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Steam Nozzles

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21.1. Introduction

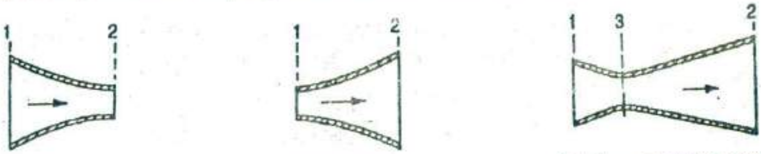
A steam nozzle is a passage of varying cross-section, which converts heat energy of steam into kinetic energy. During the first part of the nozzle, the steam increases its velocity. But in its later part, the steam gains more in volume than in velocity. Since the mass of steam, passing through any section of the nozzle remains constant, the variation of steam pressure in the nozzle depends upon the velocity, specific volume and dryness fraction of steam. A well designed nozzle converts the heat energy of steam into kinetic energy with a minimum loss.

The main use of steam nozzle in steam turbines, is to produce a jet of steam with a high velocity. The smallest section of the nozzle is called throat.

21.2. Types of Steam Nozzles

Following three types of nozzles are important from the subject point of view :

1. *Convergent nozzle.* When the cross-section of a nozzle decreases continuously from entrance to exit, it is called a convergent nozzle as shown in Fig. 21.1 (*a*).



(*a*) Convergent. (*b*) Divergent. (*c*) Convergent-divergent.

Fig. 21.1. Types of steam nozzles.

2. *Divergent nozzle.* When the cross-section of a nozzle increases continuously from entrance to exit, it is called a divergent nozzle, as shown in Fig. 21.1 (*b*).

3. *Convergent-divergent nozzle.* When the cross-section of a nozzle first decreases from its entrance to throat, and then increases from its throat to exit, it is called a convergent-divergent nozzle as shown in Fig. 21.1 (*c*). This type of nozzle is widely used these days in various types of steam turbines.

21.3. Flow of Steam through Convergent-divergent Nozzle

The steam enters the nozzle with a high pressure, but with a negligible velocity. In the converging portion (*i.e.* from the inlet to the throat), there is a drop in the steam pressure with a rise in its velocity. There is also a drop in the enthalpy or total heat of the steam. This drop of heat is not utilised in doing some external work, but is converted into kinetic energy. In the divergent portion (*i.e.* from the throat to outlet), there is further drop of steam pressure with a further rise in its velocity. Again, there is a drop in the enthalpy or total heat of steam, which is converted into kinetic energy.

It will be interesting to know that the steam enters the nozzle with a high pressure and negligible velocity. But leaves the nozzle with a high velocity and small pressure. The pressure, at which the steam leaves the nozzle, is known as back pressure. Moreover, no heat is supplied or rejected by the steam during flow through a nozzle. Therefore, it is considered as isentropic flow, and the corresponding expansion is considered as an isentropic expansion.

21.4. Friction in a Nozzle or Nozzle Efficiency

As a matter fact, when the steam flows through a nozzle, some loss in its enthalpy or total heat takes place due to friction between the nozzle surface and the flowing steam. This can be best understood with the help of *h-s* diagram or Mollier chart, as shown in Fig. 21.2., which can be completed as discussed below :

1. First of all, locate the point A for the initial conditions of the steam. It is a point, where the saturation line meets the initial pressure (p_1) line.

2. Now draw a vertical line through A to meet the final pressure (p_2) line. This is done as the flow through the nozzle is isentropic, which is expressed by a vertical line AB. The heat drop $(h_1 - h_2)$ is known as *isentropic heat drop*.

3. Due to friction in the nozzle the actual heat drop in the steam will be less than $(h_1 - h_2)$. Let this heat drop be shown as AC instead of AB.

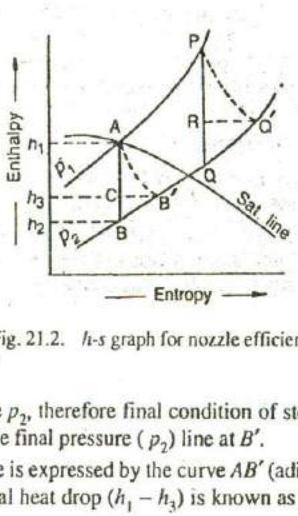


Fig. 21.2. *h-s* graph for nozzle efficiency.

4. As the expansion of steam ends at the pressure p_2, therefore final condition of steam is obtained by drawing a horizontal line through C to meet the final pressure (p_2) line at B'.

5. Now the actual expansion of steam in the nozzle is expressed by the curve AB' (adiabatic expansion) instead of AB (isentropic expansion). The actual heat drop $(h_1 - h_3)$ is known as *useful heat drop*.

Now the coefficient of nozzle or nozzle efficiency (usually denoted by K) is defined *as the ratio of useful heat drop to the isentropic heat drop*. Mathematically,

$$K = \frac{\text{Useful heat drop}}{\text{Isentropicheat drop}} = \frac{AC}{AB} = \frac{h_1 - h_3}{h_1 - h_2}$$

Notes : 1. We see from Fig. 21.2, that the dryness fraction of steam at B' is greater than that at B. It is thus obvious, that the effect of friction is to increase the dryness fraction of steam. This is due to the fact that the energy lost in friction is transferred into heat, which tends to dry or superheat the steam.

2. A similar effect is produced when the steam is superheated at the entrance of the nozzle.

Let PR = Useful heat drop, and PQ = Isentropicheat drop.

$\therefore$ Nozzle efficiency, $K = \dfrac{PR}{PQ}$

3. In general, if 15% of the heat drop is lost in friction, then efficiency of the nozzle is equal to $100 - 15 = 85\% = 0.85$.

21.5. Velocity of Steam Flowing through a Nozzle

Consider a unit mass flow of steam through a nozzle.

Let V_1 = Velocity of steam at the entrance of nozzle in m/s

V_2 = Velocity of steam at any section considered in m/s,

h_1 = Enthalpy or total heat of steam entering the nozzle in kJ/kg, and

h_2 = Enthalpy or total heat of steam at the section considered in kJ/kg.

We know that for a steady flow process in a nozzle,

$$h_1 + \frac{*1}{1000}\left(\frac{V_1^2}{2}\right) = h_2 + \frac{1}{1000}\left(\frac{V_2^2}{2}\right) + \text{Losses}$$

Neglecting losses in a nozzle,

$$\frac{1}{1000}\left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right) = h_1 - h_2$$

$$\therefore \quad V_2 = \sqrt{V_1^2 + 2000\,(h_1 - h_2)} = \sqrt{V_1^2 + 2000\,h_d} \qquad \ldots (i)$$

where h_d = Enthalpy or heat drop during expansion of steam in a nozzle

$= h_1 - h_2$

Since the entrance velocity or velocity of approach (V_1) is negligible as compared to V_2, therefore from equation (*i*),

$$V_2 = \sqrt{2000\,h_d} = 44.72\sqrt{h_d} \qquad \ldots (ii)$$

Note : In actual practice, there is always a certain amount of friction present between the steam and nozzle surfaces. This reduces the heat drop by 10 to 15 percent and thus the exit velocity of steam is also reduced correspondingly. Thus the above relation may be written as :

$$V_2 = 44.72\sqrt{K\,h_d}$$

where K is the nozzle coefficient or nozzle efficiency.

Example 21.1. *Dry saturated steam at 5 bar with negligible velocity expands isentropically in a convergent nozzle to 1 bar and dryness fraction 0.94. Determine the velocity of steam leaving the nozzle.*

Solution. Given : $p_1 = 5$ bar ; $p_2 = 1$ bar ; $x_2 = 0.94$

From steam tables, corresponding to a pressure of 5 bar, we find that enthalpy or total heat of dry saturated steam,

$$h_1 = h_{g1} = 2747.5 \text{ kJ/kg}$$

and corresponding to a pressure of 1 bar, we find that

$$h_{f2} = 417.5 \text{ kJ/kg, and } h_{fg2} = 2257.9 \text{ kJ/kg}$$

$\therefore$ Enthalpy or total heat of final steam,

$$h_2 = h_{f2} + x_2\,h_{fg2} = 417.5 + 0.94 \times 2257.9 = 2540 \text{ kJ/kg}$$

and enthalpy or heat drop, $h_d = h_1 - h_2 = 2747.5 - 2540 = 207.5$ kJ/kg

We know that velocity of steam leaving the nozzle,

$$V_2 = 44.72\sqrt{h_d} = 44.72\sqrt{207.5} = 644 \text{ m/s} \quad \textbf{Ans.}$$

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* We know that $\text{K.E.} = \frac{1}{2}mV^2 = \frac{1}{2} \times 1 \times V^2 = \frac{V^2}{2}\text{ J} = \frac{1}{1000}\left(\frac{V^2}{2}\right)\text{kJ} \qquad \ldots (\because m = 1 \text{ kg})$

Example 21.2. *Dry saturated steam at a pressure of 15 bar enters in a nozzle and is discharged at a pressure of 1.5 bar. Find the final velocity of the steam, when the initial velocity of the steam is negligible.*

If 10% of the heat drop is lost in friction, find the percentage reduction in the final velocity.

Solution. Given : $p_1 = 15$ bar ; $p_2 = 1.5$ bar

Final velocity of the steam

From steam tables, corresponding to a pressure of **15 bar**, we find that enthalpy of dry saturated steam,

$$h_1 = 2789.9 \text{ kJ/kg}$$

and corresponding to a pressure of 1.5 bar, enthalpy of dry saturated steam,

$$h_2 = 2693.4 \text{ kJ/kg}$$

$\therefore$ Heat drop, $\quad h_d = h_1 - h_2 = 2789.9 - 2693.4 = 96.5$ kJ/kg

We know that final velocity of the steam,

$$V_2 = 44.72\sqrt{h_d} = 44.72\sqrt{96.5} = 439.3 \text{ m/s } \textbf{Ans.}$$

Percentage reduction in the final velocity

We know that heat drop lost in friction

$$= 10\% = 0.1 \qquad \ldots\text{(Given)}$$

$\therefore$ Nozzle coefficient or nozzle efficiency

$$K = 1 - 0.1 = 0.9$$

We know that final velocity of the steam,

$$V_2 = 44.72\sqrt{K\,h_d} = 44.72\sqrt{0.9 \times 96.5} = 416.8 \text{ m/s}$$

$\therefore$ Percentage reduction in final velocity

$$= \frac{439.3 - 416.8}{439.3} = 0.051 \text{ or } 5.1\% \textbf{ Ans.}$$

Example 21.3. *Dry saturated steam at 10 bar is expanded isentropically in a nozzle to 0.1 bar. Using steam tables only, find the dryness fraction of the steam at exit. Also find the velocity of steam leaving the nozzle when 1. initial velocity is negligible, and 2. initial velocity of the steam is 135 m/s.*

Solution. Given : $p_1 = 10$ bar ; $p_2 = 0.1$ bar

Dryness fraction of the steam at exit

Let $\quad x_2$ = Dryness fraction of the steam at exit.

From steam tables, corresponding to a pressure of 10 bar, we find that entropy of dry saturated steam,

$$s_1 = s_{g1} = 6.583 \text{ kJ/kg K}$$

and corresponding to a pressure of 0.1 bar, we find that

$$s_{f2} = 0.649 \text{ kJ/kg K}, \text{ and } s_{fg2} = 7.502 \text{ kJ/kg K}$$

Since the expansion of steam is isentropic, therefore

Entropy of steam at inlet (s_1) = Entropy of steam at exit (s_2)

$$6.583 = s_{f2} + x_2\, s_{fg2} = 0.649 + x_2 \times 7.502$$

$$x_2 = 0.791 \textbf{ Ans.}$$

1. *Velocity of steam leaving the nozzle when initial velocity is negligible*

From steam tables, corresponding to a pressure of 10 bar, we find that enthalpy or total heat of dry saturated steam,

$$h_1 = h_{g1} = 2776.2 \text{ kJ/kg}$$

and corresponding to a pressure of 0.1 bar,

$$h_{f2} = 191.8 \text{ kJ/kg, and } h_{fg2} = 2392.9 \text{ kJ/kg}$$

∴ Enthalpy or total heat of steam of exit,

$$h_2 = h_{f2} + x_2 h_{fg2}$$

$$= 191.8 + 0.791 \times 2392.9 = 2084.6 \text{ kJ/kg}$$

and heat drop,
$$h_d = h_1 - h_2 = 2776.2 - 2084.6 = 691.6 \text{ kJ/kg}$$

We know that velocity of steam leaving the nozzle,

$$V_2 = 44.72\sqrt{h_d} = 44.72\sqrt{691.6} = 1176 \text{ m/s} \quad \textbf{Ans.}$$

2. *Velocity of steam leaving the nozzle when initial velocity, $V_1 = 135$ m/s*

We know that velocity of steam leaving the nozzle,

$$V_2 = \sqrt{V_1^2 + 2000\,h_d} = \sqrt{(135)^2 + 2000 \times 691.6} = 1184 \text{ m/s} \quad \textbf{Ans.}$$

Example 21.4. *Dry saturated steam at a pressure of 10 bar is expanded in a nozzle to a pressure of 0.7 bar. With the help of Mollier diagram find the velocity and dryness fraction of steam issuing from the nozzle, if the friction is neglected.*

Also find the velocity and dryness fraction of the steam, if 15% of the heat drop is lost in friction.

Solution. Given : $p_1 = 10$ bar ; $p_2 = 0.7$ bar

Velocity and dryness fraction of steam issuing from the nozzle, if friction is neglected

The process on the Mollier diagram, as shown in Fig. 21.3, is drawn as discussed below :

1. First of all, locate the point A on the saturation line (because the steam is initially dry saturated) where the initial pressure line (10 bar) meets it.

2. Since the expansion in the nozzle is isentropic, therefore draw a vertical line through A to meet the final pressure line (0.7 bar) at point B.

Now from the Mollier diagram, we find that

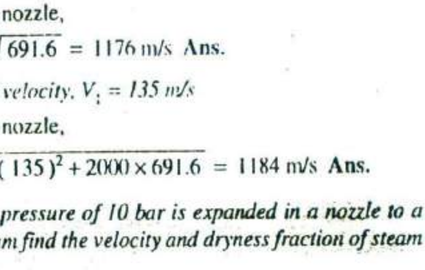


Fig 21.3

$$h_1 = 2772 \text{ kJ/kg,}$$

and
$$h_2 = 2310 \text{ kJ/kg}$$

∴ Heat drop,

$$h_d = h_1 - h_2 = 2772 - 2310 = 462 \text{ kJ/kg}$$

We know that velocity of steam issuing from the nozzle,

$$V_2 = 44.72\sqrt{h_d} = 44.72\sqrt{462} = 961 \text{ m/s} \quad \textbf{Ans.}$$

From Mollier diagram, we also find that the dryness fraction of steam issuing from the nozzle (*i.e.* at point B) is $x_2 = 0.848$. **Ans.**

Velocity and dryness fraction of steam issuing from the nozzle if 15% of the heat drop is lost in friction.

Since 15% heat drop is lost in friction, therefore nozzle coefficient or nozzle efficiency,

$$K = 100 - 15 = 85\% = 0.85$$

and heat drop due to friction $= 462 \times 0.15 = 69.3$ kJ/kg

We know that velocity of steam issuing from the nozzle,

$$V_2 = 44.72\sqrt{K h_d} = 44.72\sqrt{0.85 \times 462} = 886 \text{ m/s} \quad \textbf{Ans.}$$

Now let us complete the Mollier diagram as discussed below :

1. Locate point C on the vertical line AB, such that $BC = 69.3$ kJ/kg.
2. Now through C, draw a horizontal line CB' to meet the final pressure line (0.7 bar) at B'.

From the Mollier diagram, we find that the dryness fraction of steam issuing from the nozzle, (*i.e.* at point B') is $x_3 = 0.878$. **Ans.**

21.6. Mass of Steam Discharged through Nozzle

We have already discussed that the flow of steam, through the nozzle is isentropic, which is approximately represented by the general law:

$$pv^n = \text{Constant}$$

We know that gain in kinetic energy

$$= \frac{V_2^2}{2} \qquad \text{... (Neglecting initial velocity of steam)}$$

and Heat drop = Work done during Rankine cycle

$$= \frac{n}{n-1}(p_1 v_1 - p_2 v_2)$$

Since gain in kinetic energy is equal to heat drop, therefore

$$\frac{V_2^2}{2} = \frac{n}{n-1}(p_1 v_1 - p_2 v_2)$$

$$= \frac{n}{n-1} \times p_1 v_1 \left(1 - \frac{p_2 v_2}{p_1 v_1}\right) \qquad \text{... (i)}$$

We know that $\quad p_1 v_1^n = p_2 v_2^n$

$$\frac{v_2}{v_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = \left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}}$$

$$\therefore \quad v_2 = v_1 \left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}} \qquad \text{... (ii)}$$

Substituting, the value of v_2 / v_1 in equation (*i*),

$$\frac{V_2^2}{2} = \frac{n}{n-1} \times p_1 v_1 \left[1 - \frac{p_2}{p_1}\left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}}\right]$$

$$= \frac{n}{n-1} \times p_1 v_1 \left[1 - \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right]$$

or

$$V_2 = \sqrt{2 \times \frac{n}{n-1} \times p_1 v_1 \left[1 - \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right]} \qquad \ldots (iii)$$

Now the volume of steam flowing per second

= Cross-sectional area of nozzle × Velocity of steam $= A\, V_2$

and volume of 1 kg of steam *i.e.* specific volume of steam at pressure p_2

$= v_2$ m³/kg

∴ Mass of steam discharged through nozzle per second,

$$m = \frac{\text{Volume of steam flowing per second}}{\text{Volume of 1 kg of steam at pressure } p_2}$$

$$= \frac{AV_2}{v_2} = \frac{A}{v_2} \sqrt{2 \times \frac{n}{n-1} \times p_1 v_1 \left[1 - \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right]}$$

Substituting the value of v_2 from equation (*ii*),

$$m = \frac{A}{v_1} \left(\frac{p_1}{p_2} \right)^{-\frac{1}{n}} \sqrt{\frac{2n}{n-1} \times p_1 v_1 \left[1 - \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right]}$$

$$= \frac{A}{v_1} \left(\frac{p_2}{p_1} \right)^{\frac{1}{n}} \sqrt{\frac{2n}{n-1} \times p_1 v_1 \left[1 - \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right]}$$

$$= A \sqrt{\left(\frac{p_2}{p_1} \right)^{\frac{2}{n}} \times \frac{2n}{n-1} \times \frac{p_1}{v_1} \left[1 - \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right]}$$

$$= A \sqrt{\frac{2n}{n-1} \times \frac{p_1}{v_1} \left[\left(\frac{p_2}{p_1} \right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1} \right)^{\frac{n+1}{n}} \right]} \qquad \ldots (iv)$$

Example 21.5. *Dry air at a temperature of 27° C and pressure of 20 bar enters a nozzle and leaves at a pressure of 4 bar. Find the mass of air discharged, if the area of the nozzle is 200 mm².*

Solution. Given : $T_1 = 27°\text{C} = 27 + 273 = 300$ K ; $p_1 = 20$ bar $= 20 \times 10^5$ N/m² ; $p_2 = 4$ bar $= 4 \times 10^5$ N/m² ; $A = 200$ mm² $= 200 \times 10^{-6}$ m²

Let v_1 = Specific volume of air in m³/kg.

We know that $p_1 v_1 = m\, R\, T_1$

$$\therefore \qquad v_1 = \frac{m\, R\, T_1}{p_1} = \frac{1 \times 287 \times 300}{20 \times 10^5} = 0.043 \text{ m}^3/\text{kg} \quad \ldots (\because R \text{ for air} = 287 \text{ J/kg K})$$

We know that mass of steam discharged through the nozzle,

$$m = A\sqrt{\frac{2n}{n-1}\times\frac{p_1}{v_1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}}-\left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right]}$$

$$= 200\times10^{-6}\sqrt{\frac{2\times1.4}{1.4-1}\times\frac{20\times10^5}{0.043}\left[\left(\frac{4}{20}\right)^{\frac{2}{1.4}}-\left(\frac{4}{20}\right)^{\frac{1.4+1}{1.4}}\right]}$$

$$= 200\times10^{-6}\sqrt{3256\times10^5\,[\,0.1-0.06\,]} = 0.72\text{ kg/s}$$ **Ans.**

21.7. Condition for Maximum Discharge through a Nozzle (Critical Pressure Ratio)

A nozzle is, normally, designed for maximum discharge by designing a certain throat pressure which produces this condition.

Let p_1 = Initial pressure of steam in N/m^2,

p_2 = Pressure of steam at throat in N/m^2,

v_1 = Volume of 1 kg of steam at pressure (p_1) in m^3,

v_2 = Volume of 1 kg of steam at pressure (p_2) in m^3, and

A = Cross-sectional area of nozzle at throat, in m^2.

We have derived an equation in the previous article that the mass of steam discharged through nozzle,

$$m = A\sqrt{\frac{2n}{n-1}\times\frac{p_1}{v_1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}}-\left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right]} \quad \ldots(i)$$

There is only one value of the ratio p_2/p_1, which produces maximum discharge from the nozzle. This ratio p_2/p_1, is obtained by differentiating the right hand side of the equation. We see from this equation that except p_2/p_1, all other values are constant. Therefore, only that portion of the equation which contains p_2/p_1, is differentiated and equated to zero for maximum discharge.

$$\therefore \quad \frac{d}{d\left(\frac{p_2}{p_1}\right)}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}}-\left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right] = 0$$

or

$$\frac{2}{n}\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}-1}-\frac{n+1}{n}\left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}-1} = 0$$

$$\frac{2}{n}\left(\frac{p_2}{p_1}\right)^{\frac{2-n}{n}} = \frac{n+1}{n}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$\left(\frac{p_2}{p_1}\right)^{\frac{2-n}{n}}\times\left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}} = \frac{n+1}{n}\times\frac{n}{2}$$

$$\left(\frac{p_2}{p_1}\right)^{\frac{2-n}{n}-\frac{1}{n}} = \frac{n+1}{2}$$

$$\left(\frac{p_2}{p_1}\right)^{\frac{1-n}{n}} = \frac{n+1}{2}$$

$$\frac{p_2}{p_1} = \left(\frac{n+1}{2}\right)^{\frac{n}{1-n}} = \left(\frac{n+1}{2}\right)^{\frac{-n}{-(1-n)}}$$

$$= \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}} \qquad \ldots (ii)$$

Notes : 1. The ratio p_2 / p_1 is known as *critical pressure ratio*, and the pressure p_2 at the throat is known as *critical pressure.*

2. The maximum value of the discharge per second is obtained by substituting the value of p_2 / p_1 in equation (*i*).

$$\therefore \text{ Maximum discharge, } m_{max} = A\sqrt{\frac{2n}{n-1}\times\frac{p_1}{v_1}\left[\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}-\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\right]}$$

$$= A\sqrt{\frac{2n}{n-1}\times\frac{p_1}{v_1}\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}\left[1-\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}-\frac{2}{n-1}}\right]}$$

$$= A\sqrt{\frac{2n}{n-1}\times\frac{p_1}{v_1}\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}\left[1-\left(\frac{2}{n+1}\right)\right]}$$

$$= A\sqrt{\frac{2n}{n-1}\times\frac{p_1}{v_1}\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}\left[\frac{n-1}{n+1}\right]}$$

$$= A\sqrt{\frac{2n}{n+1}\times\frac{p_1}{v_1}\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}} \qquad \ldots (iii)$$

3. We see from the above equation that in a convergent-divergent nozzle, the discharge depends upon the area of nozzle at throat and the initial conditions of the steam (*i.e.* pressure p_1 and volume v_1). It is independent of the exit conditions of the steam. It is thus obvious, that the discharge remains constant after the throat (*i.e.* in the divergent portion of the nozzle).

4. The equations derived above are true for gases also.

21.8. Values for Maximum Discharge through a Nozzle

In the last article we have derived a relation for the maximum discharge through a nozzle, *i.e.*

$$m_{max} = A\sqrt{\frac{2n}{n+1}\times\frac{p_1}{v_1}\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}}$$

Now we shall discuss the values of maximum discharge for the following three conditions :

1. *When the steam is initially dry saturated*

We know that for dry saturated steam, $n = 1.135$. Therefore substituting the value of n in the relation for maximum discharge, we have

$$m_{max} = 0.637\,A\sqrt{\frac{p_1}{v_1}}$$

2. *When the steam is initially superheated*

We know that for superheated steam, $n = 1.3$. Therefore substituting the value of n in the relation for maximum discharge, we have

$$m_{max} = 0.666\,A\sqrt{\frac{p_1}{v_1}}$$

3. *For gases*

We know that for gases, $n = 1.4$. Therefore substituting the value of n in the relation for maximum discharge, we have

$$m_{max} = 0.685\,A\sqrt{\frac{p_1}{v_1}}$$

Example 21.6. *Dry air at a pressure of 12 bar and 300°C is expanded isentropically through a nozzle at a pressure of 2 bar. Determine the maximum discharge through the nozzle of 150 mm² area.*

Solution. Given : $p_1 = 12$ bar $= 12 \times 10^5$ N/m² ; $T_1 = 300°C = 300 + 273 = 573$ K ; $p_2 = 2$ bar ; $A = 150$ mm² $= 150 \times 10^{-6}$ m²

Let $\quad v_1$ = Specific volume of air in m³/kg.

We know that $\quad p_1 v_1 = m R T_1$

$$\therefore \quad v_1 = \frac{m R T_1}{p_1} = \frac{1 \times 287 \times 573}{12 \times 10^5} = 0.137 \text{ m}^3/\text{kg}$$

We know that maximum discharge through the nozzle,

$$m_{max} = 0.685\,A\sqrt{\frac{p_1}{v_1}} = 0.685 \times 150 \times 10^{-6}\sqrt{\frac{12 \times 10^5}{0.137}} \text{ kg/s}$$

$$= 0.304 \text{ kg/s } \textbf{Ans.}$$

Example 21.7. *Steam at a pressure of 10 bar and 210°C is supplied to a convergent divergent nozzle with a throat area of 1500 mm². The exit is below critical pressure. Find the coefficient of discharge, if the flow is 7200 kg of steam per hour.*

Solution. Given : $p_1 = 10$ bar $= 10 \times 10^5$ N/m² ; $T_1 = 210°$ C ; $A = 1500$ mm² $= 1500 \times 10^{-6}$ m² ; $m = 7200$ kg/h $= 2$ kg/s

From steam tables, for superheated steam, corresponding to a pressure of 10 bar and 210° C, we find that specific volume of steam,

$$v_1 = 0.2113 \text{ m}^3/\text{kg}$$

We know that for superheated steam, $n = 1.3$.

$$\therefore \text{ Maximum discharge, } \quad m_{max} = A\sqrt{\frac{2n}{n+1} \times \frac{p_1}{v_1}\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}}$$

$$= 1500 \times 10^{-6} \sqrt{\frac{2 \times 1.3}{1.3+1} \times \frac{10 \times 10^5}{0.2113} \left(\frac{2}{1.3+1}\right)^{\frac{2}{1.3-1}}} \text{ kg/s}$$

$$= 2.17 \text{ kg/s}$$

We know that coefficient of discharge

$$= \frac{\text{Actual discharge}}{\text{Maximum discharge}} = \frac{2}{2.17} = 0.922 \text{ Ans.}$$

Note : The maximum discharge for superheated steam may also be calculated by using the relation,

$$m_{max} = 0.666\, A \sqrt{\frac{p_1}{v_1}}$$

21.9. Values for Critical Pressure Ratio

We have also discussed in Art. 21.7 that the critical pressure ratio,

$$\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$$

where p_2/p_1 = Critical pressure ratio.

We shall now discuss the values of critical pressure ratio for the following conditions :

1. *When the steam is initially saturated*

We know that for dry saturated steam, $n = 1.135$.

$$\therefore \quad \frac{p_2}{p_1} = \left(\frac{2}{1.135+1}\right)^{\frac{1.135}{1.135-1}} = 0.577 \quad \text{or} \quad p_2 = 0.577\, p_1$$

2. *When the steam is initially superheated*

We know that for superheated steam, $n = 1.3$.

$$\therefore \quad \frac{p_2}{p_1} = \left(\frac{2}{1.3+1}\right)^{\frac{1.3}{1.3-1}} = 0.546 \quad \text{or} \quad p_2 = 0.546\, p_1$$

3. *When the steam is initially wet*

It has been experimentally found that the critical pressure ratio for wet steam,

$$\frac{p_2}{p_1} = 0.582 \quad \text{or} \quad p_2 = 0.582\, p_1$$

4. *For gases*

We know that for gases, $n = 1.4$.

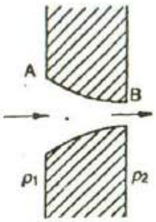
$$\therefore \quad \frac{p_2}{p_1} = \left(\frac{2}{1.4+1}\right)^{\frac{1.4}{1.4-1}} = 0.528 \quad \text{or} \quad p_2 = 0.528\, p_1$$

21.10. Physical Significance of Critical Pressure Ratio

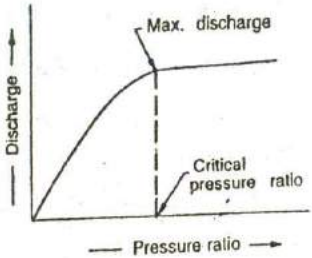
In the previous article, we discussed the values of critical pressure ratio for various forms of steam. But now we shall discuss the physical significance of the critical pressure ratio.

Now consider two vessels A and B connected by a convergent nozzle as shown in Fig. 21.4 (a). Let the vessel A contains steam at a high and steady pressure (p_1), and the vessel B contains steam at another pressure (p_2) which may be varied at will.

First of all, let the pressure (p_2) in the vessel *B* be made equal to the pressure (p_1) in the vessel *A*. In this case, there will be no flow of steam through the nozzle. Now if the pressure (p_2) in the vessel *B* is gradually reduced, the discharge through the nozzle will increase accordingly as shown in Fig. 21.4 (*b*). As the pressure (p_2) in the vessel *B* approaches the critical value, the rate of discharge will also approach its maximum value. If the pressure (p_2) in the vessel *B* is further reduced, it will not increase the rate of discharge. But the discharge will remain the same as that at critical pressure as shown in Fig. 21.4 (*b*). The ratio of exit pressure to the inlet pressure is called *critical pressure ratio.*



(*a*) Nozzle section.



(*b*) Discharge-pressure ratio curve.

Fig. 21.4

We know that the velocity of steam at any section in the nozzle [Refer Art. 21.6, equation (*iii*)],

$$V_2 = \sqrt{\frac{2n}{n-1} \times p_1 v_1 \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]} \quad \ldots (i)$$

and the critical pressure ratio for maximum discharge,

$$\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}} \text{ or } \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \frac{2}{n+1}$$

Substituting this value in equation (*i*),

$$V_2 = \sqrt{\frac{2n}{n-1} \times p_1 v_1 \left[1 - \frac{2}{n+1}\right]} = \sqrt{\frac{2n}{n-1} \times p_1 v_1 \left[\frac{n+1-2}{n+1}\right]}$$

$$= \sqrt{\frac{2n}{n+1} \times p_1 v_1} = \sqrt{\frac{2n}{n+1} \times \frac{p_1}{\rho_1}} \quad \ldots (ii)$$

$$\ldots \left[\because \text{Volume } (v) = \frac{1}{\text{Density } (\rho)}\right]$$

We also know that for isentropic expansion,

$$p_1 v_1^n = p_2 v_2^n$$

or

$$\frac{p_1}{\rho_1^n} = \frac{p_2}{\rho_2^n} \quad \ldots \left(\because v = \frac{1}{\rho}\right)$$

or

$$\frac{1}{\rho_1} = \frac{1}{\rho_2}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$\therefore \qquad \frac{p_1}{\rho_1} = \frac{p_1}{\rho_2}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} \qquad \text{(Multiplying by } p_1\text{)}$$

$$= \frac{p_2}{\rho_2} \times \frac{p_1}{p_2}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} = \frac{p_2}{\rho_2}\left(\frac{p_2}{p_1}\right)^{\frac{1-n}{n}}$$

$$= \frac{p_2}{\rho_2}\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}\times\frac{1-n}{n}} = \frac{p_2}{\rho_2}\left(\frac{n+1}{2}\right) \quad \ldots\left[\because \frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}\right]$$

Substituting the value of p_1/ρ_1, in equation (*ii*),

$$V_2 = \sqrt{2\left(\frac{n}{n+1}\right)\frac{p_2}{\rho_2}\left(\frac{n+1}{2}\right)} = \sqrt{\frac{n\,p_2}{\rho_2}} \qquad \ldots (iii)$$

This is the value of velocity of sound in the medium at pressure p_2 and is known as *sonic velocity*.

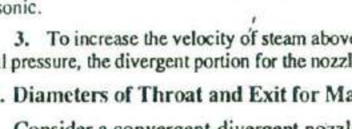
Notes : 1. The critical pressure gives the velocity of steam at the throat equal to the velocity of sound.

2. The flow in the convergent portion of the nozzle is sub-sonic and in the divergent portion it is supersonic.

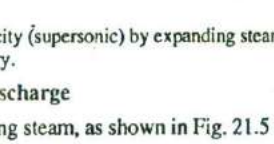
3. To increase the velocity of steam above sonic velocity (supersonic) by expanding steam below the critical pressure, the divergent portion for the nozzle is necessary.

21.11. Diameters of Throat and Exit for Maximum Discharge

Consider a convergent-divergent nozzle discharging steam, as shown in Fig. 21.5 (*a*).



