

# **Applications of Social Network Analysis to Online Social Networks**

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# **Exploring the Google+ Social Graph**

Magno et. al, IMC 2012

# Google+

- users (linked to GMail accounts)
  - real names
  - user info (personal, professional, ...)
  - interests
  - education
- links between users
  - directed relationship
  - users organized into circles
  - publish-to-circle
  - subscribe-to-all-circles (can narrow down)

# Google+ main results

- 7 of the 20 top users are IT professionals
- 1% of users share contact information
- lots of single males share home phone
- sharing differs greatly between countries
- physical distance crucial in formation of links
- global and national links differ greatly by country

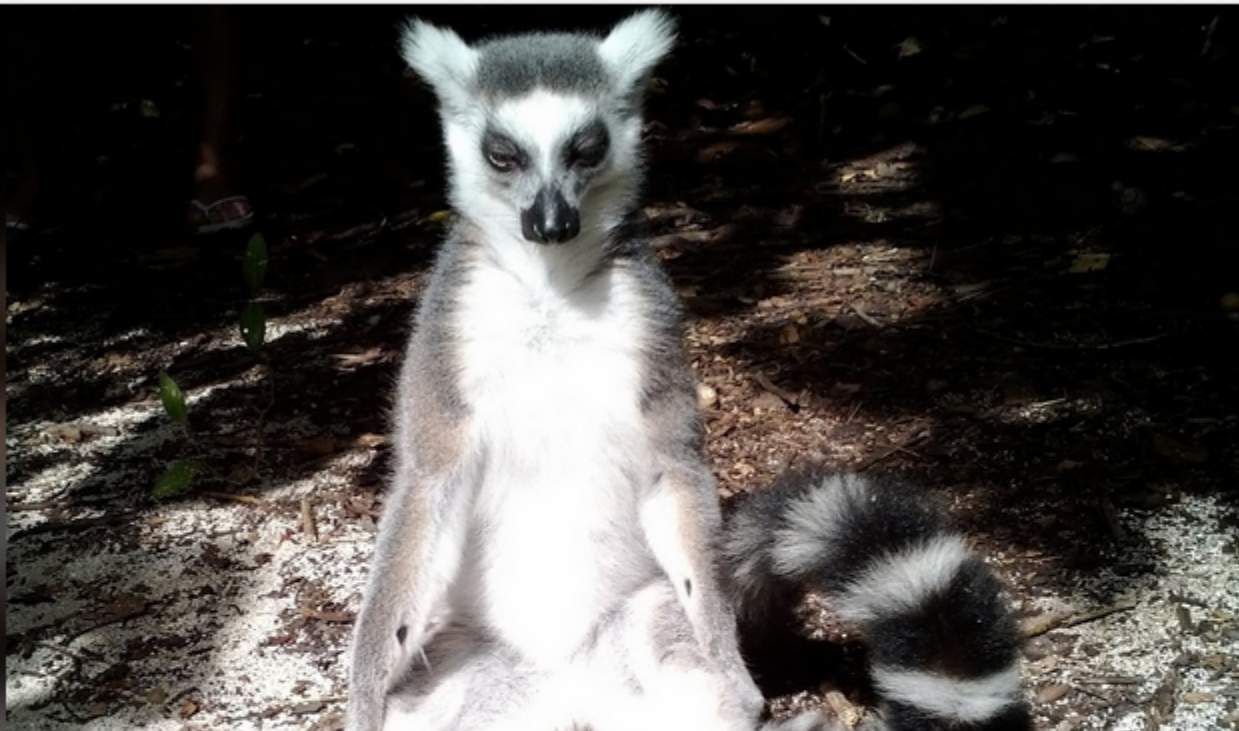


Larry Page

Works at Google  
Lives in Mountain View, CA

Add to circles

7,829,582 have him in circles



About

Posts

Photos

Videos

Reviews

## People

Have him in circles

7,829,582 people



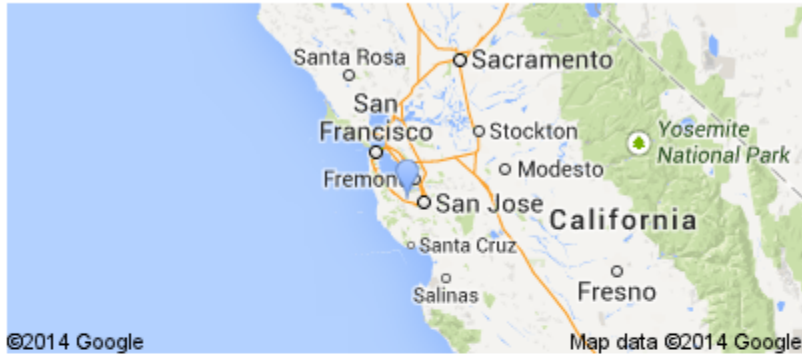
## Work

Employment

Google

CEO, present

## Places



### Currently

Mountain View, CA

## Basic Information

**Gender** Male


**Relationship** Married


## Links

### Google+ URL

[google.com/+LarryPage](https://google.com/+LarryPage)

### YouTube

 [Larry Page](#)

 Add your own contact details about Larry. Visible only to you.

Search for people, pages, or posts




+Thom...



Share




 People ▾

Find people

**Your circles**

More ▾

 Hangouts

Relevance ▾

Actions ▾

 Type a name



Add a person



**Marcus Liwicki**



**Saurav Biswas**



**Marlom Konrath**



**Christiano Gava**



**Christoph Studer**

Drag people to your circles to follow and share



Partners  
42

IUPR  
31

UniKL  
50

Interesting  
2

## [Larry Page - Google+](#)



<https://plus.google.com/+LarryPage> ▾

by Larry Page - in 7,755,216 Google+ circles

I have suggestions to you if u take it your business will increase ten folds for sure . first i will give my suggestions then implement it, if u get 2 times more profit just ...

### [I'm excited to announce Calico ...](#)

+Larry Page the difference between  
+Google and other ...

### [Here is the speech I just gave ...](#)

+Larry Page is still in the earnings  
call. Google Q2 2013 ...

### [Larry Page - About](#)

Larry Page. Works at Google. Lives  
in Mountain View, CA. 7 ...

### [Larry Page - Videos](#)

Larry Page - Google - Mountain View,  
CA. ... Larry Page. Works ...

[More results from google.com »](#)



**Table 1: Top 20 users ranked by in-degree**

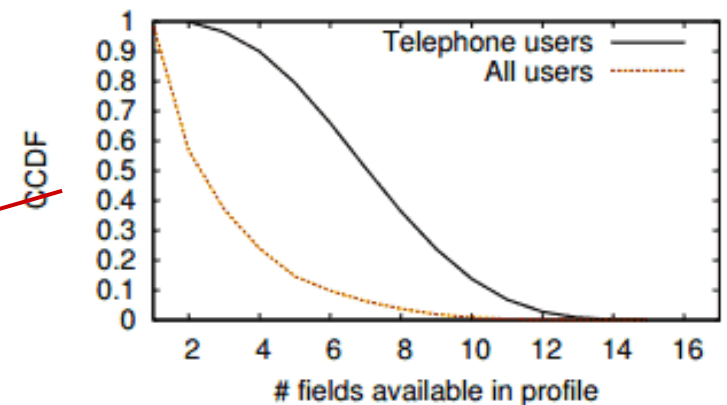
| <b>Rank</b> | <b>Name</b>     | <b>About</b>               |
|-------------|-----------------|----------------------------|
| 1           | Larry Page      | IT (Google)                |
| 2           | Mark Zuckerberg | IT (Facebook)              |
| 3           | Britney Spears  | Musician                   |
| 4           | Snoop Dogg      | Musician                   |
| 5           | Sergey Brin     | IT (Google)                |
| 6           | Tyra Banks      | Model                      |
| 7           | Vic Gundotra    | IT (Google)                |
| 8           | Paris Hilton    | Socialite                  |
| 9           | Richard Branson | Businessman (Virgin Group) |
| 10          | Dane Cook       | Comedian                   |
| 11          | Jessi June      | Model                      |
| 12          | Trey Ratcliff   | Blogger                    |
| 13          | will.i.am       | Musician                   |
| 14          | Felicia Day     | Actor                      |
| 15          | Thomas Hawk     | Blogger                    |
| 16          | Tom Anderson    | IT (Myspace)               |
| 17          | Pete Cashmore   | IT (Mashable)              |
| 18          | Guy Kawasaki    | IT (Apple) & Writer        |
| 19          | Wil Wheaton     | Actor & Writer             |
| 20          | Ron Garan       | Astronaut (NASA)           |

**Table 2: Public attributes available in Google+**

| Attribute         | Available  | %      |
|-------------------|------------|--------|
| Name              | 27,556,390 | 100.00 |
| Gender            | 26,914,758 | 97.67  |
| Education         | 7,471,191  | 27.11  |
| Places lived      | 7,371,461  | 26.75  |
| Employment        | 5,917,609  | 21.47  |
| Phrase            | 4,075,132  | 14.79  |
| Other profiles    | 3,713,546  | 13.48  |
| Occupation        | 3,656,447  | 13.27  |
| Contributor to    | 3,622,627  | 13.15  |
| Introduction      | 2,149,191  | 7.80   |
| Other names       | 1,210,760  | 4.39   |
| Relationship      | 1,186,903  | 4.31   |
| Braggin rights    | 1,074,964  | 3.90   |
| Recommended links | 1,001,349  | 3.63   |
| Looking for       | 753,704    | 2.74   |
| Work (contact)    | 60,434     | 0.22   |
| Home (contact)    | 58,876     | 0.21   |

**Table 3: Information shared by all users and *tel*-users**

|                           | All users  | Tel-users |
|---------------------------|------------|-----------|
| Total                     | 27,556,390 | 72,736    |
| Gender (N)                | 26,914,758 | 71,267    |
| Male                      | 67.65%     | 85.99%    |
| Female                    | 31.46%     | 11.26%    |
| Other                     | 0.89%      | 2.75%     |
| Relationship (N)          | 1,186,903  | 29,068    |
| Single                    | 42.82%     | 57.24%    |
| Married                   | 26.59%     | 21.03%    |
| In a relationship         | 19.80%     | 10.23%    |
| It's complicated          | 3.16%      | 3.98%     |
| Engaged                   | 4.39%      | 2.98%     |
| In an open relationship   | 1.26%      | 2.77%     |
| Widowed                   | 0.50%      | 0.58%     |
| In a domestic partnership | 1.08%      | 0.77%     |
| In a civil union          | 0.39%      | 0.41%     |
| Location (N)              | 6,621,644  | 45,676    |
| United States             | 31.38%     | 8.92%     |
| India                     | 16.71%     | 31.90%    |
| Brazil                    | 5.76%      | 4.72%     |
| United Kingdom            | 3.35%      | 2.19%     |
| Canada                    | 2.30%      | 1.52%     |
| Other                     | 40.50%     | 50.77%    |

