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Homework 6

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Problem 1

- Suppose P, Q, and R are non-empty sets. Prove that Px(Q ∩ R) = (PxQ) ∩ (PxR) by showing that each side of this equation must be a subset of the other side, and concluding that the two sides must therefore be equal.
  Px(Q ∩ R) ⊆ (PxQ) ∩ (PxR)
  let (x,y) ∈ Px(Q ∩ R)
  x ∈ P and y ∈ (Q ∩ R)
  x ∈ P and y ∈ R and Q ∈ R
  (x,y) ∈ PxR and (x,y) ∈ P x Q
  (x,y) ∈ (PxQ) ∩ (PxR)

  (PxQ) ∩ (PxR) ⊆ Px(Q ∩ R)
  let (x,y) ∈ (PxQ) ∩ (PxR)
  (x,y)∈ PxQ and (x,y)∈ PxR
  x ∈ P, y ∈ Q and x ∈ P, y ∈ R
  x ∈ P and x ∈ P, y ