More on Supervised Learning

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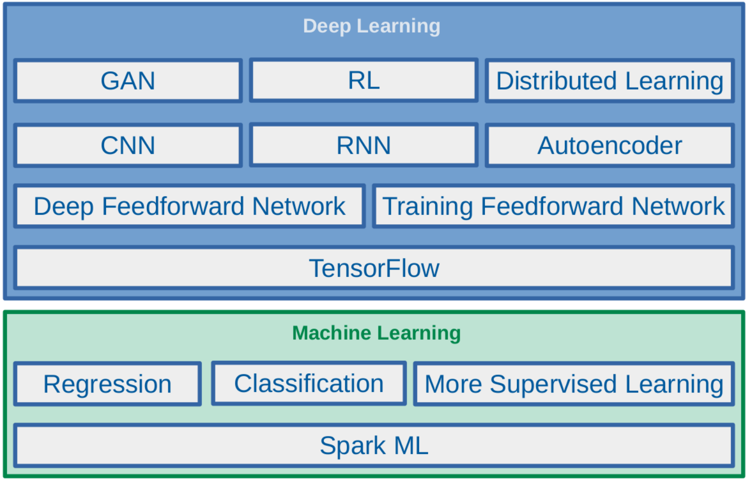
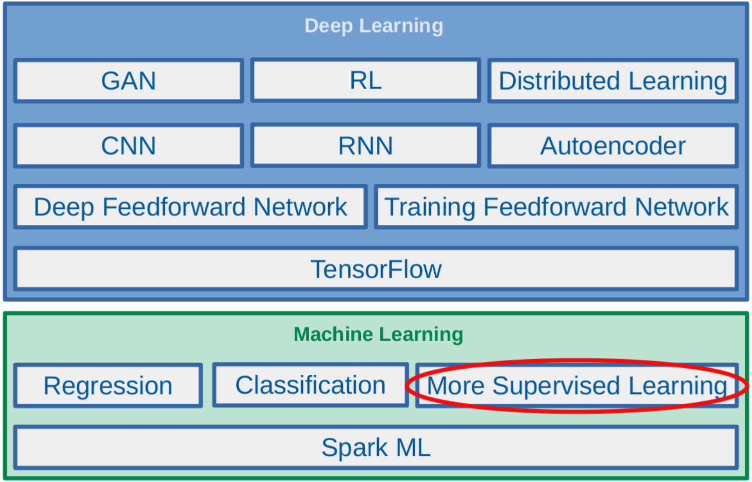
The Course Web Page

https://id2223kth.github.io

Where Are We?


Deep Learning
GAN
RL
Distributed Learning
CNN
RNN
Autoencoder
Deep Feedforward Network
Training Feedforward Network
TensorFlow
Machine Learning
Regression
Classification
More Supervised Learning
Spark ML

Where Are We?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Let's Start with an Example

Buying Computer Example (1/3)

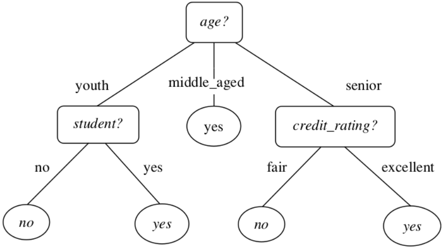
- Given the dataset of $\mathtt{m}$ people.

id	age	income	student	credit rating	buys computer
1	youth	high	no	fair	no
2	youth	high	no	excellent	no
3	middleage	high	no	fair	yes
4	senior	medium	no	fair	yes
5	senior	low	yes	fair	yes
⋮	⋮	⋮	⋮	⋮	⋮

- Predict if a new person buys a computer?
- Given an instance $\mathbf{x}^{(i)}$, e.g., $\mathtt{x}_1^{(i)} = \mathtt{senior}$, $\mathtt{x}_2^{(i)} = \mathtt{medium}$, $\mathtt{x}_3^{(i)} = \mathtt{no}$, and $\mathtt{x}_4^{(i)} = \mathtt{fair}$, then $\mathtt{y}^{(i)} = ?$