**Figure 2: Number of fields shared by users in the profile**

# degree distribution

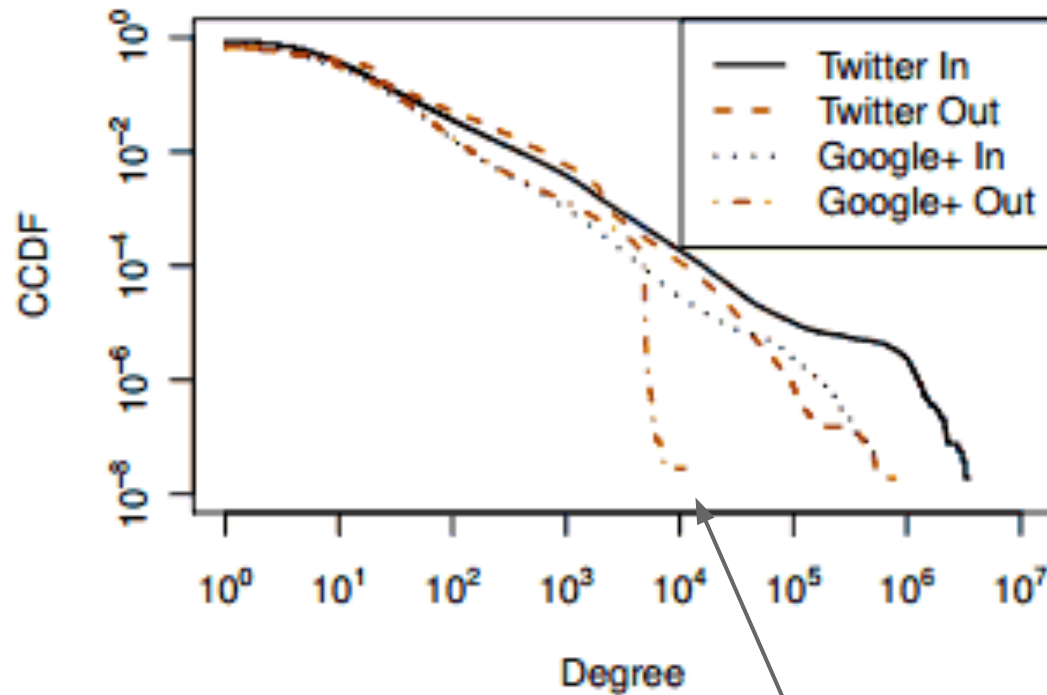


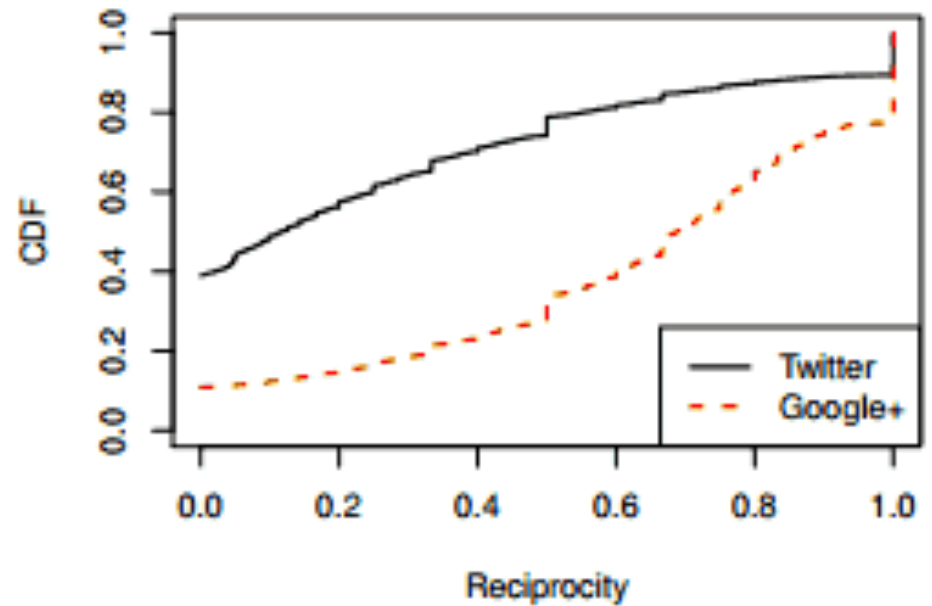
Figure 3: Degree Distributions

policy-based cutoff

power law distribution typical of human social networks

# reciprocity

$$RR(u) = \frac{|OS(u) \cap IS(u)|}{|OS(u)|}$$



(a) Reciprocal links

Which service has higher reciprocity?

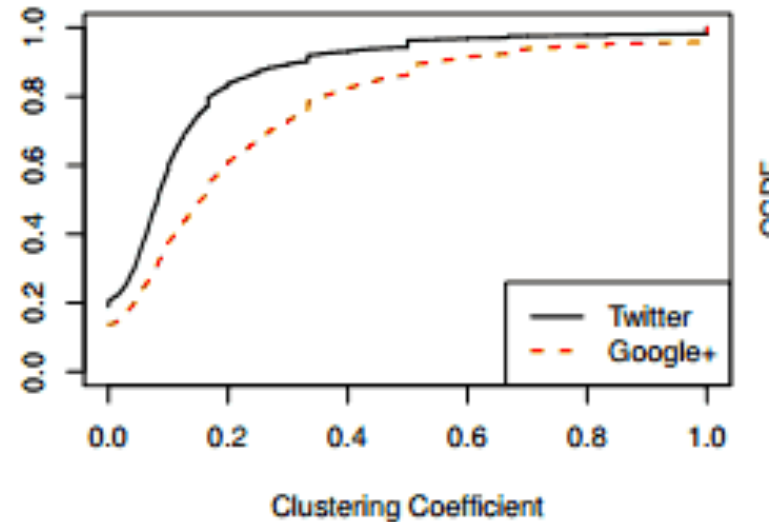
# reciprocity

- Google+ has higher reciprocity of links than Twitter
- probable cause: fewer big media outlets on Google+

# clustering coefficient

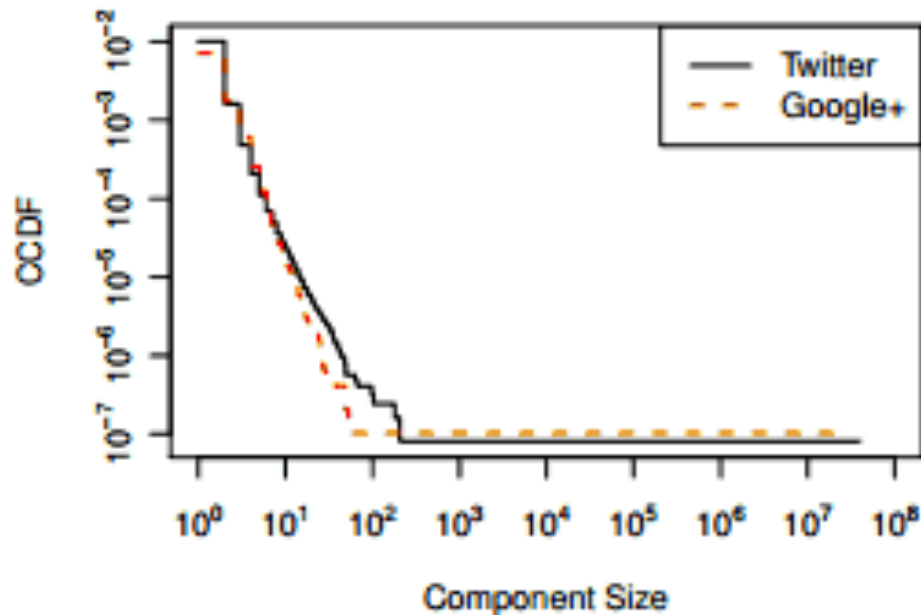
clustering = probability that two neighbors of a node are neighbors of each other (triangles)

Google+ has higher clustering coefficients  
(=more personal usage?)



# strongly connected components

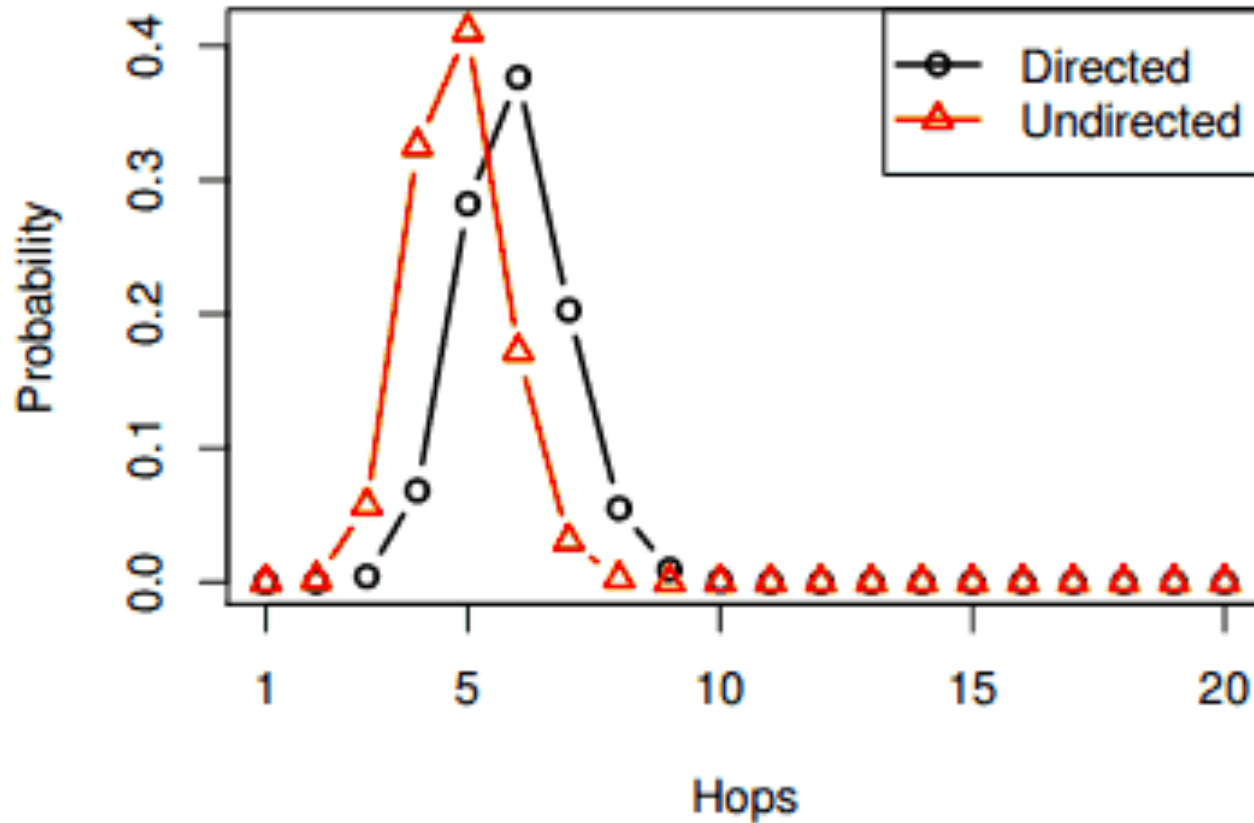
one weakly connected component from crawl  
measure number and size of SCCs



(c) Size of the strongly connected components



# degrees of separation



**Figure 5: Estimated path length distribution**

# social graph statistics

**Table 4: Comparison of topological characteristics of Google+ and other online social networks**

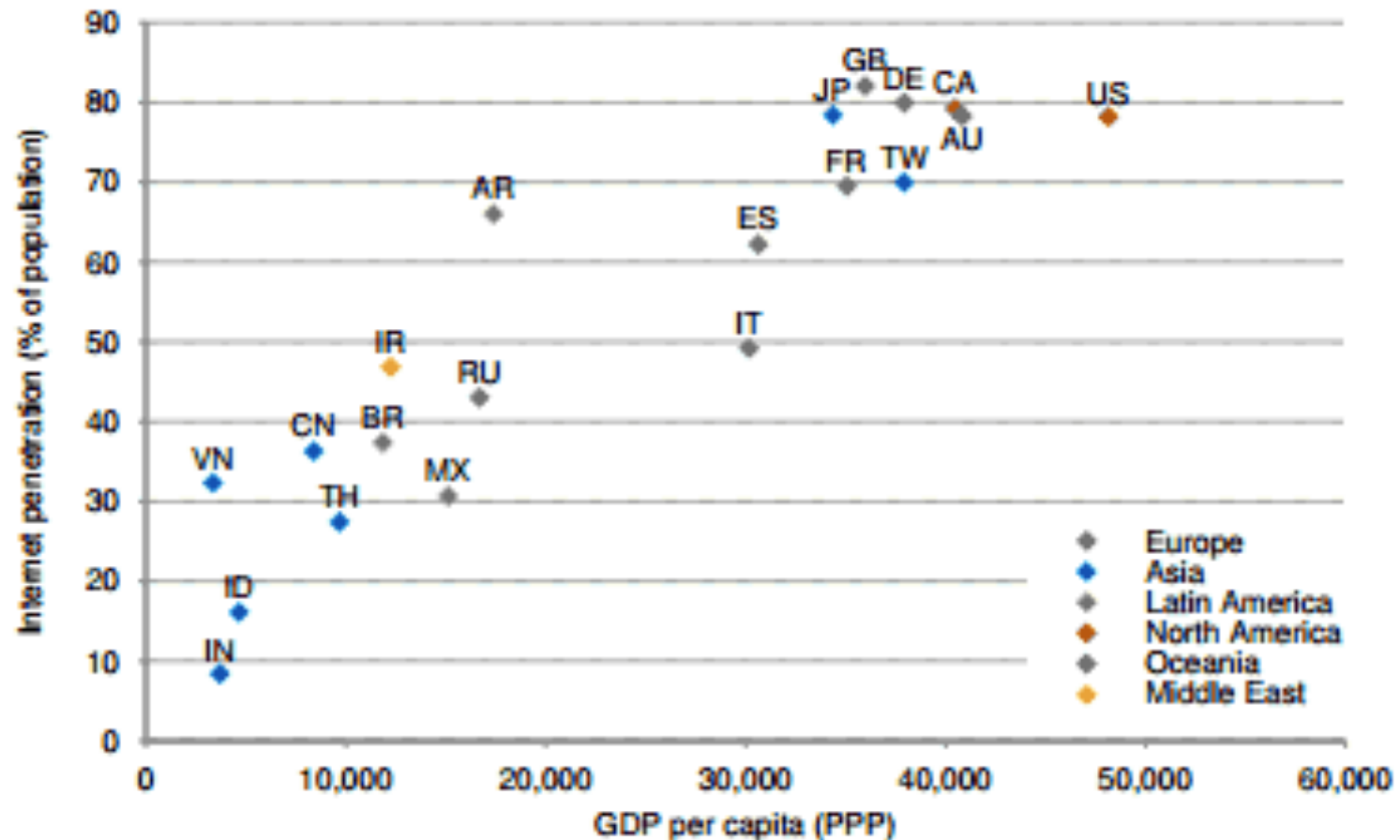
| Network  | Nodes | Edges | % Crawled | Path length | Reciprocity | Diameter | In-degree | Out-degree |
|----------|-------|-------|-----------|-------------|-------------|----------|-----------|------------|
| Google+  | 35M   | 575M  | 56%       | 5.9         | 32%         | 19       | 16.4      | 16.4       |
| Facebook | 721M  | 62G   | 100%      | 4.7         | 100%        | 41       | 190.2     | 190.2      |
| Twitter  | 41.7M | 106M  | 100%      | 4.1         | 22%         | 18       | 28.19     | 29.34      |
| Orkut    | 3M    | 223M  | 11%       | 4.3         | 100%        | 9        | -         | -          |

