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Homework 8

Problem 1

Use induction to prove that the following identities hold for all n≥1

- $\sum_{n=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
  Let p(n) be the preposition that $\sum_{n=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ is true
  Basic Step: p(1) is true, because when n=1 1^2=$\frac{1(1+1)(2+1)}{6}$=1 is true
  Inductive Hypothesis: The induction hypothesis is the statement that p(k) is true, that is, $\sum_{n=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ where k is an arbitrary nonnegative integer greater than 1. We must show that if P (k) is true,then P (k+1), which states that $\sum_{i=1}^{K}(i+1)^2 = \frac{(k+1)(k+2)(2(k+1)+1)}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$
  $1^2 + 2^2 + ... + k^2 + (K+1)^2$=$(1^2 + 2^2 + ... + k^2) + (K+1)^2$
  from our hypothesis we have
  $\frac{k(k+1)(2k+1)}{6} + (K+1)^2$
  $(k+1)\frac{k(2k+1)}{6} + (K+1)$
  $(k+1)\frac{k(2k+1)+6(K+1)}{6}$
  $\frac{(k+1)(2k+3)(K+2)}{6}$

- $\sum_{n=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$
  Let p(n) be the preposition that $\sum_{n=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$ is true
  Basic Step: p(1) is true, because when n=1 $1^3 = \frac{1^2(1+1)^2}{4}$=1 is true
  Inductive Hypothesis: The induction hypothesis is the statement that p(k) is true, that is, $\sum_{n=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$ where k is an arbitrary nonnegative integer greater than 1. We must show that if P (k) is true,then P (k+1), which states that $\sum_{i=1}^{K}(i+1)^2 = \frac{(k+1)^2(k+2)^2}{4}$
  $1^3 + 2^3 + ... + k^3 + (K+1)^3$=$(1^3 + 2^3 + ... + k^3) + (K+1)^3$
  from our hypothesis we have
  $\frac{k^2(k+1)^2}{4} + (K+1)^3 = \frac{(k+1)^2(k+2)^2}{4}$

- $(1-\frac{1}{4})(1-\frac{1}{9})...(1-\frac{1}{n^2}) = \frac{n+1}{2n}$
  Let p(n) be the preposition that $(1-\frac{1}{4})(1-\frac{1}{9})...(1-\frac{1}{n^2}) = \frac{n+1}{2n}$
  Basic Step: p(1) is true, because when n=2 $\frac{3}{4} = \frac{3}{4}$ is true
  Inductive Hypothesis: The induction hypothesis is the statement that p(k) is true, that is $(1-\frac{1}{4})(1-\frac{1}{9})...(1-\frac{1}{n^2}) = \frac{n+1}{2n}$, where k is an arbitrary nonnegative integer greater than 2. We must show that if P (k) is true,then P (k+1), which states that
  $(1-\frac{1}{4})(1-\frac{1}{9})...(1-\frac{k+1}{2k})(1-\frac{1}{(k+1)^2}) = \frac{k+2}{2(k+1)}$ from our hypothesis we have
  $(\frac{k+1}{2k})(1-\frac{1}{(k+1)^2})$=$\frac{k+2}{2(k+1)}$

Problem 2

Let the sequence Tn be defined by T1=T2=T3= 1 andTn=Tn1+Tn2+Tn3 for n≥4.
Use induction to prove that Tn¡$2^n$for n≥4
using strong induction
Let p(n) be the preposition that Tn¡2^n
Basic Step: p(4) is true, because when n=4, T4=3¡16
T1=T2=T3≤2 **Inductive Hypothesis:** The induction hypothesis is the statement that p(k) is true, that is Tk¡2^k, where k is an arbitrary nonnegative integer greater than 4. We must show that if P (k) is true,then P (k+1), which states that T(k+1)¡2^{K+1}
T(k+1)=T(k)+T(k-1)+T(k-2)
=$2^k + 2^{k-1} + 2^{k-2}$≤$2^{k-2+2} + 2^{k-2+1} + 2^{k-2}$
≤$2^k(4 + 1 + 2)$
≤2^k

Problem 3

Consider the function $f(n) = 50n^3 + 6n^3log(n^3) - nlog(n^2)$ which represents the complexity of somealgorithm

1. Find a tight big-O bound of the form g(n) =n^p for the given function f with some natural number p. What are the constants C and k from the big-O definition?
   $f(n) = 50n^3 + 6n^3log(n^3) - nlog(n^2)$
   $50n^3 \leq n^4$
   $6n^3log(n^3) = 18n^3log(n) \leq 18n^3 * n \leq 18n^4$
   $nlog(n^2) \leq 2nlog(n) \leq 2n * n \leq 2n^2 \leq 2n^4$ for n ≥ 1
   so f is a $\mathbb{O}(n^4)$ with C=21 and k=1
2. Find a tight big-Ω bound of the form g(n)=n^p for the given function f with some natural number p. What are the constants C and k from the big-Ω definition?
   $f(n) = 50n^3 + 6n^3log(n^3) - nlog(n^2)$
   $50n^3 \geq n^3$
   $6n^3log(n^3) = 18n^3log(n) \geq 18n^3forn \geq e$
   $nlog(n^2) \leq 2nlog(n) \geq 0$ for n ≥ 1
   so f is a $\Omega(n^3)$ with C=19 and k=6
3. Since $\Omega(n^3)$ and $\mathbb{O}(n^4)$ do not have the same p, we can not conclude that $\ominus(n^p)$

Problem 4

Multiply the following matrices:

1.

$$\begin{bmatrix}3 & 4\\1 & 0\\2 & 7\end{bmatrix} * \begin{bmatrix}8 & 1 & 2\\7 & 6 & 5\end{bmatrix} = \begin{bmatrix}3*8+7*4 & 3*1+4*6 & 3*2+4*5\\1*8+0*7 & 1*1+0*6 & 1*2+0*5\\2*8+7*7 & 1*2+6*7 & 2*2+7*5\end{bmatrix} = \begin{bmatrix}52 & 27 & 24\\8 & 1 & 2\\65 & 44 & 39\end{bmatrix}$$

2.

$$\begin{bmatrix}a_{11} & a_{12} & a_{13}\\a_{21} & a_{22} & a_{23}\\a_{31} & a_{32} & a_{33}\end{bmatrix} * \begin{bmatrix}b_{11} & b_{12} & b_{13}\\b_{21} & b_{22} & b_{23}\\b_{31} & b_{32} & b_{33}\end{bmatrix} =$$

$$\begin{bmatrix}a_{11}*b_{11}+a_{12}*b_{21}+a_{13}*b_{31} & a_{21}*b_{11}+a_{22}*b_{21}+a_{23}*b_{31} & a_{31}*b_{11}+a_{32}*b_{21}+a_{33}*b_{31}\\a_{11}*b_{12}+a_{12}*b_{22}+a_{13}*b_{32} & a_{21}*b_{12}+a_{22}*b_{22}+a_{23}*b_{32} & a_{31}*b_{12}+a_{32}*b_{22}+a_{33}*b_{32}\\a_{11}*b_{13}+a_{12}*b_{23}+a_{13}*b_{33} & a_{21}*b_{13}+a_{22}*b_{23}+a_{23}*b_{33} & a_{31}*b_{13}+a_{32}*b_{23}+a_{33}*b_{33}\end{bmatrix}$$