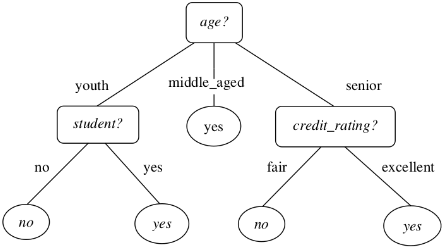
Buying Computer Example (2/3)

id	age	income	student	credit rating	buys computer
1	youth	high	no	fair	no
2	youth	high	no	excellent	no
3	middleage	high	no	fair	yes
4	senior	medium	no	fair	yes
5	senior	low	yes	fair	yes
⋮	⋮	⋮	⋮	⋮	⋮



Buying Computer Example (3/3)

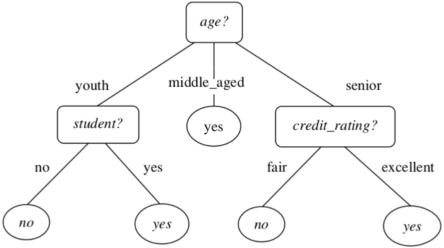
- Given an input instance $\mathbf{x}^{(i)}$, for which the class label $y^{(i)}$ is unknown.
- The attribute values of the input (e.g., age or `income`) are tested.
- A path is traced from the root to a leaf node, which holds the class prediction for that input.
- E.g., input $\mathbf{x}^{(i)}$ with $x_1^{(i)} = \texttt{senior}$, $x_2^{(i)} = \texttt{medium}$, $x_3^{(i)} = \texttt{no}$, and $x_4^{(i)} = \texttt{fair}$.



Decision Tree

Decision Tree

- A decision tree is a flowchart-like tree structure.
  - The topmost node: represents the root
  - Each internal node: denotes a test on an attribute
  - Each branch: represents an outcome of the test
  - Each leaf: holds a class label



Training Algorithm (1/2)

- Decision trees are constructed in a top-down recursive divide-and-conquer manner.
- The algorithm is called with the following parameters.
  - Data partition $\mathtt{D}$: initially the complete set of training data and labels $\mathtt{D} = (\mathbf{X}, \mathbf{y})$.
  - Feature list: list of features $\{\mathbf{x}_1^{(\mathtt{i})}, \cdots, \mathbf{x}_\mathtt{n}^{(\mathtt{i})}\}$ of each data instance $\mathbf{x}^{(\mathtt{i})}$.
  - Feature selection method: determines the splitting criterion.

Training Algorithm (2/2)

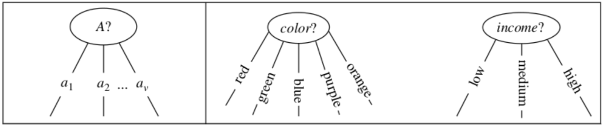
- 1. The tree starts as a single node, $\mathtt{N}$, representing the training data instances $\mathtt{D}$.
- 2. If all instances $\mathbf{x}$ in $\mathtt{D}$ are all of the same class, then node $\mathtt{N}$ becomes a leaf.
- 3. The algorithm calls feature selection method to determine the splitting criterion.
  - Indicates (i) the splitting feature $\mathtt{x_k}$, and (ii) a split-point or a splitting subset.
  - The instances in $\mathtt{D}$ are partitioned accordingly.
- 4. The algorithm repeats the same process recursively to form a decision tree.

Training Algorithm - Termination Conditions

- The training algorithm stops only when any one of the following conditions is true.
- 1. All the instances in partition D at a node N belong to the same class.
  - It is labeled with that class.
- 2. No remaining features on which the instances may be further partitioned.
- 3. There are no instances for a given branch, that is, a partition D_j is empty.
- In conditions 2 and 3:
  - Convert node N into a leaf.
  - Label it either with the most common class in D.
  - Or, the class distribution of the node tuples may be stored.