# **geographic quiz**

put the following countries in descending order of per-capita GDP

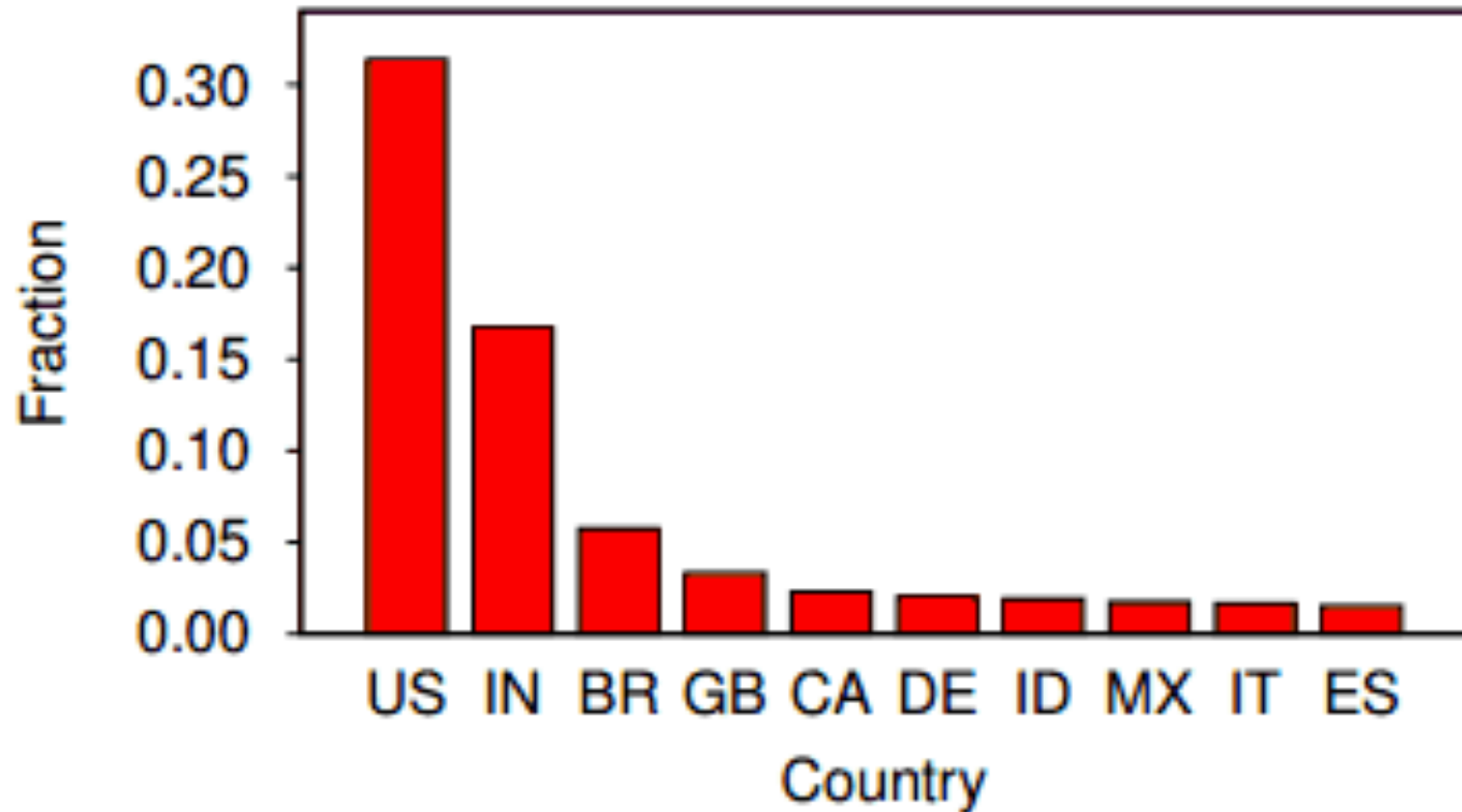
Russia, China, US, Germany, Japan

# internet penetration

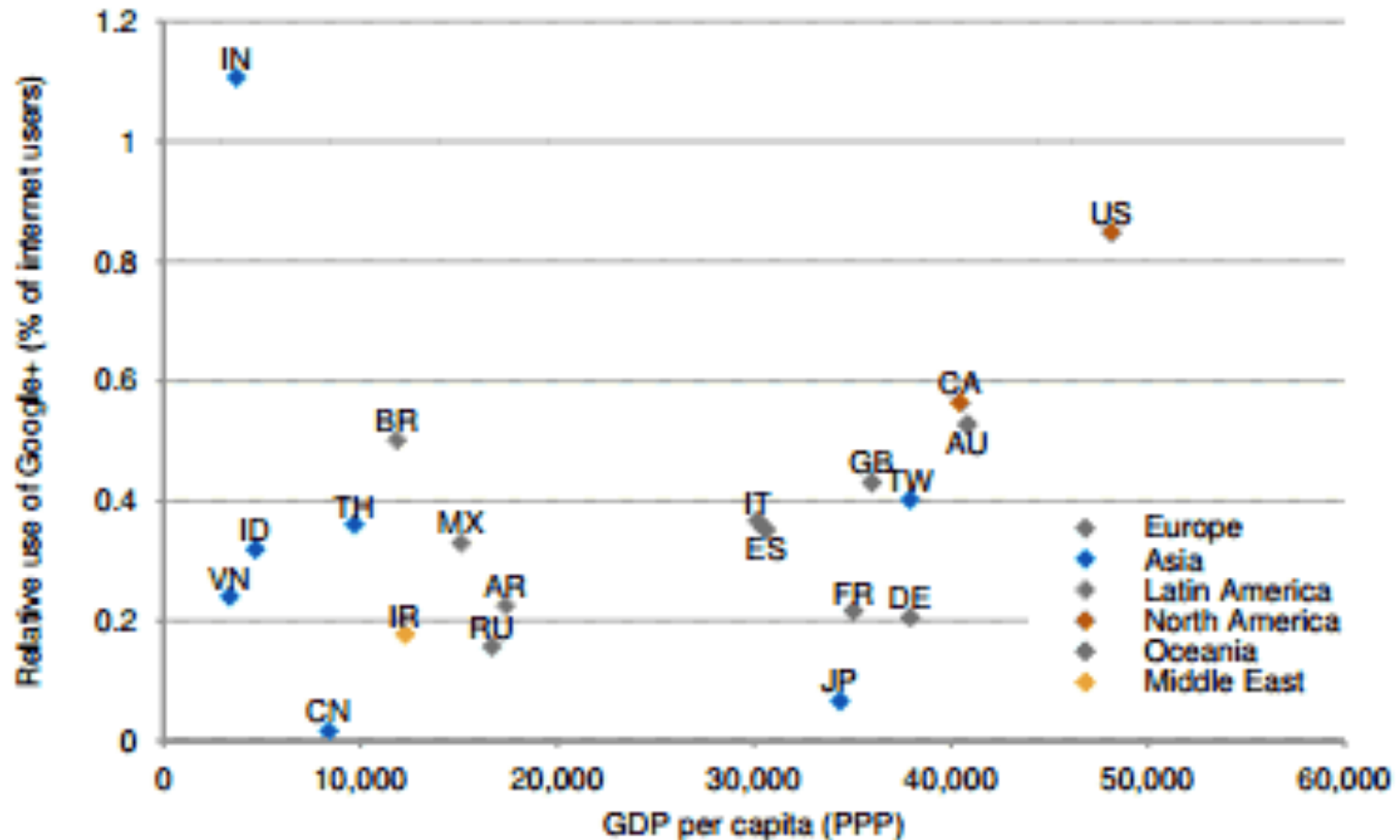


(b) GDP Per Capita and Internet Penetration

# country shares among Google+ users

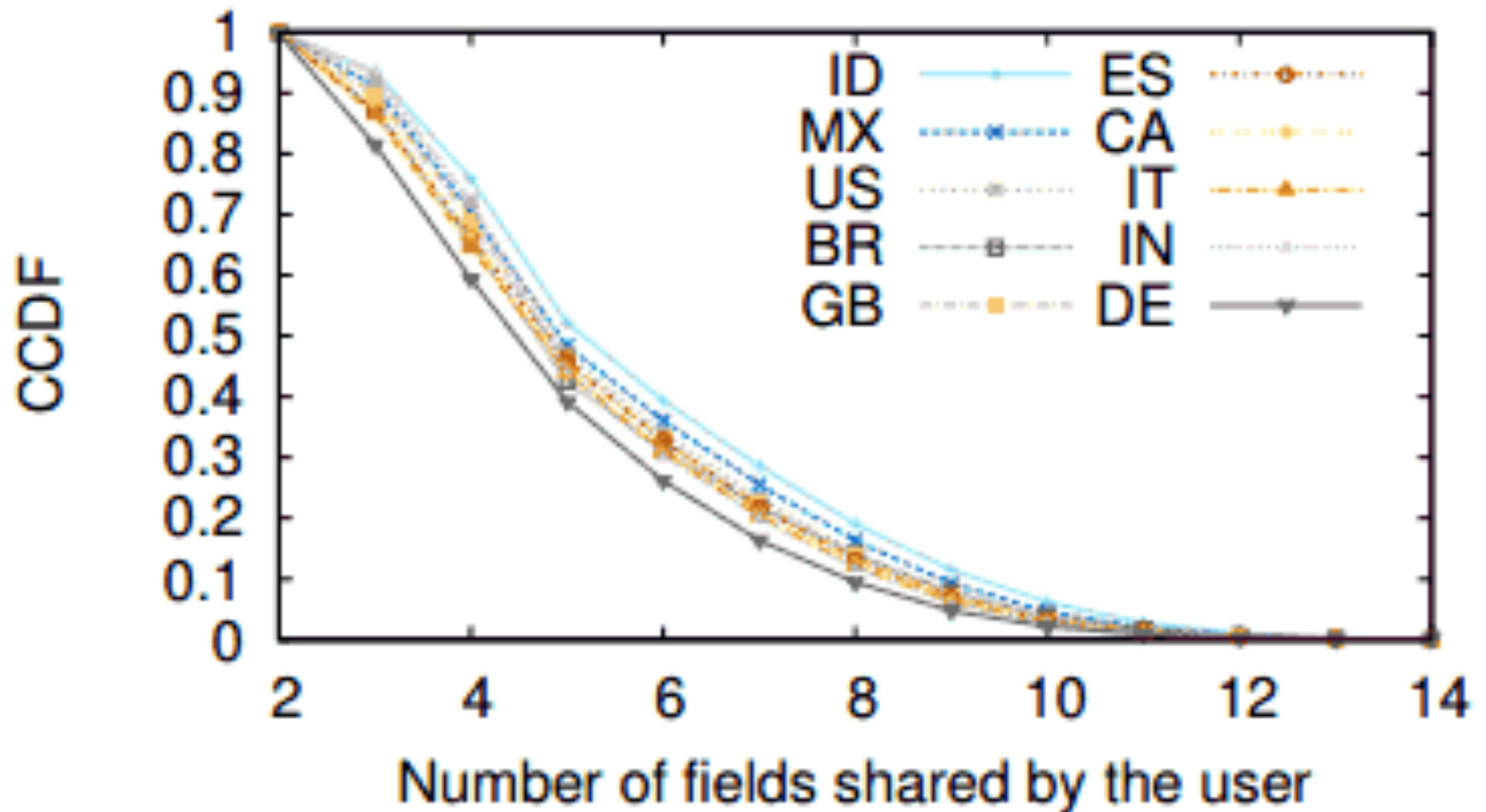


# Google+ adoption

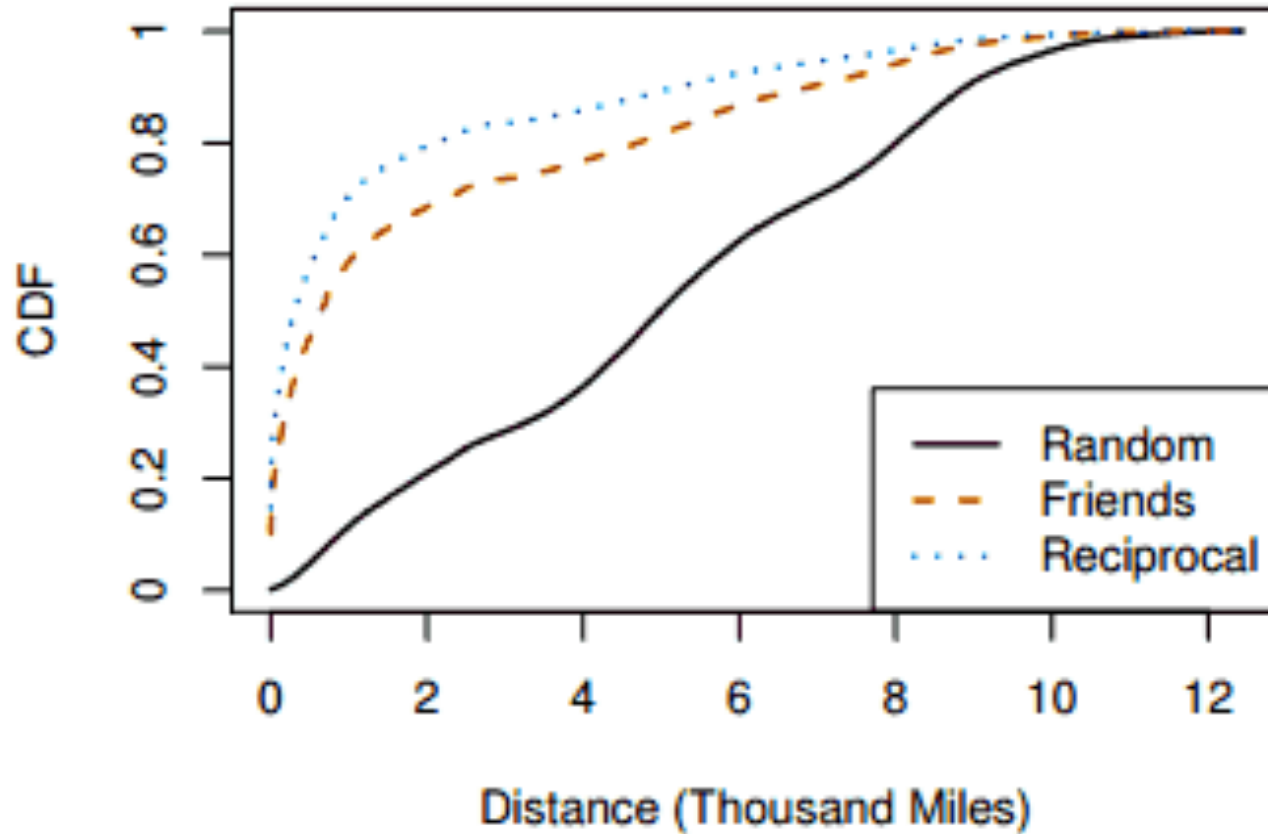


(a) GDP Per Capita and Use of Google+

# information sharing by country



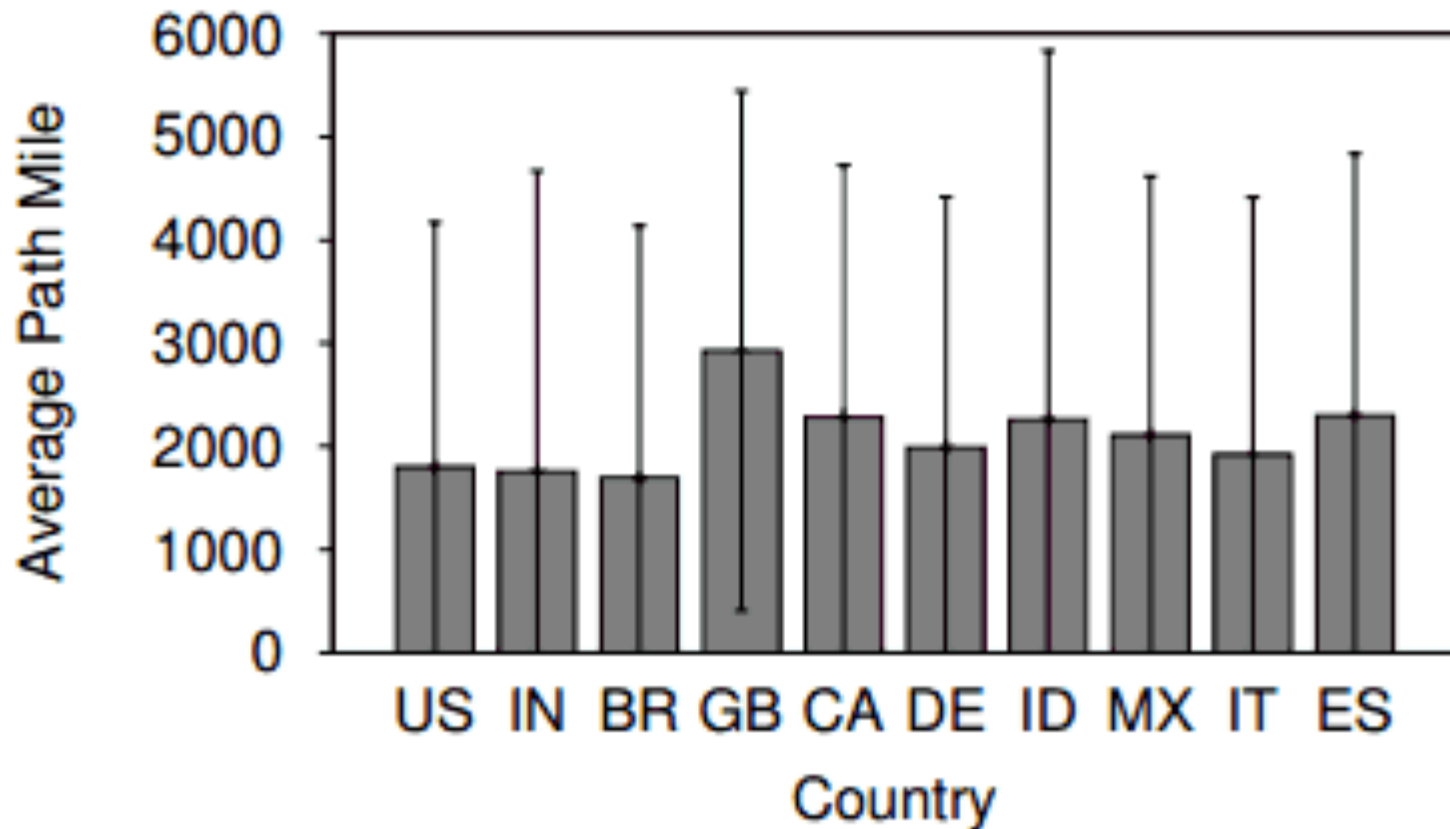
# path mile distribution of Google+ users



(a) Path Mile distribution of Google+ users

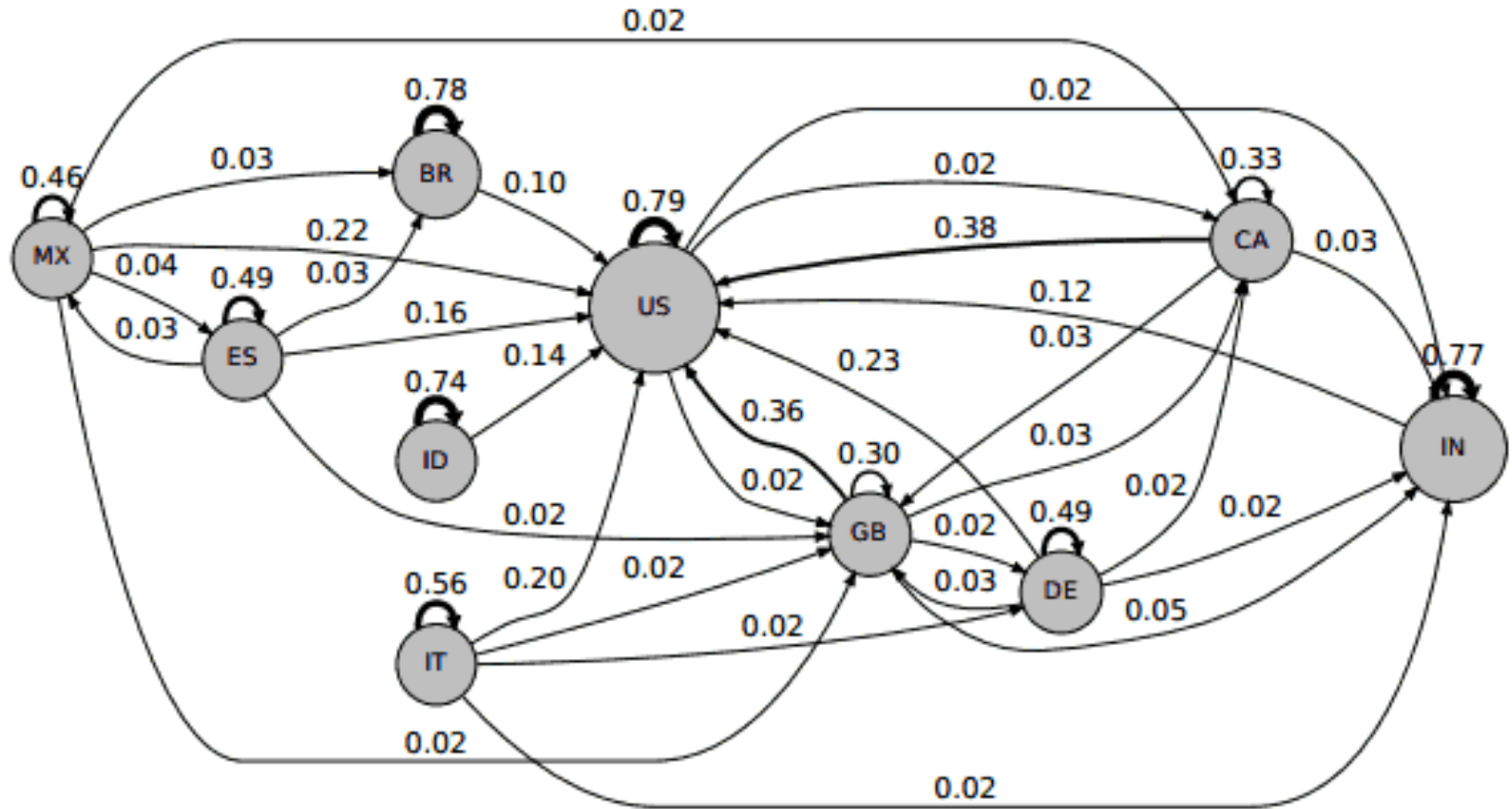


# average path mile by country



(b) Average path mile with standard deviation

# link distribution across countries



**Figure 10: Link distribution across the top countries**

# conclusions (from paper)

- shorter path lengths, but probably due to size and novelty (?)
