CSCI 2824 – Spring 2019 **Homework 2**

This assignment is due on Friday, February 1 to Gradescope by Noon. You are expected to write up your solutions neatly and **use the coverpage**. Remember that you are encouraged to discuss problems with your classmates, but you must work and write your solutions on your own.

Important: On the **official CSCI 2824 cover page** of your assignment clearly write your full name, the lecture section you belong to (001 or 002), and your student ID number. You may **neatly** type your solutions for +1 extra credit on the assignment. You will lose all 5 style/neatness points if you fail to use the official cover page.

1. You decided to go on a spring break vacation! Your destination: the island of Knights & Knaves. On this island, there are only two types of native inhabitants; Knights, who always tell the truth, and Knaves, who always lie. As you are finding a nice spot on the beach to set up a picnic, you are approached by 3 of the native inhabitants. We'll call them Aramis, Bertrand, and Charleston. Aramis says, " Bertrand is a knave." Bertrand says, " Charleston is a knave or I am a knight, but not both." Charleston says, "Aramis is a knight and Bertrand is a knave." Determine the nature of each of these three inhabitants. Justify your answer with a truth table; showing all rows.

2. (a) Show that $((p \to q) \land (q \to r)) \to (p \to r)$ is a tautology using **both** (i) a truth table and (ii) a chain of logical equivalences. Note that you may only use logical equivalences from Table 6 (p. 27 of the Rosen textbook) and the other four starred equivalences given in lecture. At each step you should cite the name of the equivalence rule you are using, and please only use one rule per step. Is this compound proposition satisfiable? Why or why not?

   (b) Show that $(p \to q) \to r$ and $p \to (q \to r)$ are not logically equivalent.

3. Suppose that the domain of the propositional function $P(x)$ consists of the integers 5, 6, 7, and 8. Express the following statements without using quantifiers, instead using negations, disjunctions, and conjunctions. [e.g. $\exists x P(x)$ would be $P(5) \lor P(6) \lor P(7) \lor P(8)$]

   (a) $\forall x P(x)$

   (b) $\neg \exists x P(x)$

   (c) $\neg \forall x P(x)$

4. We spent time in lecture talking about how to convert base-10 numbers to binary. Use the same principles to convert 163 to base-3. Make sure to show all of your steps.

5. Consider the following **satisfiability** problem: The Scooby Doo gang: Fred, Daphne, Shaggy, Velma, and Scooby are going on vacation. However, before they can book their travel, they need to all agree on where to go. Their trip may involve one or more destinations. They must all travel together to all of the places as one group (so part of the group cannot go to one location while the others go somewhere else).

   i Shaggy wants to go to Venice, or not to Shanghai.

   ii If the gang goes to Paris, then Velma does not want to go to Venice.

   iii Daphne wants to go to Brussels if and only if the gang also goes to London and Paris.

   iv Fred does not want to go to Paris.

   v Scooby just wants to leave the house and does not care where the gang goes.

   Let $T(x)$ represent the propositional function "the trip must include destination x", where the domain for x is the set of possible travel locations: Venice (V), Shanghai (S), Paris (P), Brussels (B), and London (L). Note that statements such as "Shaggy want to go to Venice" does *not* imply that Shaggy only wants to go to Venice. For example, Shaggy would be perfectly happy going to Venice and Shanghai.

   (a) Translate each of the group's travel requirements $i - iv$ from English into a proposition using the given propositional function. [No need to translate Scooby's wishes!]

   (b) Are the group's travel wishes satisfiable? If they are, provide a list of destinations that satisfies the requirements. If they are not, provide a **concise** written argument explaining why not. Do **not** use a truth table.

   (c) What travel destination should we have included in the list? [Note, this is for fun.]