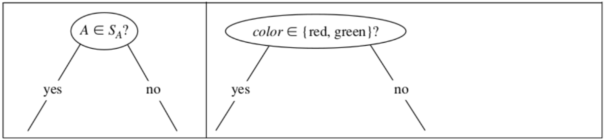
Training Algorithm - Partitioning Instances (1/3)

- Assume $\mathtt{A}$ is the splitting feature
- Three possibilities to partition instances in $\mathtt{D}$ based on the feature $\mathtt{A}$.
- 1. $\mathtt{A}$ is discrete-valued
    - Assume $\mathtt{A}$ has $\mathtt{v}$ distinct values $\{\mathtt{a}_1, \mathtt{a}_2, \cdots, \mathtt{a}_\mathtt{v}\}$
    - A branch is created for each known value $\mathtt{a}_\mathtt{j}$ of $\mathtt{A}$ and labeled with that value.
    - Partition $\mathtt{D}_\mathtt{j}$ is the subset of tuples in $\mathtt{D}$ having value $\mathtt{a}_\mathtt{j}$ of $\mathtt{A}$.



Training Algorithm - Partitioning Instances (2/3)

- 2. A is discrete-valued
  - A binary tree must be produced.
  - The test at node N is of the form $A \in S_A$?, where S_A is the splitting subset for A.
  - The left branch out of N corresponds to the instances in D that satisfy the test.
  - The right branch out of N corresponds to the instances in D that do not satisfy the test.



Training Algorithm - Partitioning Instances (3/3)

- 3. $\mathtt{A}$ is continuous-valued
  - A test at node $\mathtt{N}$ has two possible outcomes: corresponds to $\mathtt{A} \leq \mathtt{s}$ or $\mathtt{A} > \mathtt{s}$, with $\mathtt{s}$ as the split point.
  - The instances are partitioned such that $\mathtt{D}_1$ holds the instances in $\mathtt{D}$ for which $\mathtt{A} \leq \mathtt{s}$, while $\mathtt{D}_2$ holds the rest.
  - Two branches are labeled according to the previous outcomes.

Training Algorithm - Feature Selection Measures (1/2)

- Feature selection measure: how to split instances at a node `N`.
- Pure partition: if all instances in a partition belong to the same class.
- The best splitting criterion is the one that most closely results in a pure scenario.

Training Algorithm - Feature Selection Measures (2/2)

- It provides a ranking for each feature describing the given training instances.
- The feature having the best score for the measure is chosen as the splitting feature for the given instances.
- Two popular feature selection measures are:
  - Information gain (ID3)
  - Gini index (CART)

Information Gain (Entropy)

ID3 (1/7)

- ID3 (Iterative Dichotomiser 3) uses information gain as its feature selection measure.
- The feature with the highest information gain is chosen as the splitting feature for node `N`.
- The information gain is based on the decrease in entropy after a dataset is split on a feature.

ID3 (2/7)

- What's entropy?
- The average information needed to identify the class label of an instance in $\mathtt{D}$.

$$\mathtt{entropy}(\mathtt{D}) = -\sum_{\mathtt{i}=1}^{\mathtt{m}} \mathtt{p_i} \log_2(\mathtt{p_i})$$

- $\mathtt{p_i}$ is the probability that an instance in $\mathtt{D}$ belongs to class $\mathtt{i}$, with $\mathtt{m}$ distinct classes.
- $\mathtt{D}$'s entropy is zero when it contains instances of only one class (pure partition).

ID3 (3/7)

RID	*age*	*income*	*student*	*credit_rating*	*Class: buys_computer*
1	youth	high	no	fair	no
2	youth	high	no	excellent	no
3	middle_aged	high	no	fair	yes
4	senior	medium	no	fair	yes
5	senior	low	yes	fair	yes
6	senior	low	yes	excellent	no
7	middle_aged	low	yes	excellent	yes
8	youth	medium	no	fair	no
9	youth	low	yes	fair	yes
10	senior	medium	yes	fair	yes
11	youth	medium	yes	excellent	yes
12	middle_aged	medium	no	excellent	yes
13	middle_aged	high	yes	fair	yes
14	senior	medium	no	excellent	no

$$\mathtt{entropy}(\mathtt{D}) = -\sum_{\mathtt{i}=1}^{\mathtt{m}} \mathtt{p_i} \log_2(\mathtt{p_i})$$

$$\text{label} = \mathtt{buys\_computer} \Rightarrow \mathtt{m} = 2$$

$$\mathtt{entropy}(\mathtt{D}) = -\frac{9}{14}\log_2\left(\frac{9}{14}\right) - \frac{5}{14}\log_2\left(\frac{5}{14}\right) = 0.94$$

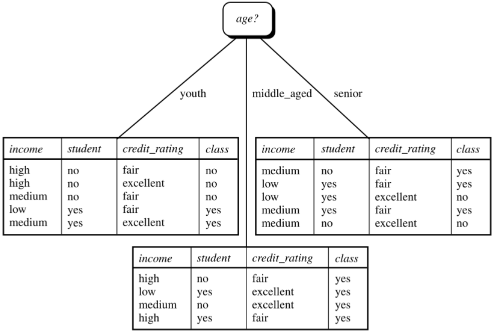
ID3 (4/7)

- Suppose we want to partition instances in $\mathtt{D}$ on some feature $\mathtt{A}$ with $\mathtt{v}$ distinct values, $\{\mathtt{a}_1, \mathtt{a}_2, \cdots, \mathtt{a}_\mathtt{v}\}$.
- $\mathtt{A}$ can split $\mathtt{D}$ into $\mathtt{v}$ partitions $\{\mathtt{D}_1, \mathtt{D}_2, \cdots, \mathtt{D}_\mathtt{v}\}$.
- The expected information required to classify an instance from $\mathtt{D}$ based on the partitioning by $\mathtt{A}$ is:

$$\mathtt{entropy}(\mathtt{A}, \mathtt{D}) = \sum_{\mathtt{j}=1}^{\mathtt{v}} \frac{|\mathtt{D}_\mathtt{j}|}{|\mathtt{D}|} \mathtt{entropy}(\mathtt{D}_\mathtt{j})$$

- $\frac{|\mathtt{D}_\mathtt{j}|}{\mathtt{D}}$ is the weight of the $\mathtt{j}$th partition.
- The smaller the expected information required, the greater the purity of the partitions.

ID3 (5/7)



income	*student*	*credit_rating*	*class*
high	no	fair	no
high	no	excellent	no
medium	no	fair	no
low	yes	fair	yes
medium	yes	excellent	yes

income	*student*	*credit_rating*	*class*
medium	no	fair	yes
low	yes	fair	yes
low	yes	excellent	no
medium	yes	fair	yes
medium	no	excellent	no

income	*student*	*credit_rating*	*class*
high	no	fair	yes
low	yes	excellent	yes
medium	no	excellent	yes
high	yes	fair	yes

$$\mathtt{entropy}(\mathtt{A},\mathtt{D}) = \sum_{\mathtt{j}=1}^{\mathtt{v}} \frac{|\mathtt{D_j}|}{|\mathtt{D}|}\mathtt{entropy}(\mathtt{D_j})$$

$$\mathtt{entropy}(\mathtt{age},\mathtt{D}) = \frac{5}{14}\mathtt{entropy}(\mathtt{D_{youth}}) + \frac{4}{14}\mathtt{entropy}(\mathtt{D_{middle\_aged}}) + \frac{5}{14}\mathtt{entropy}(\mathtt{D_{senior}})$$

$$\mathtt{entropy}(\mathtt{age},\mathtt{D}) = \frac{5}{14}(-\frac{2}{5}\log_2(\frac{2}{5}) - \frac{3}{5}\log_2(\frac{3}{5})) + \frac{4}{14}(-\frac{4}{4}\log_2(\frac{4}{4})) + \frac{5}{14}(-\frac{3}{5}\log_2(\frac{3}{5}) - \frac{2}{5}\log_2(\frac{2}{5})) = 0.694$$

ID3 (6/7)

- The information gain $\texttt{Gain}(\texttt{A}, \texttt{D})$ is defined as:

$$\texttt{Gain}(\texttt{A}, \texttt{D}) = \texttt{entropy}(\texttt{D}) - \texttt{entropy}(\texttt{A}, \texttt{D})$$

- It shows how much would be gained by branching on $\texttt{A}$.
- The feature $\texttt{A}$ with the highest $\texttt{Gain}(\texttt{A}, \texttt{D})$ is chosen as the splitting feature at node $\texttt{N}$.

ID3 (7/7)

- Now, we can compute the information gain $\mathtt{Gain(A)}$ for the feature $\mathtt{A\ =\ age}$.

$$\mathtt{Gain(age, D)} = \mathtt{entropy(D)} - \mathtt{entropy(age, D)} = 0.940 - 0.694 = 0.246$$

- Similarly we have:
  - $\mathtt{Gain(income, D)} = 0.029$
  - $\mathtt{Gain(student, D)} = 0.151$
  - $\mathtt{Gain(credit\_rating, D)} = 0.048$

- The age has the highest information gain among the attributes, it is selected as the splitting feature.

Gini Impurity

CART (1/8)

- CART (Classification And Regression Tree) considers a binary split for each feature.
- It uses the Gini index to measure the misclassification (impurity of D).

$$\mathtt{Gini}(D) = 1 - \sum_{i=1}^{m} p_i^2$$

- p_i is the probability that an instance in D belongs to class i, with m distinct classes.
- It will be zero if all partitions are pure. Why?
- We need to determine the splitting criterion: splitting feature + splitting subset.

CART (2/8)

- Assume $\mathtt{A}$ is a discrete-valued feature with $\mathtt{v}$ distinct values, $\{\mathtt{a}_1, \mathtt{a}_2, \cdots, \mathtt{a}_\mathtt{v}\}$, occurring in $\mathtt{D}$.
- $\mathtt{S}_\mathtt{A}$ will be all possible subsets of $\mathtt{A}$.
  - E.g., $\mathtt{A} = \mathtt{income} = \{\mathtt{low}, \mathtt{medium}, \mathtt{high}\}$
  - $\mathtt{S}_\mathtt{A} = \{\{\mathtt{low}, \mathtt{medium}, \mathtt{high}\}, \{\mathtt{low}, \mathtt{medium}\}, \{\mathtt{medium}, \mathtt{high}\}, \{\mathtt{low}, \mathtt{high}\}, \{\mathtt{low}\}, \{\mathtt{medium}\}, \{\mathtt{high}\}, \{\}\}$
  - The test is of the form $\mathtt{D}_1 \in \mathtt{s}_\mathtt{A}$?, where $\mathtt{s}_\mathtt{A}$ is a subset of $\mathtt{S}_\mathtt{A}$, e.g., $\mathtt{s}_\mathtt{A} = \{\mathtt{low}, \mathtt{high}\}$.

CART (3/8)

RID	age	income	student	credit_rating	Class: buys_computer
1	youth	high	no	fair	no
2	youth	high	no	excellent	no
3	middle_aged	high	no	fair	yes
4	senior	medium	no	fair	yes
5	senior	low	yes	fair	yes
6	senior	low	yes	excellent	no
7	middle_aged	low	yes	excellent	yes
8	youth	medium	no	fair	no
9	youth	low	yes	fair	yes
10	senior	medium	yes	fair	yes
11	youth	medium	yes	excellent	yes
12	middle_aged	medium	no	excellent	yes
13	middle_aged	high	yes	fair	yes
14	senior	medium	no	excellent	no

$$\mathtt{Gini}(\mathtt{D}) = 1 - \sum_{\mathtt{i}=1}^{\mathtt{m}} \mathtt{p}_{\mathtt{i}}^2$$

$$\text{label} = \mathtt{buys\_computer} \Rightarrow \mathtt{m} = 2$$

$$\mathtt{Gini}(\mathtt{D}) = 1 - \left(\frac{9}{14}\right)^2 - \left(\frac{5}{14}\right)^2 = 0.459$$

CART (4/8)

- If a binary split on $\mathtt{A}$ partitions $\mathtt{D}$ into $\mathtt{D}_1$ and $\mathtt{D}_2$, the Gini index of $\mathtt{D}$ given that partitioning is:

$$\mathtt{Gini}(\mathtt{A},\mathtt{D}) = \frac{|\mathtt{D}_1|}{\mathtt{D}}\mathtt{Gini}(\mathtt{D}_1) + \frac{|\mathtt{D}_2|}{\mathtt{D}}\mathtt{Gini}(\mathtt{D}_2)$$

- The subset that gives the minimum Gini index is selected as its splitting subset.