- penetration info useful for marketing and for guiding expansion
- significantly different privacy behavior across countries
- significantly different inward-looking/ outward-looking behaviors

# observations

- longitudinal observations would make this significantly more useful
- structure of network depends on complex cultural, social, linguistic, etc. factors
- analysis by machine learning, e.g. predict probability of a link between two users based on distance, country, languages, profession, etc.

# **Evolution of Social Attribute Networks**

Gong et. al, IMC 2012

# **main goals**

Creation of a Google+ dataset

Capture both state and evolution over time

Evaluation and evolution of metrics

Creation of a generative model

# social attribute networks

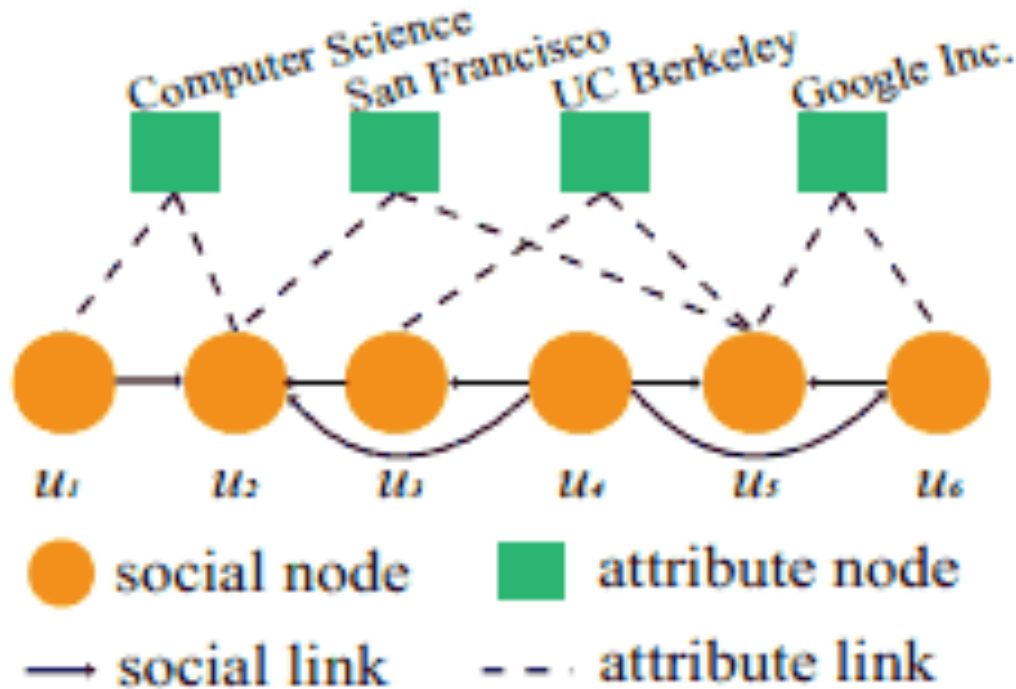
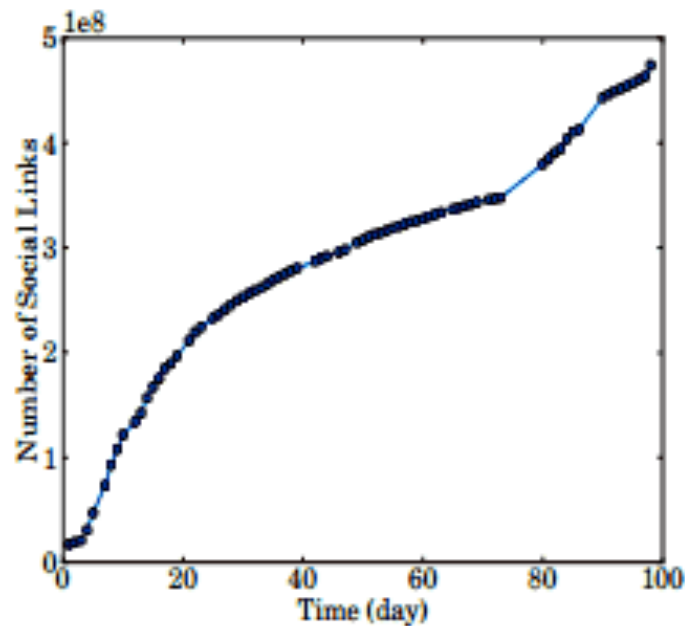