(*a*) Convergent-divergent nozzle.



(*b*) *h-s* graph for a convergent-divergent nozzle.

Fig. 21.5

Let

p_1 = Initial pressure of steam,

h_1 = Enthalpy or total heat of steam at inlet,

p_2, p_3, h_2, h_3 = Corresponding values at throat and outlet,

x_2 = Dryness fraction of steam at throat,

V_2 = Velocity of steam at throat,

v_{g2} = Specific volume of steam at throat corresponding to pressure p_2 (from steam tables),

A_2 = Area of throat,

x_3, V_3, v_{g3}, A_3 = Corresponding values at exit, and

m = Mass of steam discharged.

First of all, find the value of critical pressure (p_2) as discussed in Art. 21.7.

Now complete the *h-s* diagram, as shown in Fig. 21.5 (*b*), for the expansion of steam through the convergent-divergent nozzle as discussed below :

1. First of all, locate the point *A* for the initial conditions of steam. It is a point, where the saturation line meets the initial pressure (p_1) line.
2. Now draw a vertical line through *A* to meet the critical pressure (p_2) line at *B*. This represents the throat of the nozzle.
3. Now extend the vertical line *AB* to meet the outlet pressure (p_3) line at *C*. This represents the outlet of the nozzle.
4. Now find the values of h_1, h_2, h_3, x_2 and x_3 from the *h-s* graph.

First of all, consider the flow of steam from the inlet to the throat. We know that

Enthalphy or heat drop, $h_{d2} = h_1 - h_2$

$\therefore$ Velocity of steam at throat,

$$V_2 = 44.72\sqrt{h_{d2}} \qquad \text{. . . (Neglecting friction)}$$

We know that mass of steam discharged per second,

$$m = \frac{\text{Volume of steam flowing at throat}}{\text{Volume of 1 kg of steam at pressure } p_2}$$

$$= \frac{A_2\, V_2}{v_2} = \frac{A_2\, V_2}{x_2\, v_{g2}} \qquad \text{. . . } (\because v_2 = x_2\, v_{g2})$$

Similarly, for exit conditions,

$$m = \frac{A_3\, V_3}{x_3\, v_{g3}} = \frac{A_2\, V_2}{x_2\, v_{g2}}$$

Now knowing the value of *m*, we can determine the area or diameter of throat and exit.

Example 21.8. *Steam enters a group of nozzles of a steam turbine at 12 bar and 220°C and leaves at 1.2 bar. The steam turbine develops 220 kW with a specific steam consumption of 13.5 kg/kWh. If the diameter of nozzles at throat is 7 mm, calculate the number of nozzles.*

Solution. Given : $p_1 = 12$ bar ; $T_1 = 220°C$; $p_3 = 1.2$ bar ; Power developed = 220 kW ; $m_s = 13.5$ kg/kWh ; $d_2 = 7$ mm

We know that for superheated steam, pressure of steam at throat,

$$p_2 = 0.546\, p_1 = 0.546 \times 12 = 6.552 \text{ bar}$$

The Mollier diagram for the expansion of steam through the nozzle is shown in Fig. 21.6.

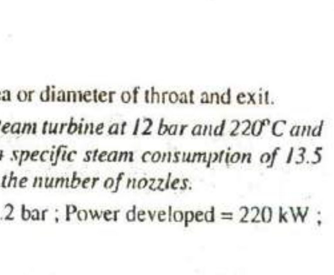


Fig. 21.6

From the Mollier diagram, we find that enthalpy of steam at entrance (*i.e.* at 12 bar and 220° C),

$$h_1 = 2860 \text{ kJ/kg}$$

Enthalpy of steam at throat (*i.e.* at pressure 6.552 bar),

$$h_2 = 2750 \text{ kJ/kg}$$

and dryness fraction of steam at throat,

$$x_2 = 0.992$$

From steam tables, we find that specific volume of dry saturated steam at throat (*i.e.* at pressure 6.552 bar),

$$v_{g2} = 0.29 \text{ m}^3/\text{kg}$$

We know that heat drop from entrance to throat,

$$h_{d2} = h_1 - h_2 = 2860 - 2750 = 110 \text{ kJ/kg}$$

$\therefore$ Velocity of steam at throat,

$$V_2 = 44.72\sqrt{h_{d2}} = 44.72\sqrt{110} = 470 \text{ m/s}$$

Area of nozzle at throat,

$$A_2 = \frac{\pi}{4}(d_2)^2 = \frac{\pi}{4} \times 7^2 = 38.5 \text{ mm}^2 = 38.5 \times 10^{-6} \text{ m}^2$$

$\therefore$ Mass flow rate per nozzle,

$$m = \frac{A_2 V_2}{v_2} = \frac{A_2 V_2}{x_2 v_{g2}} = \frac{38.5 \times 10^{-6} \times 470}{0.992 \times 0.29} = 0.063 \text{ kg/s}$$

We know that total mass flow rate

$$= 13.5 \times 220 = 2970 \text{ kg/h} = 0.825 \text{ kg/s}$$

$$\therefore \text{ Number of nozzles } = \frac{\text{Total mass flow rate}}{\text{Mass flow rate per nozzle}} = \frac{0.825}{0.063} = 13.1 \text{ say } 14 \textbf{ Ans.}$$

Example 21.9. *Estimate the mass flow rate of steam in a nozzle with the following data :*

Inlet pressure and temperature = 10 bar and 200° C ; Back pressure = 0.5 bar ; Throat diameter = 12 mm.

Solution. Given : $p_1 = 10$ bar ; $T_1 = 200°$ C ; $p_3 = 0.5$ bar ; $d_2 = 12$ mm

We know that for superheated steam, pressure of steam at throat,

$$p_2 = 0.546\, p_1 = 0.546 \times 10 = 5.46 \text{ bar}$$

The Mollier diagram for the expansion of steam through the nozzle is shown in Fig. 21.7. From the Mollier diagram, we find that

$$h_1 = 2825 \text{ kJ/kg}$$

$$h_2 = 2710 \text{ kJ/kg}$$

and $$x_2 = 0.982$$

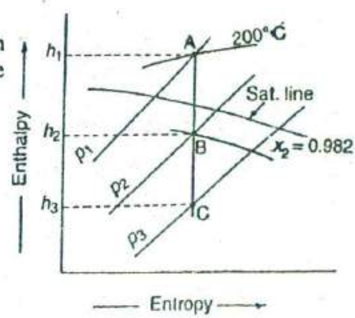


Fig. 21.7

We know that heat drop,

$$h_d = h_1 - h_2 = 2825 - 2710 = 115 \text{ kJ/kg}$$

$\therefore$ Velocity of steam at throat,

$$V_2 = 44.72\sqrt{h_d} = 44.72\sqrt{115} = 480 \text{ m/s}$$

From steam tables, corresponding to a pressure of 5.46 bar, we find that specific volume of steam at throat,

$$v_{g2} = 0.345 \text{ m}^3/\text{kg}$$

Area of nozzle at throat,

$$A_2 = \frac{\pi}{4}(d_2)^2 = \frac{\pi}{4}(12)^2 = 113 \text{ mm}^2 = 113 \times 10^{-6} \text{ m}^2$$

$\therefore$ Mass flow rate of steam,

$$m = \frac{A_2 V_2}{x_2 v_{g2}} = \frac{113 \times 10^{-6} \times 480}{0.982 \times 0.345} = 0.16 \text{ kg/s}$$

$$= 0.16 \times 3600 = 576 \text{ kg/h} \textbf{ Ans.}$$

Example 21.10. *Dry saturated steam enters a nozzle at a pressure of 10 bar and with an initial velocity of 90 m/s. The outlet pressure is 6 bar and the outlet velocity is 435 m/s. The heat loss from the nozzle is 9 kJ/kg of steam flow.*

Calculate the dryness fraction and the area at the exit, if the area at the inlet is 1256 mm².

Solution. Given : $p_1 = 10$ bar ; $V_1 = 90$ m/s ; $p_3 = 6$ bar ; $V_3 = 435$ m/s ; Loses = 9 kJ/kg ; $A_1 = 1256 \text{ mm}^2 = 1256 \times 10^{-6} \text{ m}^2$

Dryness fraction of steam

Let $\qquad x_3$ = Dryness fraction of steam at the exit.

From steam tables, corresponding to a pressure of 10 bar, we find that enthalpy of dry saturated steam,

$$h_1 = 2776.2 \text{ kJ/kg} \text{ ; and } v_{g1} = 0.1943 \text{ m}^3\text{/kg}$$

and corresponding to a pressure of 6 bar, we find that

$$h_{f3} = 670.4 \text{ kJ/kg} \text{ ; } h_{fg3} = 2085 \text{ kJ/kg} \text{ ; and } v_{g3} = 0.3155 \text{ m}^3\text{/kg}$$

We know that for a steady flow through the nozzle,

$$h_1 + \frac{1}{1000}\left(\frac{V_1^2}{2}\right) = h_3 + \frac{1}{1000}\left(\frac{V_3^2}{2}\right) + \text{Losses}$$

$$\therefore \qquad h_3 = h_1 + \frac{1}{2000}(V_1^2 - V_3^2) - \text{Losses}$$

$$= 2776.2 + \frac{1}{2000}\left[(90)^2 - (435)^2\right] - 9$$

$$= 2776.2 - 99.6 = 2676.6 \text{ kJ/kg}$$

We also know that enthalpy of wet steam (h_3),

$$2676.6 = h_{f3} + x_3 h_{fg3} = 670.4 + x_3 \times 2085$$

$$\therefore \qquad x_3 = 0.962 \textbf{ Ans.}$$

Area at exit

Let $\qquad A_3$ = Area at exit in m².

We know that $\qquad \dfrac{A_1 V_1}{x_1 v_{g1}} = \dfrac{A_3 V_3}{x_3 v_{g3}}$ or $\dfrac{1256 \times 10^{-6} \times 90}{1 \times 0.1943} = \dfrac{A_3 \times 435}{0.962 \times 0.3155}$

... (For dry saturated steam, $x_1 = 1$)

$$\therefore \qquad A_3 = 406 \times 10^{-6} \text{ m}^2 = 406 \text{ mm}^2 \textbf{ Ans.}$$

Example 21.11. *Dry saturated steam at a pressure of 8 bar enters a convergent-divergent nozzle and leaves it at a pressure of 1.5 bar, If the flow is isentropic, and the corresponding expansion index is 1.135 ; find the ratio of cross-sectional area at exit and throat for maximum discharge.*

Solution. Given : $p_1 = 8$ bar ; $p_3 = 1.5$ bar ; $n = 1.135$

Let A_2 = Cross-sectional area at throat,

A_3 = Cross-sectional area at exit, and

m = Mass of steam discharged per second.

We know that for dry saturated steam (or when n = 1.135), critical pressure ratio,

$$\frac{p_2}{p_1} = 0.577$$

$$\therefore \quad p_2 = 0.577\, p_1 = 0.577 \times 8 = 4.616 \text{ bar}$$

Now complete the Mollier diagram for the expansion of steam through the nozzle, as shown in Fig. 21.8.

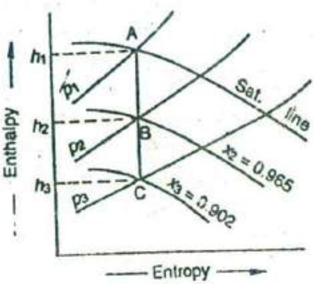


Fig. 21.8

From Mollier diagram, we find that

$h_1 = 2775$ kJ/kg ; $h_2 = 2650$ kJ/kg ; $h_3 = 2465$ kJ/kg ; $x_2 = 0.965$; and $x_3 = 0.902$

From steam tables, we also find that the specific volume of steam at throat corresponding to 4.616 bar,

$$v_{g2} = 0.405 \text{ m}^3/\text{kg}$$

and specific volume of steam at exit corresponding to 1.5 bar,

$$v_{g3} = 1.159 \text{ m}^3/\text{kg}$$

Heat drop between entrance and throat,

$$h_{d2} = h_1 - h_2 = 2775 - 2650 = 125 \text{ kJ/kg}$$

$\therefore$ Velocity of steam at throat,

$$V_2 = 44.72\sqrt{h_{d2}} = 44.72\sqrt{125} = 500 \text{ m/s}$$

and
$$m = \frac{A_2\, V_2}{x_2\, v_{g2}}$$

or
$$A_2 = \frac{m\, x_2\, v_{g2}}{V_2} = \frac{m \times 0.965 \times 0.405}{500} = 0.000\,786\, m \qquad \ldots (i)$$

Heat drop between entrance and exit,

$$h_{d3} = h_1 - h_3 = 2775 - 2465 = 310 \text{ kJ/kg}$$

$\therefore$ Velocity of steam at exit,

$$V_3 = 44.72\sqrt{h_{d3}} = 44.72\sqrt{310} = 787.4 \text{ m/s}$$

and
$$m = \frac{A_3\, V_3}{x_3\, v_{g3}}$$

or
$$A_3 = \frac{m\, x_3\, v_{g3}}{V_3} = \frac{m \times 0.902 \times 1.159}{787.4} = 0.001\,33\, m \qquad \ldots (ii)$$

$\therefore$ Ratio of cross-sectional area at exit and throat,

$$\frac{A_3}{A_2} = \frac{0.001\,33\, m}{0.000\,786\, m} = 1.7 \text{ Ans.}$$

Example 21.12. *A convergent-divergent nozzle is required to discharge 2 kg of steam per second. The nozzle is supplied with steam at 7 bar and 180° C and discharge takes place against a back pressure of 1 bar. The expansion upto throat is isentropic and the frictional resistance between the throat and exit is equivalent to 63 kJ/kg of steam. Taking approach velocity of 75 m/s and throat pressure of 4 bar, estimate* :

1. Suitable areas for the throat and exit ; and 2. Overall efficiency of the nozzle based on the enthalpy drop between the actual inlet pressure and temperature and the exit pressure.

Solution. Given : $m = 2$ kg/s ; $p_1 = 7$ bar ; $T_1 = 180°$ C ; $p_3 = 1$ bar ; Frictional resistance $= 63$ kJ/kg of steam ; $V_1 = 75$ m/s ; $p_2 = 4$ bar

1. *Suitable areas for the throat and exit*

Let A_2 = Area at the throat, and

A_3 = Area at the exit.

The expansion of steam through the nozzle on the Mollier diagram is shown in Fig. 21.9. From the Mollier diagram, we find that

$h_1 = 2810$ kJ/kg ; $h_2 = 2680$ kJ/kg ; $h_{3'} = 2470$ kJ/kg ; $x_2 = 0.97$; $x_3 = 0.934$

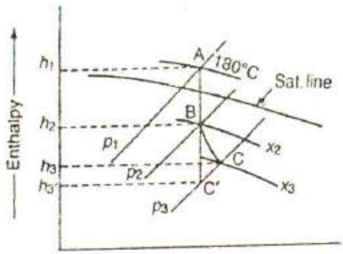


Fig. 21.9

From steam tables, we also find that the specific volume of steam at throat corresponding to 4 bar,

$$v_{g2} = 0.462 \text{ m}^3\text{/kg}$$

and specific volume of steam corresponding to 1 bar,

$$v_{g3} = 1.694 \text{ m}^3\text{/kg}$$

We know that heat drop between entrance and throat,

$$h_{d2} = h_1 - h_2 = 2810 - 2680 = 130 \text{ kJ/kg}$$

$\therefore$ Velocity of steam at throat,

$$V_2 = \sqrt{V_1^2 + 2000\, h_{d2}} = \sqrt{(75)^2 + 2000 \times 130} = 515 \text{ m/s}$$

and

$$m = \frac{A_2\, V_2}{x_2\, v_{g2}}$$

or

$$A_2 = \frac{m\, x_2\, v_{g2}}{V_2} = \frac{2 \times 0.97 \times 0.462}{515} = 1.74 \times 10^{-3} \text{ m}^2$$

$$= 1740 \text{ mm}^2 \quad \textbf{Ans.}$$

Since there is a frictional resistance of 63 kJ/kg of steam between the throat and exit, therefore

$$h_3 - h_{3'} = 63 \text{ or } h_3 = h_{3'} + 63 = 2470 + 63 = 2533 \text{ kJ/kg}$$

and heat drop between entrance and exit,

$$h_{d3} = h_1 - h_3 = 2810 - 2533 = 277 \text{ kJ/kg}$$

$\therefore$ Velocity of steam at exit,

$$V_3 = \sqrt{V_1^2 + 2000\, h_{d3}} = \sqrt{(75)^2 + 2000 \times 277} = 748 \text{ m/s}$$

and $$m = \frac{A_3 V_3}{x_3 v_{g3}}$$

or $$A_3 = \frac{m x_3 v_{g3}}{V_3} = \frac{2 \times 0.934 \times 1.694}{748} = 4.23 \times 10^{-3}\ \text{m}^2$$

$$= 4230\ \text{mm}^2\ \textbf{Ans.}$$

2. Overall efficiency of the nozzle

We know that overall efficiency of the nozzle,

$$\eta = \frac{\text{Useful heat drop}}{\text{Isentropic heat drop}} = \frac{h_1 - h_3}{h_1 - h_{3'}}$$

$$= \frac{2810 - 2533}{2810 - 2470} = 0.815 \text{ or } 81.5\ \%\ \textbf{Ans.}$$

Example 21.13. *Steam at a pressure of 10 bar and 0.9 dry discharges through a nozzle having throat area of 450 mm². If the back pressure is 1 bar, find 1. final velocity of the steam, and 2. cross-sectional area of the nozzle at exit for maximum discharge.*

Solution. Given : $p_1 = 10$ bar ; $x_1 = 0.9$; $A_2 = 450\ \text{mm}^2 = 450 \times 10^{-6}\ \text{m}^2$; $p_3 = 1$ bar

Final velocity of steam

Let V_3 = Final velocity of steam.

We know that for maximum discharge, pressure of steam at throat (for wet steam),

$$p_2 = 0.582\, p_1 = 0.582 \times 10 = 5.82\ \text{bar}$$

Now complete the Mollier diagram for the expansion of steam through the nozzle as shown in Fig. 21.10.

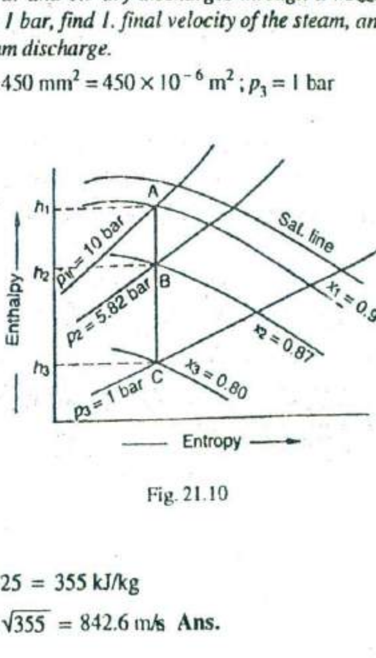


Fig. 21.10

From the Mollier diagram, we find that

$$h_1 = 2580\ \text{kJ/kg} ;\ h_2 = 2485\ \text{kJ/kg} ;$$

$$h_3 = 2225\ \text{kJ/kg} ;\ x_2 = 0.87 ;$$

and $$x_3 = 0.8$$

$\therefore$ Heat drop from entrance to exit,

$$h_{d3} = h_1 - h_3 = 2580 - 2225 = 355\ \text{kJ/kg}$$

and velocity of steam, $$V_3 = 44.72\sqrt{h_{d3}} = 44.72\sqrt{355} = 842.6\ \text{m/s}\ \textbf{Ans.}$$

Cress-sectional area of the nozzle at exit

Let A_3 = Cross-sectional area of the nozzle at exit.

From steam tables, we find that specific volume of steam at throat corresponding to a pressure of 5.82 bar,

$$v_{g2} = 0.3254\ \text{m}^3/\text{kg}$$

and specific volume of steam at exit corresponding to a pressure of 1 bar,

$$v_{g3} = 1.694\ \text{m}^3/\text{kg}$$

We know that heat drop from entrance to throat,

$$h_{d2} = h_1 - h_2 = 2580 - 2485 = 95\ \text{kJ/kg}$$

∴ Velocity of steam at throat,

$$V_2 = 44.72\sqrt{h_{d2}} = 44.72\sqrt{95} = 436 \text{ m/s}$$

*Since the mass flow rate is same at throat and exit, therefore

$$\frac{A_2 V_2}{x_2 v_{g2}} = \frac{A_3 V_3}{x_3 v_{g3}}$$

$$\frac{450 \times 10^{-6} \times 436}{0.87 \times 0.3254} = \frac{A_3 \times 842.6}{0.8 \times 1.694}$$

$$\therefore \qquad A_3 = 1114 \times 10^{-6} \text{ m}^2 = 1114 \text{ mm}^2 \text{ Ans.}$$

Example 21.14. *A gas expands in a convergent-divergent nozzle from 5 bar to 1.5 bar, the initial temperature being 700° C and the nozzle efficiency is 90%. All the losses take place after the throat. For 1 kg/s mass flow rate of the gas, find throat and exit areas.*

Take n = 1.4 and R = 287 J/kg K.

Solution. Given : $p_1 = 5$ bar ; $p_3 = 1.5$ bar ; $T_1 = 700°$ C $= 700 + 273 = 973$ K ; $K = 90\% = 0.9$; $m = 1$ kg/s ; $n = 1.4$; $R = 287$ J/kg K

Throat area

We know that pressure at the throat,

$$p_2 = 0.528 p_1 = 0.528 \times 5 = 2.64 \text{ bar}$$

and heat drop between entrance and throat,

$$h_{d2} = h_1 - h_2 = \frac{n}{n-1} \times p_1 v_1 \left[1 - \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right] \qquad \text{... (Refer Art. 21.6)}$$

$$= \frac{n}{n-1} \times m R T_1 \left[1 - \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right]$$

$$= \frac{1.4}{1.4-1} \times 1 \times 287 \times 973 \left[1 - \left(\frac{2.64}{5} \right)^{\frac{1.4-1}{1.4}} \right] \text{J/kg}$$

$$= 163\,220 \text{ J/kg} = 163.22 \text{ kJ/kg}$$

∴ Velocity of gas at the throat,

$$V_2 = 44.72\sqrt{h_{d2}} = 44.72\sqrt{163.22} = 571.3 \text{ m/s}$$

Let T_2 = Temperature of gas at the throat,

v_2 = Volume of gas at the throat, and

---

* This may also be found as discussed below :

We know that mass of steam discharged per second,

$$m = \frac{A_2 V_2}{x_2 v_{g2}} = \frac{450 \times 10^{-6} \times 436}{0.87 \times 0.3254} = 0.693 \text{ kg}$$

Similarly, $$0.693 = \frac{A_3 V_3}{x_3 v_{g3}} = \frac{A_3 \times 842.6}{0.8 \times 1.694} = 621.5 A_3$$

$$A_3 = 1114 \times 10^{-6} \text{ m}^2 = 1114 \text{ mm}^2 \text{ Ans.}$$

$$A_2 = \text{Area at the throat.}$$

We know that $$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}} = \left(\frac{5}{2.64}\right)^{\frac{1.4-1}{1.4}} = 1.2$$

$\therefore$ $$T_2 = T_1 / 1.2 = 973 / 1.2 = 810.8 \text{ K}$$

and $$p_2 v_2 = m R T_2$$

or $$v_2 = \frac{m R T_2}{p_2} = \frac{1 \times 287 \times 810.8}{2.64 \times 10^5} = 0.88 \text{ m}^3/\text{kg}$$

We know that $$m = \frac{A_2 V_2}{v_2}$$

or $$A_2 = \frac{m v_2}{V_2} = \frac{1 \times 0.88}{571.3} = 1.54 \times 10^{-3} \text{ m}^2 = 1540 \text{ mm}^2 \textbf{ Ans.}$$

Exit area

We know that heat drop between entrance and exit,

$$h_{d3} = h_1 - h_3 = \frac{n}{n-1} \times m R T_1 \left[1 - \left(\frac{p_3}{p_1}\right)^{\frac{n-1}{n}} \right]$$

$$= \frac{1.4}{1.4-1} \times 1 \times 287 \times 973 \left[1 - \left(\frac{1.5}{5}\right)^{\frac{1.4-1}{1.4}} \right] \text{J/kg}$$

$$= 284\,420 \text{ J/kg} = 284.42 \text{ kJ/kg}$$

$\therefore$ Velocity of gas at exit,

$$V_3 = 44.72 \sqrt{K h_{d3}} = 44.72 \sqrt{0.9 \times 284.42} = 715.5 \text{ m/s}$$

. . . ($\because$ losses takes place after throat)

Let $T_{3'}$ = Temperature of gas at exit when friction is neglected,

T_3 = Temperature of gas at exit when friction is considered,

v_3 = Volume of gas at exit, and

A_3 = Area at exit.

We know that $$\frac{T_1}{T_{3'}} = \left(\frac{p_1}{p_3}\right)^{\frac{n-1}{n}} = \left(\frac{5}{1.5}\right)^{\frac{1.4-1}{1.4}} = 1.41$$

$\therefore$ $$T_{3'} = T_1 / 1.41 = 973 / 1.41 = 690 \text{ K}$$

Since the nozzle efficiency is 90% (*i.e.* $K = 0.9$), therefore heat drop lost in friction is

$$= (1 - K)\, h_{d3} = (1 - 0.9)\, 284.42 = 28.442 \text{ kJ/kg}$$

$\therefore$ Increase in temperature due to friction

$$= {}^{*}28.442 \text{ K}$$

$$\therefore \qquad T_3 = 690 + 28.442 = 718.442 \text{ K}$$

and
$$p_3 v_3 = m R T_3$$

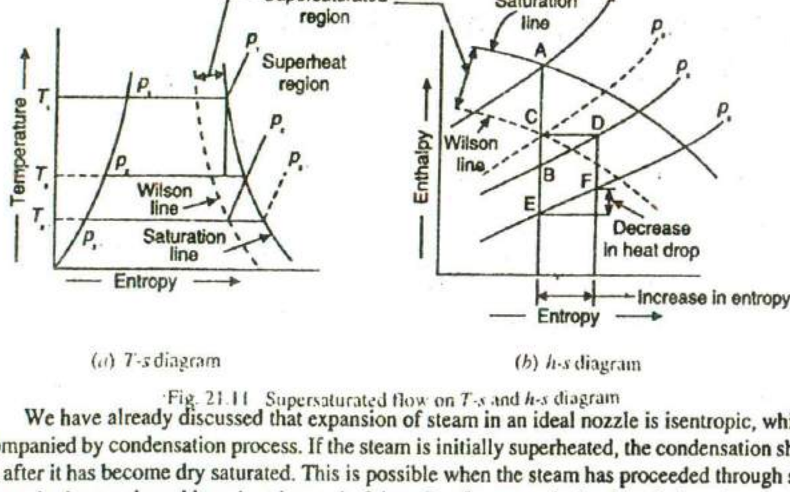
or
$$v_3 = \frac{m R T_3}{p_3} = \frac{1 \times 287 \times 718.442}{1.5 \times 10^5} = 1.375 \text{ m}^3\text{/kg}$$

We know that
$$m = \frac{A_3 V_3}{v_3}$$

or
$$A_3 = \frac{m v_3}{V_3} = \frac{1 \times 1.375}{715.5} = 1.92 \times 10^{-3} \text{ m}^2 = 1920 \text{ mm}^2 \textbf{ Ans.}$$

21.12. Supersaturated Flow or Metastable Flow through Nozzle

When dry saturated steam is expanded adiabatically or isentropically, it becomes wet and is shown by a vertical line on Mollier diagram.



(a) T-s diagram (b) h-s diagram

Fig. 21.11 Supersaturated flow on T-s and h-s diagram

We have already discussed that expansion of steam in an ideal nozzle is isentropic, which is accompanied by condensation process. If the steam is initially superheated, the condensation should start after it has become dry saturated. This is possible when the steam has proceeded through some distance in the nozzle and in a short interval of time. But from practical point of view, the steam has a great velocity (sometimes sonic and even supersonic). Thus the phenomenon of condensation does not take place at the expected rate. As a result of this, equilibrium between the liquid and vapour phase is delayed and the steam continues to expand in a dry state. The steam in such a set of conditions, is said to be *supersaturated* or in *metastable state.* It is also called *supercooled steam,* as its temperature at any pressure is less than the saturation temperature corresponding to the pressure. The flow of supersaturated steam, through the nozzle is called *supersaturated flow* or *metastable flow.*

---

* We know that heat drop lost in friction

$$= \text{Mass} \times \text{Sp. heat} \times \text{Increase in temp.}$$

$$\text{Increase in temp} = \frac{\text{Heat drop lost in friction}}{\text{Mass} \times \text{Sp. heat}} = \frac{28.442}{1 \times 1} = 28.442 \text{ K}$$

($\because$ Sp. heat is taken as 1 kJ/kg K)

Experiments of supersaturated flow of steam have shown that there is a limit to which the supersaturated flow is possible. This limit is represented by *Wilson line on $T-s$ and h-s diagram as shown in Fig. 21.11 (*a*) and (*b*) respectively. It may be noted that the Willson line closely follows the 0.97 dryness fraction line. Beyond this Wilson line, there is no supersaturation. The steam suddenly condenses and restores its normal equilibrium state.

In Fig. 21.11 (*b*) is shown the isentropic expansion of steam in a nozzle. The point A represents the position of initial dry saturated steam at pressure p_1. The line AC represents the isentropic expansion of steam in the supersaturated region. The metastable state (point C) is obtained by drawing a vertical line through A to meet the Wilson line. At C, the steam condenses suddenly. The line CD represents the condensation of steam at constant enthalpy. The point D is obtained by drawing a horizontal line through C to meet the throat pressure (p_2) of the nozzle. The line DF represents the isentropic expansion of steam in the divergent portion.

Notes : 1. The same theory is applicable, if the steam is initially superheated.

2. The difference of supersaturated temperature and saturation temperature at that pressure is known as *degree of undercooling*. Mathematically, degree of undercooling

$$= T_2 - T_2'$$

3. The ratio of pressures corresponding to temperatures T_2 and T_2' is known as *degree of supersaturation*. Mathematically, degree of supersaturation

$$= \frac{\text{Pressure corresponding to } T_2}{\text{Pressure corresponding to } T_2'} = \frac{p_2}{p_2'}$$

4. The following relations may be used in solving problem on supersaturated flow.

(*i*) $v = \dfrac{0.0023\,(h-1943)}{p}$ (*ii*) $pv^{1.3} = \text{Constant}$; and (*iii*) $\dfrac{p}{T^{13/3}} = \text{Constant}$

where v = Volume of steam in m³/kg,

p = Pressure of steam in bar,

h = Enthalpy or total heat of steam in kJ/kg, and

T = Absolute temperature of supersaturated steam in K.

21.13. Effects of Supersaturation

The following effects in a nozzle, in which supersaturation occurs, are important from the subject point of view ;

1. Since the condensation does not take place during supersaturated expansion , so the temperature at which the supersaturation occurs will be *less* than the saturation temperature corresponding to the pressure. Therefore, the density of supersaturated steam will be more** than for the equilibrium conditions, which gives the increase in the mass of steam discharged.

2. The supersaturation increases the entropy and specific volume of the steam.

3. The supersaturation reduces the heat drop (for the same pressure limits) below that for thermal equilibrium. Hence the exit velocity of the steam is reduced.

4. The supersaturation increases dryness fraction of steam.

Example 21.15. *The dry saturated steam is expanded in a nozzle from pressure of 10 bar to a pressure of 5 bar. If the expansion is supersaturated, find : 1. the degree of undercooling ; and 2. the degree of supersaturation.*

---

* The limit of supersaturated expansion was first shown by the experiments done by C.T.R. Wilson in 1857. The subsequent work by H.M. Martin has enabled a curve which was termed by him as the Wilson line.

** It has been found that the density of supersaturated steam is about *eight times* that of the ordinary saturated vapour at the corresponding pressure.

Solution. Given : $p_1 = 10$ bar ; $p_2 = 5$ bar

1. *Degree of undercooling*

From steam tables, corresponding to a pressure of 10 bar, we find that the initial temperature of steam,

$$T_1 = 179.9°\text{C} = 179.9 + 273 = 452.9 \text{ K}$$

Let T_2' = Temperatureat which supersaturation occurs.

We know that for supersaturated expansion,

$$\frac{p_1}{(T_1)^{13/3}} = \frac{p_2}{(T_2')^{13/3}} \quad \text{or} \quad \frac{T_2'}{T_1} = \left(\frac{p_2}{p_1}\right)^{3/13} = \left(\frac{5}{10}\right)^{3/13} = 0.852$$

$$\therefore \quad T_2' = T_1 \times 0.852 = 452.9 \times 0.852 = 385.9 \text{ K}$$

$$= 385.9 - 273 = 112.9° \text{C}$$

From steam tables, corresponding to a pressure of 5 bar, we find that the saturation temperature,

$$T_2 = 151.9°\text{C}$$

$\therefore$ Degree of undercooling $= T_2 - T_2' = 151.9 - 112.9 = 39°\text{C}$ **Ans.**

2. *Degree of supersaturation*

From steam tables, corresponding to a temperature of 112.9° C, we find that

$$p_2' = 1.584 \text{ bar}$$

$\therefore$ Degree of supersaturation

$$= p_2 / p_2' = 5 / 1.584 = 3.16 \text{ Ans.}$$

Example 21.16. *Find the percentage increase in discharge from a convergent-divergent nozzle expanding steam from 8.75 bar dry to 2 bar, when ; 1. the expansion is taking place under thermal equilibrium, and 2. the steam is in metastable state during part of its expansion.*

Take area of nozzle as 2500 mm².