CART (5/8)

- For a feature $\mathtt{A} = \mathtt{income}$, we consider each of the possible splitting subsets.
  - $\mathtt{S_A} = \{\{\mathtt{low}, \mathtt{medium}, \mathtt{high}\}, \{\mathtt{low}, \mathtt{medium}\}, \{\mathtt{medium}, \mathtt{high}\}, \{\mathtt{low}, \mathtt{high}\}, \{\mathtt{low}\}, \{\mathtt{medium}\}, \{\mathtt{high}\}, \{\}\}$
- Assume, we choose the splitting subset $\mathtt{s_A} = \{\mathtt{low}, \mathtt{medium}\}$.
- Consider partition $\mathtt{D_1}$ satisfies the condition $\mathtt{D_1} \in \mathtt{s_A}$, and $\mathtt{D_2}$ does not.

$$\mathtt{Gini}_{\mathtt{income} \in \{\mathtt{low}, \mathtt{medium}\}}(\mathtt{A}, \mathtt{D}) = \frac{10}{14}\mathtt{Gini}(\mathtt{D_1}) + \frac{4}{14}\mathtt{Gini}(\mathtt{D_2})$$

$$= \frac{10}{14}\mathtt{Gini}(1 - (\frac{7}{10})^2 - (\frac{3}{10})^2) + \frac{4}{14}(1 - (\frac{2}{4})^2 - (\frac{2}{4})^2) = 0.443$$

CART (6/8)

- Similarly, we calculate the Gini index values for splits on the remaining subsets.

$$\mathtt{Gini}_{\mathtt{income}\in\{\mathtt{low},\mathtt{medium}\}}(\mathtt{A},\mathtt{D}) = \mathtt{Gini}_{\mathtt{income}\in\{\mathtt{high}\}}(\mathtt{A},\mathtt{D}) = 0.443$$
$$\mathtt{Gini}_{\mathtt{income}\in\{\mathtt{low},\mathtt{high}\}}(\mathtt{A},\mathtt{D}) = \mathtt{Gini}_{\mathtt{income}\in\{\mathtt{medium}\}}(\mathtt{A},\mathtt{D}) = 0.458$$
$$\mathtt{Gini}_{\mathtt{income}\in\{\mathtt{medium},\mathtt{high}\}}(\mathtt{A},\mathtt{D}) = \mathtt{Gini}_{\mathtt{income}\in\{\mathtt{low}\}}(\mathtt{A},\mathtt{D}) = 0.450$$

- The best binary split for attribute $\mathtt{A} = \mathtt{income}$ is on $\mathtt{s}_{\mathtt{A}} = \{\mathtt{low}, \mathtt{medium}\}$ because it minimizes the Gini index.

CART (7/8)

- But, which feature?
- The reduction in impurity that would be incurred by a binary split on feature $\mathtt{A}$ is:

$$\Delta\mathtt{Gini}(\mathtt{A}) = \mathtt{Gini}(\mathtt{D}) - \mathtt{Gini}(\mathtt{A}, \mathtt{D})$$

- The feature that maximizes the reduction in impurity (has the minimum Gini index) is selected as the splitting feature.

CART (8/8)

- Now, we can compute the information gain $\texttt{Gain}(\texttt{A})$ for different features.
  - $\Delta\texttt{Gini}(\texttt{income}) = 0.459 - 0.443 = 0.016$
  - $\Delta\texttt{Gini}(\texttt{age}) = 0.459 - 0.357 = 0.102$
  - $\Delta\texttt{Gini}(\texttt{student}) = 0.459 - 0.367 = 0.092$
  - $\Delta\texttt{Gini}(\texttt{credit\_rating}) = 0.459 - 0.429 = 0.03$
- The feature $\texttt{A} = \texttt{age}$ and splitting subset $\texttt{s}_\texttt{A} = \{\texttt{youth}, \texttt{senior}\}$ gives the minimum Gini index overall.