Figure 1: *Illustration of a SAN with six social nodes and four attribute nodes. Note that the social links between users are directed whereas the attribute-user links are undirected.*

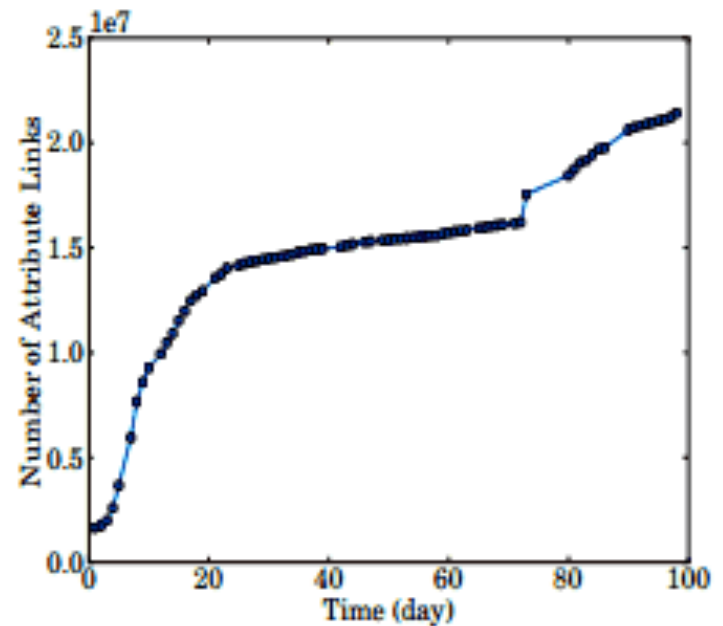
**network structure**



# growth in # links



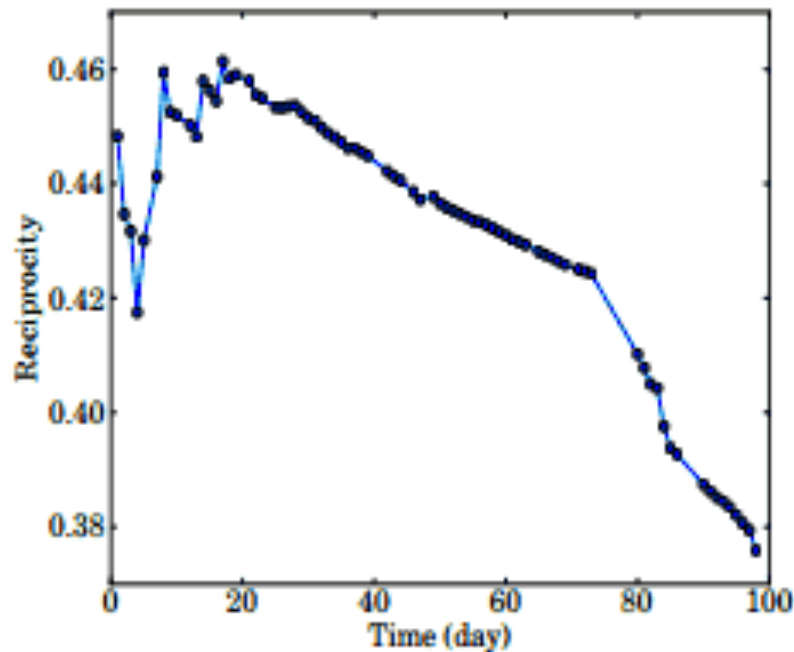
(a) Social links



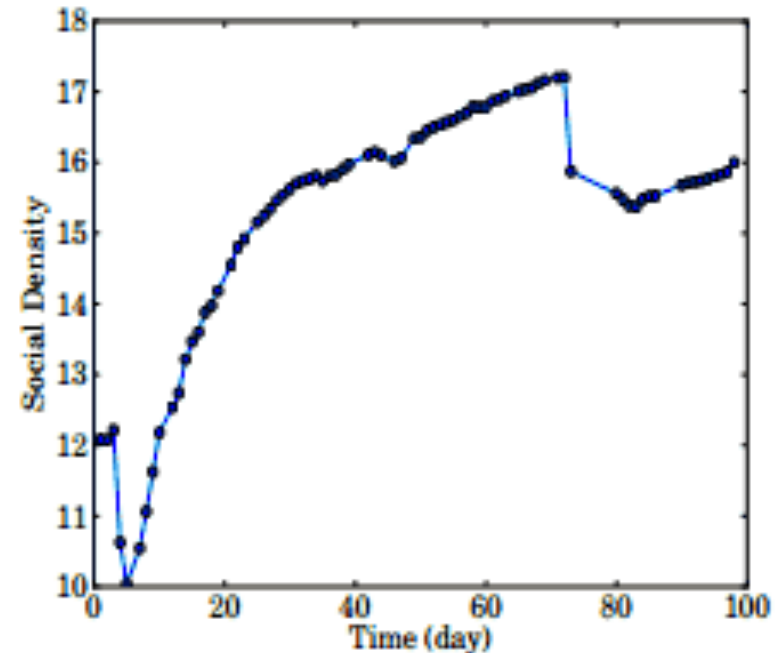
(b) Attribute links

Figure 3: *Growth in the number of social and attribute links in the Google+ dataset*

# evolution of reciprocity and social density



(a) Reciprocity



(b) Social density

Social density = ratio of links to nodes. Increasing in citation and affiliation networks, Facebook. Fluctuates on Flickr, fairly constant on E-mail.

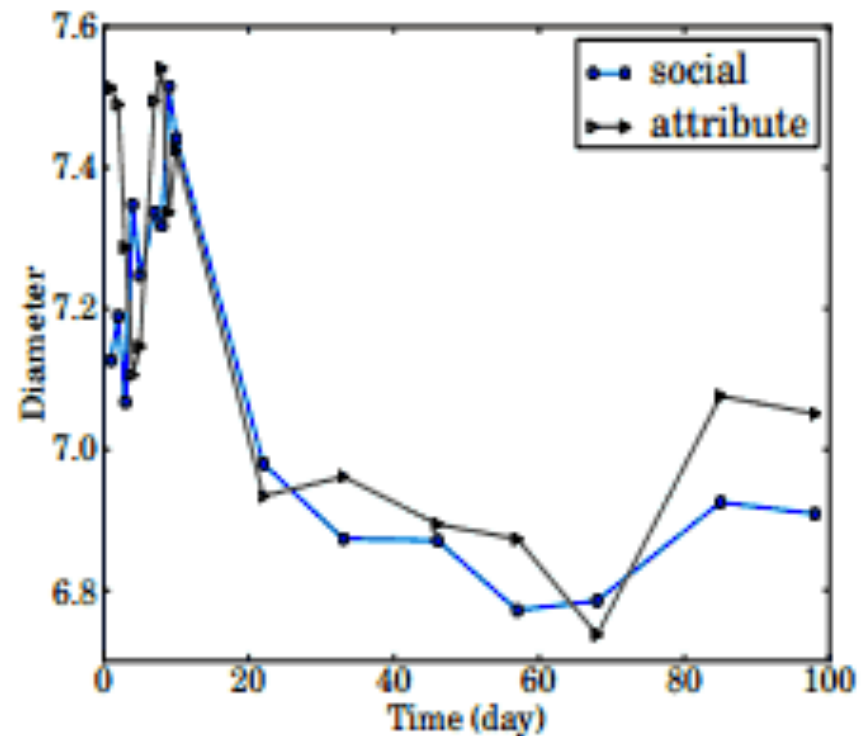
Phases: (1) many people join, but not many friends yet, (2) like social networks, (3) public release of Google+ causes drop, then rises as before.

# evolution of diameter

same three phases

not huge differences

for most networks,  
shrinks over time

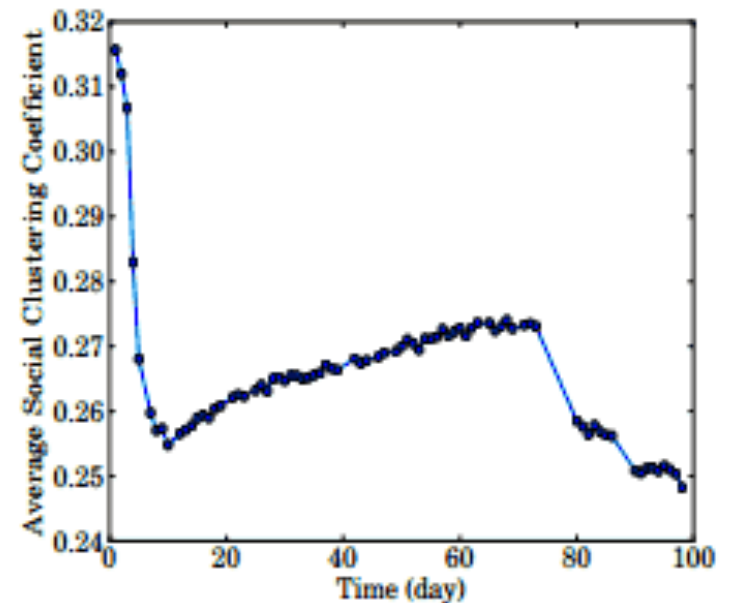


(c) Diameter

# clustering

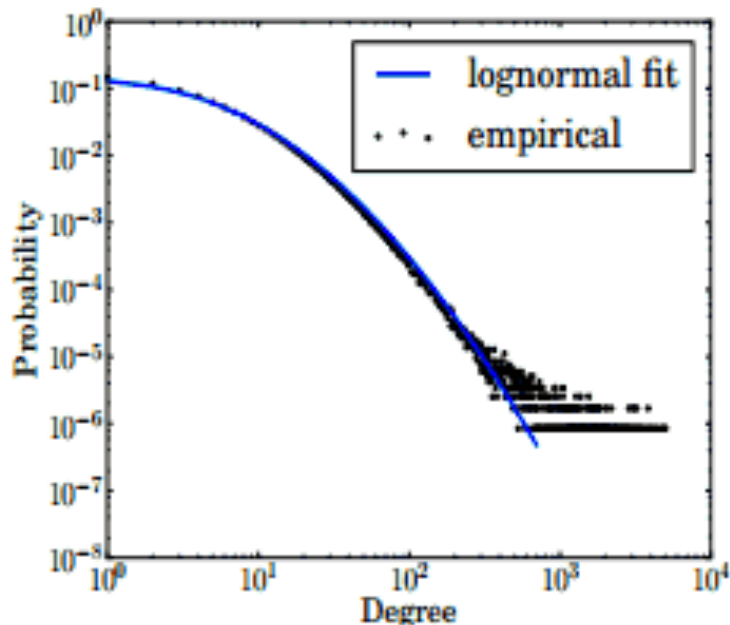
constant for E-mail,  
unknown for others