Solution. Given : $p_1 = 8.75$ bar ; $p_2 = 2$ bar ; $A_2 = 2500 \text{ mm}^2 = 2500 \times 10^{-6} \text{ m}^2$

1. *Mass of steam discharged when the expansion is under thermal equilibrium*

Let m_1 = Mass of steam discharged.

The expansion of steam under conditions of *thermal equilibrium is shown on Mollier diagram as in Fig. 21.12.

From Mollier diagram, we find that

$h_1 = 2770$ kJ/kg ; $h_2 = 2515$ kJ/kg ; and $x_2 = 0.91$

From steam tables, at a pressure of 2 bar, we find that the specific volume of steam at exit,

$$v_{g2} = 0.885 \text{ m}^3/\text{kg}$$

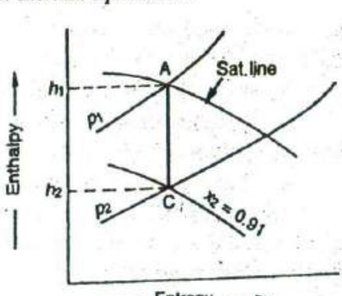


Fig. 21.12

*Thermal equilibrium means that the flow of steam of isentropic.

We know that heat drop from inlet to exit,

$$h_{d2} = h_1 - h_2 = 2770 - 2515 = 255 \text{ kJ/kg}$$

$\therefore$ Velocity of steam at exit,

$$V_2 = 44.72\sqrt{h_{d2}} = 44.72\sqrt{255} = 714 \text{ m/s}$$

and
$$m_1 = \frac{A_2 V_2}{v_2} = \frac{A_2 V_2}{x_2 v_{g2}} = \frac{2500 \times 10^{-6} \times 714}{0.91 \times 0.885} = 2.21 \text{ kg/s} \quad \textbf{Ans.}$$

2. *Mass of steam discharged when it is in *metastable state*

Let m_2 = Mass of steam discharged.

We know that volume of steam at inlet,

$$v_1 = \frac{0.0023\,(h_1 - 1943)}{p_1} = \frac{0.0023(2770 - 1943)}{8.75} = 0.217 \text{ m}^3\text{/kg}$$

and volume of steam at exit,

$$v_2 = v_1\left(\frac{p_1}{p_2}\right)^{1/1.3} = 0.217\left(\frac{8.75}{2}\right)^{1/1.3} = 0.675 \text{ m}^3\text{/kg}$$

$$\ldots(\because p_1 v_1^{1.3} = p_2 v_2^{1.3})$$

We know that volume of steam at exit (v_2),

$$0.675 = \frac{0.0023\,(h_2 - 1943)}{p_2} = \frac{0.0023\,(h_2 - 1943)}{2}$$

$$\therefore \quad h_2 = 2530 \text{ kJ/kg}$$

We know that heat drop from inlet to exit,

$$h_{d2} = h_1 - h_2 = 2770 - 2530 = 240 \text{ kJ/kg}$$

$\therefore$ Velocity of steam at exit,

$$V_2 = 44.72\sqrt{h_{d2}} = 44.72\sqrt{240} = 693 \text{ m/s}$$

and
$$m_2 = \frac{A_2 V_2}{v_2} = \frac{2500 \times 10^{-6} \times 693}{0.675} = 2.57 \text{ kg/s} \quad \textbf{Ans.}$$

$\therefore$ Percentage increase in discharge

$$= \frac{m_2 - m_1}{m_1} = \frac{2.57 - 2.21}{2.21} = 0.163 \text{ or } 16.3\,\% \quad \textbf{Ans.}$$

21.14. Steam Injector

The principle of a steam nozzle may also be applied to a steam injector. It utilises the kinetic energy of a steam jet for increasing the pressure and velocity of water. It is mostly used for forcing the feed water into steam boilers under pressure. The action of a steam injector is shown in Fig. 21.13.

The steam from the boiler is expanded to a high velocity by passing it through a convergent nozzle *A*. The steam jet enters the mixing cone and imparts its momentum to the incoming water supply from the feed tank** The cold water causes the steam to condense. The resulting jet at *B*,

* The problems on metastable flow cannot be solved by Mollier diagram unless Wilson line is drawn.

** The feed tank may be above or below the level of the steam injector.

formed by the steam and water is at atmospheric pressure, and has a large velocity. The mixture then enters delivery pipe at C through a diverging cone or diffuser, in which the kinetic energy is reduced and converted into pressure energy. This pressure energy is sufficient to overcome the boiler pressure

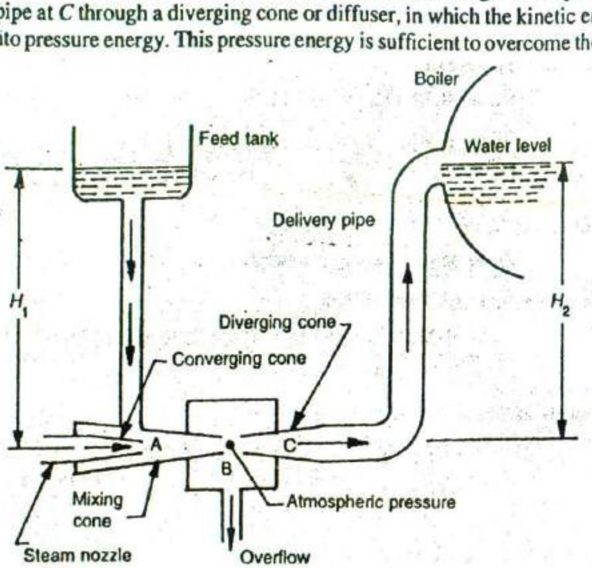


Fig. 21.13 Steam injector

and to lift the water through a height H_2. The pressure of water on leaving the delivery pipe must be about 20% higher than the boiler pressure in order to overcome all resistances. The gap between the mixing cone and diverging cone is provided with an outlet through which any excess water may overflow during the starting of the injector.

21.15. Steam Injector Calculations

The following calculations of a steam injector are important from the subject point of view :

1. *Amount of water injected*

Let m_w = Mass of water entering the mixing cone in kg/kg of steam,

V_s = Velocity of steam leaving the converging cone or nozzle,

V_w = Velocity of water entering the mixing cone, and

V_m = Velocity of the mixture leaving the mixing cone.

According to the principle of conservation of momentum,

Momentum of steam + Momentum of water = Momentum of mixture

$$1 \times V_s + m_w V_w = (1 + m_w) V_m = V_m + m_w V_m \qquad \ldots (i)$$

or
$$V_s - V_m = m_w (V_m - V_w)$$

$\therefore$
$$m_w = \frac{(V_s - V_m)}{(V_m - V_w)} \text{ kg/kg of steam}$$

Note : In case the feed tank is below the level of the steam injector, the equation (*i*) may be written as :

$$1 \times V_s - m_w V_w = (1 + m_w) V_m$$

or
$$m_w = \frac{(V_s - V_m)}{(V_m + V_w)} \text{ kg/kg ofsteam}$$

2. *Velocity of steam leaving the nozzle*

Let p_1 = Initial pressure of steam,

p_2 = Pressure of steam leaving the nozzle at A, and

h_d = Isentropic heat drop.

$\therefore$ Velocity of steam leaving the nozzle,

$$V_s = 44.72\sqrt{h_d}$$

3. *Velocity of water entering the mixing cone*

Let H_1 = Height of water in the feed tank from the level of steam injectoı

$\therefore$ Velocity of water entering the mixing cone,

$$V_w = \sqrt{2gH_1}$$

4. *Velocity of mixture leaving the mixing cone*

Let p_1 = Pressure of steam in the boiler in bar,

p_m = Pressure of the mixture leaving the mixing cone at B in bar,

ρ = Density of the mixture at B in kg/m^3, and

V_m = Velocity of mixture at B.

$\therefore$ Total energy per kg of water at B

$$= \frac{10^5 p_m}{\rho} + \frac{V_m^2}{2} \text{ (in joules)} \qquad \ldots (i)$$

This energy must be sufficient to lift the water through a height H_2 metres and inject it into ıe boiler. The final pressure head on leaving at C, must be somewhat greater than the height H_2 *plus* ıe boiler pressure. If H is the necessary excess head in metres, then

Total energy per kg of water at B

$$= \frac{10^5 p_1}{\rho} + g\,(H_2 + H) \text{ (in joules)} \qquad \ldots (ii)$$

Equating equations (i) and (ii),

$$\frac{10^5 p_m}{\rho} + \frac{V_m^2}{2} = \frac{10^5 p_1}{\rho} + g\,(H_2 + H)$$

ɔr

$$V_m = \sqrt{\frac{2 \times 10^5 (p_1 - p_m)}{\rho} + 2g\,(H_2 + H)} \qquad \ldots (iii)$$

We have already discussed that the pressure of the mixture leaving the mixing cone is atmospheric. Therefore, taking the value of p_m as 1.013 bar and assuming the density of the mixture as 1000 kg/m^3 (equal to density of water), we get

$$V_m = \sqrt{\frac{2 \times 10^5 (p_1 - 1.013)}{1000} + 2g\,(H_2 + H)}$$

$$= 4.43\sqrt{10.2\,(p_1 - 1.013) + H_2 + H}$$

Note : If V_d is the velocity in the delivery pipe, then

$$H = \frac{V_d^2}{2g}$$

5. *Nozzle areas*

Let A_a = Area of steam nozzle at A,

A_b = Area of combining nozzle (or mixing cone) at B,

V_s = Velocity of steam leaving the converging cone,

v_a = Specific volume of steam after expansion in nozzle at A,

m = Mass of water required to be delivered in kg/s, and

m_w = Mass of water entering the mixing cone in kg/kg of steam.

$\therefore$ Mass of steam supplied,

$$m_s = \frac{A_a V_s}{v_a} = \frac{m}{m_w}$$

or

$$A_a = \frac{m}{m_w} \times \frac{v_a}{V_s}$$

Also

$$m_s + m = A_b V_m \rho$$

or

$$A_b = \frac{m_s + m}{1000 V_m} = \frac{m\left(1 + \frac{1}{m_w}\right)}{1000 V_m} \quad \ldots (\because \rho = 1000 \text{ kg/m}^3)$$

6. *Heat balance per kg of steam*

Let h = Enthalpy or total heat of steam entering the injector in kJ,

t_w = Temperature of water in feed tank in °C,

h_{fw} = Sensible heat of water supplied to the injector, corresponding to a temperature of t_w in kJ/kg,

t_w = Temperature of water leaving the mixing cone at B in °C, and

h_{fm} = Sensible heat of water leaving the mixing cone at B, corresponding to a temperature of t_m in kJ/kg.

Then heat supplied in steam + Heat supplied in water + Kinetic energy of water

= Heat in mixture + Kinetic energy of mixture

$$h + m_w h_{fw} \pm \frac{m_w V_w^2}{2000} = (1 + m_w) h_{fm} + \frac{(1 + m_w) V_m^2}{2000}$$

From this equation, the value of h_{fm} may be determined and hence the temperature of the mixture t_m is known.

Note : In the above equation *plus* sign is used when feed tank is above the level of the steam injector, while *negative* sign is used when it is below the level of injector.

Example 21.17. *An injector is required to deliver 120 kg of water per minute from a tank, whose constant water level is 3 m below the level of injector, into a boiler in which the steam pressure is 15 bar. The water level in the boiler is 0.7 metre above the level of the injector. The steam for the injector is taken from the same boiler and it is assumed to be dry and saturated. The pressure of steam leaving steam nozzle is 0.6 times that of the supply pressure. The temperature of the water in the feed tank is 25° C. If the velocity in the delivery pipe is 15 m/s, find :*

1. Mass of water injected per kg of steam, 2. Area of mixing cone, 3. Area of steam nozzle, and 4. Temperature of water leaving the injector

Solution. Given : $m = 120$ kg/min $= 2$ kg/s ; $H_1 = 3$ m below the level of injector ; $p_1 = 15$ bar ; $H_2 = 0.7$ m above the level of injector ; $p_2 = 0.6\,p_1 = 0.6 \times 15 = 9$ bar ; $t_w = 25°$ C ; $V_d = 15$ m/s

1. *Mass of water injected per kg of steam*

From Mollier chart, the isentropic heat drop between pressure 15 bar dry and 9 bar,

$$h_d = h_1 - h_2 = 2795 - 2700 = 95 \text{ kJ/kg}$$

and dryness fraction of steam after expansion,

$$x_2 = 0.965$$

We know that velocity of steam leaving the nozzle,

$$V_s = 44.72\sqrt{h_d} = 44.72\sqrt{95} = 436 \text{ m/s}$$

We know that velocity of water entering the mixing cone,

$$V_w = \sqrt{2gH_1} = \sqrt{2 \times 9.81 \times 3} = 7.67 \text{ m/s}$$

and velocity of mixture leaving the mixing cone,

$$V_m = 4.43\sqrt{10.2\,(p_1 - 1.013) + H_2 + H}$$

$$= 4.43\sqrt{10.2\,(15 - 1.013) + 0.7 + 11.47} = 55 \text{ m/s}$$

$$\ldots\left[\because H = \frac{V_d^2}{2g} = \frac{15^2}{2 \times 9.81} = 11.47 \text{ m}\right]$$

$\therefore$ Mass of water injected per kg of steam,

$$m_w = \frac{V_s - V_m}{V_m + V_w} = \frac{436 - 55}{55 + 7.67} = 6.08 \text{ kg } \textbf{Ans.}$$

2. *Area of mixing cone*

We know that area of the mixing cone

$$A_b = \frac{m\left(1 + \frac{1}{m_w}\right)}{1000\,V_m} = \frac{2\left(1 + \frac{1}{6.08}\right)}{1000 \times 55} = 42.3 \times 10^{-6} \text{ m}^2$$

$$= 42.3 \text{ mm}^2 \textbf{ Ans.}$$

3. *Area of steam nozzle*

From steam tables, corresponding to a pressure of 9 bar, we find that specific volume of steam,

$$v_{ga} = 0.2148 \text{ m}^3\text{/kg}$$

We know that area of steam nozzle,

$$A_a = \frac{m}{m_w} \times \frac{v_a}{V_s} = \frac{m}{m_w} \times \frac{x_2\,v_{ga}}{V_s} = \frac{2}{6.08} \times \frac{0.965 \times 0.2148}{436}$$

$$= 156 \times 10^{-6} \text{ m}^2 = 156 \text{ mm}^2 \textbf{ Ans.}$$

4. *Temperature of water leaving the injector*

From steam tables, corresponding to a temperature of $t_w = 25°$ C, we find that

$$h_{fw} = 104.8 \text{ kJ/kg}$$

We know that $\quad h_1 + m_w h_{fw} - \dfrac{m_w V_w^2}{2000} = (1 + m_w)\,h_{fm} + \dfrac{(1 + m_w)\,V_m^2}{2000}$

$$2795 + 6.08 \times 104.8 - \frac{6.08\,(7.67)^2}{2000} = (1 + 6.08)\,h_{fm} + \frac{(1 + 6.08)\,(55)^2}{2000}$$

$$3432 = 7.08\,h_{fm} + 10.71$$

or
$$h_{fm} = 483.2 \text{ kJ/kg}$$

$\therefore$ Temperature of water leaving the injector (from steam tables corresponding to 483.2 kJ/kg),

$$t_m = 115^\circ\text{C} \ \textbf{Ans.}$$

EXERCISES

1. The dry and saturated steam at a pressure of 5 bar is expanded isentropically in a nozzle to a pressure of 0.2 bar. Find the velocity of steam leaving the nozzle. **[Ans.** 1000 m/s**]**

2. The dry and saturated steam at a pressure of 10.5 bar is expanded isentropically in a nozzle to a pressure of 0.7 bar. Determine the final velocity of the steam issuing from the nozzle, when (*a*) friction is neglected, and (*b*) 10% of the heat drop is lost in friction.

 The initial velocity of steam may be neglected. **[Ans.** (*a*) 905 m/s; (*b*) 859 m/s**]**

3. Steam at a pressure of 6.3 bar and 200° C is expanded in a nozzle to a pressure of 0.2 bar. Find the final velocity and dryness fraction of steam, if

 (*a*) friction is neglected ; and (*b*) 10% of the heat drop is lost in friction.

[Ans. (*a*) 1039 m/s, 0.83 ; (*b*) 996 m/s, 0.852**]**

4. Steam is supplied to a nozzle at 3.5 bar and 0.96 dry. The steam enters the nozzle at 240 m/s. The pressure drops to 0.8 bar. Determine the velocity and dryness fraction of the steam when it leaves the nozzle.

[Ans. 545.5 m/s ; 0.92**]**

5. Steam expands through an ideally, insulated nozzle following a reversible polytropic law $p\,v^{1.2} = C$. There is no change in potential energy but the pressure drops from 20 bar to 2 bar and the specific volume increases from 0.05 m^3 to 0.3 m^3. If the entrance velocity is 80 m/s, determine the exit velocity.

[Ans. 697.5 m/s**]**

6. Calculate the throat area of nozzle supplied with steam at 10 bar and 200°C. The rate of flow of steam is 1.2 kg/s. Neglect friction and assume the velocity at inlet to be small. **[Ans.** 837 mm^2**]**

7. Steam expands isentropically from the state of 8 bar and 250° C to 1.5 bar in a convergent-divergent nozzle. The steam flow rate is 0.75 kg/s. Find : 1. the velocity of steam at exit from the nozzle ; and 2. the exit area of nozzle. Neglect the inlet velocity of steam. **[Ans.** 800 m/s ; 1054 mm^2**]**

8. Steam enters a group of convergent-divergent nozzles at a pressure of 22 bar and with a temperature of 240° C. The exit pressure is 4 bar and 9% of the total heat drop is lost in friction. The mass flow rate is 10 kg/s and the flow upto the throat may be assumed friction less. Calculate

 1. the throat and exit velocities, and 2. the throat and exit areas.

[Ans. 529 m/s, 775 m/s : 3000 mm^2, 5500 mm^2**]**

9. The throat diameter of a nozzle is 5 mm. If dry and saturated steam at 10 bar is supplied to the nozzle, calculate the mass flow per second. The exhaust pressure is 1.5 bar. Assume friction less adiabatic flow and index of expansion, $n = 1.135$.

 If 10 percent of the isentropic heat drop is lost in friction, what should be the correct diameter at outlet for steam to issue at the same exhaust pressure ? **[Ans.** 103.3 kg/h ; 7.12 mm**]**

10. Calculate the throat and exit diameters of a convergent divergent nozzle which will discharge 820 kg of steam per hour from a pressure of 8 bar superheated to 220° C into a chamber having a pressure of 1.5 bar. The friction loss in the divergent part of the nozzle may be taken as 0.15 of the total enthalpy drop.

[Ans. 15.2 mm ; 20.6 mm**]**

11. A steam turbine develops 185 kW with a consumption of 16.5 kg/kW/h. The pressure and temperature of the steam entering the nozzle are 12 bar and 220° C. The steam leaves the nozzle at 1.2 bar. The diameter of the nozzle at throat is 7 mm. Find the number of nozzles.

If 8% of the total enthalpy drop is lost in friction in the diverging part of the nozzle, determine the diameter at the exit of the nozzle and the exit velocity of the leaving steam.

Sketch the skeleton Mollier diagram and show on it the values of pressure, temperature or dryness fraction, enthalpy and specific volume at inlet, throat and exit. **[Ans.** 14 ; 11.1 mm ; 847 m/s]

12. Steam expands in a nozzle under the following conditions : Inlet pressure = 15 bar ; Inlet temperature = 250° C; Final pressure = 4 bar ; Mass flow = 1 kg/s.

Calculate the required throat and exit areas, using Mollier diagram, when 1. the expansion is frictionless, and 2. the friction loss at any pressure amounts to 10 percent of the total heat drop down to that pressure. **[Ans.** 480 mm^2, 606 mm^2 ; 508 mm^2, 650 mm^2]

13. Gases expand in a convergent divergent nozzle from 3.6 bar and 425° C to a back pressure of 1 bar, at the rate of 18 kg/s. If the nozzle efficiency is 0.92, calculate the required throat and exit areas of the nozzle. Neglect inlet velocity and friction in the convergent part. For the gases, take c_p = 1.113 kJ/kg K and γ = 1.33.

[Ans. 0.0325 m^2 ; 0.04 m^2]

14. The dry saturated steam expands in a nozzle from a pressure of 2 bar to 1 bar. If the expansion is supersaturated, determine the degree of undercooling and the degree of supersaturation. **[Ans.** 37.63°C ; 4.58]

15. Steam at 42 bar and 260° C enters a nozzle and leaves at 28 bar. Neglecting initial kinetic energy and considering super-saturation, determine the discharge area for a flow of 10 350 kg/h and a nozzle velocity coefficient of 96%. **[Ans.** 484 mm^2]

16. Compare the mass of discharge from a convergent-divergent nozzle expanding from 8 bar and 210° C to 2 bar, when

1. the expansion takes place under thermal equilibrium, and 2. the steam is in super-saturated condition during a part of its expansion.

Take area of nozzle as 2400 mm^2. **[Ans.** 8.3%]

17. An injector is to deliver 100 kg of water per minute from a tank, whose constant water level is 1.2 m below the level of the injector into a boiler in which the steam pressure is 14 bar. The water level in the boiler is 1.5 metre above the level of the injector. The steam for the injector is taken from the same boiler and it is assumed to be dry and saturated. The pressure of steam leaving steam nozzle is 0.5 times that of the supply pressure. If the velocity in the delivery pipe is 13.5 m/s, find :

1. Mass of water pumped per kg of steam ; 2. Area of mixing cone ; 3. Area of steam nozzle ; and 4. Temperature of water leaving the injector, if the temperature of water in the feed tank is 15° C.

[Ans. 7.9 kg ; 35.5 mm^2 ; 107.4 mm^2 ; 87.8° C]

QUESTIONS

1. Explain the function of nozzles used with steam turbines.

2. Discuss the functions of the convergent portion, the throat and the divergent portion of a convergent-divergent nozzle with reference to flow of steam.

3. What is steady flow energy equation as applied to steam nozzles ? Explain its use in the calculation of steam velocity at the exit of a nozzle.

4. Discuss the effect of friction during the expansion of steam through a convergent-divergent nozzle when

   (*i*) the steam at entry to the nozzle is saturated, and

   (*ii*) the steam at entry is superheated.

   Assume the pressure of steam to be initially same in both the cases. Mark the processes on a sketch of enthalpy-entropy diagram.

5. Explain what is meant by critical pressure ratio of a nozzle.

6. Starting from fundamentals, show that for maximum discharge through a nozzle, the ratio of throat pressure to inlet pressure is given by $\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$ where n is the index for isentropic expansion through the nozzle.

7. Derive an expression for maximum discharge through convergent divergent nozzle for steam.

8. Draw the 'discharge' versus 'ratio of pressures at outlet to inlet' curve for a convergent steam nozzle. Discuss the physical significance of critical pressure ratio.

9. Explain the supersaturated or metastable flow of steam through a nozzle and the significance of Wilson's line.

10. What are the effects of supersaturation on discharge and heat drop ?

OBJECTIVE TYPE QUESTIONS

1. The steam leaves the nozzle at a
   (*a*) high pressure and low velocity (*b*) high pressure and high velocity
   (*c*) low pressure and low velocity (*d*) low pressure and high velocity
2. The effect of friction in a nozzle ...... dryness fraction of steam.
   (*a*) increases (*b*) decreases
3. The velocity of steam leaving the nozzle (V) is given by
   (*a*) $V = 44.72\, K\, h_d$ (*b*) $V = 44.72\, K \sqrt{h_d}$
   (*c*) $V = 44.72 \sqrt{K\, h_d}$ (*d*) $V = 44.72\, h_d \sqrt{K}$

where K = Nozzle coefficient, and
h_d = Enthalpy drop during expansion.

4. The critical pressure ratio for initially dry saturated steam is
   (*a*) 0.528 (*b*) 0.546 (*c*) 0.577 (*d*) 0.582
5. The critical pressure ratio for initially superheated steam is ...... as compared to initially dry saturated steam.
   (*a*) more (*b*) less
6. The flow of steam is super sonic
   (*a*) at the entrance to the nozzle (*b*) at the throat of the nozzle
   (*c*) in the convergent portion of the nozzle (*d*) in the divergent portion of the nozzle
7. The difference of supersaturated temperature and saturation temperature at that pressure is known as
   (*a*) degree of supersaturation (*b*) degree of superheat
   (*c*) degree of undercooling (*d*) none of these
8. In a nozzle, the effect of supersaturation is to
   (*a*) decrease the dryness fraction of steam (*b*) decrease the specific volume of steam
   (*c*) increase the entropy (*d*) increase the enthalpy drop
9. The density of supersaturated steam is about .... that of the ordinary saturated vapour at the corresponding pressure.
   (*a*) same as (*b*) 2 times (*c*) 4 times (*d*) 8 times
10. When the back pressure of a nozzle is below the designed value of pressure at exit of nozzle, the nozzle is said to be
   (*a*) choked (*b*) under damping (*c*) over damping

ANSWERS

1. (*d*)	2. (*a*)	3. (*c*)	4. (*c*)	5. (*b*)
6. (*d*)	7. (*c*)	8. (*c*)	9. (*d*)	10. (*b*)

22

Impulse Turbines

1. Introduction. 2. Advantages of Steam Turbines over Reciprocating Steam Engines. 3. Classification of Steam Turbines. 4. Impulse Turbine. 5. De-laval Impulse Turbine. 6. Pressure and Velocity of Steam in an Impulse Turbine. 7. Velocity Triangles for Moving Blade of an Impulse Turbine. 8. Combined Velocity Triangle for Moving Blade. 9. Power Produced by an Impulse Turbine. 10. Effect of Friction on the Combined Velocity Triangle. 11. Combined Velocity Diagram for Axial Discharge. 12. Velocity Diagram for Two Stage Impulse Turbine.

22.1. Introduction

A steam turbine is a prime mover in which rotary motion is obtained by the gradual change of momentum of the steam. We have already discussed that in a reciprocating steam engines, the steam acts on the piston, as a load or weight, *i.e.*, the action of steam is *static*. The statical pressure of steam gives to and fro motion to the piston, and conversion of energy takes place through crank and connecting rod.

In a steam turbine, the force exerted on the blades is due to the velocity of steam. This is due to the fact that the curved blades by changing the direction of steam receive a force or impulse. The action of steam in this case is said to be *dynamic*. Thus, the dynamical pressure of steam rotates the vanes, buckets or blades directly. The turbine blades are curved in such a way that the steam directed upon them enters without shock, though there is always some loss of energy by the friction upon the surface of blades. In general, a steam turbine, essentially, consists of the following two parts :

1. The nozzle in which the heat energy of high pressure steam is converted into kinetic energy, so that the steam issues from the nozzle with a very high velocity.

2. The blades which change the direction of steam issuing from the nozzle, so that a force acts on the blades due to change of momentum and propel them.

Thus, the basic principle of operation of a steam turbine is the generation of high velocity steam jet by the expansion of high pressure steam and then conversion of kinetic energy, so obtained into mechanical work on rotor blades.

22.2. Advantages of Steam Turbines over Reciprocating Steam Engines

Following are the important advantages of steam turbines over reciprocating steam engines :

1. A steam turbine may develop higher speeds and a greater steam range is possible.
2. The efficiency of a steam turbine is higher.
3. The steam consumption is less.
4. Since all the moving parts are enclosed in a casing, the steam turbine is comparatively safe.
5. A steam turbine requires less space and lighter foundations, as there are little vibrations.
6. There is less frictional loss due to fewer sliding parts.

7. The applied torque is more uniform to the driven shaft.
8. A steam turbine requires less attention during running. Moreover, the repair costs are generally less.

22.3. Classification of Steam Turbines

The steam turbines may be classified into the following types :

1. *According to the mode of steam action*

   (*i*) Impulse turbine, and (*ii*) Reaction turbine.

2. *According to the direction of steam flow*

   (*i*) Axial flow turbine, and (*ii*) Radial flow turbine.

3. *According to the exhaust condition of steam*

   (*i*) Condensing turbine, and (*ii*) Non-condensing turbine.

4. *According to the pressure of steam*

   (*i*) High pressure turbine, (*ii*) Medium pressure turbine, and (*iii*) Low pressure turbine.

5. *According to the number of stages*

   (*i*) Single stage turbine, and (*ii*) Multi-stage turbine.

In this chapter, we shall discuss impulse turbines only. All the above mentioned other steam turbines will be discussed at the appropriate places in the book.

22.4. Impulse Turbine

An *impulse turbine, as the name indicates, is a turbine which runs by the impulse of steam jet. In this turbine, the steam is first made to flow through a nozzle. Then the steam jet impinges on the turbine blades (which are curved like buckets) and are mounted on the circumference of the wheel. The steam jet after impinging glides over the concave surface of the blades and finally leave the turbine.

Note : The action of the jet of steam, impinging on the blades, is said to be an *impulse* and the rotation of the rotor is due to the impulsive forces of the steam jets.

22.5. De-Level Impulse Turbine

A **De-Level turbine is the simplest type of impulse steam turbine, and is commonly used. It has the following main components :

1. *Nozzle.* It is a circular guide mechanism, which guides the steam to flow at the designed direction and velocity. It also regulates the flow of steam. The nozzle is kept very close to the blades, in order to minimise the losses due to windage.

2. *Runner and blades.* The runner of a De-Laval impulse turbine essentially consists of a circular disc fixed to a horizontal shaft. On the periphery of the runner, a number of blades are fixed uniformly. The steam jet impinges on the buckets, which move in the direction of the jet. This movement of the blades makes the runner to rotate.

Fig. 22.1 Runner and bucket of impulse turbine.

The surface of the blades is made very smooth to minimise the frictional losses. The blades are generally

* The first impulse turbine was devised by Giovanni Branca in 1629.

** Named after the Swedish engineer De-Lavel, who devised this turbine in 1881.

made of special steel alloys. In most of the cases, the blades are bolted to the runner disc. But sometimes the blades and disc are cast as a single unit.

It has been experienced that all the blades do not wear out equally with the time. A few of them get worn out and damaged early and need replacement. This can be done only if the blades are bolted to the disc.

3. *Casing.* It is an air-tight metallic case, which contains the turbine runner and blades. It controls the movement of steam from the blades to the condenser, and does not permit it to move into the space. Moreover, it is essential to safeguard the runner against any accident.

22.6. Pressure and Velocity of Steam in an Impulse Turbine

The pressure of steam jet is reduced in the nozzle and remains constant while passing through the moving blade. The velocity of steam is increased in the nozzle, and is reduced while passing through the moving blades.

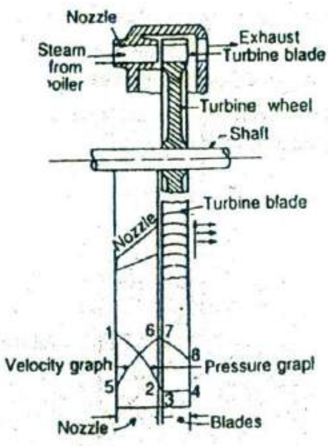


Fig. 22.2 Pressure and velocity graph of steam in a simple impulse turbine.

Fig. 22.2 shows the pressure and velocity graphs of the steam in a simple impulse turbine while it flows in the nozzle and blades. The pressure graph 1-2-3-4 represents steam pressure at entrance of the nozzle, exit of the nozzle, entrance of the blades and exit of the blades respectively. Similarly, velocity graph 5-6-7-8 represents the velocity of steam at entrance of the nozzle, exit of the nozzle, entrance of the blades and exit of the blades respectively.

22.7. Velocity Triangles for Moving Blade of an Impulse Turbine

We have already discussed that in an impulse turbine, the steam jet after leaving the nozzle impinges on one end of the blade. The jet then glides over the inside surface of the blade and finally leaves from the other edge, as shown in Fig. 22.3. It may be noted that the jet enters and leaves the blades tangentially for shockless entry and exit.

Consider a steam jet entering a curved blade after leaving the nozzle at *C*. Now let the jet glides over the inside surface and leaves the blade at *D*, as shown in Fig. 22.3. Now let us draw the velocity triangles at inlet and outlet tips of the moving blade, as shown in Fig. 22.3.

Let V_b = Linear velocity of the moving blade (*AB*),

V = Absolute velocity of steam entering the moving blade (*BC*),

V_r = Relative velocity of jet to the moving blade (AC). It is the vectorial difference between V_b and V.

V_f = Velocity of flow at entrance of the moving blade. It is the vertical component of V.

V_w = Velocity of whirl at entrance of the moving blade. It is the horizontal component of V.

θ = Angle which the relative velocity of jet to the moving blade (V_r) makes with the direction of motion of the blade.

α = Angle with the direction of motion of the blade at which the jet enters the blade.

$V_1, V_{r1}, V_{f1}, V_{w1}, \beta, \phi$ = Corresponding values at exit of the moving blade.

It may be seen from the above, that the original notations (*i.e.* V, V_r, V_f and V_w) stand for the inlet triangle. The notations with suffix 1 (*i.e.* V_1, V_{r1}, V_{f1} and V_{w1}) stand for the outlet triangle.

It may be noted that as the steam jet enters and leaves the blades without any shock (or in other words tangentially), therefore shape of the blades will be such that V_r and V_{r1} will be along the tangents to the blades at inlet and outlet respectively. The angle θ is called the blade angle at inlet and angle ϕ is the blade angle at exit of the moving blade.

