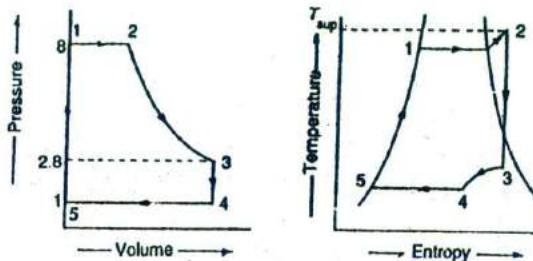
Fig. 10.16

From steam tables of dry saturated steam, corresponding to a pressure of 2.8 bar, we find that

$$h_{f3} = 551.5 \text{ kJ/kg} ; h_{fg3} = 2170 \text{ kJ/kg} ; v_{g3} = 0.646 \text{ m}^3\text{/kg} ; s_{f3} = 1.647 \text{ kJ/kg K} ; \text{ and}$$
$$s_{fg3} = 5.367 \text{ kJ/kg K}$$

and corresponding to a pressure of 1 bar, we find that

$$h_{f4} = 417.5 \text{ kJ/kg}$$

First of all, let us find the dryness fraction of steam at point 3 (*i.e.* x_3). We know that for isentropic expansion 2-3,

Entropy before expansion (s_2)

$$= \text{Entropy after expansion } (s_3)$$

or
$$6.886 = s_{f3} + x_3 s_{fg3} = 1.647 + x_3 \times 5.367$$

∴
$$x_3 = 0.976$$

Since the dryness fraction of steam at point 3 (*i.e.* x_3) is less than one, therefore the condition of steam at point 3 is wet.

∴ Enthalpy or total heat of steam at point 3,

$$h_3 = h_{f3} + x_3 h_{fg3} = 551.5 + 0.976 \times 2170 = 2669.4 \text{ kJ/kg}$$

and volume at point 3,
$$v_3 = x_3 v_{g3} = 0.976 \times 0.646 = 0.63 \text{ m}^3\text{/kg}$$

1 *Workdone per kg of steam*

We know that workdone per kg of steam

$$= (h_2 - h_3) + 100 (p_3 - p_4) v_3$$
$$= (2838.6 - 2669.4) + 100 (2.8 - 1) 0.63 \qquad \ldots (\because h_2 = h_{sup})$$
$$= 282.6 \text{ kJ/kg} = 282.6 \text{ kN-m/kg} = 282 600 \text{ N-m/kg} \textbf{ Ans.}$$

2 *Efficiency of the unit*

We know that heat supplied

$$= h_2 - h_{f4} = 2838.6 - 417.5 = 2421.1 \text{ kJ/kg}$$

∴ Efficiency of the unit,

$$\eta_{MR} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{282.6}{2421.1} = 0.117 \text{ or } 11.7\% \text{ Ans.}$$

Workdone for Rankine cycle working between the same pressure and temperature limits

The Rankine cycle working between the same pressure and temperature limits is shown in Fig. 10.17.

(a) p-v diagram.   (b) T-s diagram.
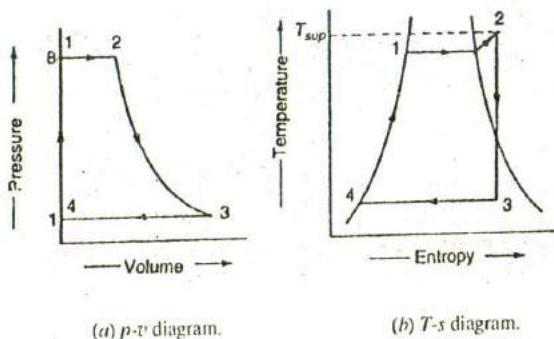
Fig. 10.17

From steam tables, corresponding to a pressure of 1 bar, we find that

$h_{f3} = 417.5 \text{ kJ/kg} ; h_{fg3} = 2258 \text{ kJ/kg} ; s_{f3} = 1.303 \text{ kJ/kg K} ;$ and $s_{fg3} = 6.057 \text{ kJ/kg K}$

First of all, let us find the dryness fraction of steam at point 3 (*i.e.* x_3). We know that for isentropic expansion 2-3,

Entropy before expansion (s_2)

= Entropy after expansion (s_3)

or $6.886 = s_{f3} + x_3 s_{fg3} = 1.303 + x_3 \times 6.057$

∴ $x_3 = 0.92$

We know that enthalpy or total heat of steam at point 3,

$$h_3 = h_{f3} + x_3 h_{fg3} = 417.5 + 0.92 \times 2258 = 2495 \text{ kJ/kg}$$

∴ Workdone during the Rankine cycle

$$= h_2 - h_3 = 2838.6 - 2495 = 343.6 \text{ kJ/kg} \qquad ...(\because h_2 = h_{sup})$$

$$= 343.6 \text{ kN-m/kg} = 343\,600 \text{ N-m/kg Ans.}$$

Efficiency of the Rankine cycle

We know that heat supplied

$$= h_2 - h_{f3} = 2838.6 - 417.5 = 2421.1 \text{ kJ/kg}$$

∴ Efficiency of the Rankine cycle,

$$\eta_R = \frac{\text{Workdone}}{\text{Heat supplied}} = \frac{343.6}{2421.1} = 0.142 \text{ or } 14.2\% \text{ Ans.}$$

Example 10.14. *The cylinder of a steam engine is 300 mm in diameter and piston stroke is 580 mm. The steam at admission is at 10 bar and 300° C. It expands isentropically to 0.7 bar and then reduced at constant volume to a condenser at 0.28 bar. Determine : 1. the modified Rankine efficiency ; 2. the new stroke if the same amount of steam from the original condition is expanded isentropically to condenser pressure ; 3 the new Rankine efficiency ; and 4. the workdone by the extraction and boiler feed pumps per kg of wate. returned to the boiler.*

Solution.   Given : $D = 300$ mm $= 0.3$ m ; $L = 580$ mm $= 0.5$ m ; $p_1 = p_2 = 10$ bar ; $T_{sup} = 300°$ C ; $p_3 = 0.7$ bar ; $p_4 = p_y = p_5 = 0.28$ bar

From steam tables for superheated steam, corresponding to a pressure of 10 bar and 300° C, we find that

$$h_2 = h_{sup} = 3052.1 \text{ kJ/kg} ; \; s_2 = s_{sup} = 7.125 \text{ kJ/kg K}$$
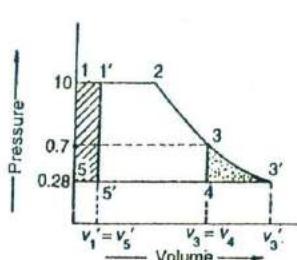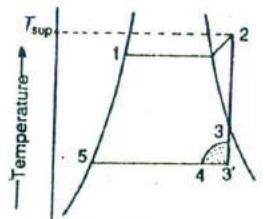
(a) p-v diagram.   (b) T-s diagram

Fig. 10.18

Similarly, from steam tables for dry saturated steam, corresponding to a pressure of 0.7 bar, we find that

$$h_{f3} = 376.8 \text{ kJ/kg} ; \quad h_{fg3} = 2283.3 \text{ kJ/kg} ; \quad s_{f3} = 1.192 \text{ kJ/kg K} ;$$

$$s_{fg3} = 6.288 \text{ kJ/kg} ; \text{ and } v_{g3} = 2.365 \text{ m}^3\text{/kg}$$

and corresponding to a pressure of 0.28 bar, we find that

$$h_{f4} = 282.7 \text{ kJ/kg}$$

1. *Modified Rankine efficiency*

First of all, let us find the dryness fraction of steam at point 3 (*i.e.* x_3). We know that for isentropic expansion 2-3, as shown in Fig. 10.18,

Entropy before expansion (s_2)

$$= \text{ Entropy after expansion } (s_3)$$

$$7.125 = s_{f3} + x_3 \, s_{fg3} = 1.192 + x_3 \times 6.288$$

∴ $x_3 = 0.943$

We know that enthalpy or total heat of steam at point 3,

$$h_3 = h_{f3} + x_3 \, h_{fg3} = 376.8 + 0.943 \times 2283.3 = 2530 \text{ kJ/kg}$$

and volume of steam at point 3,

$$v_3 = x_3 \, v_{g3} = 0.943 \times 2.365 = 2.23 \text{ m}^3\text{/kg}$$

We know that modified Rankine efficiency,

$$\eta_{MR} = \frac{(h_2 - h_3) + 100\,(p_3 - p_4)\,v_3}{h_3 - h_{f4}}$$

$$= \frac{(3052.1 - 2530) + 100\,(0.7 - 0.28)\,2.23}{3052.1 - 282.7}$$

$$= 0.2223 \text{ or } 22.23\% \text{ Ans.}$$

2. New stroke

Let $\qquad L' =$ New stroke length.

Since the same amount of steam as at point 3 is expanded isentropically from the original condition (*i.e.* at 10 bar and 300° C) to the condenser pressure $p_{3'} = 0.28$ bar, therefore let us find the dryness fraction of steam at point 3' (*i.e.* $x_{3'}$).

From steam tables, corresponding to a pressure of 0.28 bar, we find that

$$h_{f3'} = 282.7 \text{ kJ/kg} \,; \; h_{fg3'} = 2340 \text{ kJ/kg} \,; \; s_{f3'} = 0.925 \text{ kJ/kg K} \,;$$

$$s_{fg3'} = 6.868 \text{ kJ/kg K} \,; \text{ and } v_{g3'} = 5.578 \text{ m}^3\text{/kg}$$

We know that for isentropic expansion 2-3' as shown in Fig. 10.18,

$$s_2 = s_{3'}$$

$$7.125 = s_{f3'} + x_{3'}\,s_{fg3'} = 0.925 + x_{3'} \times 6.868$$

∴ $\qquad\qquad x_{3'} = 0.903$

We know that cylinder volume,

$$v = \frac{\pi}{4} \times D^2\,L = \frac{\pi}{4}\,(0.3)^2\,0.58 = 0.041 \text{ m}^3$$

∴ Mass of steam at point 3,

$$m_3 = \frac{v}{v_3} = \frac{0.041}{2.23} = 0.0184 \text{ kg/stroke}$$

and volume of steam at point 3',

$$v_{3'} = m_3\,x_{3'}\,v_{g3'} = 0.0184 \times 0.903 \times 5.578 = 0.0927 \text{ m}^3 \qquad \therefore \dots (i)$$

We also know that new stroke volume at point 3',

$$v_{3'} = \frac{\pi}{4} \times D^2 \times L' = \frac{\pi}{4}\,(0.3)^2\,L' = 0.0707\,L'\,\text{m}^3 \qquad \dots (ii)$$

From equations (i) and (ii),

$$L' = 0.0927 / 0.0707 = 1.31 \text{ m Ans.}$$

3. New Rankine efficiency

We know that enthalpy or total heat of steam at point 3',

$$h_{3'} = h_{f3'} + x_{3'}\,h_{fg3'} = 282.7 + 0.903 \times 2340 = 2395.7 \text{ kJ/kg}$$

∴ New Rankine efficiency,

$$\eta_R = \frac{h_2 - h_{3'}}{h_2 - h_{f3'}} = \frac{3052.1 - 2395.7}{3052.1 - 282.7} = 0.237 \text{ or } 23.7\% \text{ Ans.}$$

4. Workdone by the extraction and boiler feed pumps

The work is done by the extraction and boiler feed pumps in raising the pressure of water from condenser pressure (0.28 bar) to boiler pressure (10 bar). In Fig. 10.18, area 1-1'-5'-5 represents the required work done.

From steam tables, corresponding to a pressure of 0.28 bar, we find that

Volume of water, $v_5 = 0.001\ 021\ m^3/kg$

∴ Workdone by the extraction and boiler feed pumps

$$= 100\ (p_1 - p_5)\ v_5 = 100\ (10 - 0.28)\ 0.001\ 021\ kJ/kg$$

$$= 0.9924\ kJ/kg\ \textbf{Ans.}$$

EXERCISES

1. A Carnot engine works between pressure limits of 35 bar and 0.7 bar. Find the amount of work done per kg of steam and thermal efficiency of the engine. **[Ans. 518.2 kJ/kg ; 29.6%]**

2. A simple Rankine cycle uses steam as the working medium and operates between 0.5 bar and 20 bar. If the steam entering the turbine is dry at 20 bar, determine the quality of steam as it leaves the turbine.

Entropy of water at 0.5 bar = 1.091 kJ/kg K ; Entropy of saturated steam at 20 bar = 6.341 kJ/kg K ; and Entropy of saturated steam at 0.5 bar = 7.524 kJ/kg K. **[Ans. 0.816]**

3. Dry and saturated steam at a pressure of 11 bar is supplied to a turbine and is expanded isentropically to a pressure of 1 bar. Calculate 1. Heat supplied, 2. Heat rejected, and 3. Theoretical thermal efficiency.
[Ans. 2382.2 kJ/kg ; 1955.5 kJ/kg ; 21.35%]

4. A boiler supplies dry saturated steam to a steam power plant at a pressure of 12.5 bar. The back pressure is 0.2 bar. Find the efficiency of the plant, if it operates on Rankine cycle by using : 1. Steam tables only, and 2. Mollier chart. **[Ans. 25.3%]**

5. Dry and saturated steam at 15 bar is supplied to a steam engine. The exhaust takes place at 1.1 bar. Calculate 1. Rankine efficiency ; 2. Mean effective pressure ; 3. Steam consumption per I.P. hour, if the efficiency ratio is 0.65 ; and 4. Carnot cycle efficiency for the given pressure limits using steam as the working fluid.
[Ans. 18.8% ; 3.36 bar ; 12.5 kg/kWh ; 20.4%]

6. In a Rankine cycle, the maximum pressure of steam supplied is 6 bar. The dryness fraction is 0.9. The exhaust pressure is 0.7 bar. Find the theoretical workdone and Rankine efficiency. **[Ans. 325.3 kJ/kg ; 15%]**

7. Compare the efficiencies of Carnot cycle and Rankine cycle if the maximum and minimum temperatures are 400° C and 40° C. The steam in the case of Rankine cycle is supplied at 20 bar. **[Ans. 53.5% ; 33.35%]**

8. A steam engine is supplied with 90% dry steam at a pressure of 10 bar. The exhaust takes place at 1.1 bar. Determine : 1. Rankine efficiency ; 2. Percentage increase in efficiency if the steam has a temperature of 250° C before entering the cylinder. **[Ans. 16.1% ; 2.4%]**

9. Steam is supplied to a Rankine engine at 21 bar with 110° C of superheat. The back pressure is 0.20 bar. Find : 1. Rankine efficiency, and 2. Volume of steam entering the engine per hour if the steam supplied is 100 kg per hour. **[Ans. 29.1% ; 11.64 m³/h]**

10. Steam is supplied to a steam turbine at a pressure of 20 bar and 230° C. It is then expanded isentropically to a pressure of 1 bar. Determine : 1. Rankine efficiency, 2. Specific volume of steam at the end of expansion, and 3. Carnot efficiency between the same temperature limits. **[Ans. 21% ; 1.44 m³/kg]**

11. In an ideal Rankine cycle, the steam condition at turbine inlet is 20 bar and 350° C. The condenser pressure is 0.08 bar. Determine the cycle efficiency. If the steam flow rate is 2000 kg/h, what is the power out-put in kW ? **[Ans. 32.4% ; 533.5 kW]**

12. Steam at a pressure of 15 bar and at a temperature of 300° C is supplied to a steam turbine working on the Rankine cycle. If the exhaust takes place at 0.15 bar, evaluate the Rankine efficiency. Calculate the steam consumption in kg/h to develop 750 kW, if efficiency ratio is 0.6. **[Ans. 28.3% ; 5653 kg/h]**

13. A steam engine uses dry saturated steam at a pressure of 10 bar and the back pressure is 0.7 bar. The pressure at release is 3.8 bar. Assuming the pressure drop to take place at a constant volume, find the efficiency of the modified Rankine cycle. Neglect clearance. **[Ans. 13%]**

14. A steam engine receives steam at a pressure of 5.6 bar and 190° C. The pressure at release is 2.8 bar, and back pressure 1.0 bar. Determine : 1. Work done in kJ/kg of steam ; and 2. Efficiency of the cycle.