slow increase in phase 2,  
suggesting community  
formation

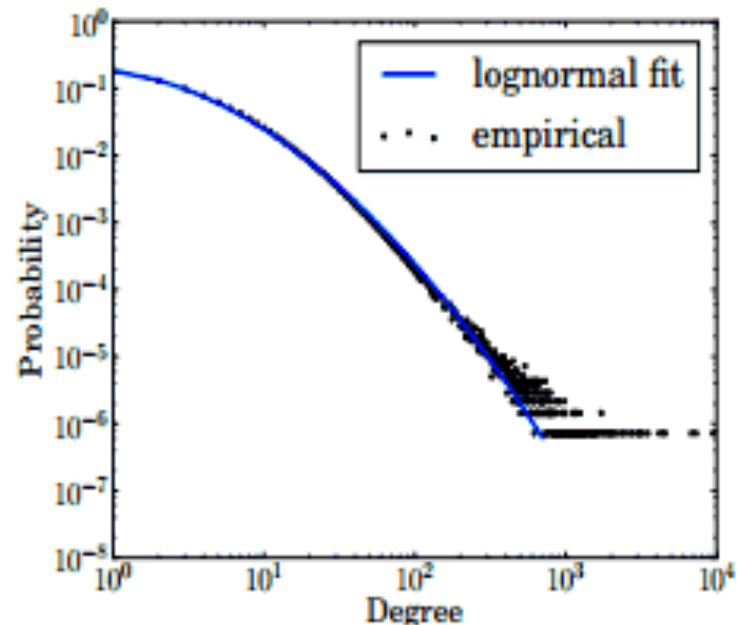


(d) Social clustering coefficient

# distribution of in/outdegrees



(a) Outdegree

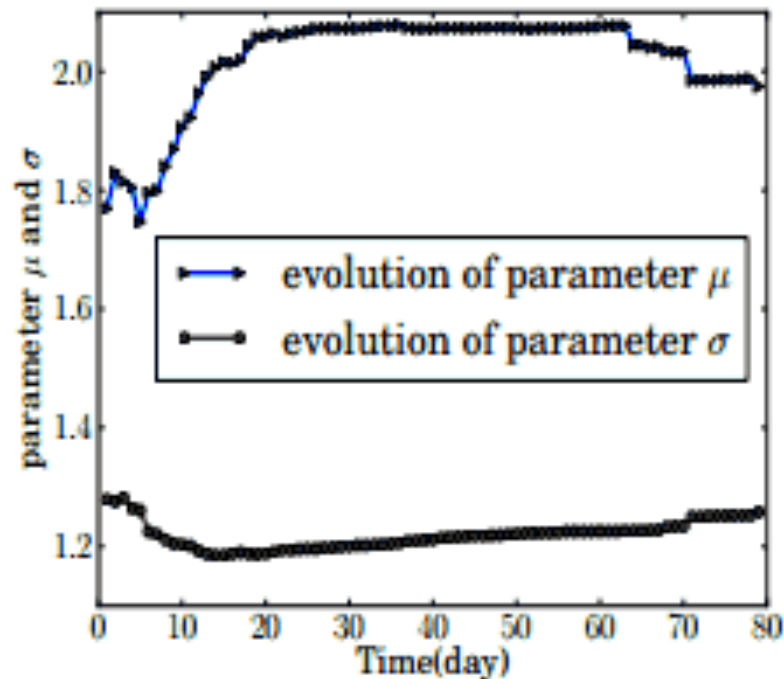


(b) Indegree

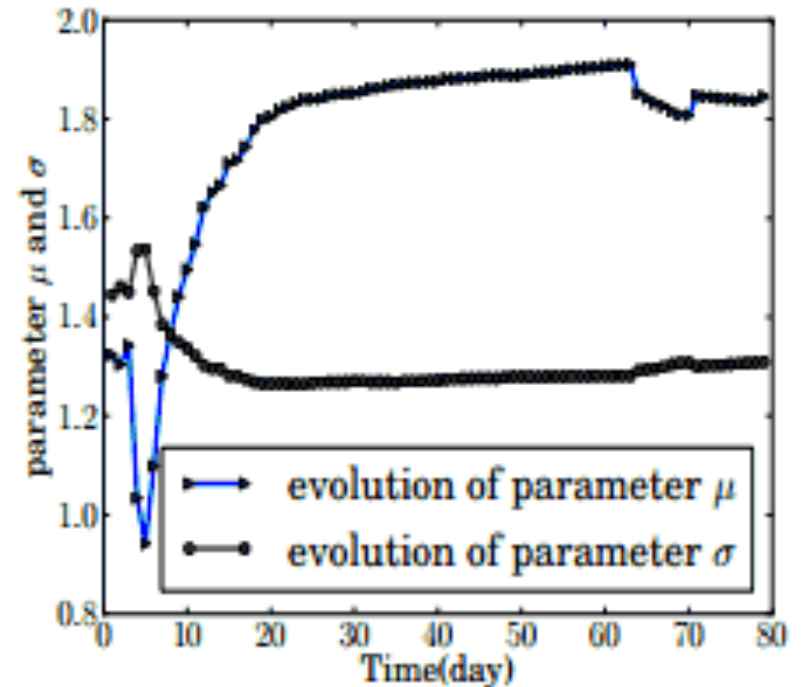
Figure 5: *Indegree and outdegree distributions for the social nodes in the Google+ SAN along with their best-fit curves. We observe that both are best modeled by a discrete lognormal distribution unlike many networks that suggest power-law distributions.*

where have we seen a log normal distribution before?

# evolution of degree distributions



(a) Outdegree



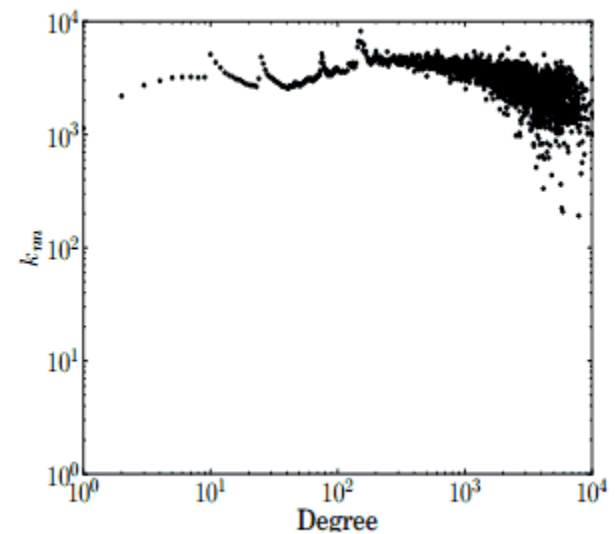
(b) Indegree

Figure 6: *Evolution of the lognormal parameters for the indegree and outdegree distributions.*

# joint degree distribution

average in-degree of nodes  
attaching to nodes with a  
given out-degree

increasing = high-degree  
tends to connect to high-  
degree



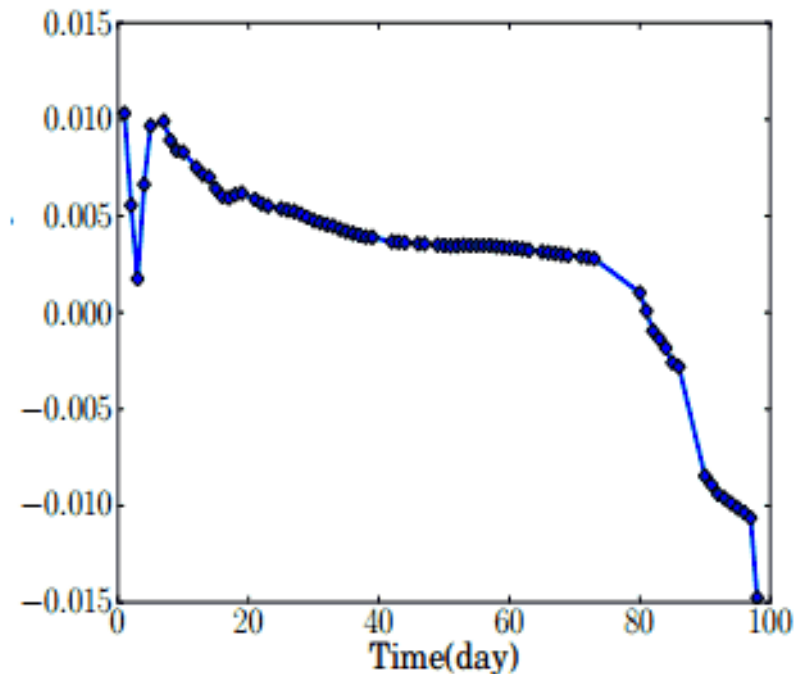
(a)  $k_{nn}$  metric

# assortativity

correlation coefficient of  
JDD

typically positive for  
social networks,  
negative for pub/sub

near 0 for Google+ =  
hybrid network?

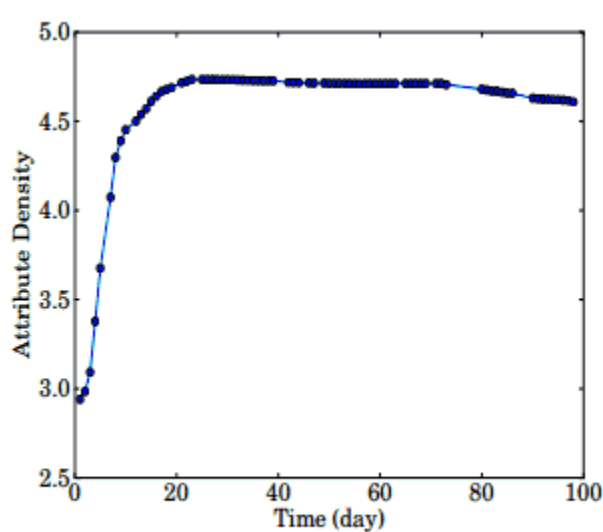


(b) Assortativity

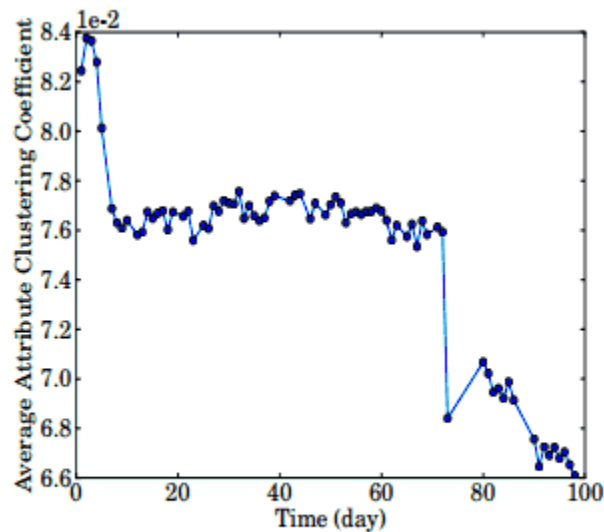


attribute structure

# attribute density & clustering



(a) Attribute density



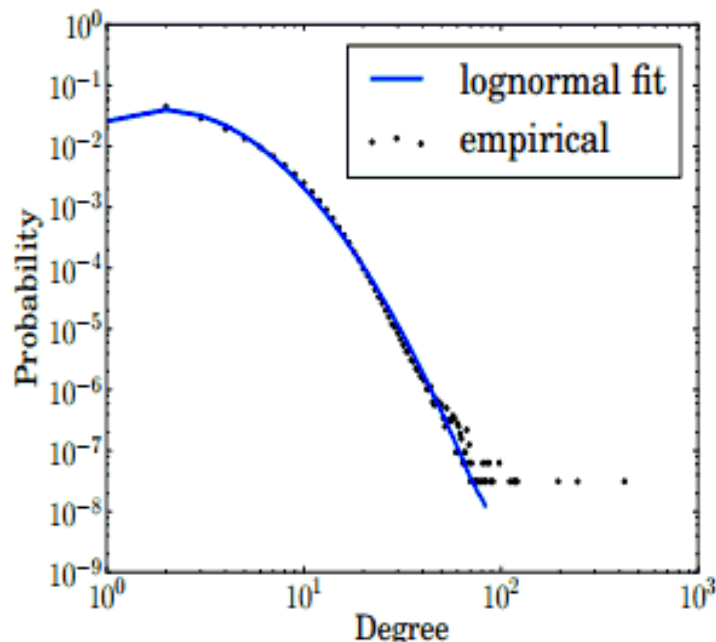
(b) Clustering coefficient

Attribute density:  $|E_a|/|V_a|$

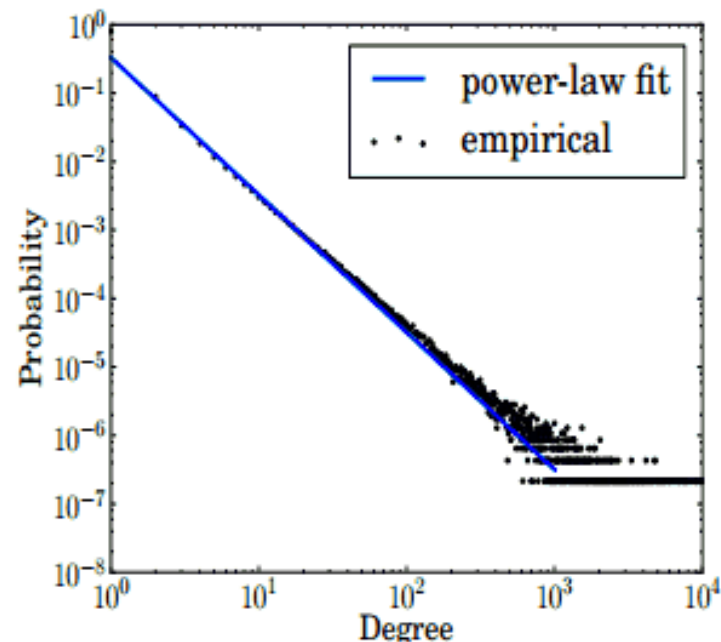
Clustering density for attributes: “third link” is social link, so propensity for attributes to form communities.

NB: this includes attributes like “city” etc.

# distributions of attribute-induced degrees



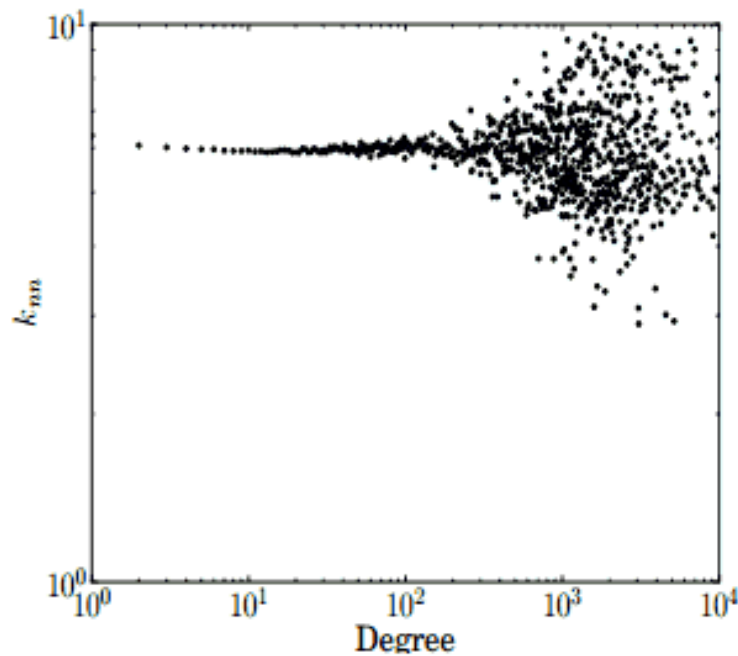
(a) Attribute degree of social nodes



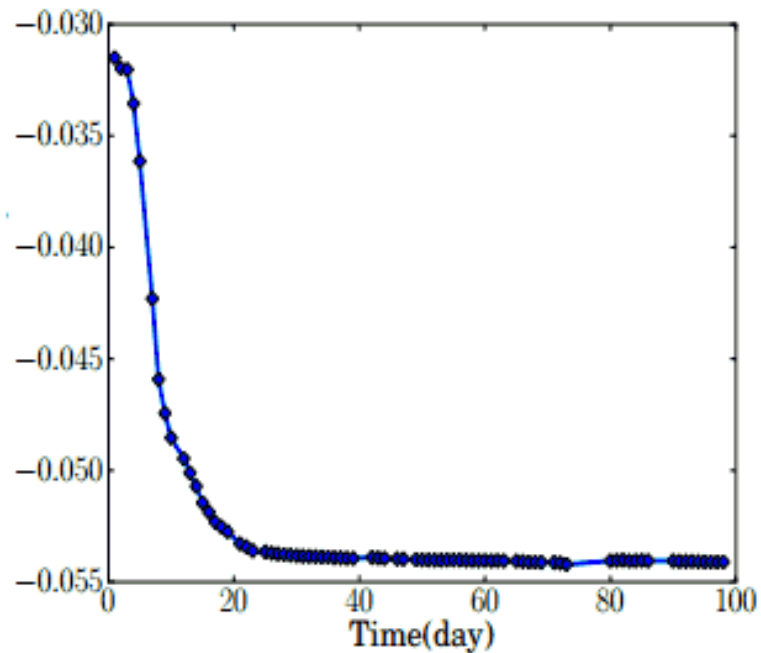
(b) Social degree of attribute nodes

Figure 10: *Distributions of attribute-induced degrees in the Google+ SAN along with their best fits. The attribute degree of social nodes is best modeled by a lognormal whereas the social degree of attribute nodes is best modeled by a power-law distribution.*

# assortativity of attributes



(a) Attribute  $k_{nn}$



(b) Evolution of assortativity

Figure 12: (a) *Joint degree of attribute nodes: Log-log plot of the social degree versus the average attribute degree of social neighbors of attribute nodes.* (b) *The evolution of the attribute assortativity coefficient.*

**comment... are these well chosen?**

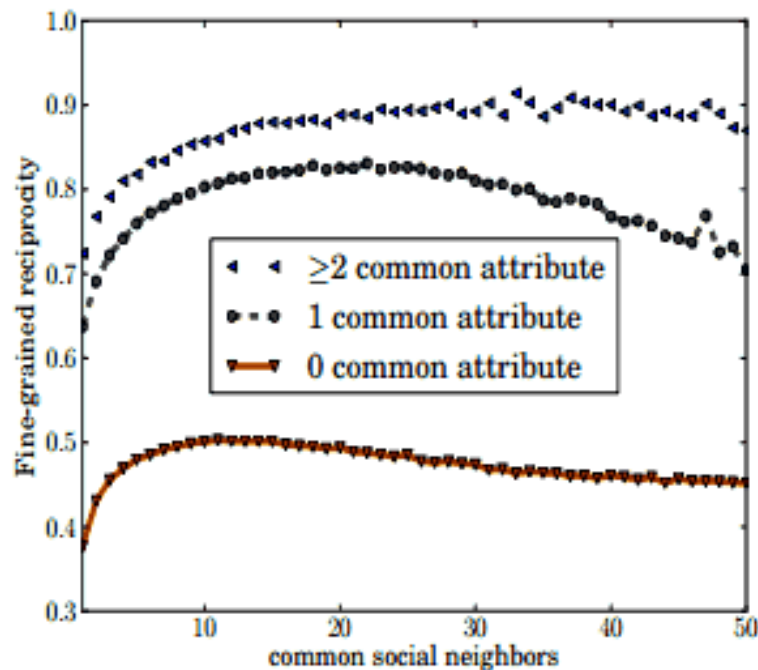
Interesting questions about the relationship between attributes and the social graph.

Why “extend” graph analysis?

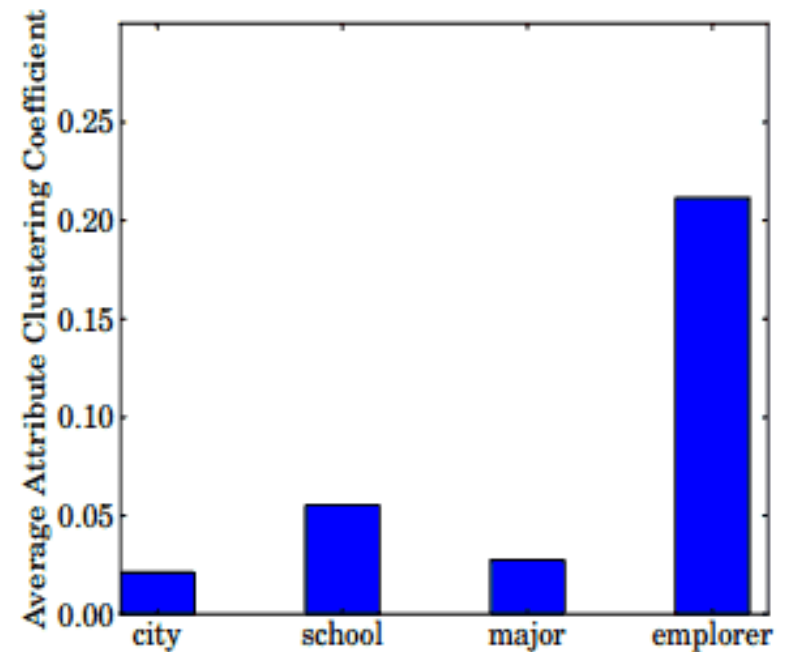
What are some of the things wrong with this approach?

What are better approaches?

# social network structure vs attributes



(a) Reciprocity



(b) Clustering coefficients

Figure 13: *Influence of attribute on reciprocity and clustering coefficients.*

# observations on Google+ measurements

- When viewed as a graph structure, the attribute graph has different kinds of distributions and behaviors from the social graph.
- The attribute graph influences the social graph in “interesting” ways.
- Some attributes have stronger influence than others.

(duh!)

generative model



# preferential attachment

Build a network by adding nodes one at a time.

Nodes attach to existing nodes with a probability related to their degree. Extension: number of shared attributes  $a$ .

- *Power Attribute Preferential Attachment (PAPA):*  
$$f(u, v) \propto d_i(v)^\alpha (1 + a(u, v)^\beta)$$
- *Linear Attribute Preferential Attachment (LAPA):*  
$$f(u, v) \propto d_i(v)^\alpha (1 + \beta \cdot a(u, v))$$

# triangle closing

Friend requests to friends of friends. Do attributes improve triangle closing?

- *Baseline*: Select a social neighbor  $v$  within a 2-hop radius uniformly at random.
- *Random-Random (RR)*: Select a social neighbor  $w \in \Gamma_s(u)$  uniformly at random, and then select a social neighbor  $v \in \Gamma_s(w)$  uniformly at random which is shown to have very good performance in previous work [29]. 14% better than baseline
- *Random-Random-SAN (RR-SAN)*: select a neighbor  $w \in \Gamma_s(u) \cup \Gamma_a(u)$  uniformly at random, and then select a social neighbor  $v \in \Gamma_s(w)$  uniformly at random.<sup>4</sup> 36% better than RR

# generative model

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**Algorithm 1:** Social-Attribute Network Model

---

```
1 T, simulated time steps
2 Initialization.
3 for  $1 \leq t \leq T$  do
4     Social node arrival. Sample a set of new social nodes  $V_{t,new}$ .
5     for  $v_{new} \in V_{t,new}$  do
6         Attribute degree sampling. Sample the number of attributes
           $n_a(v_{new})$  for  $v_{new}$  from a lognormal distribution.
7         for  $1 \leq i \leq n_a(v_{new})$  do
8             | Attribute linking.
9         end
10        First outgoing linking.
11        lifetime sampling.
12        sleep time sampling.
13    end
14    Collect woken social nodes  $V_{t,woken}$ .
15    for  $v_{woken} \in V_{t,woken}$  do
16        | Outgoing linking.
17        | sleep time sampling.
18    end
19 end
```

---

# Zheleva model

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**Algorithm 2** Co-evolution model