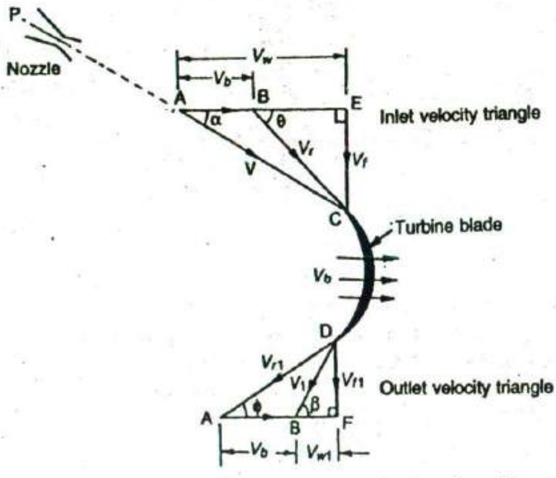


Fig. 22.3 Velcotiy triangles of an impulse turbine.

In Fig. 22.3, PC is the axis of the nozzle, which delivers the steam jet with a high velocity (V) at an angle α with the direction of motion of the blade. The jet impinges on a series of turbine blades mounted on the runner disc.

The axial component of V (*i.e.* EC) which does no work on the blade, is known as *velocity of flow* (V_f). It causes the steam to flow through the turbine and also an axial thrust on the rotor. The tangential component of V (represented by BE) is known as velocity of whirl at inlet (V_w). The linear velocity or mean velocity of the blade (*i.e.* V_b) is represented by AB in magnitude and direction. The length AC represents the relative velocity (V_r) of the steam jet with respect to the blade.

The jet now glides over and leaves the blade with a relative velocity V_{r1}, which is represented by *DA*. The absolute velocity of jet (V_1) as it leaves the blade, is represented by *DB* inclined at an angle β with the direction of the blade motion. The tangential component of V_1 (represented by *BF*) is known as velocity of whirl at exit (V_{w1}). The axial component of V_1 (represented by *DF*) is known as velocity of flow at exit (V_{f1}).

Notes : 1. The inlet triangle of velocities is represented by *BEC*, whereas the outlet triangle by *AFD*.

2. The relations between inlet and outlet velocity triangle (until and unless given) is :

$$V_r = V_{r1}$$

22.8. Combined Velocity Triangle for Moving Blades

In the last article, we have discussed the inlet and outlet velocity triangles separately. For the sake of simplification, a combined velocity triangle for the moving blade is drawn, for solving problems on steam turbines, as shown in Fig. 22.4, and as discussed below :

1. First of all, draw a horizontal line, and cut off *AB* equal to velocity of blade (V_b) to some suitable scale.

2. Now at *B*, draw a line *BC* at an angle α with *AB*. Cut off *BC* equal to *V* (*i.e.* velocity of steam jet at inlet of the blade) to the scale.

3. Join *AC*, which represents the relative velocity at inlet (V_r). Now at *A*, draw a line *AD* at an angle ϕ with *AB*.

4. Now with *A* as centre and radius equal to *AC*, draw an arc meeting the line through *A* at *D*, such that $AC = AD$ or $V_r = V_{r1}$.

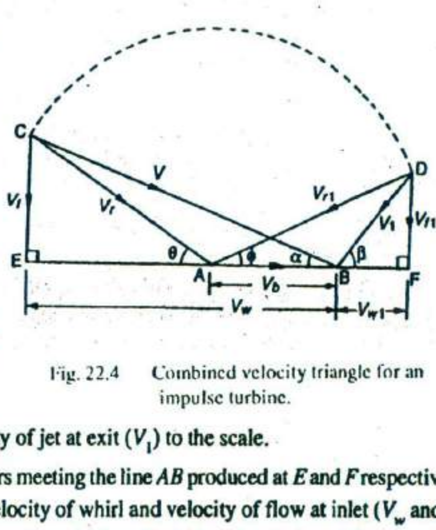


Fig. 22.4 Combined velocity triangle for an impulse turbine.

5. Join *BD*, which represents velocity of jet at exit (V_1) to the scale.

6. From *C* and *D*, draw perpendiculars meeting the line *AB* produced at *E* and *F* respectively.

7. Now *EB* and *CE* represents the velocity of whirl and velocity of flow at inlet (V_w and V_f) to the scale. Similarly, *BF* and *DF* represents the velocity of whirl and velocity of flow at outlet (V_{w1} and V_{f1}) to the scale.

22.9. Power Produced by an Impulse Turbine

Consider an impulse turbine working under the action of a steam jet. Let us draw a combined velocity triangle, for the impulse turbine as shown in Fig. 22.4.

Let m = Mass of the steam flowing through the turbine in kg/s,

$(V_w + V_{w1})$ = Change in the velocity of whirl in m/s.

We know that according to the Newton's second law of motion, force in the direction of motion of the blades,

$$F_X = \text{Mass of steam flowing per second} \times \text{Change in the velocity of whirl}$$

$$= m\,[V_w - (-V_{w1})]$$

$$= m\,[V_w + V_{w1}] = m \times EF \text{ N}$$

and work done in the direction of motion of the blades

$$= \text{Force} \times \text{Distance}$$

$$= m\,[V_w + V_{w1}]\,V_b \text{ N-m/s} \qquad \ldots (ii)$$

$$= m \times EF \times AB \text{ N-m/s}$$

∴ Power produced by the turbine,

$$P = m \times EF \times AB \text{ watts} \qquad \ldots (\because 1 \text{ N-m/s} = 1 \text{ watt})$$

$$= m\,(V_w + V_{w1})\,V_b \text{ watts}$$

Similarly, we can find out the axial thrust on the wheel which is due to the difference of velocities of flow at inlet and outlet. Mathematically, axial thrust on the wheel,

$$F_Y = \text{Mass of steam flowing per second} \times \text{Change in the velocity of flow}$$

$$= m\,(V_f - V_{f1}) = m\,(CE - DF) \text{ N} \qquad \ldots (iii)$$

Notes : 1. In equation (*i*), the value of V_{w1} is taken as *negative* because of the opposite direction of V_w with respect to the blade motion. In other words, when point F in the velocity diagram lies on the right of point B, then V_{w1} is *negative*. Thus change in velocity of whirl,

$$= V_w - (-V_{w1}) = V_w + V_{w1}$$

2. If V_{w1} is in the same direction with respect to the blade motion, then V_{w1} is taken as positive. In other words, when point F in the velocity diagram lies on the left of point B, then V_{w1} is positive. Thus change in velocity of whirl

$$= V_w - (+V_{w1}) = V_w - V_{w1} \qquad \text{(See Example 22.3)}$$

Example 22.1. *In a De-lavel turbine, the steam enters the wheel through a nozzle with a velocity of 500 m/s and at an angle of 20° to the direction of motion of the blade. The blade speed is 200 m/s and the exit angle of the moving blade is 25°. Find the inlet angle of the moving blade, exit velocity of steam and its direction and work done per kg of steam.*

Solution. Given : $V = 500$ m/s ; $\alpha = 20°$; $V_b = 200$ m/s ; $\phi = 25°$

Now let us draw the combined velocity triangle, as shown in Fig. 22.5, as discussed below :

1. First of all, draw a horizontal line and cut off AB equal to 200 m/s, to some suitable scale, representing the blade speed (V_b).

2. Now at B, draw a line BC at an angle of 20° (nozzle angle, α) and cut off BC equal to 500 m/s to the scale to represent the velocity of steam jet entering the blade (V).

3. Join AC, which represents the relative velocity at inlet (V_r).

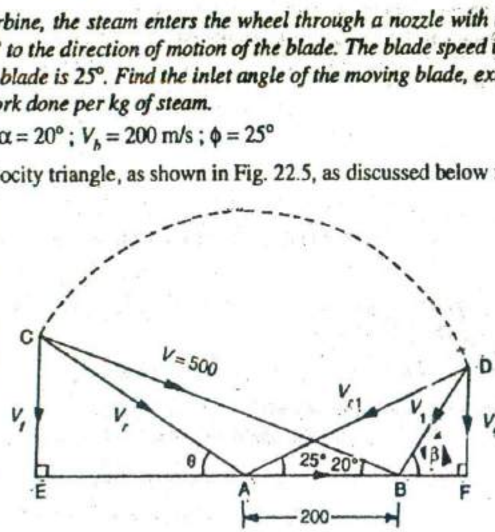


Fig. 22.5

4. At A, draw a line AD at an angle of 25° (exit angle of the moving blade, ϕ). Now with A as centre, and radius equal to AC, draw an arc meeting the line through A at D.

5. Join BD, which represents the velocity of steam jet at outlet (V_1).

6. From C and D, draw perpendiculars meeting the line AB produced at E and F respectively. CE and DF represents the velocity of flow at inlet (V_f) and outlet (V_{f1}) respectively.

The following values are *measured from the velocity diagram :

$$\theta = 32^\circ ; \beta = 59^\circ ; V_1 = BD = 165 \text{ m/s}$$

$$V_w = BE = 470 \text{ m/s and } V_{w1} = BF = 90 \text{ m/s}$$

Inlet angle of moving blade

By measurement from the velocity diagram, we find that the inlet angle of the moving blade,

$$\theta = 32^\circ \textbf{ Ans.}$$

Exit velocity of steam

By measurement from the velocity diagram, we also find that the exit velocity of steam,

$$V_1 = 165 \text{ m/s}$$

Direction of the exit steam

By measurement from the velocity diagram, we also find that the direction of the exit steam,

$$\beta = 59^\circ \textbf{ Ans.}$$

Workdone per kg of steam

We know that workdone per kg of steam

$$= m\,(V_w + V_{w1})$$

$$= 1\,(470 + 90) = 560 \text{ N-m } \textbf{Ans.} \qquad \ldots (\because m = 1 \text{ kg})$$

22.10. Effect of Friction on the Combined Velocity Triangle

In the last article, we have discussed that the relative velocity of steam jet is the same at the inlet and outlet tips of the blade. In other words, we have assumed that the inner side of the curved blade offers no resistance to the steam jet. But in actual practice, some resistance is always offered by the blade surface to the gliding steam jet, whose effect is to reduce the relative velocity of the jet. *i.e.* to make V_{r1} less than V_r. The ratio of V_{r1} to V_r is known as *blade velocity coefficient* or coefficient of velocity or friction factor, (usually denoted by K). Mathematically, blade velocity coefficient,

$$K = \frac{V_{r1}}{V_r}$$

---

* These values may also be found out from the geometry of the velocity diagram as discussed below :

$$V_w = V \cos 20^\circ = 500 \times 0.9397 = 469.9 \text{ m/s}$$

$$V_f = V \sin 20^\circ = 500 \times 0.342 = 171 \text{ m/s}$$

$$\tan\theta = \frac{V_f}{V_w - V_b} = \frac{171}{469.9 - 200} = 0.6335 \quad \text{or} \quad \theta = 32.35^\circ$$

$$V_r = V_{r1} = \frac{V_f}{\sin\theta} = \frac{171}{\sin 32.35^\circ} = \frac{171}{0.5351} = 319.4 \text{ m/s}$$

$$V_{f1} = V_{r1} \sin 25^\circ = 319.4 \times 0.4226 = 135 \text{ m/s}$$

$$V_{w1} = (V_{r1} \cos 25^\circ) - 200 = (319.4 \times 0.9063) - 200 = 89.5 \text{ m/s}$$

$$\tan\beta = \frac{V_{f1}}{V_{w1}} = \frac{135}{89.5} = 1.51 \quad \text{or} \quad \beta = 56.47^\circ$$

$$V_1 = \frac{V_{f1}}{\sin\beta} = \frac{135}{\sin 56.47^\circ} = \frac{135}{0.8336} = 162 \text{ m/s}$$

It may be noted that the effect of friction on the combined velocity triangle will be to reduce the relative velocity at outlet (V_{r1}) as shown in Fig. 22.6.

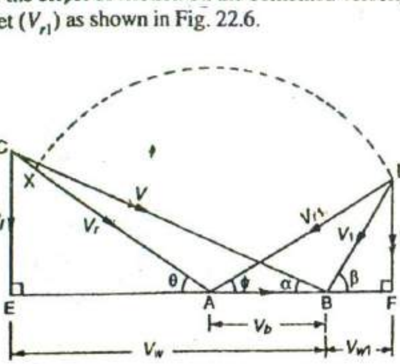


Fig. 22.6. Effect of friction on the combined velocity triangle.

Notes : 1. Since V_{r1} is decreased due to friction, therefore work done per kg of steam is also reduced.

2. The value of K varies from 0.75 to 0.85 depending upon the shape of the blades.

Example 22.2. *The velocity of steam, leaving the nozzles of an impulse turbine, is 1200 m/s and the nozzle angle is 20°. The blade velocity is 375 m/s and the blade velocity coefficient is 0.75. Assuming no loss due to shock at inlet, calculate for a mass flow of 0.5 kg/s and symmetrical blading : (a) blade inlet angle ; (b) driving force on the wheel ; (c) axial thrust on the wheel ; and (d) power developed by the turbine.*

Solution. Given : $V = 1200$ m/s ; $\alpha = 20°$; $V_b = 375$ m/s ; $K = V_{r1}/V_r = 0.75$; $m = 0.5$ kg/s ; $\theta = \phi$, for symmetrical blading.

Now draw the combined velocity triangle, as shown in Fig. 22.7, as discussed below :

1. First of all, draw a horizontal line, and cut off AB equal to 375 m/s to some suitable scale representing the velocity of blade (V_b).

2. Now at B, draw a line BC at an angle of 20° (Nozzle angle, α) and cut off BC equal to 1200 m/s to the scale to represent the velocity of steam jet entering the blade (V).

3. Join CA, which represents the relative velocity at inlet (V_r). By measurement, we find that $CA = V_r = 860$ m/s. Now cut off AX equal to $860 \times 0.75 = 645$ m/s to the scale to represent the relative velocity at exit (V_{r1}).

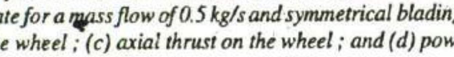


Fig. 22.7

4. At A, draw a line AD at an angle φ equal to the angle θ, for symmetrical blading. Now with A as centre, and radius equal to AX, draw an arc meeting the line through A at D, such that $AD = V_{r1}$.

5. Join BD, which represents the velocity of steam jet at outlet (V_1).

6. From C and D, draw perpendiculars meeting the line AB produced at E and F respectively. CE and DF represents the velocity of flow at inlet (V_f) and outlet (V_{f1}) respectively.

The following values are *measured from the velocity diagram :

$$\theta = 29^\circ ; V_w = BE = 1130 \text{ m/s} ; V_{w1} = BF = 190 \text{ m/s}$$

$$V_f = CE = 410 \text{ m/s and } V_{f1} = DF = 310 \text{ m/s}$$

(*a*) *Blade inlet angle*

By measurement from the velocity diagram, we find that the blade angle at inlet,

$$\theta = 29^\circ \text{ Ans.}$$

(*b*) *During force on the wheel*

We know that driving force on the wheel,

$$F_X = m(V_w + V_{w1}) = 0.5(1130 + 190) = 660 \text{ N Ans.}$$

(*c*) *Arial thrust on the wheel*

We know that axial thrust on the wheel,

$$F_Y = m(V_f - V_{f1}) = 0.5(410 - 310) = 50 \text{ N Ans.}$$

(*d*) *Power developed by the turbine*

We know that power developed by the turbine,

$$P = m(V_w + V_{w1})V_b = 0.5(1130 + 190)375 = 247\,500 \text{ W}$$

$$= 247.5 \text{ kW Ans.}$$

Example 22.3. *A steam jet entres the row of blades with a velocity of 375 m/s at an angle of 20° with the direction of motion of the moving blades. If the blade speed is 165 m/s, find the suitable inlet and outlet blade angles assuming that there is no thrust on the blades. The velocity of steam passing over the blades is reduced by 15%. Also determine power developed by the turbine per kg of steam flowing over the blades per second.*

Solution. Given : $V = 375$ m/s ; $\alpha = 20^\circ$; $V_b = 165$ m/s ; $m = 1$ kg/s

Since there is no thrust on the blades, therefore

$$V_f = V_{f1}$$

Also, the velocity of steam passing over the blades is reduced by 15%, therefore

$$V_{r1} = 0.85\,V_r$$

---

* These values may also be found out from the geometry of the velocity diagram as discussed below :

$$V_w = V\cos 20^\circ = 1200 \times 0.9397 = 1128 \text{ m/s}$$

$$V_f = V\sin 20^\circ = 1200 \times 0.3420 = 410.4 \text{ m/s}$$

$$\tan\theta = \frac{V_f}{V_w - V_b} = \frac{410.4}{1128 - 375} = 0.5450 \quad \text{or} \quad \theta = 28.6^\circ$$

$$V_r = \frac{V_f}{\sin\theta} = \frac{410.4}{\sin 28.6^\circ} = \frac{410.4}{0.4787} = 856.5 \text{ m/s}$$

$$V_{r1} = 0.75 \times 856.5 = 642.4 \text{ m/s}$$

$$V_{f1} = V_{r1}\sin 28.6^\circ = 642.4 \times 0.4787 = 307.5 \text{ m/s}$$

$$V_{w1} = (V_{r1}\cos 28.6^\circ) - 375 = (642.4 \times 0.8780) - 375 = 189 \text{ m/s}$$

Now draw the combined velocity triangle, as shown in Fig. 22.8, as discussed below :

1. First of all, draw a horizontal line and cut off AB equal to 165 m/s to some suitable scale representing the blade speed (V_b).

2. Now draw inlet velocity triangle ABC on the base AB with $\alpha = 20°$ and $V = 375$ m/s to the scale. From the velocity triangle, we find that $V_f = 30$ m/s and $V_r = 230$ m/s.

3. Similarly, draw outlet velocity triangle ABD on the same base AB with $V_{r1} = 0.85\ V_r = 0.85 \times 230 = 195.5$ m/s to the scale and $V_{f1} = V_f = 130$ m/s.

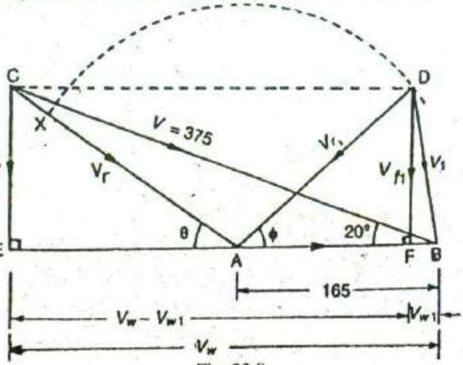


Fig. 22.8

4. From C and D draw perpendiculars to meet the line AB produced at E and F. From the geometry of the figure, we find that V_{w1} is in the opposite direction of V_w.

The following values are measured from the combined velocity triangle :

$$\theta = 34°;\ \phi = 41° \text{ and } (V_w - V_{w1}) = 320 \text{ m/s}$$

Inlet and outlet blade angle

By measurement from the velocity diagram, we find that inlet blade angle,

$$\theta = 34° \text{ Ans.}$$

and outlet blade angle,
$$\phi = 41° \text{ Ans.}$$

Power developed by the turbine

We know that power developed by the turbine,

$$P = m\,(V_w - V_{w1})\,V_b$$

$$= 1 \times 320 \times 165 = 52\,800 \text{ W} = 52.8 \text{ kW Ans.}$$

Example 22.4. *The blade speed of a single ring impulse blading is 250 m/s and nozzle angle is 20°. The heat drop is 550 kJ/kg and nozzle efficiency is 0.85. The blade discharge angle is 30° and the machine develops 30 kW, when consuming 360 kg of steam per hour. Draw the velocity diagram and calculate : 1. axial thrust on the blading, and 2. the heat equivalent per kg of steam friction of the blading.*

Solution. Given : $V_b = 250$ m/s ; $\alpha = 20°$; $h_d = 550$ kJ/kg ; $K = 0.85$; $\phi = 30°$; $P = 30$ kW $= 30 \times 10^3$ W ; $m = 360$ kg/h $= 0.1$ kg/s

We know that the velocity of steam entering the blades,

$$V = 44.72\sqrt{K\,h_d}$$

$$= 44.72\sqrt{0.85 \times 550} = 967 \text{ m/s}$$

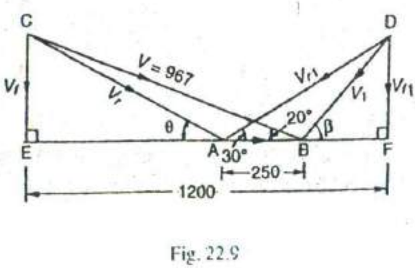


Fig. 22.9

and power developed (P),

$$30 \times 10^3 = m\,(V_w + V_{w1})\,V_b = 0.1\,(V_w + V_{w1})\,250$$

$\therefore \qquad V_w + V_{w1} = 1200$ m/s

Now draw the combined velocity triangle, as shown in Fig. 22.9, as discussed below :

1. First of all, draw a horizontal line and cut off *AB* equal to 250 m/s to some suitable scale representing the blade speed (V_b).

2. Now draw inlet velocity triangle *ABC* on the base *AB* with $\alpha = 20°$ and $V = 967$ m/s to the scale.

3. From point *C*, draw *CE* perpendicular to *AB* produced and take

$$EF = V_w + V_{w1} = 1200 \text{ m/s}$$

4. Draw a line at point *A* at an angle 30° intersecting the perpendicular drawn through *F*, at *D*. Now complete the outlet velocity triangle *ABD* as shown in the figure.

The following values are measured from the combined velocity triangle :

$$V_f = 400 \text{ m/s} \; ; \; V_{f1} = 325 \text{ m/s} \; ; \; V_r = 895 \text{ m/s and } V_{r1} = 650 \text{ m/s}$$

1. *Axial thrust on the blading*

We know that axial thrust on the blading,

$$F_Y = m(V_f - V_{f1}) = 0.1\,(400 - 325) = 7.5 \text{ N} \quad \textbf{Ans.}$$

2. *Heat equivalent per kg of steam of friction of the blading*

We know that work lost in friction of the blading

$$= \frac{(V_r)^2 - (V_{r1})^2}{2000} = \frac{(895)^2 - (650)^2}{2000} = 189.26 \text{ kJ}$$

$\therefore$ Heat equivalent per kg of steam friction of the blading

$$= 189.26 \text{ kJ} \quad \textbf{Ans.}$$

Example 22.5. *Steam at 5 bar and 200° C is first made to pass through nozzles. It is then supplied to an impulse turbine at the rate of 30 kg/minute. The steam is finally exhausted to a condenser at 0.2 bar. The blade speed is 300 m/s. The nozzles are inclined at 25° with the direction of motion of the blades and the outlet blade angle is 35°. Neglecting friction, find the theoretical power developed by the turbine.*

Solution. Given : $p_1 = 5$ bar ; $T_1 = 200°$ C ; $m = 30$ kg/min $= 0.5$ kg/s ; $p_2 = 0.2$ bar ; $V_b = 300$ m/s ; $\alpha = 25°$; $\phi = 35°$

First of all, let us draw Mollier diagram for the flow of steam through the nozzle, as shown in Fig. 22.10. From this diagram, we find that heat drop during the flow,

$$h_d = h_1 - h_2$$
$$= 2850 - 2340 = 510 \text{ kJ/kg}$$

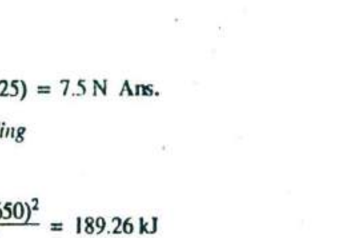


Fig. 22.10

$\therefore$ Velocity of steam at inlet of the blade,

$$V = 44.72\sqrt{510} = 1010 \text{ m/s}$$

Now draw the combined velocity triangle, as shown in Fig. 22.11, as discussed below :

1. First of all, draw a horizontal line and cut off *AB* equal to 300 m/s, to some suitable scale, to represent the blade speed (V_b).

2. Now draw inlet velocity triangle *ABC* on the base *AB* with $\alpha = 25°$ and $V = 1010$ m/s. From the velocity triangle, we find that $V_r = 850$ m/s to the scale.

3. Similarly, draw the outlet velocity triangle ABD on the same base AB with $\phi = 35°$ and $V_{r1} = V_r = 850$ m/s to the scale.

4. From C and D draw perpendiculars to meet the line AB produced at E and F.

By measurement from the velocity diagram, we find that

$$EF = V_w + V_{w1} = 1400 \text{ m/s}$$

We know theoretical power developed by the turbine,

$$P = m(V_w + V_{w1})\,V_b$$
$$= 0.5 \times 1400 \times 300$$
$$= 210\,000 \text{ W}$$
$$= 210 \text{ kW} \quad \textbf{Ans.}$$

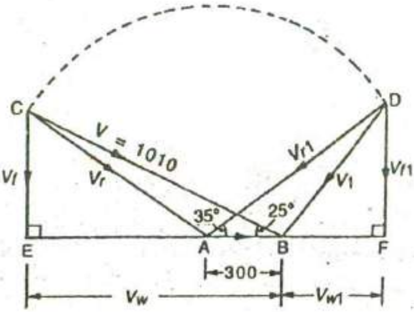


Fig. 22.11

Example 22.6. *In a certain stage of an impulse turbine, the nozzle angle is 20° with the plane of the wheel. The mean diameter of the blade ring is 2.8 metres. It develops 55 kW at 2400 r.p.m. Four nozzles, each of 10 mm diameter expand steam isentropically from 15 bar and 250° C to 0.5 bar. The axial thrust is 3.5 N. Calculate :*

1. blade angles at entrance and exit, and 2. power lost in blade friction.

Solution. Given : $\alpha = 20°$; $D = 2.8$ m ; $P = 55$ kW $= 55 \times 10^3$ W ; $N = 2400$ r.p.m. ; $n = 4$; $d = 10$ mm ; $p_1 = 15$ bar ; $T_1 = 250°$ C ; $p_2 = 0.5$ bar ; $F_Y = 3.5$ N

We know that blade speed,

$$V_b = \pi DN/60 = \pi \times 2.8 \times 2400/60 = 352 \text{ m/s}$$

1. *Blade angles at entrance and exit*

First of all, let us draw Mollier diagram for the flow of steam through the nozzle, as shown in Fig. 22.12. From this diagram, we find that heat drop during the flow,

$$h_d = h_1 - h_2 = 2920 - 2330 = 590 \text{ kJ/kg}$$

∴ Velocity of steam at inlet of the blade,

$$V = 44.72\sqrt{590} = 1086 \text{ m/s}$$

From the Mollier diagram, we also find that dryness fraction of steam at inlet of the blade,

$$x = 0.86$$

From steam tables, corresponding to a pressure of 0.5 bar, we find that specific volume of steam,

$$v_g = 3.24 \text{ m}^3\text{/kg}$$

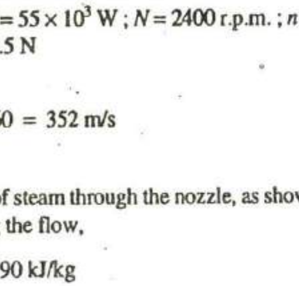


Fig. 22.12

We know that area of each nozzle

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 10^2 = 78.55 \text{ mm}^2 = 78.55 \times 10^{-6} \text{ m}^2$$

∴ Mass of steam discharged through the nozzle,

$$m = \frac{n\,A\,V}{x\,v_g} = \frac{4 \times 78.55 \times 10^{-6} \times 1086}{0.86 \times 3.24} = 0.12 \text{ kg/s}$$

We know that power developed (P),

$$55 \times 10^3 = m\,(V_w + V_{w1})\,V_b = 0.12\,(V_w + V_{w1})\,352 = 42.24\,(V_w + V_{w1})$$

$$\therefore \quad V_w + V_{w1} = 1302 \text{ m/s}$$

We also know that axial thrust (F_Y),

$$3.5 = m\,(V_f - V_{f1}) = 0.12\,(V_f - V_{f1})$$

$$\therefore \quad V_f - V_{f1} = 29.2 \text{ m/s} \quad \text{or} \quad V_{f1} = (V_f - 29.2) \text{ m/s}$$

Now draw the combined velocity triangle, as shown in Fig. 22.13, as discussed below :

1. First of all, draw a horizontal line and cut off AB equal to 352 m/s, to some suitable scale, representing the blade speed (V_b).

2. Now draw the inlet velocity triangle ABC on the base AB with $\alpha = 20°$ and $V = 1086$ m/s to the scale. By measurement, we find that $V_f = 370$ m/s.

3. Similarly, draw outlet velocity triangle ABD on the same base AB with EF (*i.e.* $V_w + V_{w1}$) = 1302 m/s to the scale and $V_{f1} = V_f - 29.2 = 370 - 29.2 = 340.8$ m/s.

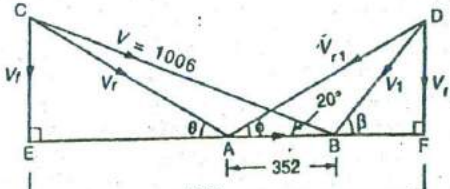


Fig. 22.13

4. From C and D, draw perpendiculars to meet the line AB produced at E and F.

By measurement from the velocity diagram, we find that

$$\theta = 29°\,; \text{ and } \phi = 28° \textbf{ Ans.}$$

2. *Power lost in friction*

By measurement from the velocity diagram, we also find that

$$V_r = 765 \text{ m/s}\,; \text{ and } V_{r1} = 730 \text{ m/s}$$

We know that power lost in blade friction

$$= \frac{(V_r)^2 - (V_{r1})^2}{2000} = \frac{(765)^2 - (730)^2}{2000} = 26.16 \text{ kJ/s}$$

$$= 26.16 \text{ kW } \textbf{Ans.} \qquad \ldots(\because 1 \text{ kJ/s} = 1 \text{kW})$$

22.11. Combined Velocity Diagram for Axial Discharge

Sometimes, the steam leaves the blade at its exit tip at 90° to the direction of the blade motion. In such a case, the turbine is said to have an axial discharge. The combined velocity diagram for axial discharge is drawn as shown in Fig. 22.14. It may be noted that in such a turbine, velocity of whirl at outlet (V_{w1}) is equal to zero. Therefore power developed by the turbine,

$$P = m \times V_w \times V_b \text{ watts} \qquad \ldots(\because V_{w1} = 0)$$

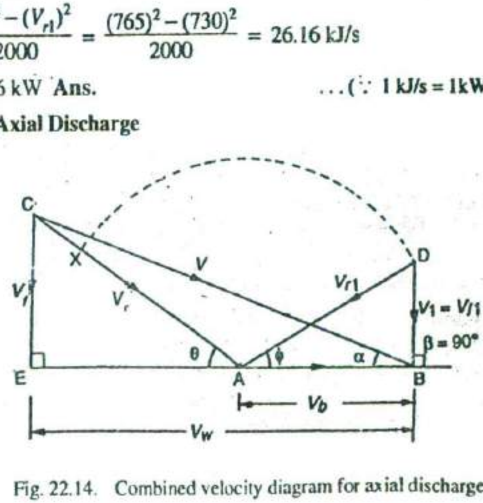


Fig. 22.14. Combined velocity diagram for axial discharge.

Example 22.7. *The steam supply to an impulse turbine with a single row of moving blades is 2 kg/s. The turbine develops 130 kW, the blade velocity being 175 m/s. The steam flows from a nozzle with a velocity of 400 m/s and the velocity coefficient of blades is 0.9. Find the nozzle angle, blade angle at entry and exit, if the steam flows axially after passing over the blades.*

Solution. Given : $m = 2$ kg/s ; $P = 130$ kW $= 130 \times 10^3$ W ; $V_b = 175$ m/s ; $V = 400$ m/s ; $K = 0.9$

Let V_w = Velocity of whirl at inlet.

We know that power developed (P),

$$130 \times 10^3 = m \times V_w \times V_b = 2 \times V_w \times 175 = 350\, V_w$$

$$\therefore \quad V_w = 371.4 \text{ m/s}$$

From the given data, we find that it is not sufficient to draw the inlet velocity triangle in the general manner. So we have to make use of the geometry of the axial discharge. Now draw the combined velocity triangle, as shown in Fig. 22.15, as discussed below :

1. First of all, draw a horizontal line and cut off AB equal to 175 m/s to some suitable scale to represent blade velocity (V_b) and EB equal to 371.4 m/s to the scale to represent velocity of whirl at inlet (V_w).

2. Now draw velocity triangle EBC on the base EB with $\angle CEB = 90°$ and $V = 400$ m/s to the scale. Join CA, which represents relative velocity at inlet (V_r). By measurement, we find that $V_r = 240$ m/s.

3. Now draw outlet velocity triangle ABD on the base AB with $\beta = 90°$ and $V_{r1} = 0.9\, V_r = 0.9 \times 240 = 216$ m/s to the scale.