[Ans. 246 kJ/kg ; 10.15%]

15. Steam at 17 bar and 250° C is supplied to a steam engine where it is expanded isentropically to a release pressure of 3.5 bar. The steam is released from the engine at constant volume into the condenser where it is condensed at a pressure of 0.5 bar and the condensate is pumped back to the boiler. The steam flow rate is 1200 kg/h. Neglecting pump work, determine with the help of steam tables : 1. the power output of the engine in kW ; and 2. the efficiency of the cycle. [Ans. 153.3 kW ; 17.87 %]

QUESTIONS

1. Describe the Carnot vapour cycle and derive the expression for its efficiency.

2. Why the Carnot cycle can not be considered as the theoretical cycle for steam power plants even though its efficiency is maximum ?

3. Show Rankine cycle on p-v and T-s diagrams and explain the processes involved. Also draw the mechanical system to show different processes of the Rankine cycle.

4. Draw the Rankine cycle on T-s diagram using dry saturated steam and obtain an expression for the Rankine cycle efficiency.

5. Describe briefly the Rankine cycle using superheated steam and show in what respect this cycle differs from Carnot cycle between the same temperatures.

6. Prove that the efficiency of a Rankine cycle using superheated steam is greater than the efficiency of a corresponding Rankine cycle using steam without superheat. Both the cycles operate between the same boiler and condenser pressure limits.

7. State the reasons for modifying the Rankine cycle for operation of steam engines.

8. Explain with the help of p-v and T-s diagrams, the sequence of operations in the modified Rankine cycle.

OBJECTIVE TYPE QUESTIONS

1. The ideal cycle on which a steam engine works, is

(a) Carnot cycle   (b) Rankine cycle   (c) Otto cycle   (d) Joule cycle

2. Rankine cycle comprises

(a) two isothermal and two isentropics   (b) two isobarics and two isothermals

(c) two isobarics and two isentropics   (d) two isothermals and two isochorics

3. The Rankine cycle, as compared to Carnot cycle, has ....'work ratio,

(a) high   (b) low

4. In a Rankine cycle with superheated steam,

(a) the workdone increases

(b) the dryness fraction of steam after isentropic expansion increases

(c) the specific steam consumption decreases

(d) all of the above

5. In the operation of steam engines, the vapour cycle adopted is

(a) Carnot cycle   (b) Rankine cycle

(c) modified Rankine cycle   (d) Regenerative cycle

ANSWERS

1. (b)   2. (c)   3. (a)   4. (d)   5. (c)

Fuels

1. Introduction. 2. Classification of Fuels. 3. Solid Fuels. 4. Liquid Fuels. 5. Merits and Demerits of Liquid Fuels over Solid Fuels. 6. Gaseous Fuels. 7. Merits and Demerits of Gaseous Fuels. 8. Requirements of a Good Fuel. 9. Calorific Value of Fuels. 10. Gross or Higher Calorific Value. 11. Net or Lower Calorific Value. 12. Experimental Determination of Higher Calorific Value. 13. Bomb Calorimeter. 14. Boy's Gas Calorimeter.

11.1. Introduction

A fuel, in general terms, may be defined as a substance (containing mostly carbon and hydrogen) which, on burning with oxygen in the atmospheric air, produces a large amount of heat. The amount of heat generated is known as calorific value of the fuel.

As the principal constituents of a fuel are carbon and hydrogen, therefore, it is also known as hydrocarbon fuel. Sometimes, a few traces of sulphur are also present in it.

11.2. Classification of Fuels

The fuels may be classified into the following three general forms :

1. Solid fuels,

2. Liquid fuels, and

3. Gaseous fuels.

Each of these fuels may be further subdivided into the following two types :

(a) Natural fuels, and

(b) Prepared fuels.

11.3. Solid Fuels

The natural solid fuels are wood, peat, lignite or brown coal, bituminous coal and anthracite coal. The prepared solid fuels are wood charcoal, coke, briquetted coal and pulverised coal.

The following solid fuels are important from the subject point of view :

1. *Wood.* At one time it was extensively used as a fuel. It consists of mainly carbon and hydrogen. The wood is converted into coal when burnt in the absence of air. It is not considered as a commercial fuel, except in industries, where a large amount of waste wood is available. The calorific value of wood varies with its kind and moisture content. The average calorific value of the wood is 19 700 kJ/kg.

2. *Peat.* It is a spongy humid substance found in boggy land. It may be regarded as the first stage in the formation of coal. It has a large amount of water contents (upto 30%) and therefore has to be dried before use. It has a characteristic odour at the time of burning, and has a smoky flame. Its average calorific value is 23 000 kJ/kg.

3. *Lignite or brown coal.* It represents the next stage of peat in the coal formation, and is an intermediate variety between bituminous coal and peat. It contains nearly 40% moisture and 60% of carbon. When dried, it crumbles and hence does not store well. Due to its brittleness, it is converted into briquettes, which can be handled easily. Its average calorific value is 25 000 kJ/kg.

4. *Bituminous coal.* It represents the next stage of lignite in the coal formation and contains very little moisture (4 to 6%) and 75 to 90% of carbon. It is weather-resistant and burns with a yellow flame. The average calorific value of bituminous coal is 33 500 kJ/kg. The bituminous coal is of the following two types :

(a) Caking bituminous coal, and (b) Non-caking bituminous coal.

(a) *Caking bituminous coal* : It softens and swells on heating and its pieces adhere together forming a pasty mass which makes firing difficult. It burns with a fairly long flame. Its specific gravity is 1.26 to 1.36. The caking variety is very useful for manufacturing gas. It is also known as soft coal. Its average calorific value is 35 000 kJ/kg.

(b) *Non-caking bituminous coal* : It burns with a shorter flame than the caking coal, and gives off little or no smoke. Its specific gravity is 1.22 to 1.42. The non-caking variety is mostly used as fuel for steam boilers, hence it is known as steam coal. Its average calorific value is 33 000 kJ/kg.

Note : The bituminous coal is non-caking, if its carbon content is 78 to 81%. If the percentage of carbon is 81 to 82.5%, it is slightly caking. In medium caking bituminous coal, the carbon content is 82.5 to 84%. But if the carbon content is 84 to 89%, it makes the coal strongly caking.

5. *Anthracite coal.* It represents the final stage in the coal formation, and contains 90% or more carbon with a very little volatile matter. It is thus obvious, that the anthracite coal is comparatively smokeless, and has very little flame. It possesses a high calorific value of about 36 000 kJ/kg and is therefore, very valuable for steam raising and general power purposes.

6. *Wood charcoal.* It is made by heating wood with a limited supply of air to a temperature not less than 280° C. It is a good prepared solid fuel, and is used for various metallurgical processes.

7. *Coke.* It is produced when coal is strongly heated continuously for 42 to 48 hours in the absence of air in a closed vessel. This process is known as *carbonisation of coal.* Coke is dull black in colour, porous and smokeless. It has a high carbon content (85 to 90%) and has a higher calorific value than coal.

If the carbonisation of coal is carried out at 500 to 700° C, the resulting coke is called *lower temperature coke* or *soft coke.* It is used as a domestic fuel. The coke produced by carbonisation of coal at 900 to 1100° C, is known as *hard coke.* The hard coke is mostly used as a blast furnace fuel for extracting pig iron from iron ores, and to some extent as a fuel in cupola furnace for producing cast iron.

8. *Briquetted coal.* It is produced from the finely ground coal by moulding under pressure with or without a binding material. The binding materials usually used are pitch, coal tar, crude oil and clay etc. The briquetted coal has the advantage of having, practically, no loss of fuel through grate openings and thus it increases the heating value of the fuel.

9. *Pulverised coal.* The low grade coal with a high ash content, is powdered to produce pulverised coal. The coal is first dried and then crushed into a fine powder by pulverising machines. The pulverised coal is widely used in the cement industry and also in metallurgical processes.

Note : Out of all the above mentioned types of solid fuels, anthracite coal is commonly used in all types of heat engines.

11.4. Liquid Fuels

Almost all the commercial liquid fuels are derived from natural petroleum (or crude oil). The crude oil is obtained from bore-holes in the earth's crust in certain parts of the world. The liquid fuels consist of hydrocarbons. The natural petroleum may be separated into petrol or gasoline, paraffin oil or kerosene, fuel oils and lubricating oils by boiling the crude oil at different temperatures and

subsequent fractional distillation* or by a process such as cracking.** The solid products like vaseline and paraffin wax are recovered from the residue in the still.

The following liquid fuels are important from the subject point of view :

1. *Petrol or gasoline.* It is the lightest and most volatile liquid fuel, mainly used for light petrol engines. It is distilled at a temperature from 65° to 220° C.

2. *Kerosene or paraffin oil.* It is heavier and less volatile fuel than the petrol, and is used as heating and lighting fuel. It is distilled at a temperature from 220° to 345° C.

3. *Heavy fuel oils.* The liquid fuels distilled after petrol and kerosene are known as heavy fuel oils. These oils are used in diesel engines and in oil-fired boilers. These are distilled at a temperature from 345° to 470° C.

11.5. Merits and Demerits of Liquid Fuels over Solid Fuels

Following are the merits and demerits of liquid fuels over solid fuels :

Merits

    1. Higher calorific value.

    2. Lower storage capacity required.

    3. Better economy in handling.

    4. Better control of consumption by using valves.

    5. Better cleanliness and freedom from dust.

    6. Practically no ashes.

    7. Non-deterioration in storage.

    8. Non-corrosion of boiler plates.

    9. Higher efficiency.

Demerits

    1. Higher cost.

    2. Greater risk of fire.

    3. Costly containers are required for storage and transport.

11.6. Gaseous Fuels

The natural gas is, usually, found in or near the petroleum fields, under the earth's surface. It, essentially, consists of marsh gas or methane (CH_4) together with small amounts of other gases such as ethane (C_2H_6), carbon dioxide (CO_2) and carbon monoxide (CO).

The following prepared gases, which are used as fuels, are important from the subject point of view :

1. *Coal gas.* It is also known as a *town gas.* It is obtained by the carbonisation of coal and consists mainly of hydrogen, carbon monoxide and various hydrocarbons. The quality of coal gas depends upon the quality of the coal used, temperature of the carbonisaiton and the type of plant. It is very rich among combustible gases, and is largely used in towns for street and domestic lighting and heating. It is also used in furnaces and for running gas engines. Its calorific value is about 21 000 to 25 000 kJ/m^3.

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*   It is the distillation by stages, *i.e.* distillation carried out in such a way so that the liquid with the lowest boiling point is first evaporated and recondensed. The liquid with the next higher boiling point is then evaporated and recondensed, and so on until all the available liquid fuels are separately recovered in the sequence of their boiling points.

**   Cracking is a special process of heating crude oil to a high temperature under a very high pressure (exceeding 50 atmospheres) to increase the yield of lighter distillates, particularly petrol. The residue left after distillation by cracking is called cracked residue pressure tar and is used in road construction.

2. *Producer gas.* It is obtained by the partial combustion of coal, coke, anthracite coal or charcoal in a mixed air-steam blast. It is, mostly, used for furnaces particularly for glass melting and also for power generation. Its manufacturing cost is low, and has a calorific value of about 5000 to 6700 kJ/m^3.

3. *Water gas.* It is a mixture of hydrogen and carbon monoxide and is made by passing steam over incandescent coke. As it burns with a blue flame, it is also known as *blue water gas.*

The water gas is usually converted into carburetted (enriched) water gas by passing it through a carburetter into which a gas oil is sprayed. It is, usually, mixed with coal gas to form town gas. The water gas is used in furnaces and for welding.

4. *Mond gas.* It is produced by passing air and a large amount of steam over waste coal at about 650° C. It is used for power generation and heating. It is also suitable for use in gas engines. Its calorific value is about 5850 kJ/m^3.

5. *Blast furnace gas.* It is a by-product in the production of pig iron in the blast furnace. This gas serves as a fuel in steel works, for power generation in gas engines, for steam raising in boilers and for preheating the blast for furnace. It is extensively used as fuel for metallurgical furnaces. The gas, leaving the blast furnace, has a high dust content the proportion of which varies with the operation of the furnace. It has a low heating value of about 3750 kJ/m^3.

6. *Coke oven gas.* It is a by-product from coke oven, and is obtained by the carbonisation of bituminous coal. Its calorific value varies from 14 500 to 18 500 kJ/m^3. It is used for industrial heating and power generation.

11.7. Merits and Demerits of Gaseous Fuels

Following are the merits and demerits of the gaseous fuels :

Merits

1. The supply of fuel gas, and hence the temperature of furnace is easily and accurately controlled.
2. The high temperature is obtained at a moderate cost by pre-heating gas and air with heat of waste gases of combustion.
3. They are directly used in internal combustion engines.
4. They are free from solid and liquid impurities.
5. They do not produce ash or smoke.
6. They undergo complete combustion with minimum air supply.

Demerits

1. They are readily inflammable.
2. They require large storage capacity.

11.8. Requirements of a Good Fuel

Though there are many requirements of a good fuel, yet the following are important from the subject point of view :

1. A good fuel should have a low ignition point.
2. It should have a high calorific value.
3. It should freely burn with a high efficiency, once it is ignited.
4. It should not produce harmful gases.
5. It should produce least quantity of smoke and gases.
6. It should be economical, easy to store and convenient for transportation.

11.9. Calorific Value of Fuels

The calorific value (briefly written as C.V.) or heat value of a solid or liquid fuel may be defined as the amount of heat given out by the complete combustion of 1 kg of fuel. It is expressed in terms of kJ/kg of fuel. The calorific value of gaseous fuels is, however, expressed in terms of kJ/m³ at a specified temperature and pressure.

Following are the two types of the calorific value of fuels :

1. Gross or higher calorific value, and 2. Net or lower calorific value.

These calorific values are discussed, in detail, in the following articles.

11.10. Gross or Higher Calorific Value

All fuels, usually, contain some percentage of hydrogen. When a given quantity of a fuel is burnt, some heat is produced. Moreover, some hot flue gases are also produced. The water, which takes up some of the heat evolved, is converted into steam. If the heat, taken away by the hot flue gases and the steam is taken into consideration, *i.e.* if the heat is recovered from flue gases and steam is condensed back to water at room temperature ($15°$ C), then the amount of total heat produced per kg is known as *gross* or *higher calorific value of fuel*. In other words, the amount of heat obtained by the complete combustion of 1 kg of a fuel, when the products of its combustion are cooled down to the temperature of supplied air (usually taken as $15°$ C), is called the gross or higher calorific value of fuel. It is briefly written as H.C.V.

If the chemical analysis of a fuel is available, then the higher calorific value of the fuel is determined by the following formula, known as Dulong's formula :

$$\text{H.C.V.} = 33\,800\,C + 144\,000\,H_2 + 9270\,S \text{ kJ/kg} \qquad \ldots (i)$$

where C, H_2 and S represent the mass of carbon, hydrogen and sulphur in 1 kg of fuel, and the numerical values indicate their respective calorific values.

If the fuel contains oxygen (O_2), then it is assumed that the whole amount is combined with hydrogen having mass equal to 1/8th of that of oxygen. Therefore, while finding the calorific value of fuel, this amount of hydrogen should be subtracted.

$$\therefore \qquad \text{H.C.V.} = 33\,800\,C + 144\,000\left(H_2 - \frac{O_2}{8}\right) + 9270\,S \text{ kJ/kg} \qquad \ldots (ii)$$

11.11. Net or Lower Calorific Value

When the heat absorbed or carried away by the products of combustion is not recovered (which is the case in actual practice), and the steam formed during combustion is not condensed, then the amount of heat obtained per kg of the fuel is known as *net* or *lower calorific value*. It is briefly written as L.C.V.

If the higher calorific value is known, then the lower calorific value may be obtained by subtracting the amount of heat carried away by products of combustion (especially steam) from H.C.V.

$\therefore \qquad$ L.C.V. = H.C.V. − Heat of steam formed during combustion

Let $\qquad m_s$ = Mass of steam formed in kg per kg of fuel = $9\,H_2$

Since the amount of heat per kg of steam is the latent heat of vaporisation of water corresponding to a standard temperature of $15°$ C, is 2466 kJ/kg, therefore

$$\text{L.C.V} = \text{H.C.V.} - m_s \times 2466 \text{ kJ/kg}$$
$$= \text{H.C.V.} - 9\,H_2 \times 2466 \text{ kJ/kg} \qquad \ldots (\because m_s = 9\,H_2)$$

Example 11.1. *A fuel consists of 85% carbon ; 12.5% hydrogen ; 2.5% residual matter by mass. Working from first principles, find the higher and lower calorific values per kg of the fuel.*

Solution. Given : $C = 85\% = 0.85$ kg ; $H_2 = 12.5\% = 0.125$ kg ; * Residual matter $= 2.5\%$ $= 0.025$ kg

Higher calorific value per kg of fuel

We know that higher calorific value per kg of fuel,

$$\text{H.C.V.} = 33\,800\,C + 144\,000\,H_2$$

$$= 33\,800 \times 0.85 + 144\,000 \times 0.125 = 46\,730 \text{ kJ/kg } \textbf{Ans.}$$

Lower calorific value per kg of fuel

We know that lower calorific value per kg of fuel,

$$\text{L.C.V.} = \text{H.C.V.} - (9H_2 \times 2466)$$

$$= 46\,730 - (9 \times 0.125 \times 2466) = 43\,956 \text{ kJ/kg } \textbf{Ans.}$$

Example 11.2. *A sample of coal has the following composition by mass :*

Carbon 75% ; hydrogen 6% ; oxygen 8% ; nitrogen 2.5% ; sulphur 1.5% ; and ash 7%. Calculate its higher and lower calorific values per kg of coal.

Solution. Given : $C = 75\% = 0.75$ kg ; $H_2 = 6\% = 0.06$ kg ; $O_2 = 8\% = 0.08$ kg ; $*N_2 = 2.5\%$ $= 0.025$ kg ; $S = 1.5\% = 0.015$ kg ; *Ash $= 7\% = 0.07$ kg

Higher calorific value per kg of coal

We know that higher calorific value per kg of coal,

$$\text{H.C.V.} = 33\,800\,C + 144\,000\left(H_2 - \frac{O_2}{8}\right) + 9270\,S$$

$$= 33\,800 \times 0.75 + 144\,000\left(0.06 - \frac{0.08}{8}\right) + 9270 \times 0.015$$

$$= 25\,350 + 7200 + 139 = 32\,689 \text{ kJ/kg } \textbf{Ans.}$$

Lower calorific value per kg of coal

We know that lower calorific value per kg of coal,

$$\text{L.C.V.} = \text{H.C.V.} - (9\,H_2 \times 2466)$$

$$= 32\,689 - 9 \times 0.06 \times 2466 = 31\,358.3 \text{ kJ/kg } \textbf{Ans.}$$

11.13. Experimental Determination of Higher Calorific Value

The method of determining higher calorific value, as explained in Art. 11.10, gives approximate results only. The most satisfactory method of obtaining the calorific value of a fuel is by actual experiment. In all these experimental methods, a known mass of fuel is burnt in a suitable calorimeter, and the heat so evolved is found by measuring the rise in temperature of the surrounding water. The calorimeters used for finding the calorific value of fuels are known as fuel calorimeters.

The following two fuel calorimeters are important from the subject point of view :

1. Bomb calorimeter, and 2. Boy's gas calorimeter.

These calorimeters are discussed, in detail, in the following pages.

Superfluous data.

11.14. Bomb Calorimeter

It is used for finding the higher calorific value of solid and liquid fuels. In this calorimeter, as shown in Fig. 11.1, the fuel is burnt at a constant volume and under a high pressure in a closed vessel called bomb.

The bomb is made mainly of acid-resisting stainless steel, machined from the solid metal, which is capable of withstanding high pressure (upto 100 bar), heat and corrosion. The cover or head of the bomb carries the oxygen valve for admitting oxygen and a release valve for exhaust gases. A cradle or carrier ring, carried by the ignition rods, supports the silica crucible, which in turn holds the sample of fuel under test. There is an ignition wire of platinum or nichrome which dips into the crucible. It is connected to a battery, kept outside, and can be sufficiently heated by passing current through it so as to ignite the fuel.

The bomb is completely immersed in a measured quantity of water. The heat, liberated by the combustion of fuel, is absorbed by this water, the bomb and copper vessel. The rise in the temperature of water is measured by a precise thermometer, known as Beckmann thermometer which reads upto 0.01° C.


Fig. 11.1. Bomb calorimeter.

Procedure

A carefully weighed sample of the fuel (usually one gram or so) is placed in the crucible. Pure oxygen is then admitted through the oxygen valve, till pressure inside the bomb rises to 30 atmosphere. The bomb is then completely submerged in a known quantity of water contained in a large copper vessel. This vessel is placed within a large insulated copper vessel (not shown in the figure) to reduce loss of heat by radiation. When the bomb and its contents have reached steady temperature (this temperature being noted), fuse wire is heated up electrically. The fuel ignites, and continues to burn till whole of it is burnt. The heat released during combustion is absorbed by the surrounding water and the apparatus itself. The rise in temperature of water is noted.

Let m_f = Mass of fuel sample bunt in the bomb in kg,

H.C.V. = Higher calorific value of the fuel sample in kJ/kg,

m_w = Mass of water filled in the calorimeter in kg,

m_e = Water equivalent of apparatus in kg,

t_1 = Initial temperature of water and apparatus in °C, and

t_2 = Final temperature of water and apparatus in °C.

We know that heat liberated by fuel

$$= m_f \times \text{H.C.V.} \qquad \qquad \ldots (i)$$

and heat absorbed by water and apparatus

$$= (m_w + m_e)\, c_w\, (t_2 - t_1) \qquad \qquad \ldots (ii)$$

Since the heat liberated is equal to the heat absorbed (neglecting losses), therefore equating equations (i) and (ii),

$$m_f \times \text{H.C.V.} = (m_w + m_e)\, c_w\, (t_2 - t_1)$$
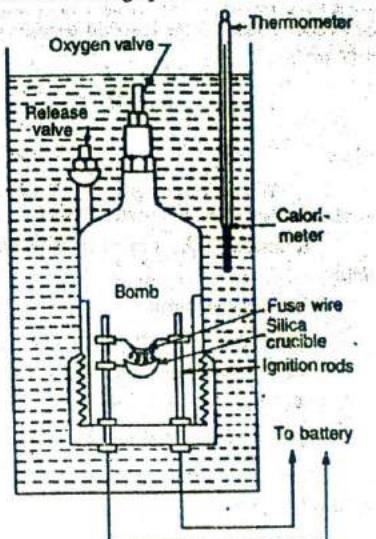
$$\therefore \qquad \text{H.C.V.} = \frac{(m_w + m_e)\, c_w\, (t_2 - t_1)}{m_f} \text{ kJ/kg}$$

Notes : 1. To compensate for the loss of heat by radiation (which cannot be totally eliminated), a cooling correction is added to the observed temperature rise. This corrected temperature rise is used in the above expression.

$$\therefore \qquad \text{H.C.V.} = \frac{(m_w + m_e)\, c_w\, [(t_2 - t_1) + t_c]}{m_f}$$

where t_c = Cooling correction.

2. This calorimeter gives H.C.V. of the fuel because any steam formed is condensed (since it cannot escape) and hence heat is recovered from it.

Example 11.3. *Calculate the higher calorific value of a coal specimen from the following data :*

Mass of coal burnt	= 1 g
Quantity of water in calorimeter	= 2.5 kg
Increase in temperature of water	= 2.6° C
Water equivalent of apparatus	= 390 g

If the fuel used contains 6% of hydrogen, calculate its lower calorific value.

Solution. Given : $m_f = 1$ g $= 0.001$ kg ; $m_w = 2.5$ kg ; $t_2 - t_1 = 2.6°$C ; $m_e = 390$ g $= 0.39$ kg ; $H_2 = 6\% = 0.06$

Higher calorific value

We know that higher calorific value,

$$\text{H.C.V.} = \frac{(m_w + m_e)\, c_w\, (t_2 - t_1)}{m_f} = \frac{(2.5 + 0.39)\, 4.2 \times 2.6}{0.001} \text{ kJ/kg}$$

$$= 31\ 560 \text{ kJ/kg Ans.} \qquad\qquad \ldots (\because\ c_w = 4.2 \text{ kJ/kg K})$$

Lower calorific value

We know that lower calorific value

$$= \text{H.C.V.} - (9H_2 \times 2466)$$

$$= 31\ 560 - 9 \times 0.06 \times 2466 = 30\ 228.3 \text{ kJ/kg Ans.}$$

11.14. Boy's Gas Calorimeter

It is primarily used for gaseous fuels, though it can be modified for liquid fuels also. It gives the higher calorific value only. But the lower calorific value may be calculated, since the amount of water produced can be collected and measured.

It consists of a suitable gas burner at *B*, in which a known volume of gas at a known pressure is burnt. The hot gases, produced by combustion, rise up in the copper chimney or combustion chamber, which is surrounded by a double metal tubing through which a continuous flow of water under a constant head is maintained. From the top of inner chamber, hot gases are deflected downwards through the space containing the inner water tubes *M*. From here the gases are deflected upwards through the space containing the outer water tubes *N*. Then the gases escape into the atmosphere from the top and their temperature is recorded just before their exit. During this process of playing up and down the water tubes, the gases give out, practically, whole of their heat so that any steam formed during combustion is condensed back into water.

The temperature of the circulating water is measured at inlet and outlet by thermometers T_1 and T_2 respectively, as shown in Fig. 11.2. After an initial warming up period during which conditions are established, simultaneous readings are taken of :

1. The volume of gas burnt in a certain time.

2. The quantity of water passing through the tube during the same time, and

3. The rise in temperature of water.

The volume of the gas consumed is reduced to some standard conditions of pressure and temperature (15° C and 760 mm of Hg), the normal temperature and pressure being 0° C and 760 mm of Hg.

Fig. 11.2. Boy's gas calorimeter.

Let v = Volume of gas burnt at standard temperature and pressure (S.T.P.) in m³,

      m_w = Mass of cooling water used in kg,

      H.C.V. = Higher calorific value of the fuel in kJ/m³,

      t_1 = Temperature of water at inlet, in °C, and

      t_2 = Temperature of water at outlet in °C.

We know that heat produced by the combustion of fuel

$$= v \times \text{H.C.V.} \qquad \ldots (i)$$

and heat absorbed by circulating water

$$= m \, c_w \, (t_2 - t_1) \qquad \ldots (ii)$$

Since the heat produced by the combustion of fuel is equal to the heat absorbed by the circulating water (neglecting losses), therefore equating equations (*i*) and (*ii*),

$$v \times \text{H.C.V.} = m_w \, c_w \, (t_2 - t_1)$$

$$\therefore \qquad \text{H.C.V.} = \frac{m_w \, c_w \, (t_2 - t_1)}{v}$$

Notes : 1. The lower calorific value may be calculated if the water produced during the combustion is drained off from the bottom of the calorimeter, collected and weighed. If this mass is *m*, then

$$\text{L.C.V.} = \text{H.C.V.} - m \times 2466 \text{ kJ/kg}$$

2. The modification of this gas calorimeter is known as Junker's calorimeter.

Example 11.4. *The following data refers to a calorific value test of a fuel by means of a gas calorimeter.*

Volume of gas used = 0.7 m³ (reckoned at S.T.P.) ; mass of water heated = 25 kg ; rise in temperature of water at inlet and outlet = 14° C ; mass of steam condensed = 0.028 kg. Find the higher and lower calorific values per m³ at S.T.P. Take the heat liberated in condensing water vapour and cooling the condensate as 2475 kJ/kg.
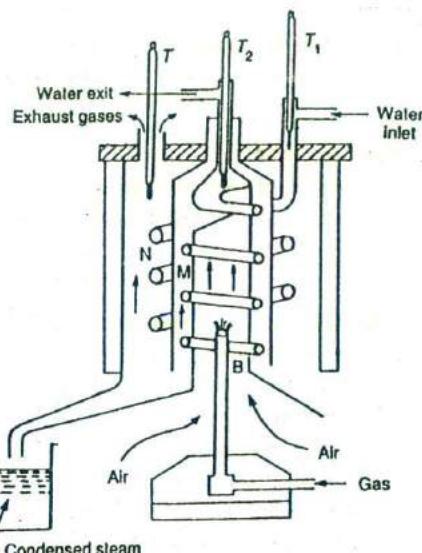
Solution. Given : $v = 0.7$ m^3 ; $m_w = 25$ kg ; $t_2 - t_1 = 14°$ C ; $m_s = 0.028$ kg ; Heat liberated $= 2475$ kJ/kg

Higher calorific value

We know that higher calorific value,

$$\text{H.C.V.} = \frac{m_w \, c_w \, (t_2 - t_1)}{v} = \frac{25 \times 4.2 \times 14}{0.7} = 2100 \text{ kJ/m}^3 \text{ Ans.}$$

Lower calorific value

We know that lower calorific value,

$$\text{L.C.V.} = \text{H.C.V.} - \text{Mass of steam condensed in kg/m}^3 \times \text{Heat liberated}$$

$$= 2100 - \frac{0.028}{0.7} \times 2475 = 2001 \text{ kJ/m}^3 \text{ Ans.}$$

Example 11.5. *The following results were obtained when a sample of gas was tested in a Junker's gas calorimeter :*

Gas burnt	$= 0.03$ m^3
Pressure of the gas	$= 54.4$ mm of water
Barometer reading	$= 750$ mm of Hg
Temperature of gas	$= 27°$ C
Temperature of water at inlet	$= 28°$ C
Temperature of water at outlet	$= 40°$ C
Mass of water passing through the calorimeter	$= 10$ kg
Steam condensed during test	$= 0.025$ kg

Determine the higher and lower calorific values of gas at 15° C and a standard barometer of 760 mm of Hg.

Solution.   Given : $v_1 = 0.03$ m^3 ; $p_1 = 54.4$ mm of water $= \frac{54.4}{13.6} + 750 = 754$ mm of Hg ; $T_1 = 27°$ C $= 27 + 273 = 300$ K ; $t_1 = 28°$ C ; $t_2 = 40°$ C ; $m_w = 10$ kg ; $m_s = 0.025$ kg ; $T_2 = 15°$ C $= 15 + 273 = 288$ K ; $p_2 = 760$ mm of Hg

First of all, let us find the volume of gas (v_2) at temperature T_2 and pressure p_2.

We know that for a perfect gas,

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad \text{or} \quad v_2 = \frac{p_1 v_1 T_2}{p_2 T_1} = \frac{754 \times 0.03 \times 288}{760 \times 300} = 0.029 \text{ m}^3$$

$\therefore$ Mass of steam condensed $= \dfrac{0.025}{0.029} = 0.86$ kg/m^3

Higher calorific value of the gas

We know that higher calorific value of the gas,

$$\text{H.C.V.} = \frac{m_w \, c_w \, (t_2 - t_1)}{v_2} = \frac{10 \times 4.2 \, (40 - 28)}{0.029} = 17\,380 \text{ kJ/m}^3 \text{ Ans.}$$

Lower calorific value of the gas

We know that lower calorific value of the gas,

$$\text{L.C.V} = \text{H.C.V.} - \text{Mass of steam condensed in kg/m}^3 \times \text{Heat liberated}$$

$$= 17\,380 - 0.86 \times 2466 = 15\,259.3 \text{ kJ/m}^3 \text{ Ans.}$$

EXERCISES

1. A sample of fuel on analysis is found to contain carbon 85% ; hydrogen 10% ; sulphur 2% and ash 3%. Find the higher calorific value of the fuel. **[Ans. 43 315.4 kJ/kg]**

2. The composition of sample of anthracite coal was found to be ; C 91%, H_2 3%, O_2 2%, N_2 0.8%, S 0.8%, and the remainder is ash.

Calculate the higher and lower calorific values of the fuel. **[Ans. 34 792 kJ/kg ; 34 126 kJ/kg]**

3. The following data were recorded during an experiment to find the calorific value of a sample of coal.

Mass of coal burnt	= 1 g
Mass of water in the calorimeter	= 1020 g
Water equivalent of the calorimeter	= 170 g
Initial temperature of water	= 23.3° C
Final temperature of water	= 26.2° C

Determine the calorific value of the sample of the coal. **[Ans. 14 494 kJ/kg]**

4. A bomb calorimeter is used to determine the calorific value of a sample of coal and the following results are obtained :

Mass of coal burnt	= 1 g
Mass of water in the calorimeter	= 2.5 kg
Water equivalent of the apparatus	= 0.75 kg
Initial temperature of water	= 17.5° C
Maximum observed temperature of water	= 20° C
Cooling correction	= + 0.015° C

If the fuel contains 4% of hydrogen, find the lower calorific value of the fuel. **[Ans. 33 442 kJ/kg]**

5. The following observations were made during a test on coal gas :

Volume of gas used	= 0.06 m³
Mass of cooling water circulated	= 9.8 kg
Mass of condensed steam collected	= 0.009 kg
Rise in temperature of cooling water	= 6.3° C
Pressure of gas tested above atmosphere	= 45 mm of water
Temperature of gas tested	= 14° C
Barometric pressure	= 750 mm of Hg

Calculate the higher and lower calorific values at N.T.P. **[Ans. 4581 kJ/m³ ; 4189 kJ/m³]**

QUESTIONS

1. What is meant by the term fuel ? What are its constituents ?

2. List out the merits and demerits of liquid fuels over solid fuels.

3. What are the advantages of gaseous fuels ?

4. Define the calorific value of a solid fuel and also that of a gaseous fuel.

5. Distinguish between higher and lower calorific value of a fuel.

6. Explain, briefly, the method used to determine the higher calorific value of the liquid fuel.

OBJECTIVE TYPE QUESTIONS

1. The principal constituents of a fuel are
   (a) carbon and hydrogen   (b) oxygen and hydrogen
   (c) sulphur and oxygen   (d) sulphur and hydrogen

2. The fuel mostly used in boilers is
   (a) brown coal   (b) peat
   (c) caking bituminous coal   (d) non-caking bituminous coal

3. Which of the following fuel has the highest calorific value ?
   (a) Peat   (b) Coke
   (c) Bituminous coal   (d) Anthracite coal

4. The fuel mostly used in blast furnace for extracting pig iron from iron ores is
   (a) hard coke   (b) soft coke
   (c) pulverised coal   (d) bituminous coal

5. Steam coal is a
   (a) pulverised coal   (b) brown coal
   (c) caking bituminous coal   (d) non-caking bituminous coal

6. A process of heating crude oil to a high temperature under a very high pressure to increase the yield of lighter distillates, is known as
   (a) cracking   (b) carbonisation
   (c) fractional distillation   (d) full distillation

7. Petrol is distilled at
   (a) $65°$ to $220°$ C   (b) $220°$ to $345°$ C   (c) $345°$ to $470°$ C   (d) $470°$ to $550°$ C

8. Which of the following gas has the highest calorific value ?
   (a) Producer gas   (b) Coal gas   (c) Mond gas   (d) Blast furnace gas

9. A bomb calorimeter is used for finding the . . . calorific value of solid and liquid fuels.
   (a) higher   (b) lower

10. Which of the following statement is incorrect ?
    (a) The liquid fuels consist of hydrocarbons.
    (b) The liquid fuels have higher calorific value than solid fuels.
    (c) The solid fuels have higher efficiency than liquid fuels.
    (d) A good fuel should have low ignition point.

ANSWERS

1. (a)	2. (d)	3. (c)	4. (a)	5. (d)
6. (a)	7. (a)	8. (b)	9. (a)	10. (c)

Combustion of Fuels

12.1. Introduction

The combustion of fuels may be defined as a chemical combination of oxygen, in the atmospheric air, and hydro-carbons. It is, usually, expressed both qualitatively and quantitatively by equations known as *chemical equations*. A chemical equation shows, in a concise form, the complete nature of the chemical action or reaction taking place.

12.2. Elements and Compounds

The elements are those substances which have so far not been resolved by any means into other substances of simpler form. About 109 such elements are known so far. The examples are hydrogen, oxygen, nitrogen, helium, iron, carbon, etc.

The compounds are formed by the combination of different elements in simple proportion. The number of compounds, that can be formed, is almost infinite. The examples are water (combination of hydrogen and oxygen), carbon dioxide (combination of carbon and oxygen), methane gas (combination of carbon and hydrogen), etc.

12.3. Atoms and Molecules

The elements are made up of minute and chemically indivisible particles known as *atoms*.

The smallest quantity of a substance, which can exist by itself, in a chemically recognizable form, is known as *molecule*. A molecule may consist of one atom, two atoms, three atoms or even more. Such a molecule is known as monoatomic, diatomic, triatomic or polyatomic molecule respectively.

12.4. Atomic Mass

Hydrogen is the lightest known substance. By taking the mass of hydrogen atom as unity, it is possible to obtain relative masses of other atoms and molecules. It may be noted that actual masses of these atoms are extremely small. The atomic mass of an element is the number of times, the atom of that element is heavier than the hydrogen atom. For example, the atomic mass of oxygen is 16. It means that the oxygen atom is 16 times heavier than the hydrogen atom.

12.5. Molecular Mass

The molecular mass of a substance is the number of times a molecule of that substance is heavier than the hydrogen atom. For example, one molecule of oxygen consists of two atoms of oxygen, each of which is 16 times heavier than hydrogen atom. It is thus obvious, that a molecule of oxygen is $2 \times 16 = 32$ times heavier than hydrogen atom. In other words, the molecular mass of oxygen is 32.

12.6. Symbols for Elements and Compounds

The elements and compounds are, generally, represented by symbols. The symbol of an atom is, usually, the initial letter of the element in capital. The symbol for carbon atom, for example, is C ; for hydrogen atom H ; for oxygen atom O ; and so on. A molecule is represented by a single expression. For example, molecule of oxygen is written as O_2, where the suffix 2 represents the number of atoms in one molecule of oxygen.

Similarly CO_2 stands for one molecule of carbon dioxide gas, which consists of one atom of carbon and two atoms of oxygen. The coefficients or prefix of an expression indicates the number of molecules of a substance, *e.g.* 7 CO_2 means 7 molecules of carbon dioxide, each of which consists of one atom of carbon and two atoms of oxygen.

The symbols, atomic mass and molecular mass of the following substances are important from the subject point of view :

Table 12.1. Symbols with atomic mass and molecular mass.
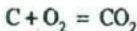
Substance	Symbol	Atomic Mass	Molecular Mass
Hydrogen	H_2	1	2
Oxygen	O_2	16	32
Nitrogen	N_2	14	28
Carbon	C	12	-
Sulphur	S	32	-
Carbon monoxide	CO	-	28
Methane or Marsh gas	CH_4	-	16
Acetylene	C_2H_2	-	26
Ethylene	C_2H_4	-	28
Ethane	C_2H_6	-	30
Carbon dioxide	CO_2		44
Sulphur dioxide	SO_2	-	64
Steam or water	H_2O	-	18

Note : If the atomic mass of a substance is known, then its molecular mass may be easily obtained, For example, molecular mass of $CO_2 = 12 + (2 \times 16) = 44$

12.7. Combustion Equations of Solid Fuels

We have already discussed in Art. 12.1 that the combustion of a fuel is the chemical combination of oxygen. Though there are many chemical equations for the chemical combination of oxygen (representing combustion), yet the following equations are important from the subject point of view :

1. When carbon burns in *sufficient quantity* of oxygen, carbon dioxide is produced along with a release of large amount of heat. This is represented by the following chemical equation :

$$C + O_2 = CO_2$$
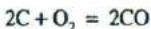
| | 1 mol. + 1 mol. = 1 mol. | ...(By volume) |
| or | 12 kg + 32 kg = 44 kg | ...(By mass) |

i.e.
$$1 \text{ kg} + \frac{8}{3} \text{ kg} = \frac{11}{3} \text{ kg}$$

It means that 1 kg of carbon requires 8/3 kg of oxygen for its complete combustion, and produces 11/3 kg of carbon dioxide gas.

2. If sufficient oxygen is not available, then combustion of carbon is incomplete. It then produces carbon monoxide instead of carbon dioxide. It is represented by the following chemical equation :

$$2C + O_2 = 2CO$$
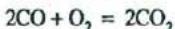
| | 2 mol. + 1 mol. = 2 mol. | ...(By volume) |
| or | 2×12 kg + 2×16 kg = 2×28 kg | ...(By mass) |

i.e.
$$1 \text{ kg} + \frac{4}{3} \text{ kg} = \frac{7}{3} \text{ kg}$$

It means that 1 kg of carbon requires 4/3 kg of oxygen, and produces 7/3 kg of carbon monoxide.

3. If carbon monoxide is burnt further, it is converted into carbon dioxide. Thus

$$2CO + O_2 = 2CO_2$$
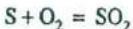
| | 2 mol. + 1 mol. = 2 mol. | ...(By volume) |
| or | 2×28 kg + 2×16 kg = 2×44 kg | ...(By mass) |

i.e.
$$1 \text{ kg} + \frac{4}{7} \text{ kg} = \frac{11}{7} \text{ kg}$$

It means that 1 kg of carbon monoxide requires 4/7 kg of oxygen, and produces 11/7 kg of carbon dioxide.

4. When sulphur burns with oxygen, it produces sulphur dioxide. This is represented by the following chemical equation :

$$S + O_2 = SO_2$$

| | 1 mol. + 1 mol. = 1 mol. | ...(By volume) |
| or | 32 kg + 2×16 kg = 64 kg | ...(By mass) |

i.e.
$$1 \text{ kg} + 1 \text{ kg} = 2 \text{ kg}$$

It means that 1 kg of sulphur requires 1 kg of oxygen for complete combustion and produces 2 kg of sulphur dioxide.

Note : We see from the above chemical equations that there is no loss of mass although changes in volume do occur. Hence the total mass on both sides of a chemical equation is the same.

12.8. Combustion Equations of Gaseous Fuels

The gaseous fuels are, generally measured by volume (in m^3) than by mass (in kg). It is thus obvious, that their combustion equations are usually stated quantitatively in volume form, though we may use masses also. Following are the important equations for the chemical combination of oxygen (representing combustion) from the subject point of view :

1.
$$2CO + O_2 = 2CO_2$$

| or | $\left[\begin{array}{c} \text{2 Volumes + 1 Volume = 2 Volumes} \\ 2 \text{ m}^3 + 1 \text{ m}^3 = 2 \text{ m}^3 \end{array} \right]$ | ...(By volume) |

i.e.
$$\left[\begin{array}{c} 2 \times 28 \text{ kg} + 1 \times 32 \text{ kg} = 2 \times 44 \text{ kg} \\ 1 \text{ kg} + \dfrac{4}{7} \text{ kg} = \dfrac{11}{7} \text{ kg} \end{array}\right] \qquad \dots \text{(By mass)}$$

It means that, 2 volumes of carbons monoxide require 1 volume of oxygen and produces 2 volumes of carbon dioxide.