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```
1: Set of nodes  $V = \emptyset$ 
2: Set of groups  $H = \emptyset$ 
3: for each time period  $t \in T$  do
4:   Set of active nodes at time  $t$ ,  $V_t = \emptyset$ 
5: end for
6: for each time period  $t \in T$  do
7:   Node arrival.  $V = V \cup V_{t,new}$ 
8:   for each new node  $v \in V_{t,new}$  do
9:     Lifetime sampling
10:    First social linking
11:  end for
12:  for each node  $v \in V_t$  do
13:    Social linking
14:    Affiliate linking.  $v$  determines  $n_h$ , the number
      of groups to join, sampled from an exponential
      distribution  $\lambda' e^{-\lambda' n_h}$  with a mean  $\mu' = \frac{1}{\lambda'} =$ 
       $\rho \cdot \text{degree}(v)^\gamma$ .
15:    for  $i = 1 : n_h$  do
16:      if  $\text{rand}() < \tau$  then
17:        Group creation.  $v$  creates group  $h$ , and forms
          edge  $e_a(v, h, t)$ .  $H = H \cup \{h_i\}$ .
18:      else
19:        Group joining.  $v$  forms edge  $e_a(v, h, t)$ . Group
           $h$  is picked through a friend with probability
           $p_v$ ; otherwise, or if no friends' groups are avail-
          able, it joins a random group with prob. propo-
          rtional to the size of  $h$ .
20:      end if
21:    end for
22:  end for
23:  for each node  $v \in V_t \cup V_{t,new}$  do
24:    Sleep time sampling
25:  end for
26: end for
```

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# statistical comparison

## Gong et al. vs Zhel

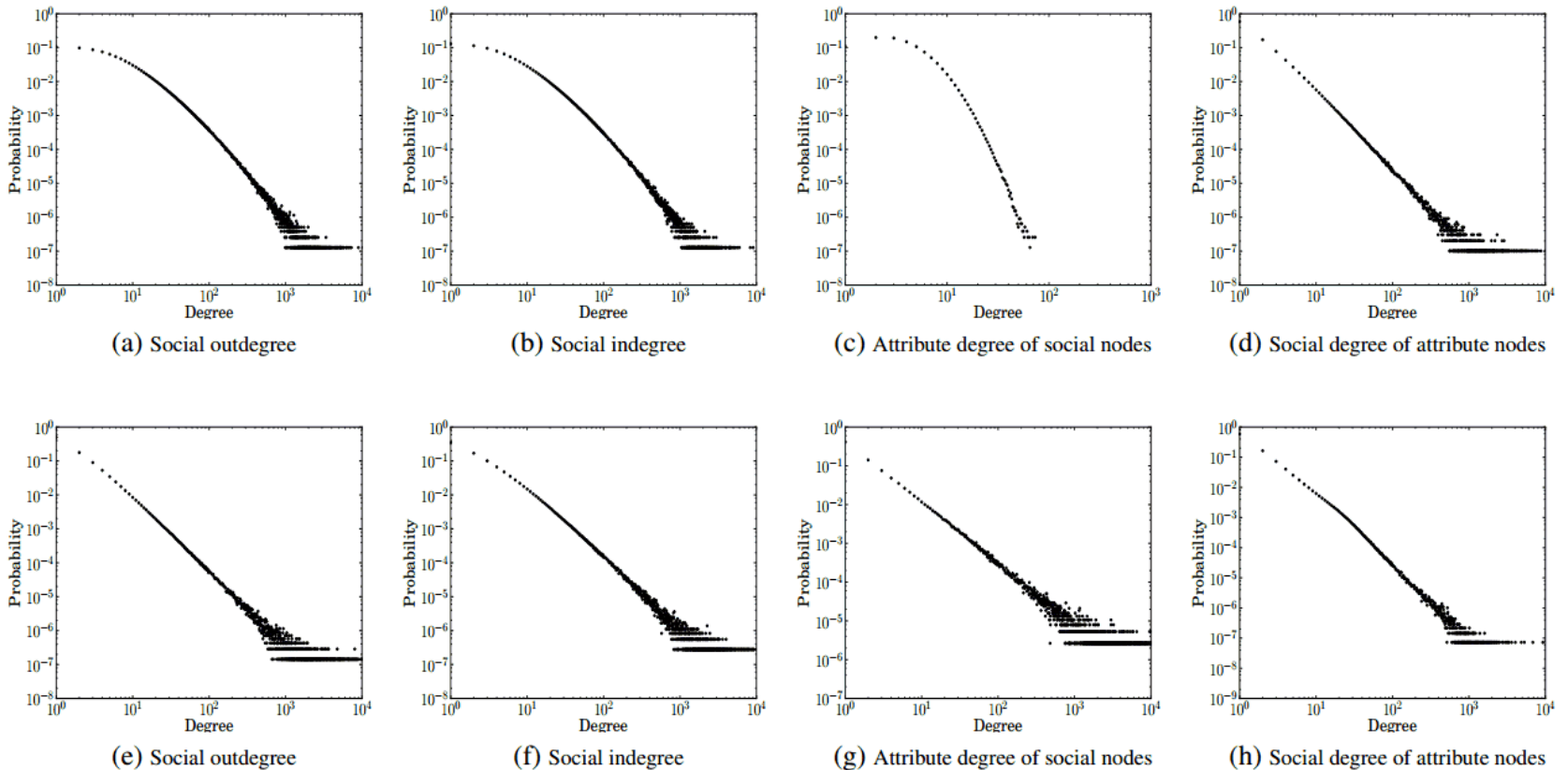


Figure 16: Degree distributions of synthetically generated SAN using our model in (a)-(d) vs. Zhel shown in (e)-(h).

# statistical comparison

## Gong et al. vs Zhel

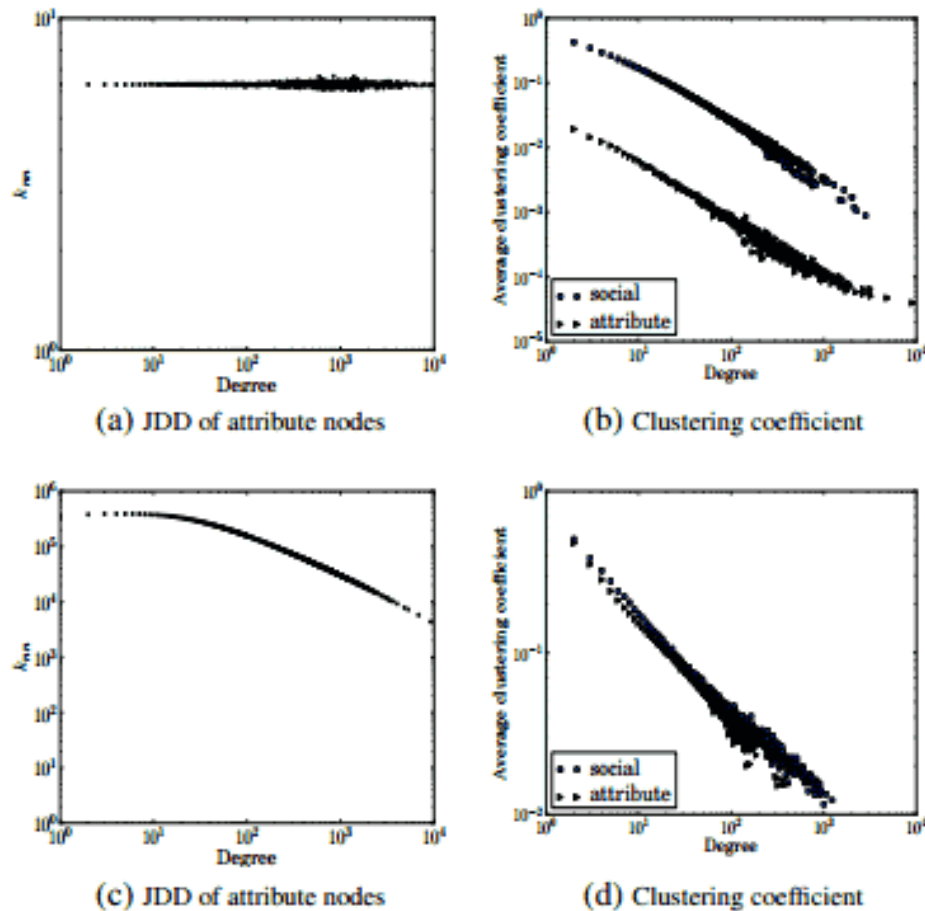


Figure 17: *Joint degree and clustering coefficient distributions of our model (a)–(b) vs. Zhel in (c)–(d).*

# application validation:

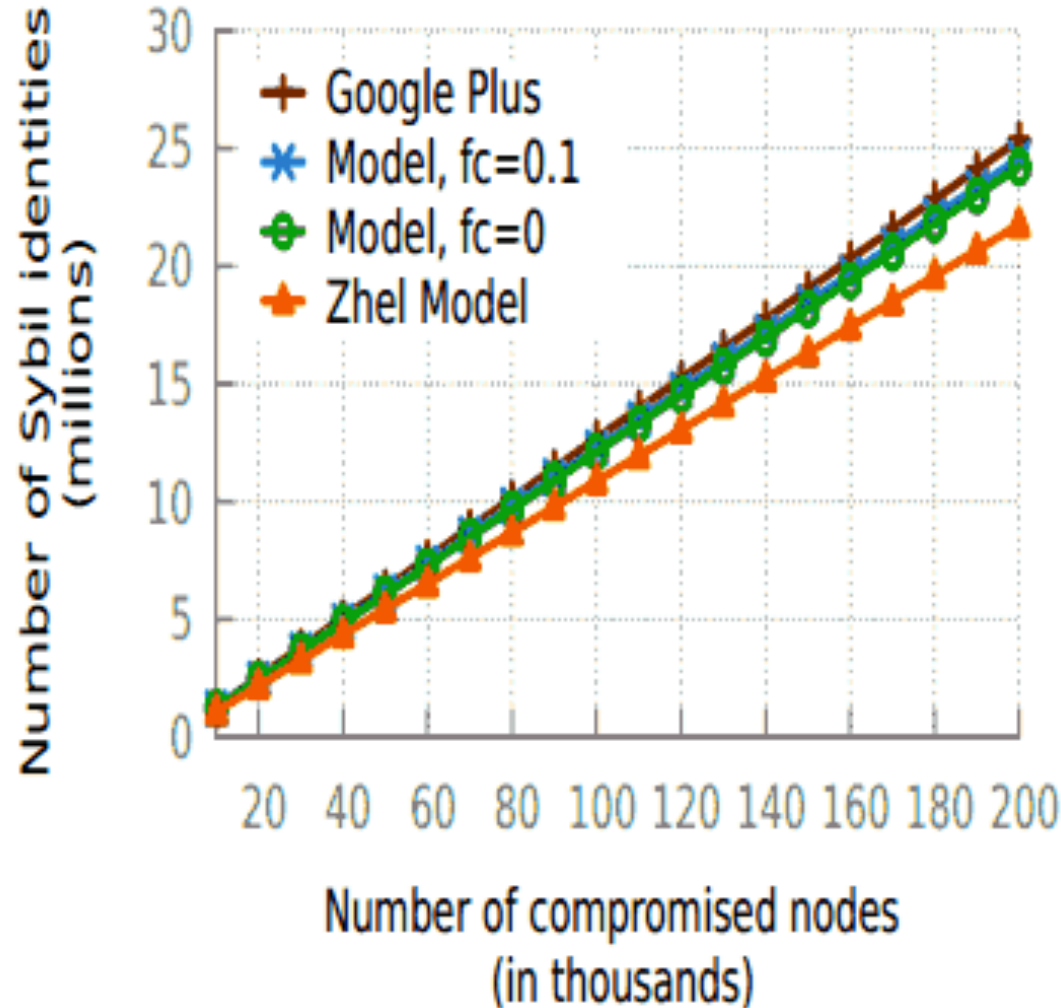
## Sybil Defense

Sybil attack (Wikipedia).

In a Sybil attack the attacker subverts the [reputation system](#) of a [peer-to-peer network](#) by creating a large number of [pseudonymous](#) identities, using them to gain a disproportionately large influence. (AKA “sockpuppets”)

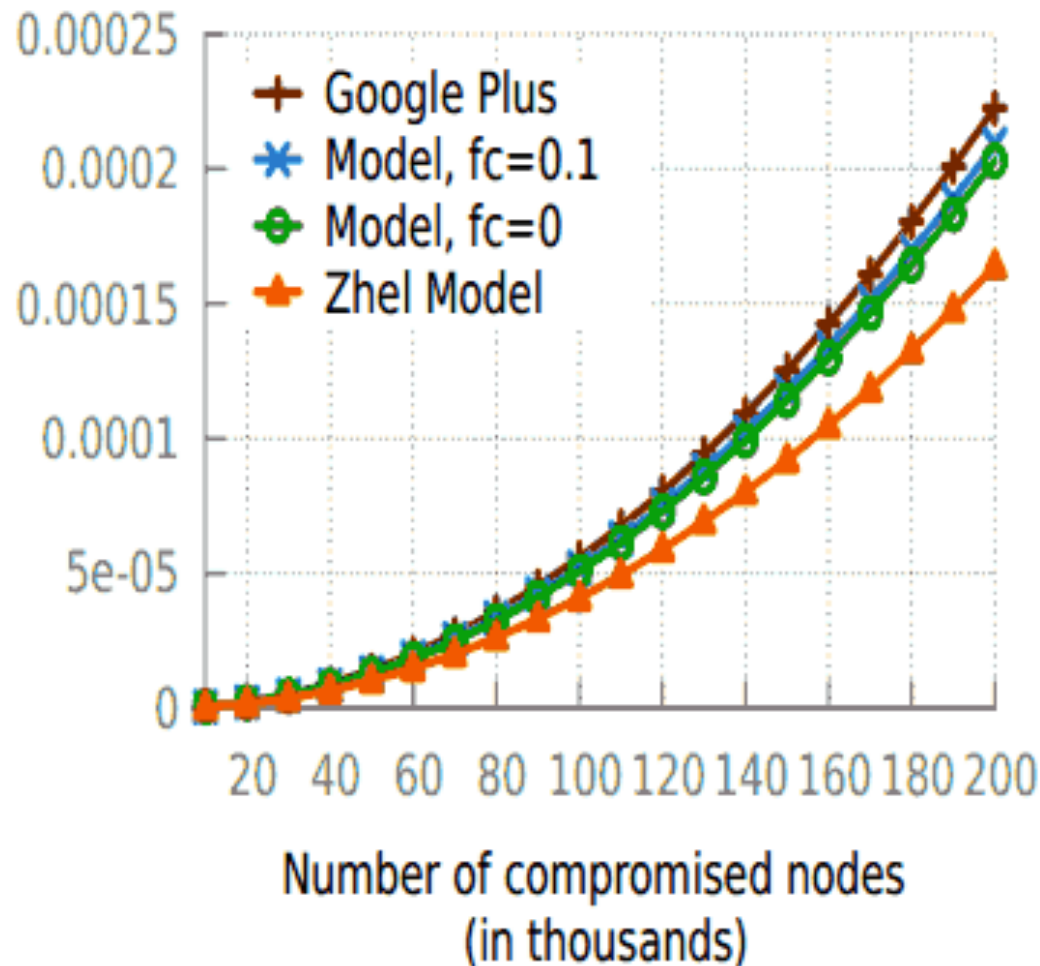
Simple Sybil defense: limit node connectivity.

# application performance prediction with different models





# application validation: subverting onion routing on social graph



# generative model summary

- model matches statistics of Google+ better than Zhel model
- model performs better for two tested applications

# **generative model - purpose**

We build generative models for several reasons:

- validate our understanding of the mechanisms building up the network
- create data for predicting application performance
- model the effects of interventions