

Problem 1

Finding the nature of the inhabitant of the island.

A: Aramis is a Knight B: Berthand is a Knight C: Charleston is a Knight

They say:

Aramis: $\neg B$

Berthand: $\neg C \oplus B$

Charleston: $A \wedge \neg B$

lets D: $\neg B \Leftrightarrow \neg C \oplus B$

E: $\neg B \Leftrightarrow A \wedge \neg B$

G: $\neg C \oplus B \Leftrightarrow A \wedge \neg B$

A	B	C	$\neg B$	$\neg C$	$\neg C \oplus B$	$A \wedge \neg B$	D	E	G	$D \wedge E \wedge G$
T	T	T	F	F	T	F	F	T	F	F
T	F	F	T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	T	T	T	T
T	F	T	T	F	F	T	F	T	F	F
F	T	T	F	F	T	F	F	T	F	F
F	F	F	T	T	T	F	T	F	F	F
F	T	F	F	T	F	F	T	T	T	T
F	F	T	T	F	F	F	F	F	T	F

Bertrand is a Night, Charleston is Knave and Aramis is a Knave

Problem 2

- Using the truth table to demonstrate that $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$ is a tautology

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$p \rightarrow r$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	F	F	F	T	F	F	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T

The statement is a tautology.

- Using chain of logical equivalent.

$$\begin{aligned}
 ((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r) &\equiv \neg((\neg p \vee q) \wedge (\neg q \vee r)) \vee (\neg p \vee r) \\
 &\equiv \neg((\neg p \vee q) \wedge (\neg q \vee r)) \vee (\neg p \vee r)
 \end{aligned}$$

$$\begin{aligned} &\equiv ((p \wedge \neg q) \wedge (q \wedge \neg r)) \vee (\neg p \vee r) \\ &\equiv ((q \wedge \neg q) \wedge (p \wedge \neg r)) \vee (\neg p \vee r) \\ &\equiv ((p \wedge \neg r)) \vee (\neg p \vee r) \end{aligned}$$

3. Showing that $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are not logical equivalent.

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \rightarrow r$	$p \rightarrow (q \rightarrow r)$
T	T	T	T	T	T	T
T	F	F	F	T	T	T
T	T	F	T	F	T	F
T	F	T	F	T	F	T
F	T	T	T	T	T	T
F	F	F	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T

The last two columns contain contradictory statements, thus the two logical statements are not equivalent.

Problem 3

- $\forall x p(x) \equiv P(5) \wedge P(6) \wedge P(7) \wedge P(8)$
- $\neg \exists x p(x) \equiv \neg P(5) \vee \neg P(6) \vee \neg P(7) \vee \neg P(8)$
- $\neg \forall x p(x) \equiv \neg P(5) \wedge \neg P(6) \wedge \neg P(7) \wedge \neg P(8)$

Problem 4

163	1	163 in base 3 is $(20001)_3$
54	0	
18	0	
6	0	
2	2	

Problem 5

- Translating each of the group's travel requirements from English into a proposition.
 - Shaggy: $V \vee \neg S$;
 - Velma: $P \rightarrow \neg V$

- (c) Daphne: $B \leftrightarrow (L \wedge P)$
(d) Fred: $\neg P$
(e) Scooby: $\exists x T(x) \equiv P \vee V \vee S \vee B$
2. Satisfaction of the gangs proposition. Lets Sat be the logical the gangs go on vacation. For Sat to be True: $(V \vee \neg S) \wedge (P \rightarrow \neg V) \wedge (B \leftrightarrow (L \wedge P)) \wedge (\neg P) \wedge (P \vee V \vee S \vee B)$ must be true.
3. The team travel which is possible. If V is true proposition a) will be true no matter the values of S. For b) to be true, P must be false. The statement c) is true only if B is false no matter the outcome of L. Because P was false, the statement d) must be true. Finally as one of the value in each is true, statement e) is true.
-Venice
-Shanghai
4. Park W in west Africa.