Or in other words, 1 kg of carbon monoxide requires 4/7 kg of oxygen and produce 11/7 kg of carbon dioxide.

2. $2H_2 + O_2 = 2H_2O$

or
$$\left[\begin{array}{c} 2 \text{ Vol.} + 1 \text{ Vol.} = 2 \text{ Vol.} \\ 2 \text{ m}^3 + 1.\text{m}^3 = 2 \text{ m}^3 \end{array}\right] \qquad \dots \text{(By volume)}$$

i.e.
$$\left[\begin{array}{c} 2 \times 2 \text{ kg} + 1 \times 32 \text{ kg} = 2 \times 18 \text{ kg} \\ 1 \text{ kg} + 8 \text{ kg} = 9 \text{ kg} \end{array}\right] \qquad \dots \text{(By mass)}$$

It means that, 2 volumes of hydrogen require 1 volume of oxygen and produce 2 volumes of water or steam.

Or in other words, 1 kg of hydrogen requires 8 kg of oxygen and produces 9 kg of water or steam.

3. $CH_4 + 2O_2 = CO_2 + 2H_2O$
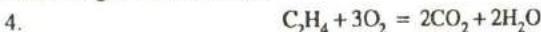
or
$$\left[\begin{array}{c} 1 \text{ Vol.} + 2 \text{ Vol.} = 1 \text{ Vol.} + 2 \text{ Vol.} \\ 1 \text{ m}^3 + 2 \text{ m}^3 = 1 \text{ m}^3 + 2 \text{ m}^3 \end{array}\right] \qquad \dots \text{(By volume)}$$

i.e.
$$\left[\begin{array}{c} 16 \text{ kg} + 2 \times 32 \text{ kg} = 44 \text{ kg} + 2 \times 18 \text{ kg} \\ 1 \text{ kg} + 4 \text{ kg} = \dfrac{11}{4} \text{ kg} + \dfrac{9}{4} \text{ kg} \end{array}\right] \qquad \dots \text{(By mass)}$$

It means that, 1 volume of methane requires 2 volumes of oxygen and produces 1 volume of carbon dioxide and 2 volumes of water or steam.

Or in other words, 1 kg of methane requires 4 kg of oxygen and produces 11/4 kg of carbon dioxide and 9/4 kg of water or steam.

4. $C_2H_4 + 3O_2 = 2CO_2 + 2H_2O$

or
$$\left[\begin{array}{c} 1 \text{ Vol.} + 3 \text{ Vol.} = 2 \text{ Vol.} + 2 \text{ Vol.} \\ 1 \text{ m}^3 + 3 \text{ m}^3 = 2 \text{ m}^3 + 2 \text{ m}^3 \end{array}\right] \qquad \dots \text{(By volume)}$$

i.e.
$$\left[\begin{array}{c} 28 \text{ kg} + 3 \times 32 \text{ kg} = 2 \times 44 \text{ kg} + 2 \times 18 \text{ kg} \\ 1 \text{ kg} + \dfrac{24}{7} \text{ kg} = \dfrac{22}{7} \text{ kg} + \dfrac{9}{7} \text{ kg} \end{array}\right] \qquad \dots \text{(By mass)}$$

It means that, 1 volume of ethyene requires 3 volumes of oxygen and produces 2 volumes of carbon dioxide and 2 volumes of water or steam.

Or in other words, 1 kg of ethyene requires 24/7 kg of oxygen and produces 22/7 kg of carbon dioxide and 9/7 kg of water or steam.

12.9. Theoretical or Minimum Mass of Air Required for Complete Combustion

We have seen in the previous articles that adequate supply of oxygen is very essential for the complete combustion of a fuel, and hence for obtaining maximum amount of heat from a fuel.

The theoretical or minimum mass (or volume) of oxygen required for complete combustion of 1 kg of fuel may be calculated from the chemical analysis of the fuel. The mass of oxygen, required by each of the constituents of the fuel, may be calculated from the chemical equations in Art. 12.8.

Now consider 1 kg of a fuel.

Let   Mass of carbon = C kg

          Mass of hydrogen = H_2 kg

          Mass of sulphur = S kg

We know that 1 kg of carbon requires 8/3 kg of oxygen for its complete combustion. Similarly, 1 kg of hydrogen requires 8 kg of oxygen and 1 kg of sulphur requires 1 kg of oxygen for its complete combustion.

∴ Total oxygen required for complete combustion of 1 kg of fuel

$$= \frac{8}{3}C + 8H_2 + S \text{ kg} \qquad \qquad \ldots (i)$$

If some oxygen (say O_2 kg) is already present in the fuel, then total oxygen for the complete ombustion of 1 kg of fuel

$$= \left[\frac{8}{3}C + 8H_2 + S\right] - O_2 \text{ kg} \qquad \qquad \ldots (ii)$$

It may be noted that the oxygen has to be obtained from atmospheric air, which mainly consists of nitrogen and oxygen along with a small amount of carbon dioxide and negligible amounts of rare gases like argon, neon, and krypton etc. But for all calculations, the composition of air is taken as :

          Nitrogen (N_2) = 77% ; Oxygen (O_2) = 23%   ... (By mass)

and   Nitrogen (N_2) = 79% ; Oxygen (O_2) = 21%   ... (By volume)

It is thus obvious, that for obtaining 1 kg of oxygen, amount of air required

$$= \frac{100}{23} = 4.35 \text{ kg} \qquad \qquad \ldots \text{(By mass)}$$

∴ Theoretical or minimum air required for complete combustion of 1 kg of fuel

$$= \frac{100}{23}\left[\left(\frac{8}{3}C + 8H_2 + S\right) - O_2\right] \text{ kg}$$

Example 12.1. A fuel has the following composition by mass :

Carbon 86%, Hydrogen 11.75%, Oxygen 2.25%.

Calculate the theoretical air supply per kg of fuel, and the mass of products of combustion per kg of fuel.

Solution. Given : C = 86% = 0.86 kg ; H_2 = 11.75 % = 0.1175 kg ; O_2 = 2.25% = 0.0225 kg

Theoretical air supply per kg of fuel

We know that theoretical air supply per kg of fuel

$$= \frac{100}{23}\left[\left(\frac{8}{3}C + 8H_2 + S\right) - O_2\right] \text{ kg}$$

$$= \frac{100}{23}\left[\left(\frac{8}{3} \times 0.86 + 8 \times 0.1175 + 0\right) - 0.0225\right] \text{ kg}$$

$$= 13.96 \text{ kg Ans.}$$

Mass of products of combustion

The chemical equations of carbon and hydrogen with oxygen are

(i) $C + O_2 = CO_2$

(ii) $2H_2 + O_2 = 2H_2O$

We know that 1 kg of carbon produces 11/3 kg of carbon dioxide and 1 kg of hydrogen produces 9 kg of water.

∴ Total mass of the products of combustion

$$= \frac{11}{3} \times C + 9H_2 \text{ kg}$$

$$= \frac{11}{3} \times 0.86 + 9 \times 0.1175 = 4.21 \text{ kg Ans.}$$

12.10. Theoretical or Minimum Volume of Air Required for Complete Combustion

We have discussed in the last article, the minimum mass of air required for complete combustion of a fuel. Similarly, the volume of oxygen required for complete combustion of 1 m³ of gaseous fuel may be calculated from the chemical analysis of the fuel. The volume of oxygen, required by each of the constituents of the fuel, may be calculated from the chemical equations as discussed in Art. 12.8. Now consider 1 m³ of a gaseous fuel.

Let volume of carbon monoxide $= CO$ m³
Volume of hydrogen $= H_2$ m³
Volume of methane $= CH_4$ m³
Volume of ethylene $= C_2H_4$ m³

We know that 2 volumes of carbon monoxide require 1 volume of oxygen. Or in other words, 1 volume of carbon monoxide requires 0.5 volume of oxygen for its complete combustion . Similarly, 2 volumes of hydrogen require 1 volume of oxygen. Or in other words, 1 volume of hydrogen requires 0.5 volume of oxygen for its complete combustion. Similarly, 1 volume of methane requires 2 volumes of oxygen and 1 volume of ethylene requires 3 volumes of oxygen for its complete combustion.

∴ Total oxygen required for complete combustion of 1 m³ of fuel

$$= 0.5 \, CO + 0.5 \, H_2 + 2CH_4 + 3 \, C_2H_4 \text{ m}^3$$

If some oxygen (say O_2 m³) is already present in the fuel, then total oxygen required for complete combustion of 1 m³ of fuel

$$= [0.5 \, CO + 0.5 \, H_2 + 2CH_4 + 3C_2H_2] - O_2 \text{ m}^3$$

Since the oxygen present in the air is 21% by volume, therefore theoretical or minimum volume of air required for complete combustion of 1 m³ of fuel

$$= \frac{100}{21} \left[\left(0.5 \, CO + 0.5 \, H_2 + 2CH_4 + 3C_2H_4 \right) - O_2 \right] \text{m}^3$$

Example 12.2. *A producer gas, used as a fuel, has the following volumetric composition :*
H_2 28% ; CO 12% ; CH_4 2% ; CO_2 16% and N_2 42%.

Find the volume of air required for complete combustion of 1 m³ of this gas. Air contains 21% by volume of oxygen.

Solution. Given : $H_2 = 28\% = 0.28$ m³ ; $CO = 12\% = 0.12$ m³ ; $CH_4 = 2\% = 0.02$ m³ ; $CO_2 = 16\% = 0.16$ m³ ; $N_2 = 42\% = 0.42$ m³

We know that theoretical air required

$$= \frac{100}{21} \left[\left(0.5 \, CO + 0.5 \, H_2 + 2CH_4 + 3C_2H_4 \right) - O_2 \right] \text{m}^3$$

$$= \frac{100}{21} [(0.5 \times 0.12 + 0.5 \times 0.28 + 2 \times 0.02 + 0) - 0] \text{ m}^3$$

$$= 1.143 \text{ m}^3 \text{ Ans.}$$

12.11. Conversion of Volumetric Analysis into Mass Analysis or Gravimetric Analysis

When the volumetric composition of any fuel gas is known, it can be converted to gravimetric composition by applying Avogadro's Law (Art. 2.9). Thus

1. Multiply the volume of each constituent by its own molecular mass. This gives the proportional mass of the constituents.

2. Add up these masses and divide each mass by this total mass, and express it as a percentage.

3. This gives percentage analysis by mass.

The conversion of volumetric analysis into mass analysis may be clearly understood by the following example.

Example 12.3. *The volumetric analysis of a gas is CO_2 14%, CO 1%, O_2 5% and N_2 80%. Calculate the fuel gas composition by mass.*

Solution. Given : $CO_2 = 14\% = 0.14$ m^3 ; $CO = 1\% = 0.01$ m^3 ; $O_2 = 5\% = 0.05$ m^3 ; $N_2 = 80\% = 0.8$ m^3

The volumetric analysis may be converted into mass analysis by completing the table as follows :

Constituent	Volume in 1 m^3 of flue gas (a)	Molecular mass (b)	Proportional mass $(c) = (a \times b)$	Mass in kg per kg of flue gas $(d) = \dfrac{(c)}{\Sigma(c)}$	% by mass $= (d) \times 100$
CO_2	0.14	44	6.16	$\dfrac{6.16}{30.44} = 0.202$	20.2%
CO	0.01	28	0.28	$\dfrac{0.28}{30.44} = 0.009$	0.9%
O_2	0.05	32	1.60	$\dfrac{1.60}{30.44} = 0.053$	5.3%
N_2	0.80	28	22.40	$\dfrac{22.40}{30.44} = 0.736$	73.6%
Total	1.00		$\Sigma(c) = 30.44$	1.0000	100.0

The fuel gas composition, by mass is given in the last column, *i.e.*

$CO_2 = 20.2\%$; $CO = 0.9\%$; $O_2 = 5.3\%$ and $N_2 = 73.6\%$ **Ans.**

12.12. Conversion of Mass Analysis into Volumetric Analysis

The conversion of mass analysis of a fuel gas into the volumetric analysis may be done by the following steps :

1. Divide the percentage mass of each constituent by its own molecular mass. This gives the proportional volumes of the constituents.

2. Add these volumes and divide each volume by this total volume, and express it as a percentage.

3. This gives percentage analysis by volume.

Example 12.4. *A fuel gas has the following percentage composition by mass :*

CO_2 13.3 % ; CO 0.95 % ; O_2 8.35% and N_2 77.4%. Convert this into volumetric analysis.

Solution. Given : $CO_2 = 13.3\%$; $CO = 0.95\%$; $O_2 = 8.35\%$; $N_2 = 77.4\%$

The mass analysis may be converted into volumetric analysis by completing the table as follows :

Constituent	% Mass analysis (a)	Molecular mass (b)	Proportional volume $(c) = \dfrac{(a)}{(b)}$	Volume in 1 m^3 of flue gas $(d) = \dfrac{(c)}{\Sigma(c)}$	% Volumetric analysis $= (d) \times 100$
CO_2	13.3	44	$\dfrac{13.3}{44} = 0.302$	$\dfrac{0.302}{3.357} = 0.090$	9.0
CO	0.95	28	$\dfrac{0.95}{28} = 0.034$	$\dfrac{0.034}{3.357} = 0.010$	1.0
O_2	8.35	32	$\dfrac{8.35}{32} = 0.261$	$\dfrac{0.261}{3.357} = 0.078$	7.8
N_2	77.4	28	$\dfrac{77.4}{28} = 2.76$	$\dfrac{2.76}{3.357} = 0.822$	82.2
Total	100.0		$\Sigma(c) = 3.357$	1.000	100.0

The fuel gas composition by volume, is given in the last column, *i.e.*

$CO_2 = 9\%$; $CO = 1\%$; $O_2 = 7.8\%$ and $N_2 = 82.2\%$ **Ans.**

12.13. Mass of Carbon in Flue Gases

The mass of carbon, contained in 1 kg of flue or exhaust gases, may be calculated from the mass of carbon dioxide and carbon monoxide present in them.

We know that 1 kg of carbon produces 11/3 kg of carbon dioxide. Hence 1 kg of carbon dioxide will contain 3/11 kg of carbon. Also 1 kg of carbon produces 7/3 kg of carbon monoxide, hence 1 kg of carbon monoxide will contain 3/7 kg of carbon.

$\therefore$ Mass of carbon per kg of flue gas

$$= \frac{3}{11} CO_2 + \frac{3}{7} CO$$

where CO_2 and CO represent the quantities of carbon dioxide and carbon monoxide present in 1 kg of flue gases.

12.14. Mass of Flue Gases per kg of Fuel Burnt

The mass of dry flue gases may be obtained by comparing the mass of carbon present in the flue gases with the mass of carbon in the fuel, since there is no loss of carbon during combustion.

Mathematically, mass of flue gas per kg of fuel

$$= \frac{\text{Mass of carbon in 1 kg of fuel}}{\text{Mass of carbon in 1 kg of flue gas}}$$

Example 12.5. *The ultimate analysis of dry coal burnt in a boiler is C 84%, H_2 9% and incombustibles 7% by mass Determine the mass of dry flue gases per kg of coal burnt, if the volumetric composition of the flue gas is : CO_2 8.75 %, CO 2.25 %, O_2 8% and N_2 81 %.*

Solution. Given : $C = 84\% = 0.84$ kg ; $H_2 = 9\% = 0.09$ kg ; Incombustibles $= 7\% = 0.07$ kg ; $CO_2 = 8.75\% = 0.0875$ m^3 ; $CO_2 = 2.25\% = 0.0225$ m^3 ; $O_2 = 8\% = 0.08$ m^3 ; $N_2 = 81\% = 0.81$ m^3

First of all, let us convert the volumetric analysis of flue gas to mass analysis for the mass of carbon present in 1 kg of flue gas as given in the following table :

Constituent	Volume in 1 m³ of flue gas (a)	Molecular mass (b)	Proportional mass (c) = a × b	Mass of constituent in kg per kg of flue gas $d = \dfrac{(c)}{\Sigma(c)}$
CO_2	0.0875	44	3.85	$\dfrac{3.85}{29.72} = 0.130$
CO	0.0225	28	0.63	$\dfrac{0.63}{29.72} = 0.021$
O_2	0.08	32	2.56	$\dfrac{2.56}{29.72} = 0.086$
N_2	0.81	28	22.68	$\dfrac{22.68}{29.72} = 0.763$
Total	1.0000		$\Sigma(c) = 29.72$	1.000

We know that mass of carbon in 1 kg of flue gas

$$= \frac{3}{11} CO_2 + \frac{3}{7} CO = \frac{3}{11} \times 0.130 + \frac{3}{7} \times 0.021 = 0.044 \text{ kg}$$

∴ Mass of dry flue gas per kg of coal burnt

$$= \frac{\text{Mass of carbon in 1 kg of coal}}{\text{Mass of carbon in 1 kg flue gas}} = \frac{0.84}{0.044} = 19.1 \text{ kg Ans.}$$

Example 12.6. *A blast furnace gas has the following volumetric composition :*

$CO_2 = 11\%$; $CO = 27\%$; $H_2 = 2\%$ and $N_2 = 60\%$.

Find the theoretical volume of air required for the complete combustion of 1 m³ of the gas. Find the percentage composition of dry flue gases by volume. Assume that air contains 21 % of O_2 and 79% of N_2 by volume.

Solution. Given : $CO_2 = 11\% = 0.11$ m³ ; $CO = 27\% = 0.27$ m³ ; $H_2 = 2\% = 0.02$ m³ ; $N_2 = 60\% = 0.6$ m³

Theoretical volume of air required

We know that theoretical volume of air required

$$= \frac{100}{21} \left[(0.5 \, CO + 0.5 \, H_2 + 2 \, CH_4 + 3 \, C_2H_4) - O_2 \right] m^3$$

$$= \frac{100}{21} [0.5 \times 0.27 + 0.5 \times 0.02] = 0.69 \text{ m}^3 \text{ Ans.}$$

. . . (∵ CH_4, C_2H_4 and O_2 are equal to zero)

Percentage composition of dry flue gases

We know that 1 m³ of CO produces 1 m³ of CO_2, therefore

Volume of CO_2 obtained from the given 0.27 m³ of CO

$$= 0.27 \text{ m}^3$$

and volume of CO_2 already present in the fuel,

$$= 0.11 \text{ m}^3 \qquad \qquad \text{. . . (Given)}$$

.. Total volume of CO_2 in the flue gas

$$= 0.11 + 0.27 = 0.38 \text{ m}^3$$

Now volume of N_2 from the theoretical air supplied

$$= \frac{79}{100} \times 0.69 = 0.545 \text{ m}^3$$

and volume of N_2 already present in the fuel

$$= 0.6 \text{ m}^3 \qquad \qquad \qquad \text{. . (Given)}$$

∴ Total volume of N_2 in the flue gas

$$= 0.6 + 0.545 = 1.145 \text{ m}^3$$

and total volume of the dry flue gas

$$= \text{Total volume of } CO_2 + \text{Total volume of } N_2$$

$$= 0.38 + 1.145 = 1.525 \text{ m}^3$$

∴ Percentage of CO_2 in the dry flue gas

$$= \frac{0.38}{1.525} \times 100 = 24.92\% \text{ Ans.}$$

and percentage of N_2 in the dry flue gas

$$= \frac{1.145}{1.525} \times 100 = 75.08 \% \text{ Ans.}$$

12.15. Excess Air Supplied

In the last two articles, we have discussed the theoretical or minimum air required for complete combustion. But in actual practice, to ensure complete and rapid combustion of a fuel, some quantity of air, in excess of the theoretical or minimum air, is supplied. It is due to the fact, that the excess air does not come in contact with the fuel particles. If just minimum amount of air is supplied, a part of the fuel may not burn properly.

The amount of excess air supplied varies with the type of fuel and firing conditions. It may approach to a value of 100 per cent, but the modern tendency is to use 25 to 50 per cent excess air.

12.16. Mass of Excess Air Supplied

· The mass of excess air supplied may be determined by the mass of unused oxygen, found in the flue gases. We know that in order to supply one kg of oxygen, we need 100/23 kg of air.

·   Similarly, mass of excess air supplied

$$= \frac{100}{23} \times \text{Mass of excess oxygen}$$

∴ Total mass of air supplied

$$= \text{Mass of necessary air} + \text{Mass of excess air}$$

Notes : 1. The mass of excess air supplied per kg of fuel may approximately be obtained from the following formula :

*Excess air supplied per kg of fuel

$$= \frac{79 \times O_2 \times C}{21 \times 33 \, (CO_2 + CO)}$$

where O_2, CO_2 and CO represents the percentages of oxygen, carbon dioxide and carbon monoxide respectively in flue gases (by volume) in the percentage of carbon in fuel by mass.

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*   The excess air supplied per kg of fuel may also be obtained as discussed in Example 12.9 (Note 2).

2. The amount of air actually required per kg of fuel may approximately be obtained from the following formula :

Actual air required per kg of fuel

$$= \frac{N_2 \times C}{33 \, (CO_2 + CO)}$$

where N_2 is percentage of nitrogen in the flue gases by volume.

3. The richness of air-fuel mixture supplied to an internal combustion engine is measured by the ratio of the air to the fuel by mass.

∴ Air-fuel ratio = Mass of air supplied per kg of fuel.

Example 12.7. *A steam boiler uses pulverised coal in the furnace. The ultimate analysis of coal (by mass) as received is :*

C 78% ; H_2 3% ; O_2 3% ; S 1% ; ash 10% ; and moisture 5%.

Excess air supplied is 30%. Calculate the mass of air to be supplied and mass of gaseous product formed per kg of coal burnt.

Solution.   Given : C = 78% = 0.78 kg ; O_2 = 3% = 0.03 kg ; S = 1% = 0.01 kg ; Ash = 10% = 0.1 kg ; Moisture = 5% = 0.05 kg ; Excess air supplied = 30%

Mass of air to be supplied per kg of coal burnt

We know that theoretical air required to burn 1 kg of coal

$$= \frac{100}{23} \left[\left(\frac{8}{3} C + 8H_2 + S \right) - O_2 \right] kg$$

$$= \frac{100}{23} \left[\left(\frac{8}{3} \times 0.78 + 8 \times 0.03 + 0.01 \right) - 0.03 \right] = 10.1 \text{ kg}$$

∴ Excess air supplied per kg of coal

$$= \frac{30 \times 10.1}{100} = 3.03 \text{ kg}$$

and mass of air to be supplied per kg of coal burnt

= Theoretical air + Excess air

= 10.1 + 3.03 = 13.13 kg **Ans.**

Mass of gaseous products formed per kg of coal

The gaseous products formed are carbon dioxide (CO_2), water (H_2O), sulphur dioxide (SO_2), excess oxygen (O_2) and nitrogen (N_2). We know that 1 kg of carbon produces 11/3 kg of carbon dioxide, 1 kg of hydrogen produces 9 kg of water and 1 kg of sulphur produces 2 kg of sulphur dioxide.

∴ Mass of CO_2 produced per kg of coal

$$= \frac{11}{3} \times 0.78 = 2.86 \text{ kg} \qquad \qquad \qquad \text{... (i)}$$

Mass of H_2O produced per kg of coal

$$= 9 \times 0.03 = 0.27 \text{ kg} \qquad \qquad \qquad \text{... (ii)}$$

Mass of SO_2 produced per kg of coal

$$= 2 \times 0.01 = 0.02 \text{ kg} \qquad \qquad \qquad \text{... (iii)}$$

Mass of excess O_2 produced per kg of coal

$$= \frac{23}{100} \times \text{Excess air supplied} = \frac{23}{100} \times 3.03 = 0.70 \text{ kg} \qquad \text{... (iv)}$$

Mass of N_2 per kg of coal

$$= \frac{77}{100} \times \text{Actual mass of air supplied} = \frac{77}{100} \times 13.13 = 10.1 \text{ kg} \quad \dots (v)$$

Hence the mass of the gaseous products of combustion per kg of coal are : $CO_2 = 2.86$ kg, $H_2O = 0.27$ kg, $SO_2 = 0.02$ kg, excess $O_2 = 0.70$ kg and $N_2 = 10.1$ kg. **Ans.**

Example 12.8. *A gas fuel has the following percentage composition by volume :*

CO 10% ; H_2 50% ; CH_4 26% ; O_2 3% ; CO_2 2% ; and N_2 9%.

Estimate the minimum volume of air required for complete combustion of 1 m^3 of the gas. If 50% excess air is supplied, give the volumes of each of the dry constituents of the flue gas. Air contains 21% by volume of oxygen.

Solution. Given : $CO = 10\% = 0.1 \text{ m}^3$; $H_2 = 50\% = 0.5 \text{ m}^3$, $CH_4 = 26\% = 0.26 \text{ m}^3$; $O_2 = 3\%$ $= 0.03 \text{ m}^3$; $CO_2 = 2\% = 0.02 \text{ m}^3$; $N_2 = 9\% = 0.09 \text{ m}^3$; Excess air supplied = 50%

Minimum volume of air required for complete combustion

We know that minimum volume of air required

$$= \frac{100}{21} [(0.5 \text{ CO} + 0.5 \text{ H}_2 + 2CH_4) - O_2] \text{ m}^3$$

$$= \frac{100}{21} [(0.5 \times 0.1 + 0.5 \times 0.5 + 2 \times 0.26) - 0.03] = 3.76 \text{ m}^3 \text{ Ans.}$$

Volume of dry constituents of the flue gas

Since 50% excess air is supplied, therefore actual amount of air supplied

$$= 3.76 \times 1.5 = 5.64 \text{ m}^3$$

∴ Excess air supplied

$$= 5.64 - 3.76 = 1.88 \text{ m}^3$$

and excess oxygen supplied

$$= 1.88 \times \frac{21}{100} = 0.395 \text{ m}^3$$

The dry constituents of the flue gas contain CO_2 from the combustion of fuel as well as from the combustion of CO and CH_4, excess O_2 and N_2 from the combustion of the fuel as well as from actual air supplied.

Now volume of CO_2 contained in the fuel (as given)

$$= 0.02 \text{ m}^3 \quad \dots (i)$$

We know that 1 m^3 of CO produces 1 m^3 of CO_2. Therefore volume of CO_2 obtained from 0.1 m^3 of CO

$$= 0.10 \text{ m}^3 \quad \dots (ii)$$

We also know that 1 m^3 of CH_4 produces 1 m^3 of CO_2. Therefore volume of CO_2 obtained from 0.26 m^3 of CH_4

$$= 0.26 \text{ m}^3 \quad \dots (iii)$$

∴ Total volume of CO_2 obtained from the flue gas

$$= 0.02 + 0.10 + 0.26 = 0.38 \text{ m}^3$$

Volume of excess O_2 in flue gas

$$= 0.395 \text{ m}^3 \quad \dots \text{(As above)}$$

Volume of N_2 in the fuel $= 0.09 \, m^3$

Volume of N_2 from the actual air supplied

$$= \frac{79}{100} \times \text{Volume of actual air} = \frac{79}{100} \times 5.64 = 4.46 \, m^3$$

$\therefore$ Total volume of N_2 in the flue gas

$$= 0.09 + 4.46 = 4.55 \, m^3$$

Hence the volume of dry constituents of the flue gas are :

*$CO_2 = 0.38 \, m^3$, Excess $O_2 = 0.395 \, m^3$, and $N_2 = 4.55 \, m^3$ Ans.

Example 12.9. *During a boiler trial, the dry flue gas analysis by volume was reported as*

$$CO_2 \, 13\% \; ; \; CO \, 0.3\% \; ; \; O_2 \, 6\% \; ; \; N_2 \, 80.7\%$$

The coal analysis by mass was reported as

$$C \, 62.4\% \; ; \; H_2 \, 4.2\% \; ; \; O_2 \, 4.5\% \; ; \; O_2 \, 4.5\% \; ; \; moisture \, 15\% \; ; \; ash \, 13.9\%.$$

Calculate : 1. Minimum air required to burn 1 kg of coal, 2. Mass of air actually supplied per kg of coal, 3. The amount of excess air supplied per kg of coal burnt.