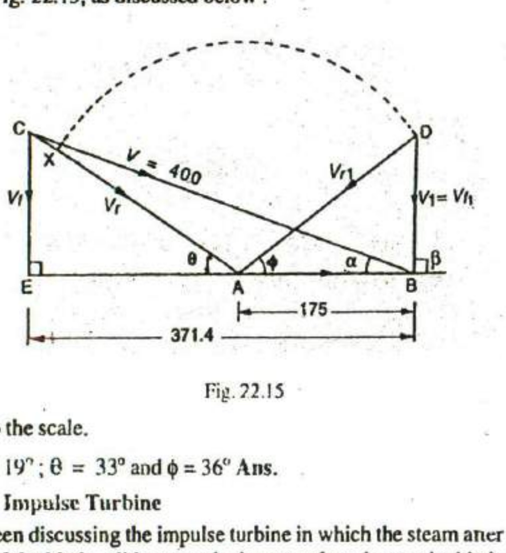


Fig. 22.15

By measurement, we find that $\alpha = 19°$; $\theta = 33°$ and $\phi = 36°$ **Ans.**

22.12. Velocity Diagram for Two Stage Impulse Turbine

In the previous articles, we have been discussing the impulse turbine in which the steam after leaving the nozzle impinges on one end of the blade, glides over the inner surface, leaves the blade and then exhausts into the condenser. But sometimes, the steam after leaving the moving blade is made to flow through a fixed blade ring (in order to make the steam to flow at a designed angle) and again impinges on second moving blade. This type of turbine is called *two-stage impulse turbine*, whose velocity triangles are shown in Fig. 22.16.

The separate velocity diagrams for the first moving, first fixed and second moving blade rings for a two stage impulse turbine compounded for velocity are shown in Fig. 22.16. It may be noted, that the blade velocity (V_b) is constant for both the stages. The inlet and outlet velocity diagram for the first moving blade is the same as shown in Fig. 22.16. It may also be noted, that the absolute velocity at exit from the first moving blade is the entrance velocity to the fixed blade ring. Similarly, the exit velocity from the fixed ring is the entrance velocity to the second moving blade ring.

The combined velocity triangle for a two stage impulse turbine may be drawn, as shown in Fig. 22.17, as discussed below :

1. First of all, draw a horizontal line and cut off AB equal to the given blade velocity (V_b) to some suitable scale.

2. Now draw the inlet velocity triangle *ABC* for the first moving ring on the base *AB* with the help of nozzle angle of the first moving ring (α) and velocity of steam entering the turbine (V).

3. Now cut off *CX* equal to the friction of the blades on the first moving ring. The length *AX* will give the value of relative velocity at exit of the first moving ring (V_{r1}).

4. Now draw the outlet velocity triangle *ABD* for the first moving ring on the same base *AB* with the help of exit blade angle for the first moving ring (ϕ) and relative velocity at exit for the first moving ring (V_{r1}).

5. Now cut off *DY* equal to the friction of the blades of the fixed ring. The length *BY* will give the exit velocity of steam from the fixed ring. It will also be equal to the velocity of steam entering the second moving ring (V_1).

6. Now draw the inlet velocity triangle *ABC'* for the second moving ring on the same base *AB* with the help of nozzle angle of the second moving ring (α') and velocity of steam entering the second moving ring (V')

7. Now cut off *C'Z* equal to the friction of blades on the second moving ring. The length *AZ* will give the value of relative velocity at exit of the second moving ring (V_r').

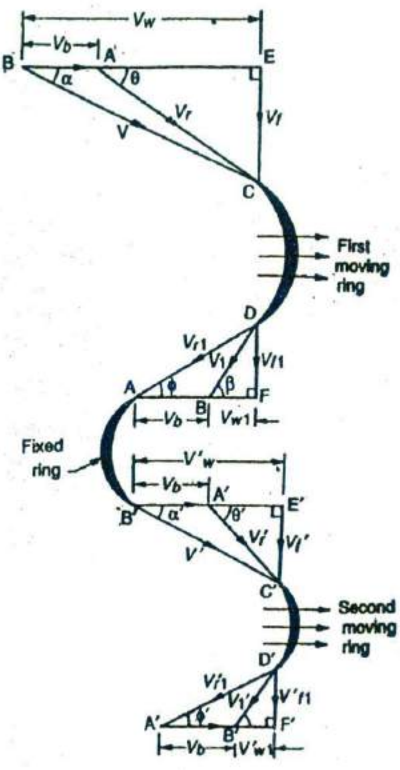


Fig. 22.16. Two-stage impulse turbine.

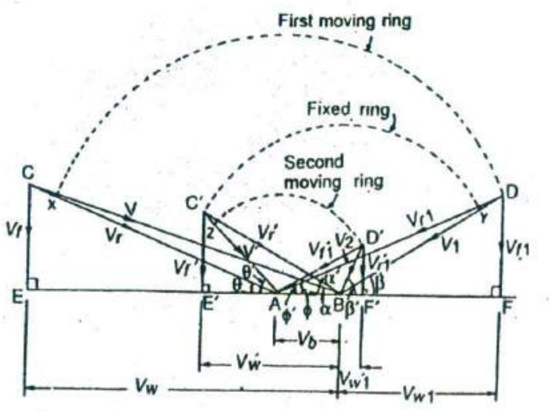


Fig. 22.17. Combined velocity triangle for two stage impulse turbine.

8. Now draw the outlet velocity triangle ABD' for the second moving ring on the same base AB with the help of exit blade angle for the second moving ring (ϕ') and exit velocity of the second moving ring (V_2).

We know that power developed by a two stage impulse turbine,

$$P = m\,(EF + E'F')\,V_b \text{ watts}$$

where m is the mass of steam supplied in kg/s.

Example 22.8. *A velocity compounded impulse turbine has two rows of moving blades with a fixed row of guide blades. The steam leaves the nozzle at 900 m/s in a direction at 18° to the plane of rotation. The blade speed is 150 m/s and the blade outlet angles are 24°, 26° and 30° for the first moving, first fixed and second moving respectively. The friction factor is 0.9 for all rows. The steam supply is 4500 kg per hour. Determine :*

(a) Tangential force on the rotor ; (b) Total work done on the blades ; and (c) Power developed by the turbine.

Solution. Given : $V = 900$ m/s ; $\alpha = 18°$; $V_b = 150$ m/s ; $\phi = 24°$; $\alpha' = 26°$; $\phi' = 30°$; $K = V_{r1}/V_r = V'/V_1 = V_{r1}'/V_r' = 0.9$; $m = 4500$ kg/h $= 1.25$ kg/s

Now draw the combined velocity triangle for the two stage impulse turbine, as shown in Fig. 22.18, as discussed below :

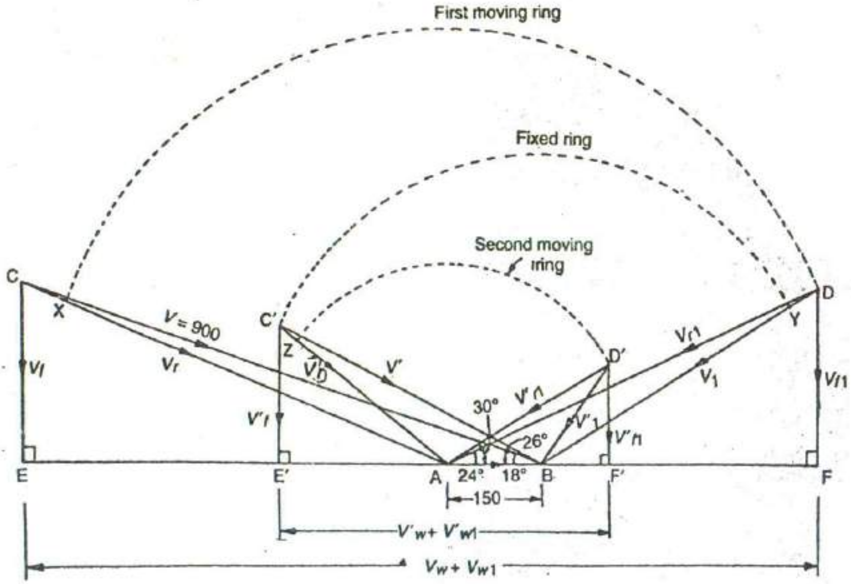


Fig. 22.18

1. First of all, draw a horizontal line and cut off AB equal to 150 m/s, to some suitable scale, to represent the blade speed (V_b).

2. Now draw the inlet velocity triangle ABC for the first moving ring on the base AB with nozzle angle for the first moving ring ($\alpha = 18°$) and $V = 900$ m/s, to the scale. By measuring the side AC, we find that $V_r = 750$ m/s.

3. Now draw the outlet velocity triangle *ABD* for the first moving ring on the same base *AB* with $\phi = 24°$ and $V_{r1} = 0.9\ V_r = 0.9 \times 750 = 675$ m/s, to the scale. By measuring the side *BD*, we find that $V_1 = 540$ m/s.

4. Now draw the inlet velocity triangle ABC' for the second moving ring on the same base *AB* with $\alpha' = 26°$ and $V' = 0.9\ V_1 = 0.9 \times 540 = 486$ m/s. By measuring the side AC', we find that $V_r' = 350$ m/s.

5. Now draw the outlet velocity triangle ABD' for the second moving ring on the same base *AB* with $\phi' = 30°$ and $V_{r1}' = 0.9\ V_r' = 0.9 \times 350 = 315$ m/s.

The following values are measured from the combined velocity diagram

$$EF = (V_w + V_{w1}) = 1400 \text{ m/s}$$

and

$$E'F' = (V_w' + V_{w1}') = 690 \text{ m/s}$$

(*a*) *Tangential force on the rotor*

We know that tangential force on the rotor,

$$F_X = m\,(EF + E'F') = 1.25\,(1400 + 690) = 2612.5 \text{ N} \textbf{ Ans.}$$

(*b*) *Total workdone on the blades*

We know that total workdone on the blades,

$$W.D. = m\,(EF + E'F')\,V_b = 1.25\,(1400 + 690)\,150 = 391\,880 \text{ N-m/s}$$

$$= 391.88 \text{ kN-m/s} \textbf{ Ans.}$$

(*c*) *Power developed by the turbine*

We know that power developed by the turbine,

$$P = m\,(EF + E'F')\,V_b = 391.88 \text{ kW} \textbf{ Ans.} \quad \ldots(\because 1 \text{ kN-m/s} = 1 \text{ kW})$$

Example 22.9. ***Steam issuing from a nozzle at 600 m/s enters the first set of blades of a two row wheel impulse turbine. The tips of both the set of moving blades are inclined at 30° to the plane of motion. Find the speed of the blades, so that the steam is finally discharged axially. Neglect friction.***

Also find the power developed by the turbine, if the mass of steam supplied to the turbine is 3 kg/s.

Solution. Given : $V = 600$ m/s ; $\theta = \theta' = \phi = 30°$; $\beta = 90°$ (for axial discharge) ; $m = 3$ kg/s

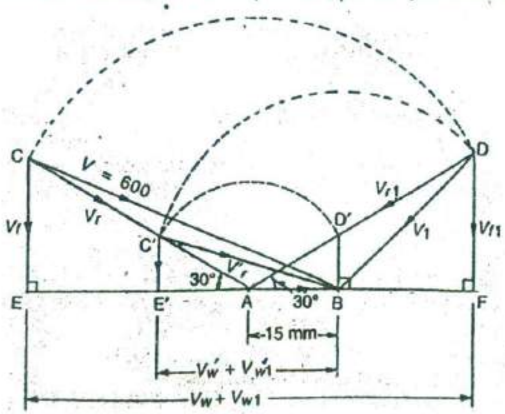


Fig. 22.19

From the given data, we find that it is not sufficient to draw inlet velocity triangle for the first moving ring. Therefore, we have to start the diagram from the outlet velocity triangle for the second moving ring. Now draw the combined velocity diagram for the two stages of the turbine, as shown in Fig. 22.19, as discussed below :

1. First of all, draw a horizontal line and mark AB equal to 15 mm to represent the blade velocity (which is required to be found out).

2. Now draw the outlet velocity triangle ABD' for the second moving ring, on the base AB with $\phi = 30°$ and $\beta = 90°$. From the geometry of the figure, we find that $D'B$ represents V_{f1}' and V_1'. Similarly, $D'A$ represents V_{r1}'. Since there is no friction in the turbine, therefore $V_{r1}' = V_r'$.

3. Now draw the inlet velocity triangle ABC' for the second moving ring on the same base AB with $\theta' = 30°$ and $V_r' = V_{r1}'$ (*i.e.* length $AC' = AD'$). From the geometry of the figure, we find that $C'A$ represents V_{r1}'. Since there is no friction in the turbine, therefore $V_1 = V_r'$.

4. Now draw the outlet velocity triangle ABD for the first moving ring on the same base AB with $\phi = 30°$ and $V_r' = V_1$ (*i.e.* length $BC' = BD$). From the geometry of the figure, we find that AD represents V_{r1}. Since there is no friction in the turbine, therefore $V_r = V_{r1}$.

5. Now draw the inlet velocity triangle ABC for the first moving ring on the same base AB with $\theta = 30°$ and $V_{r1} = V_r$ (*i.e.* length $AD = AC$).

From the combined velocity diagram, we find that the length $BC = 56$ mm.

$\therefore$ Scale, $\quad 1 \text{ mm} = 600/56 = 10.7$ m/s

and blade speed, $\quad V_b = 15 \times 10.7 = 160.5$ m/s **Ans.**

From the figure, we find that length $(EF + E'B)$

$$= 103 \text{ mm} = 103 \times 10.7 = 1102 \text{ m/s}$$

We know that power developed by the turbine,

$$P = m\,(EF + E'B)\,V_b = 3 \times 1102 \times 160.5 = 530\,610 \text{ W}$$

$$= 530.61 \text{ kW} \quad \textbf{Ans.}$$

EXERCISES

1. The following data relates to a single stage impulse turbine :

Steam velocity = 600 m/s ; Blade speed = 250 m/s ; Nozzle angle = 20° ; Blade outlet angle = 25°.

Neglecting the effect of friction, calculate the absolute velocity of steam leaving the blade and the work developed by the turbine for the steam flow rate of 20 kg/s. Also calculate the axial thrust on the bearings.

[**Ans.** 180 m/s ; 3250 kN-m/s ; 1000 N]

2. The mean diameter of the blades of a single row impulse turbine is 2 metres and the speed is 3000 r.p.m. The nozzle angle is 18° and the blade to steam speed at inlet is 0.42. The ratio of the relative velocity at outlet to inlet of the blade is 0.84. The blade outlet angle is 3° less than the inlet angle. The steam flow rate is 7 kg/s. Draw the velocity diagram for the blade and calculate the power developed and the axial thrust.

[**Ans.** 1671.5 kW ; 490 N]

3. In a stage of an impulse turbine provided with a single row wheel, the mean diameter of the blade ring is 800 mm and the speed of rotation is 3000 r.p.m. The steam issues from the nozzles with a velocity of 300 m/s and the nozzle angle is 20°. The inlet and outlet blade angles are equal and the blade friction factor is 0.86. What is the power developed in the blading when the axial thrust on the blades is 140 N. [**Ans.** 358 kW]

4. Steam with absolute velocity of 400 m/s is supplied through a nozzle to a single stage impulse turbine. The nozzle angle is 25°. The mean diameter of blade rotor is 1 m and it has a speed of 2000 r.p.m. Find suitable blade angles for zero axial thrust. If blade velocity coefficient is 0.9 and the steam flow rate is 10 kg/s, calculate the power developed. [**Ans.** 27°, 32° ; 525 kW]

5. The steam enters an impulse wheel having a nozzle angle of 20° at a velocity of 450 m/s. The exit angle of the moving blade is 20° and the relative velocity of the steam may be assumed to remain constant over the moving blades. If the blade speed is 180 m/s, determine :

1. Blade angle at inlet, 2. Work done per kg of steam, and 3. Power of the wheel, when the turbine is supplied with 1.8 kg of steam per second. [Ans. 33° ; 92.7 kN-m ; 167 kW]

6. A De-Lavel turbine it supplied with dry steam and works on a pressure range from 10.5 bar to 0.3 bar. The nozzle angle is 20° and the blade exit angle is 30°. The mean blade speed is 270 m/s. If there is a 10% loss due to friction in the nozzle and blade velocity coefficient 0.82, find the thrust on the shaft per kW power developed. [Ans. 0 154 N/kW]

7. Steam issues from the nozzles of single stage impulse turbine at 1000 m/s and the nozzles are inclined at 24° to the direction of motion of the blades, which have a speed of 400 m/s. The blade angles at inlet and outlet are equal. If the steam enters and leaves the blades without shock and the flow over the blades is frictionless, find the inlet blade angle. Also determine the force exerted on the blades in the direction of their motion and power developed when the steam flows at the rate of 4000 kg/h. [Ans. 39° ; 1.135 kN ; 454 kW]

8. The following particulars refer to a velocity compounded impulse turbine having two rows of moving blades with a fixed row of guide blades between them :

The velocity of steam leaving the nozzle is 1250 m/s, nozzle angle is 20° and blade speed is 300 m/s. The blade angles of the first moving blade are symmetrical and the blade output angle of the second moving blade is 30°. The friction factor for all rows is 0.9.

Draw the velocity diagram and determine the power developed and the axial thrust on the rotor for a steam rate of 5000 kg/h. [Ans. 871 kW ; 173.6 N]

QUESTIONS

1. What is a turbine ? How does it differ from a steam engine ?
2. Give the classification of steam turbines.
3. Explain the principle of impulse turbine.
4. Show by graphical method, variation in the pressure and velocity of steam in an impulse turbine.
5. Describe the use of combined velocity triangle of an impulse turbine.
6. Describe a relation for the power developed by an impulse turbine.
7. What do you understand by the term 'friction' in an impulse turbine. How does it effect the combined velocity triangle.
8. Define two-stage impulse turbine. How will you draw the combined velocity triangle for such a turbine ?

OBJECTIVE TYPE QUESTIONS

1. The action of steam in a steam turbine is

   (*a*) static (*b*) dynamic

   (*c*) static and dynamic (*d*) neither static nor dynamic

2. In an impulse turbine

   (*a*) the steam is expanded in nozzles only and there is a pressure drop and heat drop

   (*b*) the steam is expanded both in fixed and moving blades continuously

   (*c*) the steam is expanded in moving blades only

   (*d*) the pressure and temperature of steam remains constant.

3. De-Laval turbines are mostly used

   (*a*) where low speeds are required

   (*b*) for small power purposes and low speeds (*c*) for small power purposes and high speeds

(*d*) for large power purposes

4. In impulse turbines, when friction is neglected, the relative velocity of steam at outlet tip of the blade is ..... the relative velocity of steam at inlet tip of the blade.

(*a*) equal to (*b*) less than (*c*) greater than

5. The blade friction in the impulse turbine reduces the velocity of steam by ..... while it passes over the blades.

(*a*) 10 to 15% (*b*) 15 to 20% (*c*) 20 to 30% (*d*) 30 to 40%

ANSWERS

1. (*b*) 2. (*a*) 3. (*c*) 4. (*a*) 5 (*a*)

23

Reaction Turbines

1. Introduction. 2. Parson's Reaction Turbine. 3. Pressure and Velocity in a Reaction Turbine. 4. Comparison between Impulse Turbine and Reaction Turbine. 5. Velocity Triangles for Moving Blades of a Reaction Turbine. 6. Combined Velocity Triangle for Moving Blades. 7. Power Produced by a Reaction Turbine. 8. Degree of Reaction. 9. Height of Blades of a Reaction Turbine.

23.1. Introduction

In a reaction turbine, the steam enters the wheel under pressure and flows over the blades. The steam, while gliding, propels the blades and make them to move. As a matter of fact, the turbine runner is rotated by the reactive forces of steam jets. The backward motion of the blades is similar to the recoil of a gun. It may be noted that an absolute reaction turbine is rarely used in actual practice.

23.2. Parson's Reaction Turbine

A *Parson's turbine is the simplest type of reaction steam turbine, and is commonly used. It has the following main components :

1. *Casing.* It is an air-tight metallic case, in which the steam from the boiler, under a high pressure and temperature, is distributed around the fixed blades (guide mechanism) in the casing. The casing is designed in such a way that the steam enters the fixed blades with a uniform velocity.

2. *Guide mechanism.* It is a mechanism, made up with the help of guide blades, in the form of a wheel. This wheel is, generally, fixed to the casing; that is why these guide blades are also called fixed blades. The guide blades are properly designed in order to :

(*a*) allow the steam to enter the runner without shock. This is done by keeping the relative velocity at inlet of the runner tangential to the blade angle.

(*b*) allow the required quantity of steam to enter the turbine. This is done by adjusting the openings of the blades.

The guide blades may be opened or closed by rotating the regulating shaft, thus allowing the steam to flow according to the need. The regulating shaft is operated by means of a governor whose function is to govern the turbine (*i.e.* to keep to speed constant at varying loads).

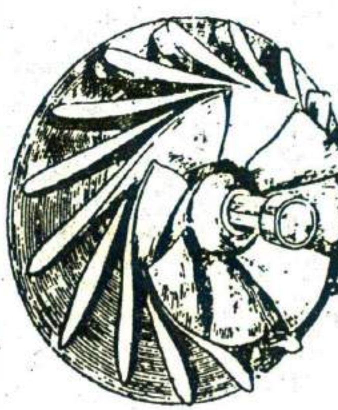


Fig. 23.1. Turbine runner.

* A Parson's turbine is also known as 50% reaction turbine. (See Art. 23.8)

3. *Turbine runner.* The turbine runner of a Parson's reaction turbine essentially consists of runner blades fixed to a shaft or rings, depending upon the type of turbine. The blades, fixed to the runner, are properly designed in order to allow the steam to enter and leave the runner without shock, as shown in Fig. 23.1.

The surface of the turbine runner is made very smooth to minimise the frictional losses. The turbine runner is, generally, cast in one piece. But sometimes, it is made up of separate steel plates welded together.

4. *Draft tube.* The steam, after passing through the runner, flows into the condenser through a tube called draft tube. It may be noted that if this tube is not provided in the turbine, then the steam will move freely and will cause steam eddies.

23.3. Pressure and Velocity of Steam in a Reaction Turbine

It will be interesting to know that the pressure in a reaction turbine is reduced in the fixed blades as well as in moving blades. The velocity of steam is increased in the fixed blades, and is reduced while passing through the moving blades.

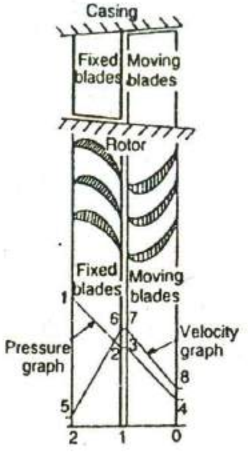


Fig. 23.2. Pressure and velocity graphs of a reaction turbine.

Fig. 23.2 shows the pressure and velocity graphs of the steam while it flows in the fixed and moving blades of a reaction turbine. The pressure graph 1-2-3-4 represents steam pressure at entrance of the fixed blades, exit of the fixed blades, entrance of the moving blades and exit of the moving blades respectively. Similarly, velocity graph 5-6-7-8 represents the velocity of steam at entrance of the fixed blades, exit of the fixed blades, entrance of the moving blades and exit of the moving blades respectively.

23.4. Comparison between Impulse Turbine and Reaction Turbine

Following are the few points of comparison between an impulse turbine and a reaction turbine :

S. No.	Impulse turbine	Reaction turbine
1.	The steam flows through the nozzles and impinges on the moving blades.	The steam flows first through guide mechanism and then through the moving blades.
2.	The steam impinges on the buckets with kinetic energy.	The steam glides over the moving vanes with pressure and kinetic energy.
3.	The steam may or may not be admitted over the whole circumference.	The steam must be admitted over the whole circumference.
4.	The steam pressure remains constant during its flow through the moving blades.	The steam pressure is reduced during its flow through the moving blades.
5.	The relative velocity of steam while gliding over the blades remains constant (assuming no friction).	The relative velocity of steam while gliding over the moving blades increases (assuming no friction).
6.	The blades are symmetrical.	The blades are not symmetrical.
7.	The number of stages required are less for the same power developed.	The number of stages required are more for the same power developed.

23.5. Velocity Triangles for Moving Blades of a Reaction Turbine

We have already discussed that in a reaction turbine, the steam enters at one end (say *C*) of the moving blades from the guide mechanism. The jet then glides over the inside surface of the blades and finally leaves from the other edge (say *D*), as shown in Fig. 23.3. It may be noted that the jet enters and leaves the blades tangentially for shockless entry and exit.

Consider steam, in the form of a jet, entering the curved blade (after leaving the fixed blade) at *C*. Let the jet glides over the inside surface and leaves the blade at *D*, as shown in Fig. 23.3. Now let us draw the velocity triangles at inlet and outlet tips of the moving blade, as shown in Fig. 23.3.

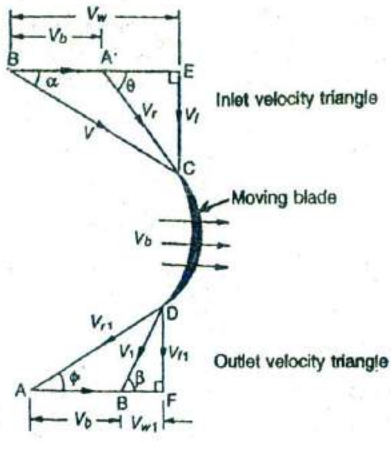


Fig. 23.3. Velocity triangles for a reaction turbine.

Let V_b = Linear velocity of the moving blade (*AB*).

V = Absolute velocity of steam entering the moving blade (*BC*).

V_r = Relative velocity of jet to the moving blade (*AC*). It is the vectorial difference of V_b and V.

V_f = Velocity of flow at entrance (*EC*). It is the vertical component of V.

V_w = Velocity of whirl at entrance (*BE*). It is the horizontal component of V.

α = Angle with the direction of motion of the blade at which the steam enters the blade.

θ = Angle which the relative velocity of jet (V_r) makes with the direction of motion of the blade.

$V_1, V_{r1}, V_{f1}, V_{w1}, \beta, \phi$ = Corresponding values at exit of the moving blade.

It may be seen, from the above, that original notations (*i.e.* V, V_r, V_f, V_w) stand for inlet triangle. The notations with suffix 1 (*i.e.* $V_1, V_{r1}, V_{f1}, V_{w1}$) stand for outlet triangle. It may be noted that as the steam enters and leaves the blades without any shock (or in other words tangentially), therefore shape of the blades will be such that V_r and V_{r1} will be along the tangents to the blades at inlet and outlet respectively.

The steam jet enters the blades along *BC* with a velocity (V) at an angle α with the direction of motion of the blade. The axial component of V (*i.e.* *CE*) which does no work on the blade, is known as *velocity of flow* (V_f). It causes the steam to flow through the blade and also an axial thrust on the rotor. The linear velocity or mean velocity of the blades (V_b) is represented by *AB* in magnitude and direction. The length *AC* represents the relative velocity (V_r) of the steam with respect to the blade. The jet now glides over and leaves the blade with a relative velocity (V_{r1}) which is represented by *DA*. The absolute velocity of jet (V_1) as it leaves the blade is represented by *DB* inclined at an angle

β with the direction of blade motion. The tangential component of V_1 (represented by BF) is known as *velocity of whirl* at exit (V_{w1}). The axial component of V_1 (represented by DF) is known as *velocity of flow* at exit (V_{f1}).

Note : The inlet triangle of velocities is represented by BEC whereas the outlet triangle by AFD.

23.6. Combined Velocity Triangle for Moving Blades

In the last article, we have discussed the inlet and outlet velocity triangles separately. For the sake of simplification, a combined velocity triangle for the moving blade is drawn, for solving problems on steam turbines, as shown in Fig. 23.4, as discussed below :

1. First of all, draw a horizontal line and cut off AB equal to the velocity of blade (V_b), to some suitable scale.

2. Now at B, draw a line BC at angle α with AB. Similarly at A, draw a line AC at angle θ with EA meeting the first line at C. Now CA and CB represent the relative velocity (V_r) and absolute velocity (V) of steam at inlet, to the scale.

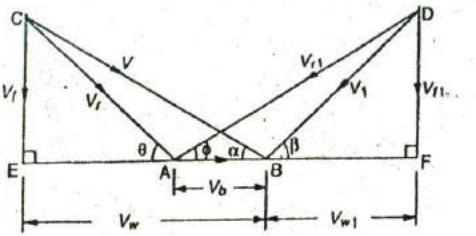


Fig. 23.4. Combined velocity triangle for a Parson's reaction turbine.

3. At A, draw a line AD at an angle ϕ (such that $\phi = \alpha$) with AB. Similarly at B draw a line BD at an angle β (such that $\beta = \theta$) with AB meeting the first line at D. Now DA and DB represent the relative velocity (V_{r1}) and absolute velocity (V_1) of steam at outlet, to the scale.

4. From C and D draw perpendiculars meeting the line AB produced at E and F.

5. Now EB and CE represent the velocity of whirl and velocity of flow at inlet (V_w and V_f) to the scale. Similarly BF and DF represent the velocity of whirl and velocity of flow at outlet (V_{w1} and V_{f1}), to the scale.

Note : A careful study of the combined velocity diagram of Parson's reaction turbine will reveal that it is symmetrical about the central line. Therefore following relations exist in the combined velocity diagram :

$$V_f = V_{f1}\,;\; V = V_{r1}\,;\; V_r = V_1\,;\; EA = BF$$

23.7. Power Produced by a Reaction Turbine

Consider a reaction turbine working under the action of steam pressure. Let us draw a combined velocity triangle for the reaction turbine, as shown in Fig. 23.4.

Let m = Mass of the steam flowing through the turbine in kg/s, and

$(V_w + V_{w1})$ = Change in the velocity of whirl in m/s.

We know that according to the Newton's second law of motion, force in the direction of motion of the blades,

$$F_X = \text{Mass of steam flowing/second} \times \text{Change in the velocity of whirl}$$

$$m\,[V_w - (-V_{w1})] = m\,[V_w + V_{w1}] = m \times EF \text{ N} \qquad \ldots (i)$$

and work done in the direction of motion of the blades

$$= \text{Force} \times \text{Distance}$$

$$= m\,(V_w + V_{w1})\,V_b = m \times EF \times AB \text{ N-m/s} \qquad \ldots (ii)$$

∴ Power produced by the turbine,

$$P = m\,(V_w + V_{w1})\,V_b \text{ watts} \qquad \ldots (\because 1 \text{ N-m/s} = 1 \text{ watt})$$

Similarly, we can find out the axial thrust on the wheel, which is due to difference of velocities of flow at inlet and outlet. Mathematically, axial thrust,

$$F_Y = \text{Mass of steam flowing /second} \times \text{Change in the velocity of flow}$$

$$= m\,(V_f - V_{f1}) = m\,(CE - DF)\text{ N} \qquad \ldots (iii)$$

Note : In equation (*i*), the value of V_{w1} is taken as *negative* because of the opposite direction of V_w with respect to the blade motion.

23.8. Degree of Reaction

We have already discussed in Art. 23.3 that in a reaction turbine, the pressure drop takes place in both the fixed and moving blades. In other words, there is an enthalpy drop in both the fixed and moving blades as shown on *h-s* diagram in Fig. 23.5. The ratio of the enthalpy or heat drop in the moving blades to the total enthalpy or heat drop in the stage is known as *degree of reaction.* Mathematically,

Degree of reaction

$$= \frac{\text{Enthalpy or heat drop in the moving blades}}{\text{Total enthalpy or heat drop in the stage}} = \frac{h_2 - h_3}{h_1 - h_3}$$

The enthalpy drop in the fixed blades per kg of steam is given by

$$h_1 - h_2 = \frac{V^2 - V_1^2}{2000}\text{ kJ/kg}$$

and enthalpy drop in the moving blades,

$$h_2 - h_3 = \frac{V_{r1}^2 - V_r^2}{2000}\text{ kJ/kg}$$

∴ Total enthalpy drop in the stage,

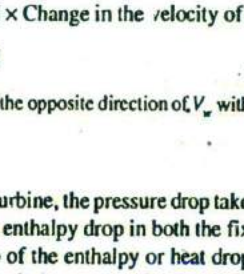


Fig. 23.5. Degree of reaction.

$$h_1 - h_3 = (h_1 - h_2) + (h_2 - h_3)$$

$$= \frac{V^2 - V_1^2}{2000} + \frac{V_{r1}^2 - V_r^2}{2000} = \frac{2\,(V_{r1}^2 - V_r^2)}{2000} = 2\,(h_2 - h_3)\text{ kJ/kg}$$

$$\ldots (\because \text{For Parson's reaction turbine, } V = V_{r1} \text{ and } V_1 = V_r)$$

We know that degree of reaction

$$= \frac{h_2 - h_3}{h_1 - h_3} = \frac{h_2 - h_3}{2\,(h_2 - h_3)} = \frac{1}{2} = 0.5 \text{ or } 50\%$$

Thus we see that a Parson's reaction turbine is a 50 percent reaction turbine.

Example 23.1. *In one stage of a reaction steam turbine, both the fixed and moving blades have inlet and outlet blade tip angles of 35° and 20° respectively. The mean blade speed is 80 m/s and the steam consumption is 22 500 kg per hour. Determine the power developed in the pair, if the isentropic heat drop for the pair is 23.5 kJ per kg.*

Solution. Given : $\theta = \beta = 35°$; $\phi = \alpha = 20°$; $V_b = 80$ m/s ; $m = 22\,500$ kg/h = 6.25 kg/s ; $^*h_d = 23.5$ kJ/kg

Now let us draw the combined velocity triangle, as shown in Fig. 23.6, as discussed below :

1. First of all, draw a horizontal line and cut off *AB* equal to 80 m/s (V_b) to some suitable scale.

2. Now at *B*, draw a line *BC* at an angle $\alpha = 20°$, with *AB*. Similarly, at *A* draw a line *AC* at an angle $\theta = 35°$ with *BA* meeting the first line at *C*.