Decision Tree in Spark (1/4)

- Two classes in `spark.ml`.
- Regression: `DecisionTreeRegressor`

```
val dt_regressor = new DecisionTreeRegressor().setLabelCol("label").setFeaturesCol("features")
val model = dt_regressor.fit(trainingData)
val predictions = model.transform(testData)
predictions.select("prediction", "rawPrediction", "probability", "label", "features").show(5)
```

- Classifier: `DecisionTreeClassifier`

```
val dt_classifier = new DecisionTreeClassifier().setLabelCol("label").setFeaturesCol("features")
val model = dt_classifier.fit(trainingData)
val predictions = model.transform(testData)
predictions.select("prediction", "rawPrediction", "probability", "label", "features").show(5)
```

Decision Tree in Spark (2/4)

- Input and output columns
- `labelCol` and `featuresCol` identify label and features column's names.
- `predictionCol` indicates the predicted label.
- `rawPredictionCol` is a vector of length of number of classes, with the counts of training instance labels at the tree node which makes the prediction.
- `probabilityCol` is a vector of length of number of classes equal to `rawPrediction` normalized to a multinomial distribution.

Decision Tree in Spark (3/4)

- Tunable parameters
- maxBins: number of bins used when discretizing continuous features.
- impurity: impurity measure used to choose between candidate splits, e.g., entropy and gini.

```
val maxBins = ...
val dt_classifier = new DecisionTreeClassifier().setMaxBins(maxBins).setImpurity("gini")
```

Decision Tree in Spark (4/4)

- Stopping criteria that determines when the tree stops building.
- maxDepth: maximum depth of a tree.
- minInstancesPerNode: for a node to be split further, each of its children must receive at least this number of training instances.
- minInfoGain: for a node to be split further, the split must improve at least this much (in terms of information gain).

```
val maxDepth = ...
val minInstancesPerNode = ...
val minInfoGain = ...
val dt_classifier = new DecisionTreeClassifier()
                    .setMaxDepth(maxDepth)
                    .setMinInstancesPerNode(minInstancesPerNode)
                    .setMinInfoGain(minInfoGain)
```

Ensemble Methods

Wisdom of the Crowd

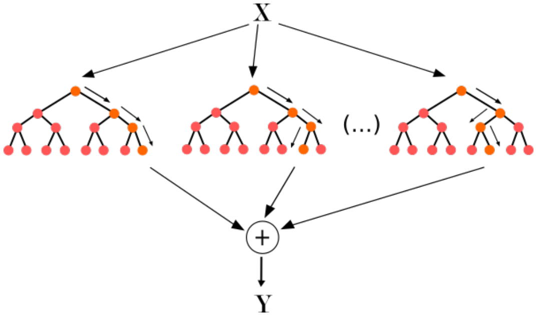
- Ask a complex question to thousands of random people, then aggregate their answers.
- In many cases, this aggregated answer is better than an expert's answer.
- This is called the wisdom of the crowd.
- Similarly, the aggregated estimations of a group of estimators (e.g., classifiers or regressors), often gets better estimations than with the best individual estimator.
- A group of estimators is an ensemble, and this technique is called Ensemble Learning.

Ensemble Learning

- Two main categories of ensemble learning algorithms.
- Bagging
  - Use the same training algorithm for every estimator, but to train them on different random subsets of the training set.
  - E.g., random forest
- Boosting
  - Train estimators sequentially, each trying to correct its predecessor.
  - E.g., adaboost and gradient boosting

Random Forest

- Random forest builds multiple decision trees that are most of the time trained with the bagging method.
- It, then, merges the trees together to get a more accurate and stable prediction.



Random Forest in Spark (1/2)

- Two classes in `spark.ml`.
- Regression: `RandomForestRegressor`

```
val rf_regressor = new RandomForestRegressor().setLabelCol("label")
                                              .setFeaturesCol("features").setNumTrees(10)
val model = rf_regressor.fit(trainingData)
val predictions = model.transform(testData)
predictions.select("prediction", "label", "features").show(5)
```

- Classifier: `RandomForestClassifier`

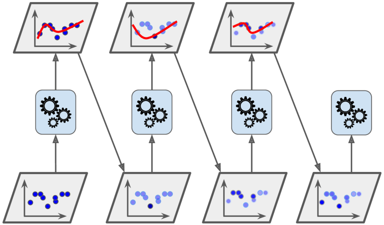
```
val rf_classifier = new RandomForestClassifier().setLabelCol("label")
                                                .setFeaturesCol("features").setNumTrees(10)
val model = rf_classifier.fit(trainingData)
val predictions = model.transform(testData)
predictions.select("prediction", "label", "features").show(5)
```

Random Forest in Spark (2/2)

- `numTrees`: number of trees in the forest.
- `subsamplingRate`: specifies the size of the dataset used for training each tree in the forest, as a fraction of the size of the original dataset.
  - Default is 1.0 and decreasing it can speed up training.
- `featureSubsetStrategy`: number of features to use as candidates for splitting at each tree node, as a fraction of the total number of features.
  - Possible values: `auto`, `all`, `onethird`, `sqrt`, `log2`, `n`

AdaBoost

- AdaBoost: train a new estimator by paying more attention to the training instances that the predecessor underfitted.
- Each estimator is trained on a random subset of the total training set.
- AdaBoost assigns a weight to each training instance, which determines the probability that each instance should appear in the training set.



Gradient Boosting (1/3)

- Just like AdaBoost, Gradient Boosting works by sequentially adding estimators to an ensemble, each one correcting its predecessor.
- However, instead of tweaking the instance weights at every iteration, this method tries to fit the new estimator to the residual errors made by the previous estimator.

Gradient Boosting (2/3)

- Let's go through a regression example using Gradient Boosted Regression Trees.
- Fit the first estimator on the training set.

```
tree_reg1 = DecisionTreeRegressor(max_depth=2)
tree_reg1.fit(X, y)
```

- Now train the second estimator on the residual errors made by the first estimator.

```
y2 = y - tree_reg1.predict(X)
tree_reg2 = DecisionTreeRegressor(max_depth=2)
tree_reg2.fit(X, y2)
```

Gradient Boosting (3/3)

- Then we train the third estimator on the residual errors made by the second estimator.

```
y3 = y2 - tree_reg2.predict(X)
tree_reg3 = DecisionTreeRegressor(max_depth=2)
tree_reg3.fit(X, y3)
```

- Now we have an ensemble containing three trees.
- It can make predictions on a new instance simply by adding up the predictions of all the trees.

```
y_pred = sum(tree.predict(X_new) for tree in (tree_reg1, tree_reg2, tree_reg3))
```

Gradient Boosting in Spark (1/2)

- Two classes in `spark.ml`.

- Regression: `GBTRegressor`

```
val gbt = new GBTRegressor().setLabelCol("label").setFeaturesCol("features")
                             .setMaxIter(10).setFeatureSubsetStrategy("auto")

val model = gbt.fit(trainingData)
val predictions = model.transform(testData)
```

- Classifier: `GBTClassifier`

```
val gbt = new GBTClassifier().setLabelCol("label").setFeaturesCol("features")
                              .setMaxIter(10).setFeatureSubsetStrategy("auto")

val model = gbt.fit(trainingData)
val predictions = model.transform(testData)
```

Summary

Summary

- Decision tree
  - Top-down training algorithm
  - Termination condition
  - Feature selection: entropy, gini
- Ensemble models
  - Bagging: random forest
  - Boosting: AdaBoost, Gradient Boosting

Reference

- Aurélien Géron, Hands-On Machine Learning (Ch. 5, 6, 7)
- Matei Zaharia et al., Spark - The Definitive Guide (Ch. 27)

Questions?