Solution. Given : For dry flue gas (by volume) : $CO_2 = 13\% = 0.13 \, m^3$; $CO = 0.3\% = 0.003$ m^3 ; $O_2 = 6\% = 0.06 \, m^3$; $N_2 = 80.7\% = 0.807 \, m^3$; For coal (by mass) : $C = 62.4\% = 0.624 \, kg$; $H_2 = 4.2\% = 0.042 \, kg$; $O_2 = 4.5\% = 0.045 \, kg$; Moisture $15\% = 0.15 \, kg$; Ash $13.9\% = 0.139 \, kg$

1. *Minimum air required to burn 1 kg of coal*

We know that minimum air required to burn 1 kg of coal.

$$= \frac{100}{23}\left[\left(\frac{8}{3}C + 8H_2 + S\right) - O_2\right] kg$$

$$= \frac{100}{23}\left[\left(\frac{8}{3} \times 0.624 + 8 \times 0.042\right) - 0.045\right] = 8.5 \, kg \text{ Ans.}$$

2. *Mass of air actually supplied per kg of coal*

First of all, let us convert the volumetric analysis of flue gas to mass analysis as given in the following table :

Constituent	Volume in 1 m^3 of flue gas (a)	Molecular mass (b)	Proportional mass in kg (c) = a × b	Mass of constituent in kg per kg of flue gas $(d) = \frac{(c)}{\Sigma(c)}$
CO_2	0.13	44	5.720	$\frac{5.720}{30.319} = 0.189$
CO	0.003	28	0.084	$\frac{0.084}{30.319} = 0.003$
O_2	0.06	32	1.920	$\frac{1.920}{30.319} = 0.063$
N_2	0.807	28	22.595	$\frac{22.595}{30.319} = 0.745$
Total	1.000		$\Sigma(c) = 30.319$	1.000

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* It may be noted that carbon dioxide and nitrogen do not require any oxygen.

We know that mass of carbon in 1 kg of flue gas

$$= \frac{3}{11} CO_2 + \frac{3}{7} CO = \frac{3}{11} \times 0.189 + \frac{3}{7} \times 0.003 = 0.053 \text{ kg}$$

∴ Total mass of flue gas per kg of fuel burnt

$$= \frac{\text{Mass of carbon in 1 kg of coal}}{\text{Mass of carbon in 1 kg of flue gas}} = \frac{0.624}{0.053} = 11.77 \text{ kg}$$

and total mass of N_2 present in 11.77 kg of flue gas

$$= 0.745 \times 11.77 = 8.77 \text{ kg / kg of coal}$$

Since the given coal does not contain nitrogen, therefore the nitrogen is supplied by air only.

∴ Mass of air actually supplied per kg of coal

$$= \frac{100}{77} \times 8.77 = 11.4 \text{ kg Ans.}$$

3. *Amount of excess air supplied per kg of fuel*

We know that the amount of excess air supplied per kg of fuel

$$= \text{Mass of air actually supplied} - \text{Minimum air}$$

$$= 11.4 - 8.5 = 2.9 \text{ kg Ans.}$$

Notes : 1. The mass of air actually supplied and the amount of excess air supplied may also be obtained as discussed below :

We know that mass of air actually supplied per kg of coal

$$= \frac{N_2 \times C}{33(CO_2 + CO)} = \frac{80.7 \times 62.4}{33(13 + 0.3)} = 11.47 \text{ kg Ans.}$$

and the amount of excess air supplied per kg of coal

$$= \frac{79 \times O_2 \times C}{21 \times 33(CO_2 + CO)} = \frac{79 \times 6 \times 62.4}{21 \times 33(13 + 0.3)} = 3.2 \text{ kg Ans.}$$

2. The amount of excess air supplied per kg of coal may also be obtained by the following two methods :

(a) The oxygen in the flue gas is unused oxygen.

∴ Mass of unused O_2 per kg of coal

$$= 0.063 \times \text{Total mass of flue gas} = 0.063 \times 11.77 = 0.7415 \text{ kg}$$

and mass of CO per kg of coal $= 0.003 \times 11.77 = 0.0353 \text{ kg}$

We know that 1 kg of CO requires 4/7 kg of oxygen and produces 1/7 kg of CO_2. Therefore Oxygen (O_2) required to burn CO

$$= 0.0353 \times \frac{4}{7} = 0.0202 \text{ kg}$$

∴   Excess $O_2 = 0.7415 - 0.0202 = 0.7213 \text{ kg}$

and excess air $= 0.7213 \times \frac{100}{23} = 3.136 \text{ kg Ans.}$

(b) We know that oxygen in the flue gas is 6% by volume.

∴ Amount of N_2 associated with this oxygen

$$= 6 \times \frac{79}{21} = 22.57\%$$

N_2 already present in the flue gas

$$= 80.7\%$$

$$\therefore \text{ Excess air} = \frac{22.57}{80.7} \times \text{Mass of actual air supplied}$$

$$= \frac{22.57}{80.7} \times 11.4 = 3.2 \text{ kg Ans.}$$

Example 12.10. *The percentage composition by mass of a sample of coal as found by analysis is given as :*

C 90, H_2 3.3, O_2 3.0, N_2 0.8, S 0.9 and ash 2.0.

Calculate the minimum mass of air required for the complete combustion of 1 kg of this fuel.

If 50% excess air is supplied, find the total mass of dry flue gases per kg of fuel and the percentage composition of the dry flue gases by volume.

Solution. Given : C = 90% = 0.9 kg ; H_2 = 3.3% = 0.033 kg ; O_2 = 3% = 0.03 kg ; N = 0.8% = 0.008 kg ; S = 0.9% = 0.009 kg ; Ash = 2% = 0.02 kg ; Excess air supplied = 50 %

Minimum mass of air required for complete combustion

We know that minimum mass of air required for complete combustion of 1 kg of fuel

$$= \frac{100}{23}\left[\left(\frac{8}{3}C + 8H_2 + S\right) - O_2\right] \text{ kg}$$

$$= \frac{100}{23}\left[\left(\frac{8}{3} \times 0.9 + 8 \times 0.033 + 0.009\right) - 0.03\right] = 11.5 \text{ kg Ans.}$$

Total mass of dry flue gases per kg of fuel

Since 50% excess air is supplied, therefore actual amount of air supplied per kg of coal

$$= 11.5 \times 1.5 = 17.25 \text{ kg}$$

$$\therefore \text{ Excess air supplied} = 17.25 - 11.5 = 5.75 \text{ kg}$$

The products of combustion are represented by the following chemical equations :

(i) $\qquad\qquad C + O_2 = CO_2$

(ii) $\qquad\qquad 2H_2 + O_2 = 2H_2O$

(iii) $\qquad\qquad S + O_2 = SO_2$

In addition to carbon dioxide, water and sulphur dioxide, the excess oxygen and nitrogen will be available in the products of combustion. It may be noted that H_2O (water vapour) is a wet gas, therefore the dry flue gases are only carbon dioxide, sulphur dioxide, excess oxygen and nitrogen. Let us now find the mass of each of these flue gases per kg of fuel.

We know that 1 kg of carbon produces 11/3 kg of carbon dioxide and 1 kg of sulphur produces 2 kg of sulphur dioxide.

$\therefore$ Mass of CO_2 contained in 0.9 kg of carbon per kg of fuel

$$= \frac{11}{3} \times 0.9 = 3.3 \text{ kg} \qquad\qquad \ldots(i)$$

and mass of SO_2 in 0.009 kg of sulphur per kg of fuel

$$= 2 \times 0.009 = 0.018 \text{ kg} \qquad\qquad \ldots(ii)$$

We also know that the mass of excess O_2 per kg of fuel

$$= \frac{23}{100} \times \text{Excess air supplied} = \frac{23}{100} \times 5.75 = 1.323 \text{ kg} \qquad \ldots(iii)$$

and mass of nitrogen in the products of combustion per kg of fuel

$$= \frac{77}{100} \times \text{Actual air supplied}$$

$$= \frac{77}{100} \times 17.25 = 13.283 \text{ kg} \qquad \qquad \dots (iv)$$

∴ Total mass of dry flue gases per kg of fuel

$$= 3.3 + 0.018 + 1.323 + 13.283 \text{ kg} = 17.924 \text{ kg Ans.}$$

Percentage composition of dry flue gases by volume

First of all, let us find out the percentage composition of the dry flue gases from the above d ita by mass. We know that the composition of :

$$CO_2 = \frac{3.3}{17.924} = 0.184 = 18.4\%$$

$$SO_2 = \frac{0.018}{17.924} = 0.001 = 0.1\%$$

$$\text{Excess } O_2 = \frac{1.323}{17.924} = 0.074 = 7.4\%$$

$$N_2 = \frac{13.283}{17.924} = 0.741 = 74.1\%$$

Now let us convert this mass analysis of dry flue gases into volumetric analysis as given below :

Constituent	% Mass analysis (a)	Molecular mass (b)	Proportional volume $(c) = \frac{(a)}{(b)}$	Volume in 1 m^3 of flue gas $(d) = \frac{(c)}{\Sigma(c)}$	% Volumetric analysis $(d) \times 100$
CO_2	18.4	44	$\frac{18.4}{44} = 0.418$	$\frac{0.418}{3.2966} = 0.1268$	12.68
SO_2	0.1	64	$\frac{0.1}{64} = 0.0016$	$\frac{0.0016}{3.2966} = 0.0005$	0.05
Excess O_2	7.4	32	$\frac{7.4}{32} = 0.231$	$\frac{0.231}{3.2966} = 0.0701$	7.01
N_2	74.1	28	$\frac{74.1}{28} = 2.646$	$\frac{2.646}{3.2966} = 0.8026$	80.26
Total	100.0		$\Sigma(c) = 3.2966$		100.00

The percentage composition of dry flue gases (by volume) is given in last column, *i.e.*
$CO_2 = 12.68\%$, $SO_2 = 0.06\%$; Excess $O_2 = 7.01\%$; and $N_2 = 80.25\%$. Ans.

Example 12.11. *A fuel oil has following analysis by mass :*

C 85%, H_2 12.5%, O_2 2% and the residue 0.5%.

The dry flue has the following composition by volume :

CO_2 9%, CO 1 %, O_2 7.77% and N_2 82.23 %.

Determine the air fuel ratio.

Solution. Given : For fuel oil (by mass) $C = 85\% = 0.85$ kg ; $H_2 = 12.5\% = 0.125$ kg ; $O_2 = 2\% = 0.02$ kg ; Residue $= 0.5\% = 0.005$ kg

For dry flue (by volume) : $CO_2 = 9\% = 0.09$ m^3 ; $CO = 1\% = 0.01$ m^3 ; $O_2 = 7.77\% = 0.0777$ m^3 ; $N_2 = 82.23\% = 0.8223$ m^3

We know that minimum air required per kg of fuel

$$= \frac{100}{23}\left[\left(\frac{8}{3}C + 8H_2 + S\right) - O_2\right] \text{kg}$$

$$= \frac{100}{23}\left[\left(\frac{8}{3}\times 0.85 + 8\times 0.125\right) - 0.02\right] = 14.1 \text{ kg} \quad \ldots(\because S = 0)$$

First of all, let us convert the volumetric analysis of dry flue gas into mass analysis as given in the following table.

Constituent	Volume in 1 m³ of the flue gas (a)	Molecular mass (b)	Proportional mass (c) = (a) × (b)	Mass in 1 kg per kg of flue gas $(d) = \frac{(c)}{\Sigma(c)}$	% Mass analysis = (d) × 100
CO_2	0.09	44	3.96	$\frac{3.96}{29.75} = 0.133$	13.3
CO	0.01	28	0.28	$\frac{0.28}{29.75} = 0.009$	0.9
O_2	0.0777	32	2.49	$\frac{2.49}{29.75} = 0.084$	8.4
N_2	0.8223	28	23.02	$\frac{23.02}{29.75} = 0.774$	77.4
Total	1.0000		$\Sigma(c) = 29.75$	1.000	100.00

We know that 1 kg of CO_2 contains 3/11 kg of carbon and 1 kg of CO contains 3/7 kg of carbon. Therefore, mass of carbon per kg of flue gas

$$= \frac{3}{11}CO_2 + \frac{3}{7}CO = \frac{3}{11}\times 0.133 + \frac{3}{7}\times 0.009 = 0.04 \text{ kg}$$

We also know that the mass of flue gas per kg of fuel

$$= \frac{\text{Mass of carbon in 1 kg of fuel}}{\text{Mass of carbon in 1 kg of flue gas}} = \frac{0.85}{0.04} = 21.25 \text{ kg}$$

Since 1 kg of CO requires 4/7 kg of oxygen, therefore mass of excess oxygen per kg of flue gas

$$= \text{Oxygen already present in flue gas} - \frac{4}{7}CO$$

$$= 0.084 - \frac{4}{7}\times 0.009 = 0.079 \text{ kg}$$

and mass of excess oxygen per kg of fuel

$$= \text{Mass of excess } O_2 \text{ per kg of the gas} \times \text{Mass of flue gas per kg of fuel}$$

$$= 0.079 \times 21.25 = 1.679 \text{ kg}$$

∴ Mass of excess air per kg of fuel

$$= \frac{100}{23}\times \text{Mass of excess oxygen} = \frac{100}{23}\times 1.679 = 7.3 \text{ kg}$$

Total air required per kg of fuel

$$= \text{Minimum air} + \text{Excess air} = 14.1 + 7.3 = 21.4 \text{ kg}$$

$\therefore$ Air-fuel ratio $= 21.4 : 1$ **Ans.**

Example 12.12. *A sample of coal with C = 0.78 ; H_2 = 0.05 ; O_2 = 0.08 ; S = 0.02 ; N_2 = 0.02 and ash = 0.05 is burnt in a furnace with 50% excess air. The flue gases enter the chimney at 325° C and the atmospheric temperature is 15° C. Take c_p for O_2, N_2 and air = 1.008 kJ/kg K and c_p for CO_2 and SO_2 from the flue gas = 1.05 kJ/kg K. Assume that the heat carried away per kg of moisture in flue gases is 2940 kJ. Calculate the quantity of heat carried away by the flue gases in kJ/kg of coal.*

Solution. Given : *C = 0.78 kg ; H_2 = 0.05 kg ; O_2 = 0.08 kg ; S = 0.02 kg ; N_2 = 0.02 kg , Ash = 0.05 kg ; Excess air = 50% ; t_g = 325° C ; t_a = 15° C ; c_p for O_2, N_2 and air = 1.008 kJ/kg K ; c_p for CO_2 and SO_2 = 1.05 kJ/kg K ; Heat carried away per kg of moisture = 2940 kJ

First of all, let us find the mass of the flue gases produced per kg of coal.

We know that minimum air required to burn 1 kg of coal

$$= \frac{100}{23} \left[\left(\frac{8}{3} C + 8H_2 + S \right) - O_2 \right] \text{kg}$$

$$= \frac{100}{23} \left[\left(\frac{8}{3} \times 0.78 + 8 \times 0.05 + 0.02 \right) - 0.08 \right] = 10.52 \text{ kg}$$

$\therefore$ Excess air supplied per kg of coal

$$= \frac{50}{100} \times 10.52 = 5.26 \text{ kg}$$

and mass of air to be supplied per kg of coal burnt

$$= \text{Minimum air} + \text{Excess air} = 10.52 + 5.26 = 15.78 \text{ kg}$$

The flue gases produced by the combustion of coal are CO_2, H_2O, SO_2, excess O_2 and N_2.