3. At *A*, draw a line *AD* at angle $\phi = 20°$ (because $\phi = \alpha$) with *AB*. Similarly, at *B* draw a line *BD* at an angle $\beta = 35°$ (because $\beta = \theta$) with *AB* meeting the first line at *D*.

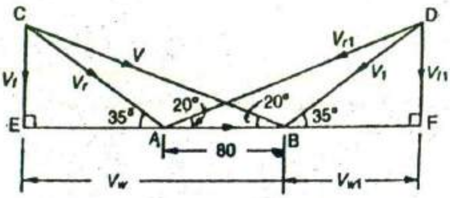


Fig. 23.6

4. From *C* and *D* draw perpendiculars meeting the line *AB* produced at *E* and *F*.

By **measurement, we find that the change in the velocity of whirl,

$$(V_w + V_{w1}) = 235 \text{ m/s}$$

We know that power developed in the pair,

$$P = m\,(V_w + V_{w1})\,V_b = 6.25 \times 235 \times 80 = 117\,500 \text{ W}$$

$$= 117.5 \text{ kW} \quad \textbf{Ans.}$$

Example 23.2. *A Parson's reaction turbine, while running at 400 r.p.m. consumes 30 tonnes of steam per hour. The steam at a certain stage is at 1.6 bar with dryness fraction of 0.9 and the stage develops 10 kW. The axial velocity of flow is constant and equal to 0.75 of the blade velocity. Find mean diameter of the drum and the volume of steam flowing per second. Take blade tip angles at inlet and exit as 35° and 20° respectively.*

Solution. Given : $N = 400$ r.p.m. ; $m = 30$ t/h = 8.33 kg/s ; $p = 1.6$ bar ; $x = 0.9$; $P = 10$ kW $= 10 \times 10^3$ W ; $V_f = 0.75\ V_b$; $\theta = \beta = 35°$; $\phi = \alpha = 20°$

* Superfluous data

** This value may also be found out from the geometry of the velocity diagram as discussed below :

$$V_f = V \sin 20° = 0.3240\ V$$

Similarly, $$V_f = V_r \sin 35° = 0.5736\ V_r$$

$\therefore$ $$0.3240\ V = 0.5736\ V_r \quad \text{or} \quad V = 1.77\ V_r$$

$$V_w = V \cos 20° = 0.9397\ V$$

Similarly, $$V_w = V_r \cos 35° + 80 = 0.8192\ V_r + 80$$

$\therefore$ $$0.9397\ V = 0.8192\ V_r + 80$$

or $$0.9397\,(1.77\ V_r) = 0.8192\ V_r + 80$$

$\therefore$ $$1.663\ V_r = 0.8192\ V_r + 80$$

$$0.8438\ V_r = 80 \quad \text{or} \quad V_r = 94.8 \text{ m/s}$$

and $$EA = V_r \cos 35° = 94.8 \times 0.8192 = 77.7$$

$\therefore$ $$(V_w + V_{w1}) = EA + AB + BF = 77.7 + 80 + 77.7 = 235.4 \text{ m/s}$$

Mean diameter of the drum

Let D = Mean diameter of the durm.

Now let us draw the combined velocity triangle, as shown in Fig. 23.7, as discussed below :

1. First of all, draw a horizontal line and cut off AB equal to 25 mm to represent the blade velocity (which is required to be found out).

2. Now at B, draw a line BC at an angle $\alpha = 20°$ with AB. Similarly at A, draw a line AC at an angle $\theta = 35°$ meeting the first line at C.

3. At A, draw a line AD at an angle $\phi = 20°$ with AB. Similarly at B, draw a line BD at an angle $\beta = 35°$ with AB meeting the first line at D.

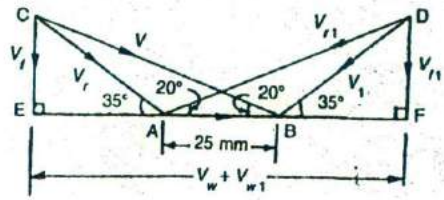


Fig. 23.7

4. From C and D draw perpendiculars meeting the line AB produced at E and F.

By *measurement, we find that the change in the velocity of whirl,

$$(V_w + V_{w1}) = 73.5 \text{ mm}$$

$$\therefore \quad \frac{(V_w + V_{w1})}{V_b} = \frac{73.5}{25} = 2.94$$

or $$(V_w + V_{w1}) = 2.94\, V_b$$

We know that the power developed (P),

$$10 \times 10^3 = m\,(V_w + V_{w1})\, V_b = 8.33 \times 2.94\, V_b \times V_b = 24.49 (V_b)^2$$

$$\therefore \quad (V_b)^2 = 408.3 \text{ or } V_b = 20.2 \text{ m/s}$$

We know that the blade velocity (V_b),

$$20.2 = \frac{\pi D N}{60} = \frac{\pi D \times 400}{60} = 20.94\, D$$

$$\therefore \quad D = 0.965 \text{ m} = 965 \text{ mm} \quad \textbf{Ans.}$$

Volume of steam flowing per second

From steam tables, corresponding to a pressure of 1.6 bar, we find that specific volume of steam,

$$v_g = 1.091 \text{ m}^3/\text{kg}$$

$\therefore$ Volume of steam flowing per second

$$= m\,x\,v_g = 8.33 \times 0.9 \times 1.091 = 8.18 \text{ m}^3/\text{s} \quad \textbf{Ans.}$$

23.9. Height of Blades of a Reaction Turbine

We have already discussed that in a reaction turbine, the steam enters the moving blades over the whole circumference. As a result of this, the area through which the steam flows is always full of steam. Now consider a reaction turbine whose end view of the blade ring in shown in Fig. 23.8.

---

* It may also be found out analytically in the same way as in the last example. Since the blade angles in this example as well as in the last example are same, therefore

$$(V_w + V_{w1}) = \frac{235.4}{80} \times 25 = 73.56 \text{ mm}$$

Let d = Diameter of rotor drum,

*h = Height of blades, and

V_{f1} = Velocity of flow at exit.

∴ Total area available for the steam to flow,

$$A = \pi (d+h) h$$

and volume of steam flowing $= \pi (d+h) h V_{f1}$

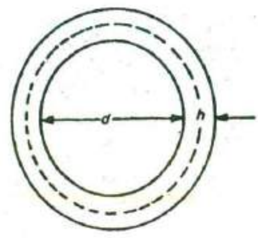


Fig. 23.8. Height of blades for reaction turbines.

We know that volume of 1 kg of steam at the given pressure is v_g (from steam tables). Therefore mass of steam flowing,

$$m = \frac{\pi (d+h) h V_{f1}}{v_g} \text{ kg/s}$$

If the steam has a dryness fraction of x, then mass of steam flowing,

$$m = \frac{\pi (d+h) h V_{f1}}{x v_g} = \frac{\pi d_m h V_{f1}}{x v_g} \text{ kg/s}$$

where d_m is the mean blade diameter and is equal to $(d+h)$.

Note : In most of the reaction turbines, the velocity of flow is constant at inlet and outlet (*i.e.* $V_f = V_{f1}$). Therefore, we can use the value of V_f instead of V_{f1} in the above relation.

Example 23.3. *In a reaction turbine, the blade tips are inclined at 35° and 20° in direction of motion. The guide blades are of the same shape as the moving blades, but reversed in direction. At a certain place in the turbine, the drum diameter is 1 metre and the blades are 100 mm high. At this place, steam has a pressure of 1.7 bar and dryness 0.935. If the speed of the turbine is 250 r.p.m. and the steam passes through the blades without shock, find the mass of steam flow and the power developed in the ring of the moving blades.*

Solution. Given : $\theta = \beta = 35°$; $\phi = \alpha = 20°$; $d = 1$ m ; $h = 100$ mm $= 0.1$ m ; $p = 1.7$ bar ; $x = 0.935$; $N = 250$ r.p.m.

We know that blade speed,

$$V_b = \frac{\pi (d+h) N}{60} = \frac{\pi (1+0.1) 250}{60} = 14.4 \text{ m/s}$$

Now let us draw the combined velocity triangle, as shown in Fig. 23.9, as discussed below :

1. First of all, draw a horizontal line, and cut off AB equal to 14.4 m/s to some suitable scale to represent the velocity of blade (V_b).

2. Now draw inlet velocity triangle ABC on the base AB with $\alpha = 20°$ and $\theta = 35°$.

3. Similarly draw outlet velocity triangle ABD on the same base AB with $\phi = 20°$ and $\beta = 35°$.

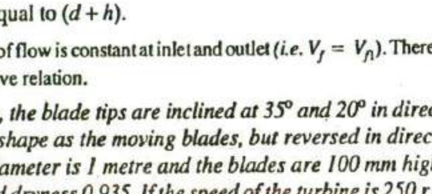


Fig. 23.9

4. From C and D draw perpendiculars to meet the line AB produced at E and F.

* Generally the height of blades is taken as $k\,d$, where k is a design constant, whose value is usually taken as 1/12 and d is the diameter of the rotor drum.

By measurement from velocity triangle, we find that

Change in the velocity of whirl, $(V_w + V_{w1}) = EF = 42.5$ m/s

and velocity of flow at outlet, $V_{f1} = DF = 10$ m/s

Mass of steam flow

From steam tables, corresponding to a pressure of 1.7 bar, we find that the specific volume of steam, $v_g = 1.031$ m^3/kg.

We know that mass of steam flow,

$$m = \frac{\pi (d+h)\, h\, V_{f1}}{x\, v_g} = \frac{\pi (1+0.1)\, 0.1 \times 10}{0.935 \times 1.031} = 3.58 \text{ kg/s} \quad \textbf{Ans.}$$

Power developed in the ring of the moving blades

We know that power developed in the ring of the moving blades,

$$P = m\,(V_w + V_{w1})\, V_b = 3.58 \times 42.5 \times 14.4 = 2191 \text{ W}$$

$$= 2.191 \text{ kW} \quad \textbf{Ans.}$$

Example 23.4. *A reaction turbine runs at 300 r.p.m. and its steam consumption is 15 400 kg/h. The pressure of steam at a certain pair is 1.9 bar ; its dryness 0.93 and power developed by the pair is 3.5 kW. The discharging blade tip angle is 20° for both fixed and moving blades and the axial velocity of flow is 0.72 of the blade velocity. Find the drum diameter and blade height. Take the tip leakage steam as 8%, but neglect blade thickness.*

Solution. Given : $N = 300$ r.p.m. ; $m_1 = 15\,400$ kg/h $= 4.28$ kg/s ; $p = 1.9$ bar ; $x = 0.93$; $P = 3.5 \text{ kW} = 3.5 \times 10^3$ W ; $\alpha = \phi = 20°$; $V_f = 0.72\, V_b$

Since the tip leakage steam is 8%, therefore actual mass of steam flowing over the blades,

$$m = 4.28 - (4.28 \times 0.08) = 3.94 \text{ kg/s}$$

Blade height

t $\quad h$ = Blade height, and

d_m = Mean diameter of the blades.

We know that blade velocity,

$$V_b = \frac{\pi\, d_m\, N}{60} = \frac{\pi\, d_m \times 300}{60} = 15.71\, d_m \text{ m/s}$$

$$\therefore \quad V_f = 0.72 \times 15.71\, d_m = 11.3\, d_m \text{ m/s}$$

Now let us draw the combined velocity triangle, as shown in Fig. 23.10, as discussed below

1. First of all, draw a horizontal line, and cut off AB equal to $15.71\, d_m$, to some suitable scale representing the blade velocity (V_b).

2. Now draw inlet velocity triangle ABC on the base AB with $\alpha = 20°$ and $BC = V_f / \sin 20° = 11.3\, d_m / 0.342 = 33\, d_m$, to the scale.

3. Similarly, draw outlet velocity triangle on the same base AB with $\phi = 20°$ and $V_{r1} = V_{f1} / \sin 20° = 11.3\, d_m / 0.342 = 33\, d_m$, to the scale.

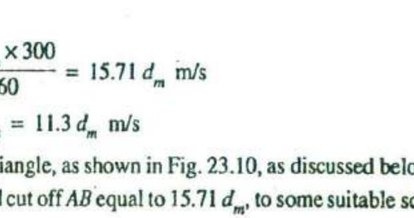


Fig. 23.10

4. From C and D draw perpendiculars to meet the line AB produced at E and F.

By measurement from velocity triangle, we find that change in the velocity of whirl,

$$(V_w + V_{w1}) = 46\, d_m \text{ m/s}$$

We know that power developed (P),

$$3.5 \times 10^3 = m\,(V_w + V_{w1})\, V_b = 3.94 \times 46\, d_m \times 15.71\, d_m = 2845\, d_m^2$$

$$\therefore \quad d_m^2 = 1.23 \quad \text{or} \quad d_m = 1.11 \text{ m}$$

and

$$V_{f1} = V_f = 11.3\, d_m = 11.3 \times 1.11 = 12.54 \text{ m/s}$$

From steam tables, corresponding to a pressure of 1.9 bar, we find that specific volume of steam,

$$v_g = 0.929 \text{ m}^3/\text{kg}$$

We know that mass of steam flow (m),

$$3.94 = \frac{\pi\, d_m\, h\, V_{f1}}{x\, v_g} = \frac{\pi \times 1.11 \times h \times 12.54}{0.93 \times 0.929} = 50.6\, h$$

$$\therefore \quad h = 0.078 \text{ m} = 78 \text{ mm} \quad \textbf{Ans.}$$

Drum diameter

We know that drum diameter,

$$d = d_m - h = 1.11 - 0.078 = 1.032 \text{ m} \quad \textbf{Ans.}$$

Example 23.5. *At a particular stage of a reaction steam turbine, the mean blade speed is 60 m/s. Steam is at a pressure of 3 bar with a temperature of 200° C. If the fixed and moving blades, at this stage, have inlet angle 30° and exit angle 20°, determine : (a) blade height at this stage, if the blade height is 1/10 of the mean blade ring diameter and the steam flow is 10 kg/s, (b) power developed by a pair of fixed and moving blade rings at this stage, and (c) the heat drop required by the pair if the steam expand with an efficiency of 85%.*

Solution. Given : $V_b = 60$ m/s ; $p = 3$ bar ; $T = 200°$ C ; $\theta = \beta = 30°$; $\phi = \alpha = 20°$

(*a*) *Blade height*

Let d = Mean diameter of blade ring,

h = Blade height = $d/10$, and ... (Given)

m = Mass of steam flow = 10 kg/s ... (Given)

Now let us draw the combined velocity triangle, as shown in Fig. 23.11, as discussed below :

1. First of all, draw a horizontal line and cut off AB equal to 60 m/s, to some suitable scale, to represent the blade speed (V_b).

2. Now draw inlet velocity triangle ABC on the base AB with $\alpha = 20°$ and $\theta = 30°$.

3. Similarly draw outlet velocity triangle ABD on the same base AB with $\phi = 20°$ and $\beta = 30°$.

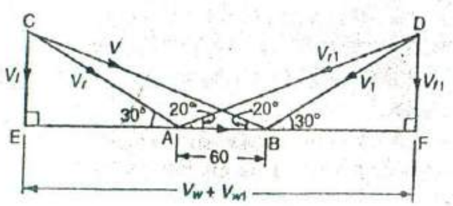


Fig. 23.11

4. From C and D draw perpendiculars to meet the line AB produced at E and F.

By measurement* from the velocity diagram, we find that

Change in the velocity of whirl, $(V_w + V_{w1}) = EF = 265$ m/s

and velocity of flow at exit, $V_{f1} = DF = 60$ m/s

From steam tables of superheated steam, corresponding to a pressure of 3 bar and 200° C, we find that the specific volume of steam,

$$v_{sup} = 0.7164 \text{ m}^3/\text{kg}$$

We know that mass of steam flow (m),

$$10 = \frac{\pi (d+h)\, h\, V_{f1}}{v_{sup}} = \frac{\pi (10\, h + h)\, h \times 60}{0.7164} = 2894\, h^2$$

$$\therefore \quad h = 0.059 \text{ m} = 59 \text{ say } 60 \text{ mm} \quad \textbf{Ans.}$$

(b) Power developed

We know that power developed by a pair of fixed and moving blade rings,

$$P = m\,(V_w + V_{w1})\, V_b = 10 \times 265 \times 60 = 159\,000 \text{ W}$$

$$= 159 \text{ kW} \quad \textbf{Ans.}$$

(c) Heat drop required by the pair

Since the steam expands with an efficiency of 85%, therefore heat drop required by the pair

$$= \frac{159}{0.85} = 187 \text{ kJ/s} \quad \textbf{Ans.} \qquad \ldots (\because 1 \text{ kW} = 1 \text{ kJ/s})$$

Example 23.6. *At a certain pair in a reaction turbine, the steam leaves the fixed blade at a pressure of 3 bar with a dryness fraction of 0.98 and a velocity of 130 m/s. The blades are 20 mm high and dicharge angle for both the rings is 20°. The ratio of axial velocity of flow to the blade velocity is 0.7 at inlet and 0.76 at exit from the moving blade. If the turbine uses 4 kg of steam per second with 5% tip leakage, find the mean blade diameter and the power developed in the ring.*

Solution. Given : $p = 3$ bar ; $x = 0.98$; $V = 130$ m/s ; $h = 20$ mm $= 0.02$ m ; $\alpha = \phi = 20°$; $V_f = 0.7\, V_b$; $V_{f1} = 0.76\, V_b$; $m_1 = 4$ kg/s

Since the tip leakage is 5%, therefore actual mass of steam flow in the turbine,

$$m = 4 - (4 \times 0.05) = 3.8 \text{ kg/s}$$

Mean blade diameter

Let d_m = Mean blade diameter.

---

* These values may also be found out analytically by the geometry of the velocity diagram by method discussed on page 526 or by the sine rule as discussed below :

$$\frac{\text{[illegible]}}{\sin 150^\circ} = \frac{V_r}{\sin 20^\circ} = \frac{60}{\sin 10^\circ}$$

$$\therefore \quad V_r = \frac{60}{\sin 10^\circ} \times \sin 20^\circ = \frac{60}{0.1736} \times 0.3420 = 118.2 \text{ m/s}$$

and $$V_f = V_r \sin 30^\circ = 118.2 \times 0.5 = 59.1 \text{ m/s}$$

$$\therefore \quad EA = V_r \cos 30^\circ = 118.2 \times 0.866 = 102.4 \text{ m/s}$$

and $$(V_w + V_{w1}) = EA + AB + BF = 102.4 + 60 + 102.4 = 264.8 \text{ m/s}$$

From the combined velocity of triangle, we find that the velocity of flow at inlet,

$$V_f = V \sin 20° = 130 \times 0.342 = 44.46 \text{ m/s}$$

and blade velocity,
$$V_b = \frac{V_f}{0.7} = \frac{44.46}{0.7} = 63.5 \text{ m/s}$$

∴ Velocity of flow at exit,

$$V_{f1} = 0.76\, V_b = 0.76 \times 63.5 = 48.3 \text{ m/s}$$

From steam tables, corresponding to a pressure of 3 bar, we find that specific volume of steam,

$$v_g = 0.6055 \text{ m}^3/\text{kg}$$

We know that mass of steam flow (m),

$$3.8 = \frac{\pi d_m \times h\, V_{f1}}{x\, v_g} = \frac{\pi d_m \times 0.02 \times 48.3}{0.98 \times 0.6055} = 5.1\, d_m$$

$$\therefore \quad d_m = 0.745 \text{ m} = 745 \text{ mm} \quad \textbf{Ans.}$$

Power developed in the ring

Now let us draw the combined velocity triangle, as shown in Fig. 23.12, as discussed below :

1. First of all, draw a horizontal line and cut off AB equal to 63.5 m/s, to some suitable scale, representing the blade velocity (V_b).

2. Now draw inlet velocity triangle ABC on the base AB with $\alpha = 20°$ and $V = 130$ m/s to the scale.

3. Similarly, draw outlet velocity triangle ABD on the same base AB with $\phi = 20°$ and V_{r1} equal to $V_{f1}/\sin 20°$ $= 48.3 / 0.342 = 141.2$ m/s, to the scale.

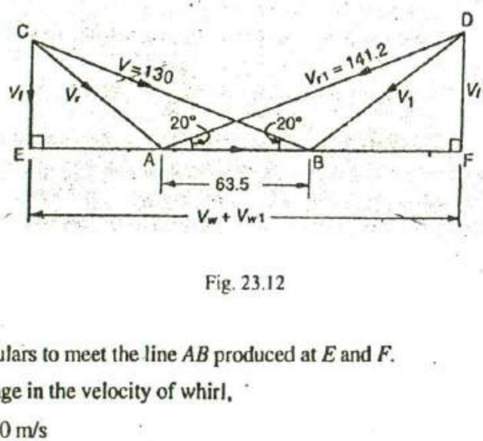


Fig. 23.12

4. From C and D draw perpendiculars to meet the line AB produced at E and F.

By measurement, we find that change in the velocity of whirl,

$$(V_w + V_{w1}) = EF = 190 \text{ m/s}$$

We know that power developed in the ring,

$$P = m\,(V_w + V_{w1})\, V_b = 3.8 \times 190 \times 63.5 = 45\,850 \text{ W}$$

$$= 45.85 \text{ kW} \quad \textbf{Ans.}$$

EXERCISES

1. The following particulars refer to a stage of a Parson's steam turbine, comprising one ring of fixed blades and one ring of moving blades :

Mean diameter of blade ring = 700 mm ; R.P.M. = 3000 ; Steam velocity at exit of blades = 160 m/s ; Blade outlet angle = 20° ; Steam flow through blades = 7 kg/s.

Draw a neat velocity diagram and find (*a*) Blade inlet angle ; (*b*) Tangential force on the ring of moving blades ; and (*c*) Power developed in the stage. **[Ans. 33.5° ; 2625 N ; 288.75 kW]**

2. A reaction turbine running at 360 r.p.m. consumes 5 kg of steam per second. The leakage is 10%. The discharge blade tip angle for both moving and fixed blades is 20°. The axial velocity of flow is 0.75 times blade velocity. The power developed by a certain pair is 4.8 kW where the pressure is 2 bar and dryness fraction is 0.95. Find the drum diameter and blades height. [Ans. 0.931 m ; 83 mm]

3. A 50% reaction turbine (with symmetrical velocity triangles) running at 400 r.p.m. has the exit angle of the blades as 20° and the velocity of steam relative to the blades at exit is 1.35 times the mean blade speed. The steam flow rate is 8.33 kg/s and at a particular stage, the specific volume is 1.381 m^3/kg. Calculate for this stage 1. a suitable blade height, assuming the rotor mean diameter 12 times the blade height, and 2. the diagram work. [Ans. 138 mm ; 153.14 N-m/s]

4. The outlet angle of a blade of Parson's turbine is 20° and the axial velocity of flow of steam is 0.5 times the mean blade velocity. Draw the velocity diagram for a stage consisting of one fixed and one moving row of blades.

It is given that mean diameter = 710 mm and speed of rotation = 3000 r.p.m. Find the inlet angle of blades if the steam is to enter the blade channels without shock.

If the blade height is 64 mm, the mean steam pressure 5.6 bar, the steam dry saturation (v_g = 0.3434 m^3/kg) ; find the power developed in the stage. [Ans. 50° ; 516.9 kW]

5. At a particular ring of a reaction turbine, the blade speed is 66 m/s and the flow of steam is 4 kg/s dry saturated at 1.4 bar. Both fixed and moving blades have inlet and exit angles of 35° and 20° respectively. Calculate : 1. the required blade height which is to be one-tenth of the mean blade ring diameter, 2. the power developed by the pair of rings, and 3. the heat drop required by the pair if the steam expand with an efficiency of 80 percent.

[Ans. 55 mm ; 57 kW ; 71.3 kJ/s]

6. The blade angles of both fixed and moving blades of a reaction steam turbine are 35° at the receiving tips and 20° at the discharging tips. At a certain point in the turbine, the drum diameter is 1.37 m and the blade height is 127 mm. The pressure of steam supply to a ring of fixed blades at this point is 1.25 bar and the dryness fraction is 0.925. Find the workdone in next row of moving blades for 1 kg of steam at 600 r.p.m., the steam passing through the blades without shock.

Assuming an efficiency of 85% for the pair of rings of fixed and moving blades, find the heat drop in the pair and the state of steam at entrance to the next row of fixed blades. [Ans. 6.77 kN-m/s ; 7.962 kJ/s]

QUESTIONS

1. Distinguish between impulse and reaction turbine.

2. Explain the functions of the blading of a reaction turbine.

3. Draw the combined velocity triangle for a single stage reaction turbine and derive an expression for workdone per stage.

4. Define the term 'degree of reaction' as applied to a reaction turbine. Show that for a Parson's reaction turbine, the degree of reaction is 50 percent.

5. What do you understand by the term 'height of blades' as applied to a reaction turbine.

OBJECTIVE TYPE QUESTIONS

1. In a reaction turbine
   - (*a*) the steam is allowed to expand in the nozzle, where it gives a high velocity before it enters the moving blades
   - (*b*) the expansion of steam takes place partly in the fixed blades and partly in the moving blades
   - (*c*) the steam is expanded from a high pressure to a condenser pressure in one or more nozzles
   - (*d*) the pressure and temperature of steam remains constant.

2. The Parson's reaction turbine has
   (*a*) only moving blades (*b*) only fixed blades
   (*c*) identical fixed and moving blades (*d*) fixed and moving blades of different shape
3. The degree of reaction is defined as the ratio of
   (*a*) heat drop in the fixed blades to the heat drop in tne moving blades
   (*b*) heat drop in the moving blades to the heat drop in the fixed blades
   (*c*) heat drop in the moving blades to the total heat drop in the fixed and moving blades
   (*d*) total heat drop in the fixed and moving blades to the heat drop in the moving blades
4. For a Parson's reaction turbine, the degree of reaction is
   (*a*) 20% (*b*) 30% (*c*) 40% (*d*) 50%
5. In a reaction turbine, when the degree of reaction is zero, then there is
   (*a*) no heat drcp in the moving blades
   (*b*) no heat drop in the fixed blades
   (*c*) maximum heat drop in the moving blades
   (*d*) maximum heat drop in the fixed blades

ANSWERS

1. (*b*) 2. (*c*) 3. (*c*) 4. (*d*) 5. (*a*)

24

Performance of Steam Turbines

1. Introduction. 2. Efficiencies of Steam Turbine. 3. Condition for Maximum Efficiency of an Impulse Turbine. 4. Condition for Maximum Efficiency of a Reaction Turbine. 5. Compounding of Impulse Steam Turbines (Methods of Reducing Rotor Speeds). 6. Velocity Compounding of an Impulse Turbine. 7. Pressure Compounding of an Impulse Turbine. 8. Pressure-velocity Compounding of an Impulse Turbine. 9. Internal Losses in Turbines. 10. Governing of Steam Turbines. 11. Throttle Governing of Steam Turbines.

24.1. Introduction

In the last two chapters, we have discussed impulse and reaction steam turbines. In these chapters, we have discussed power generated in these turbines. But in this chapter, we shall discuss their performance *i.e.* efficiencies and governing.

24.2. Efficiencies of Steam Turbine

The following efficiencies of impulse as well as reaction steam turbines are important from the subject point of view :

1. *Diagram or blading efficiency.* It is the ratio of the work done on the blades to the energy supplied to the blades.

Let V = Absolute velocity of inlet steam in m/s, and

m = Mass of steam supplied in kg/s.

$\therefore$ Energy supplied to the blade per second,

$$= \frac{mV^2}{2} \text{ J/s}$$

We know that work done on the blades per second

$$= m\,(V_w + V_{w1})\,V_b \text{ J/s}$$

$\therefore$ Diagram* or blading efficiency,

$$\eta_b = \frac{m\,(V_w + V_{w1})\,V_b}{mV^2/2} = \frac{2\,(V_w + V_{w1})\,V_b}{V^2}$$

The work done on the turbine blades may also be obtained from the kinetic energy at inlet and exit as discussed below :

Let V_1 = Absolute velocity of exit steam in m/s.

We know that kinetic energy at inlet per second

$$= \frac{mV^2}{2} \text{ J/s}$$

* It is called diagram efficiency because the quantities involved are obtained from velocity diagram.

and kinetic energy at exit per second

$$= \frac{mV_1^2}{2} \text{ J/s}$$

$\therefore$ Work done on the blades per second,

$$= \text{Loss of kinetic energy}$$

$$= \frac{mV^2}{2} - \frac{mV_1^2}{2} = \frac{m}{2}(V^2 - V_1^2) \text{ J/s}$$

and power developed, $$P = \frac{m(V^2 - V_1^2)}{2} \text{ watts} \quad \ldots (\because 1 \text{ J/s} = 1 \text{ watt})$$

$\therefore$ Blading efficiency, $$\eta_b = \frac{\frac{m}{2}(V^2 - V_1^2)}{\frac{mV^2}{2}} = \frac{V^2 - V_1^2}{V^2}$$

2. *Gross or stage efficiency.* It is the ratio of the work done on the blades per kg of steam to the total energy supplied per stage per kg of steam.

Let h_1 = Enthalpy or total heat of steam before expansion through the nozzle in kJ/kg of steam, and

h_2 = Enthalpy or total heat of steam after expansion through the nozzle in kJ/kg of steam.

$\therefore$ Enthalpy or heat drop in the nozzle ring of an impulse wheel,

$$h_d = h_1 - h_2 \text{ (in kJ/kg)}$$

and total energy supplied per stage $= 1000\, h_d$ J/kg of steam

We know that work done on the blade per kg of steam

$$= 1\,(V_w + V_{w1})\, V_b \text{ J/kg of steam}$$

$\therefore$ Gross or stage efficiency,

$$\eta_s = \frac{(V_w + V_{w1})\, V_b}{1000\, h_d} = \frac{(V_w + V_{w1})\, V_b}{1000\,(h_1 - h_2)}$$

3. *Nozzle efficiency.* It is the ratio of energy supplied to the blades per kg of steam to the total energy supplied per stage per kg of steam.

We know that energy supplied to the blades per kg of steam

$$= V^2/2 \text{ (in joules)}$$

$\therefore$ Nozzle efficiency, $$\eta_n = \frac{V^2/2}{1000\, h_d} = \frac{V^2}{2000\, h_d}$$

Note : We know that stage efficiency,

$$\eta_s = \frac{(V_w + V_{w1})\, V_b}{1000\, h_d} = \frac{2\,(V_w + V_{w1})\, V_b}{V^2} \times \frac{V^2}{2000\, h_d}$$

$$= \eta_b \times \eta_n$$

i.e. Stage efficiency = Blading efficiency × Nozzle efficiency

Example 24.1. *The velocity of steam at inlet to a simple impulse turbine is 1000 m/s and the nozzle angle is 20°. The mean blade speed is 400 m/s and the blades are symmetrical. The mass flow*

rate of steam is 0.75 kg/s. The friction effects on the blades are negligible. Estimate : (a) the blade angles ; (b) the tangential force on the blades; (c) the axial thrust; (d) the diagram power; and (e) the diagram efficiency.

Solution. Given : $V = 1000$ m/s ; $\alpha = 20°$; $V_b = 400$ m/s ; $\theta = \phi$ for symmetrical blades, $m = 0.75$ kg/s

(a) Blade angles

Now draw the combined velocity triangle, as shown in Fig. 24.1, as discussed below :

1. First of all, draw a horizontal line and cut off AB equal to 400 m/s, to some suitable scale, to represent the blade speed (V_b).

2. Now draw inlet velocity triangle ABC on the base AB with $\alpha = 20°$ and $BC = V = 1000$ m/s. By measurement, we find that blade angle at inlet,

$$\theta = 33° \text{ Ans.}$$

3. Similarly, draw outlet velocity triangle ABD on the same base AB with $\phi = \theta = 33°$ and $V_{r1} = V_r$

4. From C and D, draw perpendiculars to meet the line AB produced at E and F.

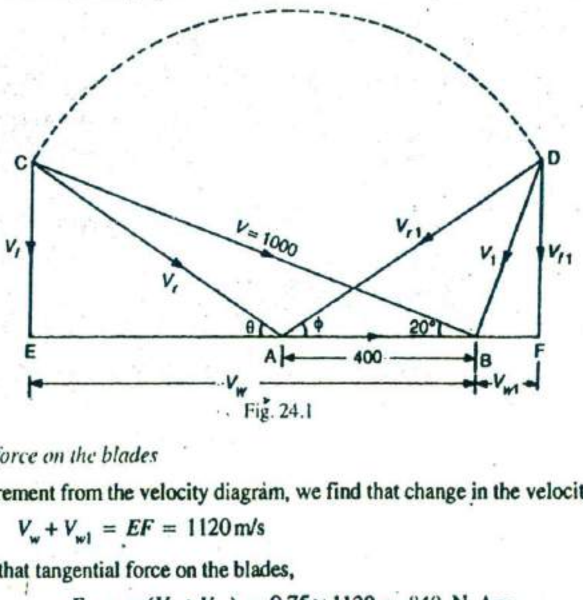


Fig. 24.1

(b) Tangential force on the blades

By measurement from the velocity diagram, we find that change in the velocity of whirl,

$$V_w + V_{w1} = EF = 1120 \text{ m/s}$$

We know that tangential force on the blades,

$$F_X = m(V_w + V_{w1}) = 0.75 \times 1120 = 840 \text{ N Ans.}$$

(c) Axial thrust

We know that axial thrust,

$$F_Y = m(V_f - V_{f1}) = 0 \text{ Ans.}$$

Note : Since the blades are symmetrical and friction effects on the blades are neglected, therefore $V_f = V_{f1}$.

