This assignment is due on Friday March 15 to Gradescope by Noon. You are expected to write or type up your solutions neatly. Remember that you are encouraged to discuss problems with your classmates, but you must work and write your solutions on your own.

Important: Make sure to clearly write your full name, the lecture section you belong to (001 or 002), and your student ID number at the top of your assignment. You may **neatly** type your solutions in LaTex for +1 extra credit on the assignment.

1. Use induction to prove that the following identities hold for all $n \geq 1$.

   (a) $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

   (b) $\displaystyle\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

   (c) $\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{9}\right) \ldots \left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}$ (for $n \geq 2$)

   Be sure to clearly state your induction hypothesis, and state whether you're using weak induction or strong induction for each part.

2. Let the sequence T_n be defined by $T_1 = T_2 = T_3 = 1$ and $T_n = T_{n-1} + T_{n-2} + T_{n-3}$ for $n \geq 4$. Use induction to prove that

   $$T_n < 2^n \quad \text{for} \quad n \geq 4$$

   Be sure to clearly state your induction hypothesis, and state whether you're using weak induction or strong induction for each part.

3. Consider the function $f(n) = 50n^3 + 6n^3 \log(n^3) - n \log(n^2)$ which represents the complexity of some algorithm.

   (a) Find a tight big-**O** bound of the form $g(n) = n^p$ for the given function f with some natural number p. What are the constants C and k from the big-**O** definition?

   (b) Find a tight big-$\boldsymbol{\Omega}$ bound of the form $g(n) = n^p$ for the given function f with some natural number p. What are the constants C and k from the big-$\boldsymbol{\Omega}$ definition?

   (c) Can we conclude that f is big-$\boldsymbol{\Theta}(\mathbf{n^p})$ for any natural number p?

4. Multiply the following matrices:

   (a)

   $$\begin{bmatrix} 3 & 4 \\ 1 & 0 \\ 2 & 7 \end{bmatrix} \cdot \begin{bmatrix} 8 & 1 & 2 \\ 7 & 6 & 5 \end{bmatrix} =$$

   (b) Leave your answer in terms of $a_{ij} \cdot b_{ij}$

   $$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} =$$