We know that 1 kg of carbon produces 11/3 kg of carbon dioxide (CO_2) ; 1 kg of hydrogen produces 9 of water (H_2O) and 1 kg of sulphur produces 2 kg of sulphur dioxide (SO_2).

$\therefore$ Mass of CO_2 produced by 0.78 kg of carbon per kg of coal

$$= \frac{11}{3} \times 0.78 = 2.86 \text{ kg}$$

Mass of H_2O produced by 0.05 kg of hydrogen per kg of coal

$$= 9 \times 0.05 = 0.45 \text{ kg}$$

Mass of SO_2 produced by 0.02 kg of sulphur per kg of coal

$$= 2 \times 0.02 = 0.04 \text{ kg}$$

Mass of excess O_2 produced per kg of coal

$$= \frac{23}{100} \times \text{Excess air supplied} = \frac{23}{100} \times 5.26 = 1.21 \text{ kg}$$

and mass of N_2 produced per kg of coal

$$= \frac{77}{100} \times \text{Actual mass of air supplied} = \frac{77}{100} \times 15.78 = 12.15 \text{ kg}$$

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* A sample of 1 kg of coal is considered.

We know that heat carried away by CO_2

$$= \text{Mass} \times \text{Specific heat} \times \text{Rise in temperature}$$
$$= 2.86 \times 1.05\,(325 - 15) = 931\ \text{kJ/kg}$$

Heat carried away by SO_2

$$= 0.04 \times 1.05 \times (325 - 15) = 13.02\ \text{kJ/kg}$$

Heat carried away by excess O_2

$$= 1.21 \times 1.008\,(325 - 15) = 378\ \text{kJ/kg}$$

Heat carried away by N_2 2.15 $\times$ 1.008 $(325 - 15) = 3797\ \text{kJ/kg}$

Since the heat carried away by moisture is given as 2940 kJ/kg, therefore

Heat carried away by H_2O

$$= 0.45 \times 2940 = 1323\ \text{kJ/kg}$$

∴ Total heat carried away by flue gases

$$= 931 + 13.02 + 378 + 3797 + 1323 = 6442.02\ \text{kJ/kg of coal}\ \textbf{Ans.}$$

12.17. Flue Gas Analysis by Orsat Apparatus

To check the combustion efficiency of boilers, it is considered essential to determine the constituents of the flue gases. Such an analysis is carried out with the help of Orsat apparatus as shown in Fig. 12.1.

It consists of a graduated measuring glass tube (known as eudiometer tube) and three flasks *A, B* and *C*, each containing different chemicals for absorbing carbon dioxide, carbon monoxide and oxygen. An aspirator bottle containing water is connected to the bottom of the eudiometer tube by means of a rubber tube. It can be moved up and down, at will, for producing a suction or pressure effect on the sample of the flue gas.
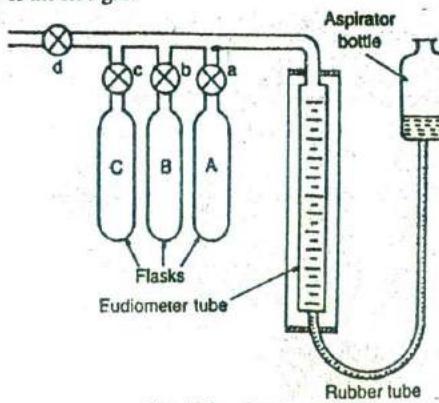
Fig. 12.1. Orsat apparatus.

The flask *A* contains caustic soda (NaOH) and is used for absorbing carbon dioxide in the sample of the flue gas.

The flask *B* contains caustic soda (NaOH) and pyrogallic acid, which absorbs oxygen from the sample of the flue gas.

The flask *C* contains a solution of cuprous chloride (Cu_2Cl_2) in hydrochloric acid (HCl). It absorbs carbon monoxide (CO) from the sample of the flue gas.

Each of the three flasks has stop cock '*a*', '*b*' and '*c*' respectively and a three-way cock '*d*' which can be opened to either atmosphere or flue gas.

The sample of the flue gas to be analysed is first sucked in the eudiometer tube, and its volume is noted. It can take usually 100 cm³ of gas. By mainipulating the level of aspirator bottle, the flue gas can, in turn, be forced into either of the flasks *A*, *B* and *C* by opening the respective cocks *a*, *b* and *c*. The flue gas is left there for sometime and then sucked back into eudiometer tube. The chemicals, in the three flasks absorb carbon dioxide, and carbon monoxide and the resulting contraction in volume enables the percentage of each gas present in the sample to be read on the eudiometer tube.

Since the flue is collected over water in the tube, therefore any steam present in it will be condensed. Similarly, the sulphur dioxide present in it will also be absorbed. Hence the percentages of dry flue gases are only obtained by Orsat apparatus.

EXERCISES

1. The percentage composition of a sample of coal is found by analysis as : C 91%, H_2 3%, O_2 = 2%, N_2 0.8%, S 0.8% and the remaining is ash.

    Calculate the minimum mass of air for complete combustion of 1 kg of coal.   [Ans. 11.5 kg]

2. A sample of coal was found on analysis to have following composition by mass :

    Carbon 72.2%, hydrogen 3.2%, oxygen 18.5%, sulphur 2.4%, and the remainder being incombustible matter.

    Calculate the theoretical mass of air required for the complete combustion of 1 kg of this coal.
    [Ans. 8.51 kg]

3. A coal contains by mass 81% carbon, 6% hydrogen and the remainder ash. Find : 1. minimum mass of air required to burn 1 kg of coal, and 2. the mass of produces of combustion.

    [Ans. 11.48 kg ; CO = 2.97 kg. H_2O = 0.54 kg]

4. The volumetric analysis of a flue gas is CO_2 15% ; CO 2.2% ; O_2 1.6% and N_2 81.2%. Convert this volumetric analysis into percentage analysis by mass.   [Ans. CO_2 21.6% : CO 2.2 % ; O_2 1.7% : N_2 74.5%]

5. The percentage composition of a certain fuel by mass is

    C = 87.1 ; H = 4.4 ; O = 1.2 and ash 7.3.

    The percentage volumetric composition of dry flue gases is

    CO_2 = 15 ; CO = 2.2 ; O = 1.6 ; N_2 = 81.2.

    Estimate the mass of flue gases produced per kg of fuel burnt.   [Ans. 12.85 kg]

6. A fuel has the following composition by mass :

    C = 86; H = 12; O = 1; S = 1.

    Estimate the minimum volume of air required at N.T.P. for complete combustion. Determine also the percentage composition by mass of the products of combustion. The constituents of air by mass is 77% N_2 and 23% O_2. Air measures 0.773 m³/kg at N.T.P.

    [Ans. 10.93 m³/kg of fuel ; CO_2 22.42%, SO_2 0.142%, N_2 77.43%]

7. A certain fuel has the following composition by mass :

    C 80%, H_2 10 %, and S 10%.

    The volumetric analysis of the fuel gas is : CO_2 10%, CO 1%, O_2 10% and N_2 79%. Find per kg of coal : 1. the minimum air required, 2. the actual air supplied, and 3. the excess air supplied.

    [Ans. 13.2 kg ; 17.4 kg ; 4.2 kg]

8. The percentage composition (by mass) of a certain fuel is C 88%, H_2 3.6%, O_2 4.8% and ash 3.6%. The percentage composition (by volume) of the flue gases are CO_2 10.9%, CO 1%, O_2 7.1% and N_2 81%.

    Determine : 1. the mass of air actually supplied per kg of coal ; and 2. the percentage excess air supplied.   [Ans. 18.15 kg, 38.3%]

9. The mass analysis of a fuel is carbon 75%, hydrogen 8%, oxygen 6% and remainder incombustible. If 16 kg of air were supplied per kg of coal ; find ;

    1. The percentage of excess air ; and

    2. The percentage (by mass) of CO_2 in the dry products of combustion. Take the percentage of oxygen (by mass) in air as 23.1.   [Ans. 43.6% ; 12.35%]

10. A producer gas has the following percentage analysis by volume :

H_2 15, CH_4 2, CO 20, CO_2 6, O_2 3 and N_2 54. If 50% of excess air is supplied for the combustion, determine :

     1. the volume of air supplied per m^3 of the gas, and·

     2. the volumetric analysis of the dry products of combustion.

                                [Ans. 1.322 m^3 ; CO_2 14.3%, N_2 80.8%, O_2 4.7%]

11. The ultimate analysis (by mass) of a fuel used in a boiler is : carbon = 75% ; hydrogen = 14% and the remaining being incombustible. The air supplied is 52% in excess of that required for complete combustion. If 2.4% of carbon in the fuel is burnt only to CO and the rest to CO_2, estimate the volumetric analysis of dry flue gases. Air contains 23% by mass and 21% by volume of O_2.

                                [Ans. CO_2 8.93% ; CO 0.3% ; O_2 7.44% ; N_2 83.33%]

12. Calculate the stoichiometric air-fuel ratio for the combustion of a sample of dry coal of the following by mass:

     C = 88% ; H_2 = 4% ; O_2 = 2.5% ; S = 0.5% and rest ash.

     Also determine the volumetric compositir of the dry products of combustion if 20% excess air is supplied. Assume that air contains 23.3% O_2 and rest N_2 by mass.

                        [Ans. 11.36 ; CO_2 15.84%, SO_2 0.03% ; O_2 3.56% ; N_2 80.57%]

13. A water gas at 21°C and 760 mm of Hg is burnt with dry air supplied at 21°C and 760 mm of Hg. The fuel gas composition is

     CO_2 = 6% ; N_2 = 5.5% ; H_2 = 48 % ; O_2 = 0.5% ; CH_4 = 2% and CO = 38%.

     The Orsat analysis of the flue gas showed CO_2 = 15.5% ; O_2 = 4.76% and CO = 0.2%. Find the percentage of excess air supplied for combustion and the volume of the flue gas at 232° C and 1.013 bar formed per m^3 of the fuel.   [Ans. 29.52% ; 5.025 m^3]

14. . The coal supplied to boiler furnace has the following composition by mass :

     Carbon = 82% ; Hydrogen = 5% ; Oxygen = 7% ; Nitrogen = 1% and rest ash. The volumetric analysis of dry flue gas is found to be as

     CO_2 = 10% ; CO = 1.3% ; O_2 = 7.5% and N_2 = 81.2.

     If the flue gas temperature is 200°C and the boiler room temperature is 50°C, calculate 1. percentage of excess air supplied to the boiler furnace, and 2. heat carried away by the dry flue gas per kg of coal.

     The specific heats at constant pressure for the gases are :

     CO_2 = 0.882 kJ/kg K ; CO = 1.05 kJ/kg K ; O_2 = 1.05 kJ/kg K and N_2 = 1.025 kJ/kg K.

     Assume that air contains 23% O_2 and 77% N_2 by mass.   [Ans. 53% ; 2750 kJ]

QUESTIONS

1. Define the following terms :

     (*a*) element,   (*b*) compound,   (*c*) atom,   (*d*) molecule,

     (*e*) atomic mass, and (*f*) molecular mass

2. What do you understand by 'minimum air' and 'excess air' in context of combustion ?

3. Fill in the following blanks :

     (*i*)   1 kg of carbon requires ...... kg of oxygen and produces ...... kg of carbon dioxide.

     (*ii*) 1 kg of carbon requires ...... kg of oxygen and produces ...... kg of carbon monoxide.

     (*iii*) 1 kg of carbon mono xide requires ...... kg of oxygen and produces ...... kg of carbon dioxide.

     (*iv*) 1 kg of sulphur requires ...... kg of oxygen and produces ...... kg of sulphur dioxide.

     (*v*)   1 kg of hydrogen requires .... kg of oxygen and produce .... kg of water.

     (*vi*) .... m^3 of hydrogen requires .... m^3 of oxygen and produces ...... m^3 of water.

     (*vii*) 1 m^3 of .... requires .... m^3 of oxygen and produces .... m^3 of CO_2 and .... m^3 of H_2O.

4. Give chemical reactions and numerical values for estimating the air requirement for complete combustion of coal.

5. Lay down the procedure for determination of minimum air required for complete combustion of coal.

6. Sketch and explain the use of Orsat apparatus used in determining the percentage of flue or exhaust gases. Does this help in controlling combustion ?

OBJECTIVE TYPE QUESTIONS

1. The smallest quantity of a substance, which can exist by itself in a chemically recognizable form is known as
   (a) element   (b) compound   (c) atom   (d) molecule

2. The molecular mass of oxygen is
   (a) 12   (b) 14   (c) 16   (d) 32

3. The molecular mass of nitrogen is ...... oxygen.
   (a) equal to   (b) less than   (c) more than

4. Which of the following has minimum molecular mass ?
   (a) Oxygen   (b) Nitrogen   (c) Hydrogen   (d) Water

5. One kg of carbon monoxide (CO) requires 4/7 kg of oxygen and produces
   (a) 11/3 kg of CO_2 (b) 7/3 kg of CO   (c) 11/7 kg of CO_2 (d) 8/3 kg of CO

6. One kg of carbon requires ...... of oxygen and produces 7/3 kg of carbon monoxide.
   (a) 4/3   (b) 7/3.   (c) 8/3   (d) 11/3

7. One kg of ethylene (C_2H_4) requires 2 kg of oxygen and produces 22/7 kg of carbon dioxide. and
   (a) 9/7 kg of water   (b) 11/7 kg of water
   (c) 7/4 kg of water   (d) 11/4 kg of water

8. The mass of carbon per kg of flue gas is given by
   (a) $\dfrac{11}{3} CO_2 + \dfrac{3}{7} CO$ (b) $\dfrac{3}{7} CO_2 + \dfrac{11}{3} CO$
   (c) $\dfrac{7}{3} CO_2 + \dfrac{3}{11} CO$ (d) $\dfrac{3}{11} CO_2 + \dfrac{7}{3} CO$

9. The mass of flue gas per kg of fuel is the ratio of the
   (a) mass of oxygen in 1 kg of flue gas to the mass of oxygen in 1 kg of fuel
   (b) mass of oxygen in 1 kg of fuel to the mass of oxygen in 1 kg of flue gas
   (c) mass of carbon in 1 kg of flue gas to the mass of carbon in 1 kg of fuel
   (d) mass of carbon in 1 kg of fuel to the mass of carbon in 1 kg of flue gas

10. The mass of excess air supplied is equal to
    (a) $\dfrac{23}{100} \times$ Mass of excess carbon   (b) $\dfrac{23}{100} \times$ Mass of excess oxygen
    (c) $\dfrac{100}{23} \times$ Mass of excess carbon   (d) $\dfrac{100}{23} \times$ Mass of excess oxygen

ANSWERS

1. (a)	2. (d)	3. (b)	4. (c)	5. (c)
6. (a)	7. (a)	8. (a)	9. (c)	10. (d)