(d) Diagram power

We know that diagram power,

$$P = m(V_w + V_{w1})V_b = 0.75 \times 1120 \times 400 = 336\,000 \text{ W}$$

$$= 336 \text{ kW Ans.}$$

(*e*) *Diagram efficiency*

We know that diagram efficiency,

$$\eta_b = \frac{2(V_w + V_{w1})V_b}{V^2} = \frac{2 \times 1120 \times 400}{(1000)^2} = 0.896 \quad \text{or} \quad 89.6\% \text{ Ans.}$$

Example 24.2. *The following particulars refer to a single row impulse turbine :*

Mean diameter of blade ring = 2.5 m ; Speed = 3000 r.p.m. ; Nozzle angle = 20° ; Ratio of blade velocity to steam velocity = 0.4 ; Blade friction factor = 0.8 ; Blade angle at exit = 3° less than that at inlet ; Steam flow = 36 000 kg/h.

Draw velocity diagram for moving blade and estimate (a) Power developed ; (b) Blade efficiency ; and (c) Steam consumption is kg/kW h.

Solution. Given : $d_m = 2.5$ m ; $N = 3000$ r.p.m. ; $\alpha = 20°$; $V_b / V = 0.4$; $K = V_{r1} / V_r = 0.8$; $\phi = \theta - 3°$; $m = 36\,000$ kg/h = 10 kg/s

We know that blade velocity,

$$V_b = \frac{\pi d_m N}{60} = \frac{\pi \times 2.5 \times 3000}{60} = 393 \text{ m/s}$$

$\therefore$ Steam velocity, $V = V_b / 0.4 = 393 / 0.4 = 982.5$ m/s

Now draw the combined velocity triangle, as shown in Fig. 24.2, as discussed below :

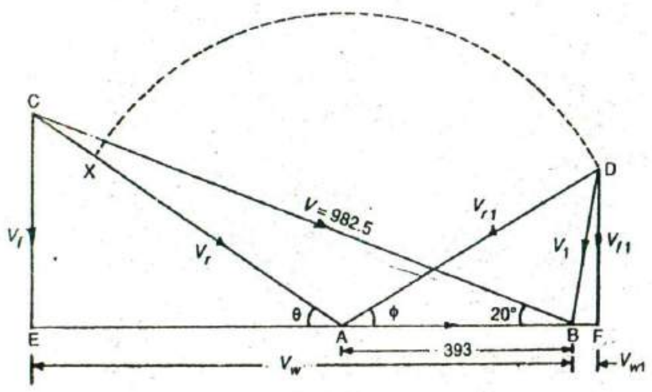


Fig. 24.2

1. First of all, draw a horizontal line and cut off AB equal to 393 m/s, to some suitable scale, to represent the blade velocity (V_b).

2 Now draw inlet velocity triangle ABC on the base AB with $\alpha = 20°$ and $BC = V = 982.5$ m/s. By measurement, we find that blade angle at inlet (θ) = 32.5°, and relative velocity of steam at inlet (V_r) = 626.7 m/s.

3. Similarly, draw outlet velocity triangle ABD on the same base AB with $\phi = \theta - 3° = 32.5 - 3 = 29.5°$ and $V_{r1} = AX = 0.8\ V_r = 0.8 \times 626.7 = 501.4$ m/s.

4. From C and D, draw perpendiculars to meet the line AB produced at E and F.

By measurement from velocity triangle, we find that change in the velocity of whirl,

$$V_w + V_{w1} = EF = 967 \text{ m/s}$$

(*a*) *Power developed*

We know that power developed,

$$P = m(V_w + V_{w1})V_b = 10 \times 967 \times 393 = 3800 \times 10^3 \text{ W}$$
$$= 3800 \text{ kW } \textbf{Ans.}$$

(*b*) *Blade efficiency*

We know that blade efficiency,

$$\eta_b = \frac{2(V_w + V_{w1})V_b}{V^2} = \frac{2 \times 967 \times 393}{(982.5)^2} = 0.787 \quad \text{or} \quad 78.7\% \textbf{ Ans.}$$

(*c*) *Steam consumption in kg/kWh*

We know that steam consumption

$$= \frac{m}{P} = \frac{36\,000}{3800} = 9.47 \text{ kg/kWh } \textbf{Ans.}$$

Example 24.3. *A single row impulse turbine receives 3 kg/s steam with a velocity of 425 m/s. The ratio of blade speed to jet speed is 0.4 and the stage output is 170 kW. If the internal losses due to disc friction etc. amount to 15 kW, determine the blading efficiency and the blade velocity coefficient. The nozzle angle is 16° and the blade exit angle is 17°.*

Solution. Given : $m = 3$ kg/s ; $V = 425$ m/s ; $V_b / V = 0.4$; Stage output = 170 kW ; Internal losses = 15 kW ; $\alpha = 16°$; $\phi = 17°$

Blading efficiency

We know that blade speed,

$$V_b = V \times 0.4 = 425 \times 0.4 = 170 \text{ m/s}$$

and total power developed, P = Stage output + Internal losses

$$= 170 + 15 = 185 \text{ kW} = 185 \times 10^3 \text{ W}$$

Let $V_w + V_{w1}$ = Change in the velocity of whirl.

We know that power developed (P),

$$185 \times 10^3 = m(V_w + V_{w1})V_b = 3(V_w + V_{w1})170$$

$$\therefore \quad V_w + V_{w1} = 185 \times 10^3 / 3 \times 170 = 363 \text{ m/s}$$

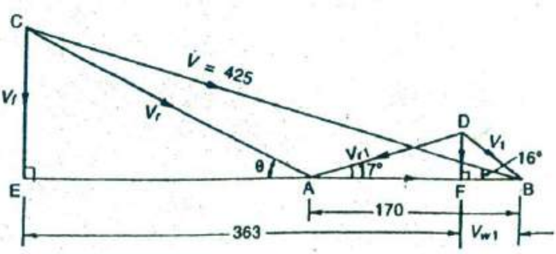


Fig. 24.3

Now draw the combined velocity triangle, as shown in Fig. 24.3, as discussed below :

1. First of all, draw a horizontal line and cut off AB equal to 170 m/s, to some suitable scale, to represent the blade speed (V_b).

2. Now draw the inlet velocity triangle ABC on the base AB with $\alpha = 16°$ and $V = 425$ m/s, to the scale.

3. Similarly, draw the outlet velocity triangle ABD on the same base AB with $\phi = 17°$ and $(V_w + V_{w1}) = 363$ m/s to the scale.

4. From C and D draw perpendiculars to meet the line AB at E and F. From the geometry of the figure, we find that V_{w1} is in the opposite direction of V_w. Therefore $(V_w - V_{w1}) = 363$ m/s.

By *measurement from the velocity diagram, we find that

Relative velocity at inlet, $V_r = 265$ m/s

and relative velocity at outlet, $V_{r1} = 130$ m/s

We know that blading efficiency,

$$\eta_b = \frac{2(V_w - V_{w1})V_b}{V^2} = \frac{2 \times 363 \times 170}{(425)^2} = 0.683 \quad \text{or} \quad 68.3\ \% \ \textbf{Ans.}$$

Blade velocity coefficient

We know that blade velocity coefficient,

$$K = \frac{V_{r1}}{V_r} = \frac{130}{265} = 0.49 \ \textbf{Ans.}$$

Example 24.4. *A two row curtis wheel operates at a blade speed of 150 m/s, when receiving 3 kg of steam per second at 10.5 bar dry and saturated. The ratio of blade speed to the steam speed at exit from the nozzle is 0.21 and nozzle efficiency is 90%. The nozzles are inclined at 16° to the plane of the wheel. The outlet angles of the first row moving, fixed and second row moving blades are respectively 20°, 24° and 32° with respective blade velocity coefficients 0.79, 0.83 and 0.88. Determine : (a) The pressure of steam at exhaust; (b) Diagram efficiency ; and (c) Stage efficiency.*

Solution. Given : $V_b = 150$ m/s; $m = 3$ kg/s ; $p = 10.5$ bar ; $V_b/V = 0.21$; $k = 90\% = 0.9$; $\alpha = 16°$; $\phi = 20°$; $\alpha' = 24°$; $\phi' = 32°$; $V_{r1}/V_r = 0.79$ for first moving blades ; $V'/V_1 = 0.83$ for fixed blades ; $V_{r1}'/V_r = 0.88$ for second moving blades

We know that steam speed at exit from the nozzle,

$$V = V_b / 0.21 = 150 / 0.21 = 714.3 \text{ m/s}$$

---

* These values may also be obtained from the geometry of the combined velocity triangle, as discussed below :

$$V_f = 425 \sin 16° = 425 \times 0.2756 = 117.1 \text{ m/s}$$

Now $$\tan\theta = \frac{117.1}{425\cos 16° - 170} = \frac{117.1}{(425 \times 0.9613) - 170} = 0.4909$$

$\therefore$ $$\theta = 26.15° \text{ and } \angle ACB = 26.15° - 16° = 10.15°$$

Now in triangle ABC,

$$\frac{V_r}{\sin 16°} = \frac{170}{\sin 10.15°}$$

$\therefore$ $$V_r = \frac{170}{\sin 10.15°} \times \sin 16° = \frac{170}{0.1763} \times 0.2756 = 265.8 \text{ m/s}$$

We know that $$AF = 361 - V_r \cos 26.15° = 361 - (265.8 \times 0.8976) = 122.4 \text{ m/s}$$

$$V_{r1} = \frac{122.4}{\cos 17°} = \frac{122.4}{0.9563} = 128 \text{ m/s}$$

(*a*) *Pressure of steam at exhaust*

Let p_2 = Pressure of steam at exhaust, and

h_d = Enthalpy or heat drop from pressure p_1 to p_2.

We know that the velocity of steam at exit from nozzle (V),

$$714.3 = 44.72\sqrt{k\,h_d} = 44.72\sqrt{0.9\,h_d}$$

$$\therefore \quad h_d = \left(\frac{714.3}{44.72}\right)^2 \times \frac{1}{0.9} = 283.5 \text{ kJ/kg}$$

Now let us complete the Mollier diagram for the steam flow through the nozzle, as shown in Fig. 24.4. From the Mollier diagram, we find that

$$p_2 = 2 \text{ bar } \textbf{Ans.}$$

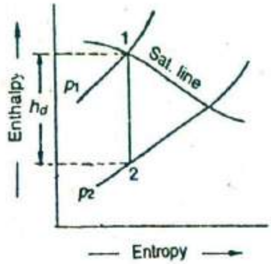


Fig. 24.4

(*b*) *Diagram efficiency*

Now draw the combined velocity triangle for the two stage impulse turbine, as shown in Fig. 24.5, as discussed below :

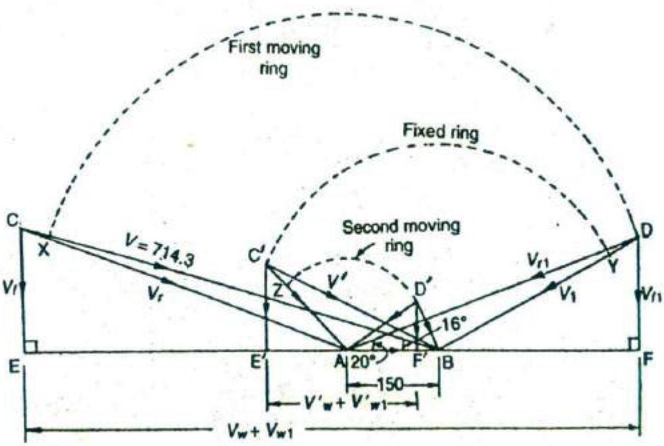


Fig. 24.5

1. First of all, draw a horizontal line, and cut off AB equal to 150 m/s to a suitable scale to represent the blade velocity (V_b).

2. Now draw the inlet velocity triangle ABC for the first moving ring on the base AB with nozzle angle for the first moving ring ($\alpha = 16°$) and $V = 714.3$ m/s. By measuring the side AC, we find that $V_r = 570$ m/s.

3 Now draw the outlet velocity triangle ABD for the first moving ring on the same base AB with $\phi = 20°$ and $V_{r1} = 0.79\ V_r = 0.79 \times 570 = 450.3$ m/s, to the scale. By measuring the side BD, we find that $V_1 = 375$ m/s.

4. Now draw the inlet velocity triangle ABC' for the second moving ring on the same base AB with $\alpha' = 24°$ and $V' = 0.83 \times 375 = 311.2$ m/s, to the scale. By measuring the side AC', we find that $V_r' = 185$ m/s.

5. Now draw the outlet velocity triangle ABD' for the second moving ring on the same base AB with $\phi' = 32°$ and $V_{r1}' = 0.88 \times V_r' = 0.88 \times 185 = 162.8$ m/s to the scale.

The following values are measured from the combined velocity diagram :

$$EF = (V_w + V_{w1}) = 1020 \text{ m/s}$$

$$E'F' = (V_w' + V_{w1}') = 275 \text{ m/s}$$

We know that diagram efficiency,

$$\eta_b = \frac{2\,AB\,(EF + E'F')}{V^2} = \frac{2 \times 150\,(1020 + 275)}{(714.3)^2} = 0.761 \text{ or } 76.1\% \text{ Ans.}$$

(c) *Stage efficiency*

We know that stage efficiency,

$$\eta_s = \frac{AB\,(EF + E'F')}{1000\,h_d} = \frac{150\,(1020 + 275)}{1000 \times 283.5} = 0.685 \text{ or } 68.5\% \text{ Ans.}$$

Example 24.5. *In an impulse turbine, there are two rings of moving blades separated by fixed blades. The tips of the moving blades are inclined at 30° to the plane of motion. The steam enters the wheel chamber through a nozzle to which the steam is supplied at 240° C and 11.5 bar. The pressure in the wheel chamber is 5 bar. Find the speed of the blades, so that the steam finally discharged is axial. Assume a 10% loss in velocity, while passing through a blade ring. Also determine the blade tip angles of the fixed blade and diagram efficiency.*

Solution. Given : ϕ, ϕ', θ and $\theta' = 30°$; $T = 240°$ C ; $p_1 = 11.5$ bar ; $p_2 = 5$ bar ; Loss in velocity = 10%

From the Mollier diagram, as shown in Fig. 24.6, we find that enthalpy drop from 11.5 bar and 240° C to 5 bar,

$$h_d = h_1 - h_2 = 2915 - 2760 = 155 \text{ kJ/kg}$$

$\therefore$ Velocity of steam at the wheel chamber,

$$V = 44.72\sqrt{155} = 556.6 \text{ m/s}$$

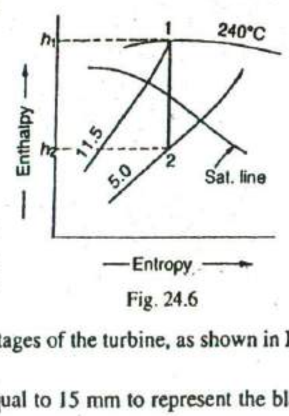


Fig. 24.6

Blade speed

From the given data, we find that it is not sufficient to draw inlet velocity triangle for the first moving ring. Therefore we have to start the diagram from the outlet velocity triangle for the moving ring.

Now draw the combined velocity diagram for the two stages of the turbine, as shown in Fig. 24.7, as discussed below :

1. First of all, draw a horizontal line and mark AB equal to 15 mm to represent the blade velocity (which is required to be found out).

2. Now draw the outlet velocity triangle ABD' for the second moving ring on the base AB with $\phi' = 30°$ and $\beta' = 90°$. From the geometry of the figure, we find that $D'B$ represents V_f' and $D'A$ represents V_{r1}'. By measurement, we find that $V_{r1}' = 17.3$ mm. Since there is a 10% loss in velocity due to friction, therefore, $V_r' = V_{r1}' / 0.9 = 17.3 / 0.9 = 19.2$ mm.

3. Now draw the inlet velocity triangle ABC' for the second moving ring on the same base AB with $\theta' = 30°$ and $V_r' = 19.2$ mm. From the geometry of the figure, we find that $C'A$ and $C'B$ represents V_r' and V' respectively to the scale. By measurement, we find $C'B = 33$ mm. Since there is a 10% loss in velocity due to friction, therefore $V_1 = V'/0.9 = 33/0.9 = 36.7$ mm.

4. Now draw the outlet velocity triangle *ABD* for the first moving ring on the same base *AB* with $\phi = 30°$ and $V_1 = 36.7$ mm. From the geometry of the figure, we find that *DF* and *DA* represent V_{f1} and V_{r1} respectively to the scale. By measurement, we find that $V_{r1} = 49$ mm. Since there is 10% loss in velocity due to friction, therefore $V_r = V_{r1} / 0.9 = 49 / 0.9 = 54.4$ mm.

5. Now draw the inlet velocity triangle *ABC* for the first moving ring on the same base *AB* with $\theta = 30°$ and $V_r = 54.4$ mm.

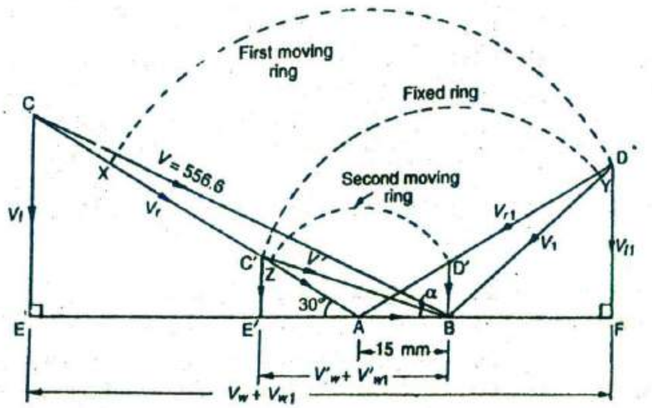


Fig. 24.7

From the combined velocity diagram, we find that the length of $BC = 67$ mm.

$\therefore$ Scale, $1 \text{ mm} = \dfrac{556.6}{67} = 8.31$ m/s

and blade speed, $V_b = 15 \times 8.31 = 124.6$ m/s

Blade tip angles

By measurement from the velocity diagram, we find that

$$\alpha' = 17° \text{ and } \beta = 43° \text{ Ans.}$$

Diagram efficiency

By measurement from the velocity diagram, we find that $EF = V_w + V_{w1} = 88$ mm, and $E'F' = V_w' + V_{w1}' = 31$ mm.

We know that diagram efficiency,

$$\eta_b = \frac{2\,AB\,(EF + E'F')}{V^2} = \frac{2 \times 15\,(88 + 31)}{67^2} = 0.795 \text{ or } 79.5\% \text{ Ans.}$$

Example 24.6. *A reaction turbine with a mean blade diameter of 1 m runs at a speed of 50 rev/second. The blades are designed with exit angles of 50° and inlet angles of 30°. If the turbine is supplied with steam at the rate of 20 kg/s and gross efficiency is 85%, determine : (a) power output of the stage; (b) specific enthalpy drop in the stage; and (c) percentage increase in relative velocity in the moving blades due to steam expansion.*

Solution. Given : $d_m = 1$ m ; $N = 50$ r.p.s. ; $\theta = \beta = 50°$; $\phi = \alpha = 30°$; $m = 20$ kg/s ; $\eta_s = 85\% = 0.85$

Power output of the stage

We know that blade velocity,

$$V_b = \pi d_m N = \pi \times 1 \times 50 = 157 \text{ m/s}$$

Now let us draw the combined velocity triangle, as shown in Fig. 24.8, as discussed below :

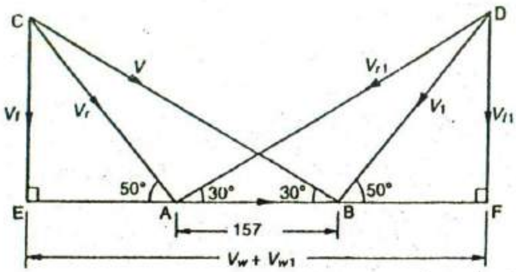


Fig. 24.8

1. First of all, draw a horizontal line and cut off AB equal to 157 m/s, to some suitable scale, to represent the blade velocity (V_b).
2. Now draw inlet velocity triangle ABC on the base AB with $\alpha = 30°$ and $\theta = 50°$.
3. Similarly, draw outlet velocity triangle ABD on the same base AB with $\phi = 30°$ and $\beta = 50°$.
4. From C and D, draw perpendiculars to meet the line AB produced at E and F.

By *measurement from the velocity triangle, we find that change in the velocity of whirl,

$$V_w + V_{w1} = EF = 450 \text{ m/s}$$

Relative velocity at inlet, $V_r = CA = 230$ m/s

and relative velocity at outlet, $V_{r1} = DA = 350$ m/s

We know that power output of the stage,

$$P = m(V_w + V_{w1})\,V_b = 20 \times 450 \times 157 = 1\,413\,000 \text{ W}$$

$$= 1413 \text{ kW} \quad \textbf{Ans.}$$

---

* These values may also be obtained from the geometry of the combined velocity triangle as discussed below :

In the inlet velocity triangle ABC,

$$\frac{V_r}{\sin 30°} = \frac{V}{\sin 130°} = \frac{157}{\sin 20°}$$

$\therefore$
$$V_r = \frac{157}{\sin 20°} \times \sin 30° = \frac{157}{0.342} \times 0.5 = 229.5 \text{ m/s}$$

and
$$V \,(\text{or } V_{r1}) = \frac{157}{\sin 20°} \times \sin 130° = \frac{157}{\sin 20°} \times \sin 50°$$

$$= \frac{157}{0.342} \times 0.766 = 351.6 \text{ m/s}$$

$\therefore$
$$EA = V_r \cos 50° = 229.5 \times 0.6428 = 147.5 \text{ m/s}$$

and
$$(V_w + V_{w1}) = EA + AB + BF = 147.5 + 157 + 147.5 = 452 \text{ m/s}$$

(*b*) *Specific enthalpy drop in the stage*

Let $\qquad U$ = Specific enthalpy drop in the stage.

We know that stage efficiency (η_s),

$$0.85 = \frac{(V_w + V_{w1})\, V_b}{1000\, h_d} = \frac{450 \times 157}{1000\, h_d} = \frac{70.65}{h_d}$$

$$\therefore \qquad h_d = 70.65/0.85 = 83.1 \text{ kJ } \textbf{Ans.}$$

(*c*) *Percentage increase in the relative velocity*

We know that increase in the relative velocity in the moving blades due to steam expansion

$$= \frac{V_{r1} - V_r}{V_r} = \frac{351.6 - 229.5}{229.5} = 0.532 \text{ or } 53.2\,\% \textbf{ Ans.}$$

24.3. Condition for Maximum Efficiency of an Impulse Turbine

We have already discussed in Art. 24.2 that the blading efficiency (or diagram efficiency) of an impulse turbine,

$$\eta_b = \frac{V^2 - V_1^2}{V^2} = \frac{2\,(V_w + V_{w1})V_b}{V^2}$$

It may be noted that the blading efficiency will be maximum when V_1 is minimum. From the combined velocity triangle, we see that the value of V_1 will be minimum, when β is equal to 90°. In other words, for maximum efficiency, the steam should leave the turbine blades at right angles to their motion. The modified combined velocity triangle for maximum efficiency is shown in Fig. 24.9. It may also be noted that for maximum efficiency, V_{w1} is zero.

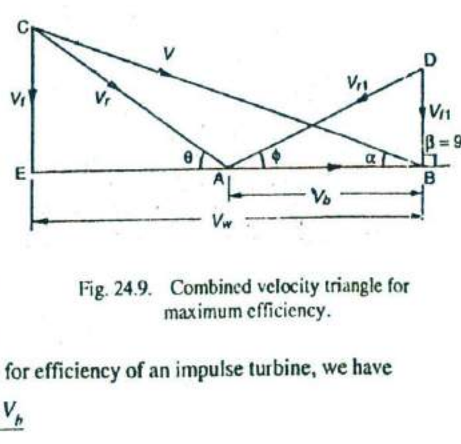


Fig. 24.9. Combined velocity triangle for maximum efficiency.

$\therefore$ For maximum efficiency, substituting $V_{w1} = 0$ in the general expression for efficiency of an impulse turbine, we have

$$\eta_{max} = \frac{2 \times V_w \times V_b}{V^2}$$

We know that in De-lavel turbine, $\theta = \phi$, considering $V_r = V_{r1}$ (or in other words, neglecting blade friction),

$$V_b = \frac{1}{2} V_w = 0.5\, V_w \qquad \ldots (\because \Delta EAC = \Delta ADB)$$

$$= 0.5\, V \cos \alpha \qquad \ldots (\because V_w = V \cos \alpha)$$

$$\therefore \qquad \eta_{max} = \frac{2 \times V_w \times 0.5\, V_w}{V^2} = \frac{V_w^2}{V^2} = \cos^2 \alpha \qquad \ldots (\because V_w/V = \cos \alpha)$$

Example 24.7. *The nozzles of the impulse stage of a turbine receive steam at 15 bar and 300° C and discharge it at 1 bar. The nozzle efficiency is 95% and the nozzle angle is 20°. The blade speed is that required for maximum efficiency and the entry of steam to the blades is without shock.*

The blade exit angle is 5° less than the blade inlet angle. The blade velocity coefficient is 0.9 and the steam flow rate is 1350 kg/h.

Calculate (a) the diagram power, and (b) the stage efficiency.

Solution. Given : $p_1 = 15$ bar ; $T_1 = 300°$ C ; $p_2 = 1$ bar ; $K = 95\% = 0.95$; $\alpha = 20°$; $\phi = \theta - 5°$; $K = V_{r1}/V_r = 0.9$; $m = 1350$ kg/h $= 0.375$ kg/s

From Mollier diagram, as shown in Fig. 24.10, we find that enthalpy drop from 15 bar and 300° C to 1 bar,

$$h_d = h_1 - h_2$$

$$= 3040 - 2520 = 520 \text{ kJ/kg}$$

∴ Velocity of steam at inlet to the blade,

$$V = 44.72\sqrt{K\, h_d}$$

$$= 44.72\sqrt{0.95 \times 520} = 994 \text{ m/s}$$

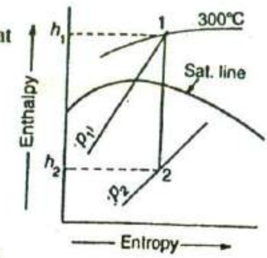


Fig. 24.10

We know that for maximum efficiency, blade velocity,

$$V_b = 0.5\, V\cos\alpha = 0.5 \times 994\cos 20° = 467 \text{ m/s}$$

Now draw the combined velocity triangle, as shown in Fig. 24.11, as discussed below :

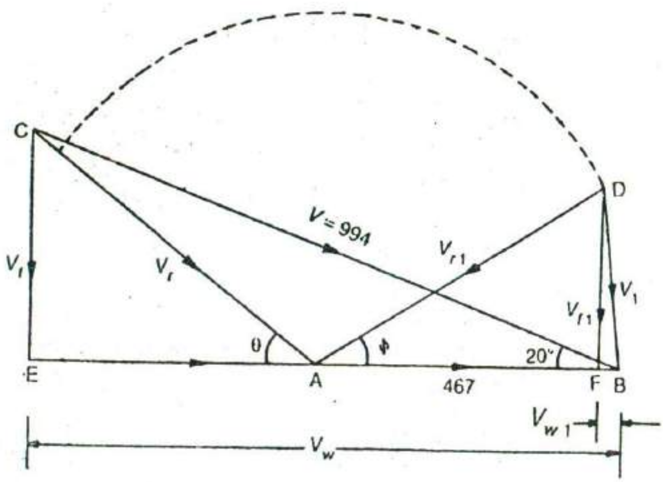


Fig. 24.11

1. First of all, draw a horizontal line and cut off AB equal to 467 m/s, to some suitable scale, to represent the blade velocity (V_b).

2. Now draw the velocity triangle ABC on the base AB with $\alpha = 20°$ and $V = 994$ m/s, to the scale. By measurement, we find that blade inlet angle $(\theta) = 36°$ and relative velocity of steam at inlet $(V_r) = 555$ m/s.

3. Similarly, draw outlet triangle ABD on the same base AB with $\phi = \theta - 5° = 36 - 5 = 31°$ and $V_{r1} = 0.9\, V_r = 0.9 \times 555 = 499.5$ m/s, to the scale.

4. From C and D, draw perpendiculars to meet the line AB produced at E and F.

By measurement from the velocity diagram, we find that change in the velocity of whirl,

$$^{*}V_w - V_{w1} = EF = 880 \text{ m/s}$$

(*a*) *Diagram power*

We know that diagram power,

$$P = m(V_w - V_{w1})\,V_b = 0.375 \times 880 \times 467 = 154\,110 \text{ W}$$

$$= 154.11 \text{ kW } \textbf{Ans.}$$

(*b*) *Stage efficiency*

We know that stage efficiency,

$$\eta_s = \frac{(V_w - V_{w1})\,V_b}{1000\,h_d} = \frac{880 \times 467}{1000 \times 520} = 0.79 \text{ or } 79\% \textbf{ Ans.}$$

Example 24.8. *A De-Laval steam turbine receives dry and saturated steam at a pressure of 15 bar and discharges at a pressure of 2 bar. The nozzle is inclined at an angle of 20° to the direction of motion of the blades and 10% of the available enthalpy drop is lost in friction of the nozzle. The mean diameter of the blade ring is 1 m and the rotational speed is 6000 r.p.m.*

Assuming a blade friction of 0.8 and taking the blade to be symmetrical, determine (a) absolute velocity of the steam as it leaves the blade, (b) diagram (blade) efficiency, and (c) maximum efficiency of the turbine.

Solution. Given : $p_1 = 15$ bar ; $p_2 = 2$ bar ; $\alpha = 20°$; Enthalpy drop lost in friction = 10% ; $d_m = 1$ m ; $N = 6000$ r.p.m. ; $V_{r1}/V_r = 0.8$; $\theta = \phi$ for symmetrical blades

(*a*) *Absolute velocity of steam as it leaves the blade*

We know that blade velocity,

$$V_b = \frac{\pi\,d_m\,N}{60} = \frac{\pi \times 1 \times 6000}{60} = 314.2 \text{ m/s}$$

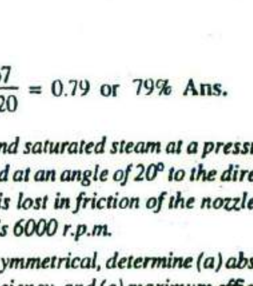


Fig. 24.12

From the Mollier diagram, as shown in Fig. 24.12, we find that enthalpy drop from 15 bar and dry saturated to 2 bar,

$$h_d = h_1 - h_2$$

$$= 2790 - 2440 = 350 \text{ kJ/kg}$$

Since the enthalpy drop lost in friction is 10 percent, therefore nozzle coefficient,

$$K = 1 - 0.1 = 0.9$$

We know that velocity of steam at inlet of the blade,

$$V = 44.72\sqrt{K\,h_d} = 44.72\sqrt{0.9 \times 350} = 793.7 \text{ m/s}$$

Now draw the combined velocity triangle, as shown in Fig. 24.13, as discussed below :

1. First of all, draw a horizontal line and cut off AB equal to 314.2 m/s, to some suitable scale, to represent the blade velocity (V_b).

2. Now draw the inlet velocity triangle ABC on the base AB with $\alpha = 20°$ and $V = 793.7$ m/s to the scale. By measurement, we find that $V_r = 510$ m/s and $\theta = 28°$.

* Since V_w and V_{w1} are in the same direction with respect to blade motion, therefore change in the velocity of whirl

$$= V_w - (+V_{w1}) = V_w - V_{w1}$$ ...(See chapter 22, Art. 22.8 Note 2)

3. Similarly draw the outlet velocity triangle ABD on the same base AB with $\phi = \theta = 28°$ and $V_{r1} = 0.8\,V_r = 0.8 \times 510 = 408$ m/s to the scale.

4. From C and D draw perpendiculars to meet the line AB produced at E and F.

By measurement from the velocity diagram, we find that the absolute velocity of steam as it leaves the blades,

$$V_1 = BD = 200 \text{ m/s } \textbf{Ans.}$$

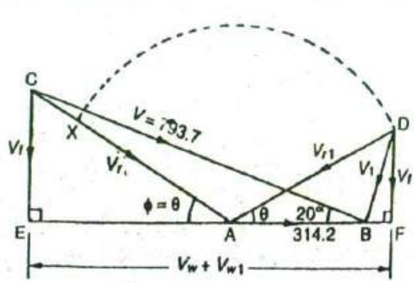


Fig. 24.13

(*b*) *Diagram (blade) efficiency*

By measurement, from the velocity diagram, we find that

$$V_w + V_{w1} = 810 \text{ m/s}$$

We know that diagram (blade) efficiency,

$$\eta_b = \frac{2\,(V_w + V_{w1})\,V_b}{V^2} = \frac{2 \times 810 \times 314.2}{(793.7)^2} = 0.808 \text{ or } 80.8\% \textbf{ Ans.}$$

(*c*) *Maximum efficiency of the turbine*

We know that maximum efficiency of the turbine,

$$\eta_{max} = \cos^2 \alpha = \cos^2 20° = (0.9397)^2 = 0.883 \text{ or } 88.3\% \textbf{ Ans.}$$

24.4. Condition for Maximum Efficiency of a Reaction Turbine

We have already discussed in Art. 23.7 that work done by a reaction turbine per kg of steam

$$= AB \times EF = V_b\,(EB + AF - AB)$$

$$= V_b\,(V \cos \alpha + V_{r1} \cos \phi - V_b)$$

We know that in a Parson's reaction turbine, $\alpha = \phi$; $V = V_{r1}$; and $V_1 = V_r$

$\therefore$ Work done per kg of steam

$$= V_b\,(2V \cos \alpha - V_b)$$

$$= V_b \times V^2 \left(\frac{2 \cos \alpha}{V} - \frac{V_b}{V^2} \right) \qquad \ldots \text{ (Multiplying and dividing by } V^2\text{)}$$

$$= V^2 \left(\frac{2\,V_b \cos \alpha}{V} - \frac{V_b^2}{V^2} \right)$$

$$= V^2\,(2\,\rho \cos \alpha - \rho^2) \qquad \ldots \left(\text{Substituting } \frac{V_b}{V} = \rho \right)$$

We know that kinetic energy supplied to the fixed blade per kg of steam

$$= \frac{V^2}{2}$$

and kinetic energy supplied to the moving blade per kg of steam

$$= \frac{(V_{r1})^2 - (V_r)^2}{2} = \frac{V^2 - V_1^2}{2} \qquad \ldots (\because V_{r1} = V;\ \text{and } V_r = V_1)$$

$\therefore$ Total energy supplied to the turbine

$$= \frac{V^2}{2} + \frac{V^2 - V_1^2}{2} = \frac{2V^2 - V_1^2}{2}$$

From the combined velocity triangle, we find that

$$V_1 = V_r = \sqrt{V^2 + V_b^2 - 2V\,V_b \cos\alpha}$$

$\therefore$ Total energy supplied to the turbine,

$$= \frac{2V^2 - (V^2 + V_b^2 - 2V\,V_b\cos\alpha)}{2} = \frac{V^2 - V_b^2 + 2V\,V_b\cos\alpha}{2}$$

$$= \frac{V^2}{2}\left(1 - \frac{V_b^2}{V^2} + \frac{2V_b}{V}\cos\alpha\right)$$

$$= \frac{V^2}{2}(1 - \rho^2 + 2\rho\cos\alpha) \qquad \ldots\left(\because \frac{V_b}{V} = \rho\right)$$

We know that diagram or blading efficiency,

$$\eta_b = \frac{\text{Work done}}{\text{Energy supplied}} = \frac{V^2(2\rho\cos\alpha - \rho^2)}{\frac{V^2}{2}(1 - \rho^2 + 2\rho\cos\alpha)}$$

$$= \frac{2(2\rho\cos\alpha - \rho^2)}{(1 - \rho^2 + 2\rho\cos\alpha)} \qquad \ldots(i)$$

It may be noted that the efficiency of the turbine will be maximum when $(1 - \rho^2 + 2\rho\cos\alpha)$ is minimum. Now for $(1 - \rho^2 + 2\rho\cos\alpha)$ to be minimum, differentiate this expression with respect to ρ and equate the same to zero, *i.e.*

$$\frac{d}{d\rho}(1 - \rho^2 + 2\rho\cos\alpha) = 0$$

or $$2\rho - 2\cos\alpha = 0$$

$$\therefore \quad \rho = \cos\alpha \text{ or } \frac{V_b}{V} = \cos\alpha \qquad \ldots\left(\because \rho = \frac{V_b}{V}\right)$$

$$\therefore \quad V_b = V\cos\alpha$$

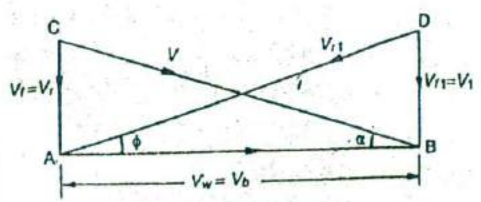


Fig. 24.14

It may be noted that when $V_b = V\cos\alpha$, the value of V_{w1} will be zero (a condition for maximum efficiency of impulse turbine also) as shown in Fig. 24.14. In such a case $V_b = V_w$.

Now substituting $\rho = \cos\alpha$ in equation (*i*) for maximum efficiency,

$$\eta_{max} = \frac{2(2\cos^2\alpha - \cos^2\alpha)}{(1 - \cos^2\alpha + 2\cos^2\alpha)} = \frac{2\cos^2\alpha}{1 + \cos^2\alpha}$$

Example 24.9. *A certain stage of a Parson's turbine consists of one row of fixed blades and one row of moving blades. The details of the turbine are as follows :*

Mean diameter of the blades = 680 mm ; R.P.M. of the turbine = 3000 ; Mass of steam passing per second = 13.5 kg ; Steam velocity at exit from fixed blades = 143.7 m/s ; Blade outlet angle = 20°.

Calculate the power developed in the stage and the gross efficiency assuming carry over coefficient as 0.74 and the efficiency of conversion of heat energy into kinetic energy in the blade channels as 0.92.

Solution. Given : $D = 680$ mm $= 0.68$ m ; $N = 3000$ r.p.m. ; $m = 13.5$ kg/s ; $V = 143.7$ m/s ; $\alpha = 20^\circ$; $K = 0.74$; $\eta = 0.92$

We know that blade velocity,

$$V_b = \frac{\pi DN}{60} = \frac{\pi \times 0.68 \times 3000}{60} = 106.8 \text{ m/s}$$

Now draw the combined velocity triangle, as shown in Fig. 24.15, as discussed below :

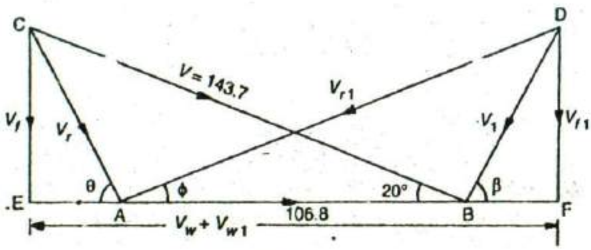


Fig. 24.15

1. First of all, draw a horizontal line and cut off AB equal to 106.8 m/s, to some suitable scale, to represent the blade velocity (V_b).

2. Now draw inlet velocity triangle ABC on the base AB with $\alpha = 20^\circ$ and $V = 143.7$ m/s, to the scale.

3. Now draw outlet velocity triangle ABD on the same base AB with $\phi = \alpha = 20^\circ$ and $V_{r1} = V = 143.7$ m/s, to the scale.

Note : In a Parson's turbine, the blades are symmetrical. Therefore

$$\phi = \alpha \,;\, \theta = \beta \,;\, V_{r1} = V \text{ and } V_1 = V_r .$$

4. From C and D, draw perpendiculars to meet the line AB produced at E and F.

By measurement from velocity diagram, we find that the change in the velocity of whirl,

$$V_w + V_{w1} = EF = 164 \text{ m/s}$$

Power developed in the stage

We know that power developed in the stage,

$$P = m\,(V_w + V_{w1})\,V_b = 13.5 \times 164 \times 106.8 = 236\,460 \text{ W} = 236.46 \text{ kW} \quad \textbf{Ans.}$$

Gross efficiency

By measurement from velocity diagram, we find that

$$V_1 = V_r = 58.7 \text{ m/s}$$

We know that heat drop,

$$h_d = 2\left(\frac{V^2 - KV_1^2}{2\,\eta}\right) = 2\left[\frac{(143.7)^2 - 0.74\,(58.7)^2}{2 \times 0.92}\right] = 19\,674 \text{ J} = 19.674 \text{ kJ}$$

∴ Gross efficiency,

$$\eta_s = \frac{(V_w + V_{w1})\,V_b}{1000\,h_d} = \frac{164 \times 106.8}{1000 \times 19.674} = 0.89 \text{ or } 89\,\% \quad \textbf{Ans.}$$

Example 24.10. *At a stage of a reaction turbine, the rotor diameter is 1.4 m and speed ratio 0.7. If the blade outlet angle is 20° and the rotor speed 3000 r.p.m., find the blade inlet angle and diagram efficiency.*

Also find the percentage increase in diagram efficiency and rotor speed, if the turbine is designed to run at the best theoretical speed.

Solution. Given : $D = 1.4$ m ; $\rho = V_b / V = 0.7$; $\alpha = \phi = 20°$; $N = 3000$ r.p.m.

We know that blade velocity,

$$V_b = \frac{\pi DN}{60} = \frac{\pi \times 1.4 \times 3000}{60} = 220 \text{ m/s}$$

and velocity of steam at inlet to the blade,

$$V = V_b / 0.7 = 220 / 0.7 = 314.3 \text{ m/s}$$

Blade inlet angle

Now draw the combined velocity triangle, as shown in Fig. 24.16, as discussed below :

1. First of all, draw a horizontal line and cut off AB equal to 220 m/s, to some suitable scale, to represent the blade velocity (V_b).

2. Now draw inlet velocity triangle ABC on the base AB with $\alpha = 20°$ and $V = 314.3$ m/s, to the scale.

3. Similarly, draw outlet velocity triangle ABD on the same base AB with $\phi = 20°$ and $V_{r1} = 314.3$ m/s, to the scale.

4. From C and D, draw perpendiculars to meet the line AB produced at E and F.

By measurement from velocity diagram, we find that the blade inlet angle,

$$\theta = 55° \textbf{ Ans.}$$

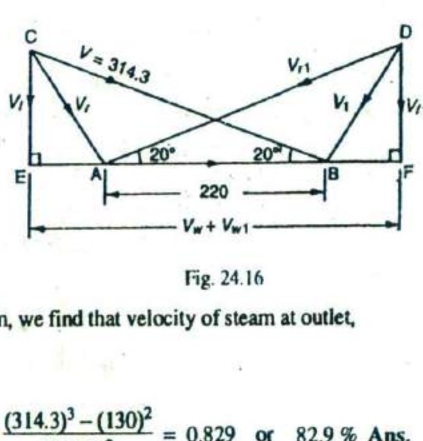


Fig. 24.16

Diagram efficiency

By measurement from velocity diagram, we find that velocity of steam at outlet,

$$V_1 = 130 \text{ m/s}$$

We know that diagram efficiency,

$$\eta_b = \frac{V^2 - V_1^2}{V^2} = \frac{(314.3)^3 - (130)^2}{(314.3)^2} = 0.829 \quad \text{or} \quad 82.9\ \% \textbf{ Ans.}$$

Percentage increase in diagram efficiency for best theoretical speed

We know that maximum efficiency of the turbine,

$$\eta_{max} = \frac{2\cos^2\alpha}{1+\cos^2\alpha} = \frac{2\times\cos^2 20°}{1+\cos^2 20°} = \frac{2\,(0.9397)^2}{1+(0.9397)^2} = 0.938$$

$\therefore$ Percentage increase in diagram efficiency

$$= \frac{0.938 - 0.829}{0.829} = 0.131 \quad \text{or} \quad 13.1\% \textbf{ Ans.}$$

Percentage increase in rotor speed for best theoretical speed

Let $\qquad N_1$ = Maximum rotor speed.

We know that for best theoretical speed (or in other words, for maximum efficiency),the blade velocity,

$$V_b = V\cos\alpha = 314.3\cos 20° = 314.3 \times 0.9397 = 295.3 \text{ m/s}$$

We also know that blade velocity (V_b),

$$\therefore \quad 295.3 = \frac{\pi D N_1}{60} = \frac{\pi \times 1.4 N_1}{60} = 0.073 N_1$$

or

$$N_1 = 4044 \text{ r.p.m.}$$

$\therefore$ Percentage increase in rotor speed

$$= \frac{4044 - 3000}{3000} = 0.348 \quad \text{or} \quad 34.8\ \% \ \textbf{Ans.}$$

24.5. Compounding of Impulse Steam Turbines (Methods of Reducing Rotor Speeds)

In the recent years, high pressure (100 to 140 bar) and high temperature steam is used in the power plants to increase their thermal efficiency. If the entire pressure drop (from boiler pressure to condenser pressure (say from 125 bar to 1 bar) is carried out in one stage only, then the velocity of steam entering into the turbine will be extremely high. It will make the turbine rotor to run at a very high speed (even up to 30 000 r.p.m.). From practical point of view, such a high speed of the turbine rotor is bound to have a number of disadvantages.

In order to reduce the rotor speed, various methods are employed. All of these methods consist of a multiple system of rotors, in series, keyed to a common shaft and the steam pressure or the jet velocity is absorbed in stages as it flows over the rotor blades. This process is known as *compounding*. The following three methods are commonly employed for reducing the rotor speed :

1. Velocity compounding, 2. Pressure compounding, and 3. Pressure-velocity compounding.

24.6. Velocity Compounding of an Impulse Turbine

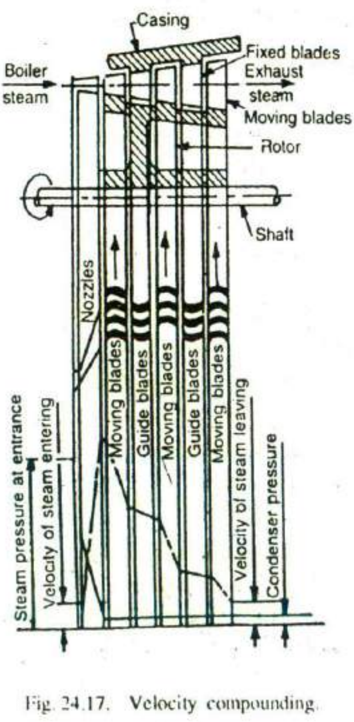


Fig. 24.17. Velocity compounding.

In velocity compounding of an impulse turbine, the expansion of steam takes place in a nozzle or a set of nozzles from the boiler pressure to the condenser pressure. The impulse wheel carries two or three rows of moving blades. Fig. 24.17 shows the three rings of moving blades, separated by two rings of fixed or guide blades in the reverse manner.

The steam, after expanding through nozzles, enters the first ring of moving blades at a high velocity. A portion of this high velocity is absorbed by this blade ring and the remaining is passed on to the next ring of fixed blades. The fixed blades change the direction of steam and direct it to the second ring of moving blades, without altering the velocity appreciably. After passing through this second ring of moving blades, a further portion of velocity is absorbed. The steam is now directed by the second ring of fixed blades to the third ring of moving blades and then enters into the condenser.

In Fig. 24.17, the curves of velocity and pressure on a base representing the axis of the turbine are shown. It may be noted, from the figure, that no pressure drop occurs either in the fixed or moving blades. All the pressure drop occures in the nozzles. This turbine can run at about one-third of the speed of De-Laval turbine, for the same pressure drop and diameter of the wheel.

24.7. Pressure Compounding of an Impulse Turbine

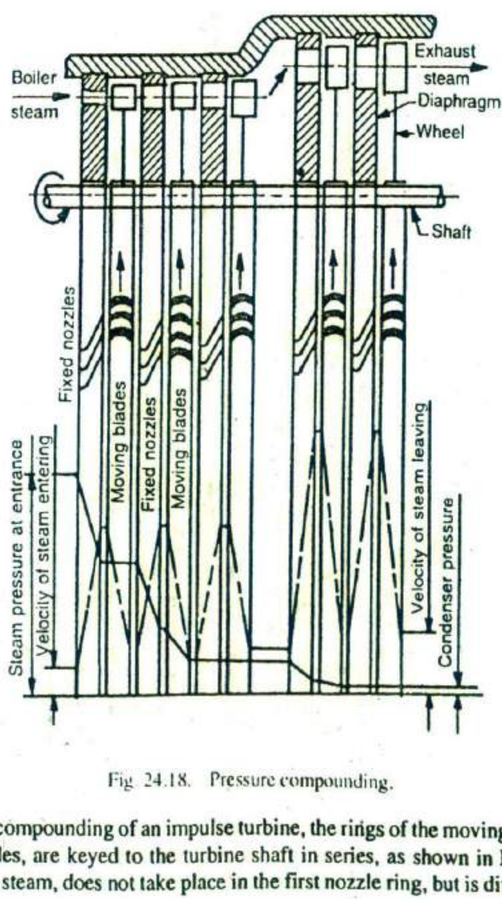


Fig. 24.18. Pressure compounding.

In a pressure compounding of an impulse turbine, the rings of the moving blades, each having a ring of fixed nozzles, are keyed to the turbine shaft in series, as shown in Fig. 24.18. The total pressure drop, of the steam, does not take place in the first nozzle ring, but is divided equally among all the nozzle rings.

The steam from the boiler is passed through the first nozzle ring, where only a small pressure drop occurs with an increase in velocity of steam. The steam is now directed on the first moving blade ring, where the pressure of steam does not alter, but the velocity decreases. This constitutes one stage. It may be noted that a stage consists of a fixed nozzle ring and a moving blade ring. The steam from the first moving blade ring enters the second nozzle ring, where its pressure is further reduced. A little consideration will show, that the pressure drop per stage in the nozzle rings is not the same. but the number of heat units, converted into velocity energy in each stage, is the same. The process is repeated in the remaining rings, until the condenser pressure is reached.

In Fig. 24.18, the curves of velocity and pressure on a base representing the axis of the turbine are also shown. It may be noted, from the figure, that by arranging a small pressure drop per stage, the velocity of steam entering the moving blades, and hence the speed of rotor is reduced.

The Rateau and Zoelly turbines are the examples of pressure compounded turbines.

24.8. Pressure-velocity Compounding of an Impulse Turbine

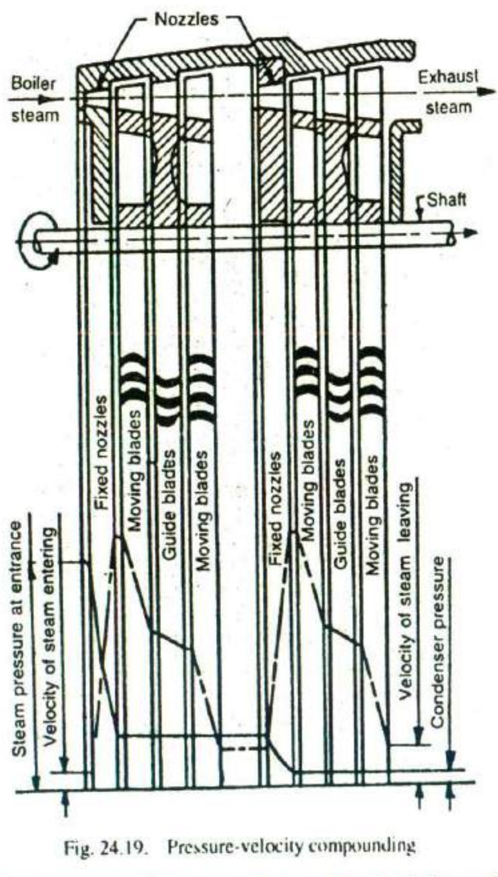


Fig. 24.19. Pressure-velocity compounding

In a pressure-velocity compounding of an impulse turbine, both the previous two methods are utilised. The total pressure drop of the steam is divided into stages, and velocity obtained in each stage is also compounded. A little consideration will show, that a pressure velocity compounded impulse turbine allows a bigger pressure drop, and hence less number of stages are required.

Fig. 24.19 shows the curves of pressure and velocity for this type of turbine. It may be noted that the diameter of the turbine is increased at each stage, to allow the increasing volume of steam at the lower pressures. A ring of nozzles is fixed at the commencement of each stage as shown in Fig. 24.19. A curtis turbine is an example of pressure-velocity compounded impulse turbine.

24.9. Internal Losses in Turbines

Strictly speaking, an ideal turbine (having 100 per cent gross efficiency) will do the work equivalent to the isentropic enthalpy or heat drop of the steam used in the turbine. But in actual practice, the work done by a turbine is much less than isentropic heat drop of the steam used. There are several factors, which effect the performance of a steam turbine. All these factors, which reduce the output of the turbine, are known as *internal losses*. Though there are many internal losses in a steam turbine, yet the following are important from the subject point of view :

1. *Nozzle loss.* It is an important loss in impulse turbines, which occurs when the steam flows through the nozzle. This loss takes place due to friction in the nozzle and the formation of eddies.

2. *Blade friction loss.* It is an important loss in both the impulse and reaction turbines, which occurs when the steam glides over the blades. This loss takes place due to friction of the surface of blades. As a result of the blade friction, the relative velocity of the steam is reduced while gliding over the blade.

3. *Wheel friction loss.* It is another important loss in both the impulse and reaction turbines, which occurs when the turbine wheel rotates in the steam. This loss takes place due to the resistance offered by the steam to the moving turbine wheel or disc. As a result of this loss, the turbine wheel rotates at a lower speed.

4. *Mechanical friction loss.* It is a loss in both the turbines, which occurs due to friction between the shaft and wheel bearing as well as regulating the valves. This loss can be reduced by lubricating the moving parts of the turbine.

5. *Leakage loss.* It is a loss in both the turbines, which occurs due to leakage of the steam at each stage of the turbine, blade tips and glands.

6. *Residual velocity loss.* It is a loss in both the turbines, which occurs due to the kinetic energy of the steam, as it leaves the turbine wheel. This loss is reduced by using multi-stage wheels.

7. *Moisture loss.* It is a loss in both the turbines, which takes place due to moisture present in the steam. This loss occurs when the steam, passing through lower stages, becomes wet. The velocity of water particles is less than that of steam. As a result of this, the steam has to drag the water particles, which reduce the kinetic energy of the steam.

8. *Radiation losses.* It is a loss in both the turbines, which takes place due to difference of the temperatures between the turbine casing and the surrounding atmosphere. This is reduced by properly insulating the turbine.

9. *Governing loss.* It is a loss in both the turbines, which occur due to throttling of the steam at the main stop valve of the governor.

24.10. Governing of Steam Turbines

We have already discussed in Art. 19.7, the necessity and processes of governing of steam engines. In the same way, we provide an arrangement for the governing of steam turbines also. Though there are many methods of governing steam turbines, yet the throttle governing is important from the subject point of view :

24.11. Throttle Governing of Steam Turbines

The throttle governing of a steam turbine is a method of controlling the turbine output by varying the quantity of steam entering into the turbine. This method is also known as *servometer method*, whose operation is given below :

The centrifugal governor is driven from the main shaft of turbine by belt or gear arrangement. The control valve controls the direction of flow of the oil (which is pumped by gear pump) either in the pipe *AA* or *BB*. The servometer or relay valve has a piston whose motion (towards left or right depends upon the pressure of the oil flowing through the pipes *AA* or *BB*) is connected to a spear or needle which moves inside the nozzle, as shown in Fig. 24.20.

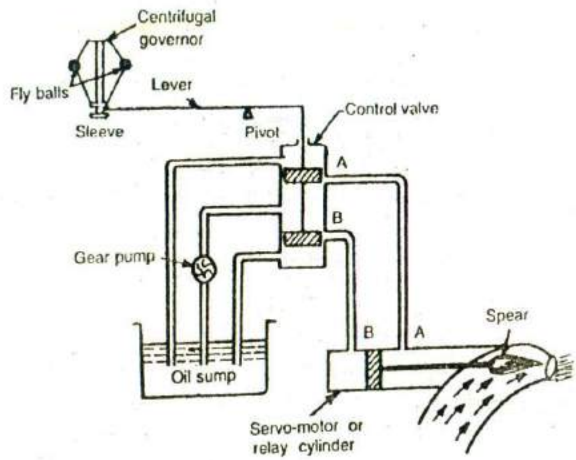


Fig. 24.20. Throttle governing of steam turbines.

We know that when the turbine is running at its normal speed, the positions of piston in the servometer, control valve, fly balls of centrifugal governor will be in their normal positions as shown in the figure. The oil pumped by the gear pump into the control valve, will come back into the oil sump as the mouths of both pipes *AA* or *BB* are closed by the two wings of control valve.

Now let us consider an instant, when load on the turbine increases. As a result of load increase, the turbine speed will be decreased. This decrease in the speed of the turbine wheel will also decrease the speed of centrifugal governor. As a result of this, the fly balls will come down (due to decrease in centrifugal force) thus decreasing their amplitude. As the sleeve is connected to the central vertical bar of the centrifugal governor, therefore coming down of the flyballs will also bring down the sleeve. This downward movement of the sleeve will raise the control valve rod, as the sleeve is connected to the control valve rod through a lever pivoted on the fulcrum. Now, a slight upward movement of the control valve rod will open the mouth of the pipe *AA* (still keeping the mouth of the pipe *BB* closed). Now, the oil under pressure will rush from the control valve to the right-side of the piston in the servometer through the pipe *AA*. This oil, under pressure, will move the piston as well as spear towards left, which will open more area of the nozzle. This increase in the area of flow will increase the rate of steam flow into the turbine. As a result of increase in the steam flow, there will be an increase in the turbine output as well as its speed. When speed of the turbine wheel will come up to its normal range, fly balls will move up. Now the sleeve as well as control valve rod will occupy its normal position, and the turbine will run at its normal speed.

It may be noted that when load on the turbine decreases, it will increase its speed. As a result of this, the fly balls will go up (due to increase in centrifugal force) and the sleeve will also go up. This will push the control valve rod downwards. This downward movement of the control valve rod will open the mouth of the pipe *BB* (still keeping the mouth of the pipe *AA* closed). Now, the oil under pressure, will rush from the control valve to the left side of the piston in the servometer through the pipe *BB*. This oil under pressure, will move the piston and spear towards right, which will decrease the area of the nozzle. This decrease in the area of the flow will decrease the rate of steam flow into the turbine. As a result of decrease in the steam flow, there will be a decrease in the turbine output as well as its speed. When the speed of the turbine is reduced to its normal range, the flyballs will come down. Now the sleeve as well as control valve rod will occupy its normal positions, and the turbine will run at its normal speed.

EXERCISES

1. The following data refer to one stage of an impulse turbine :

Steam velocity = 500 m/s ; Blade speed = 200 m/s ; Nozzle angle = 20° ; Exit angle of moving blades = 25° ; Steam flow = 5 kg/s.

Neglecting the effect of friction when passing through passages, determine : 1. the inlet angle of moving blade ; 2. the power developed ; 3. the axial thrust, and 4. the diagram efficiency.

[**Ans.** 33° ; 560 kW ; 200 N ; 89.6%]

2. In a De-Lavel turbine, the blade angles are equal. The steam enters at a velocity of 300 m/s with a nozzle angle of 16°. The blade friction factor is 0.84. If the blade speed and steam speed ratio is 0.47, estimate : 1. blade efficiency ; 2. power developed for a mass flow rate of 2 kg/s. [**Ans.** 84.6% ; 76.14 kW]

3. Following is the data pertaining to a single-row impulse turbine :

Mean diameter of blade ring = 2 m ; Speed = 3000 r.p.m. ; Nozzle angle = 18° ; Ratio of blade velocity to steam velocity = 0.5 ; Blade friction factor = 0.9 ; Blade angle at exit = 3° more than at inlet ; Steam supply = 30 000 kg/h.

Draw velocity diagram for the moving blade and estimate : 1. power developed ; 2. diagram efficiency ; and 3. steam consumption in kg/kWh. [**Ans.** 1387.16 kW ; 84.3% ; 21.6 kg/kWh]

4. A steam turbine running at 3600 r.p.m. takes 4.5 kg of steam per second. The nozzle angle is 16° and the mean diameter of the blade ring 1.2 metre. The blade outlet angle is 18° and the isentropic heat drop in the nozzle is 165 kJ/kg of steam. The shaft power is 485 kW. Assuming a nozzle efficiency as 92% and blade velocity coefficient 0.85, find 1. Blading efficiency, 2. Stage efficiency, and 3. Power lost in friction.

[**Ans.** 85.7% ; 78.8% ; 100.3 kW]

5. Steam is supplied to a simple impulse turbine at 10 bar, 250° C. The pressure in the wheel casing is 1.2 bar and the nozzle efficiency is 91%. Determine the exit area required for a steam flow rate of 5000 kg/h.

The nozzles are inclined to the plane of rotation at 20°. The blades are equiangular and have a velocity coefficient of 0.8. Assuming that the ratio of mean blade speed to steam speed at nozzle exit to be 0.45, calculate 1. the blade angles ; 2. the power developed ; and 3. the stage efficiency.

[**Ans.** 35° ; 442.26 kW ; 74%]

6. The velocity of steam as it flows out of a nozzle is 440 m/s which is compounded in an impulse turbine by passing it successively through moving, fixed and finally through a second ring of moving blades. The tip angles of the moving blades throughout the turbine are 30°. Assume a loss of 10% in velocity due to friction when the steam passes over a blade ring. Find the velocity of the moving blades in order to have final discharge of steam axial. Also determine the diagram efficiency. [**Ans.** 97 m/s ; 76%]

7. The first stage of an impulse turbine is compounded for velocity and has two rings of moving blades and one ring of fixed blades. The nozzle angle is 20° and the leaving angles of the blades are respectively as follows :

First moving 20°, fixed 25° and second moving 30°.

The velocity of steam leaving the nozzle is 600 m/s and the steam velocity relative to the blade is reduced by 10 percent during the passage through each ring. Find the diagram efficiency and the power developed for a steam flow of 4 kg/s. The blade speed may be taken as 125 m/s. [**Ans.** 78.5% ; 565 kW]

8. The following particulars relate to a two row velocity compounded impulse wheel :

Steam velocity at nozzle outlet = 650 m/s ; Mean blade velocity = 125 m/s ; Nozzle outlet angle = 16° ; Outlet angle – first row of moving blades = 18° ; Outlet angle – fixed guide blades = 22° ; Outlet angle-second row of moving blades = 36° ; Steam flow = 2.5 kg/s.

If the ratio of relative velocity at outlet to that inlet is 0.84 for all blades, find the following :

1. the axial thrust on the blades ; 2. the power developed ; and 3. the efficiency of the wheel.

[Ans. 142.5 N ; 390 kW ; 73.8%]

9. An impulse stage of a turbine has two rows of moving blades separated by fixed blades. The steam leaves the nozzles at an angle of 20° with the direction of the motion of blades. The blade exit angles are : first moving 30°, fixed 22° and second moving 30°.

If the isentropic heat drop for the nozzle is 186 kJ/kg and the nozzle efficiency 90%, find the blade speed necessary if the final velocity of steam is to be axial. Assume a loss of 15 percent in relative velocity for all blade passages. Find also the blade efficiency and the stage efficiency. [Ans. 117.2 m/s ; 71.9% ; 64.7%]

10. In a reaction turbine pair, the fixed and moving blades are of the same shape but reversed in direction. The angles of the receiving tips are 35° and of discharging tip, 20°. The mean velocity of the blades is 37.5 m/s and steam flows at the rate of 64 kg/s. If the isentropic heat drop for this turbine pair is 6 kJ/kg, calculate the diagram power and efficiency of the pair. [Ans. 285.6 kW ; 74.3%]

QUESTIONS

1. Define the following terms :

(a) Diagram efficiency, and (b) Stage efficiency.

2. Derive the condition for maximum efficiency of an impulse turbine and show that the maximum efficiency is $\cos^2 \alpha$, where α is the angle at which the steam enters the blade.

3. Determine the condition for maximum efficiency of a 50 percent reaction turbine and show that the maximum efficiency for such a turbine is $2 \cos^2 \alpha / (1 + \cos^2 \alpha)$, where α is the angle at which the steam enters the blades.

4. Explain the term 'Compounding of steam turbine'. What are the different methods of reducing rotor speed ?

5. Discuss the method of velocity compounding of an impulse turbine for achieving rotor speed reduction.

6. Enumerate the different losses in a steam turbine.

7. What are the methods of governing a steam turbine ? Describe any one method of governing steam turbines.

OBJECTIVE TYPE QUESTIONS

1. The ratio of workdone on the blades per kg of steam to the energy supplied to the blades is called

(a) diagram or blading efficiency (b) nozzle efficiency

(c) gross or stage efficiency (d) mechanical efficiency

2. The stage efficiency (η_s) is given by

(a) η_b / η_n (b) η_n / η_b (c) $\eta_b \eta_n$

where η_b = Blading efficiency, and

η_n = Nozzle efficiency.

3. The maximum efficiency of a De-Laval turbine is

(a) $\sin^2 \alpha$ (b) $\cos^2 \alpha$ (c) $\tan^2 \alpha$ (d) $\cot^2 \alpha$

where α = Nozzle angle.

4. The maximum efficiency of a reaction turbine is

(a) $\dfrac{2 \sin^2 \alpha}{1 + \sin^2 \alpha}$ (b) $\dfrac{1 + \sin^2 \alpha}{2 \sin^2 \alpha}$ (c) $\dfrac{2 \cos^2 \alpha}{1 + \cos^2 \alpha}$ (d) $\dfrac{1 + \cos^2 \alpha}{2 \cos^2 \alpha}$

5. The pressure-velocity compounded impulse turbine allows a bigger pressure drop and hence ..... number of stages are required.

   (*a*) more (*b*) less

6. Rateu turbine is a

   (*a*) simple impulse turbine (*b*) simple reaction turbine

   (*c*) velocity compounded turbine (*d*) pressure compounded turbine

7. Curtis turbine is a

   (*a*) velocity compounded turbine (*b*) pressure compounded turbine

   (*c*) pressure-velocity compounded turbine (*d*) none of these

8. De-Laval turbine is a

   (*a*) impulse reaction turbine (*b*) single rotor impulse turbine

   (*c*) multi-rotor impulse turbine (*d*) none of these

9. The compounding of turbines is done is order to

   (*a*) reduce speed of rotor (*b*) improve efficiency

   (*c*) reduce exit losses (*d*) all of these

10. The purpose of governing in steam turbines is to

    (*a*) reduce the effective heat drop

    (*b*) reheat the steam and improve its quality

    (*c*) completely balance against end thrust

    (*d*) maintain the speed of the turbine

ANSWERS

1. (*a*)	2. (*c*)	3. (*b*)	4. (*c*)	5. (*b*)
6. (*d*)	7. (*c*)	8. (*b*)	9. (*a*)	10. (*d*)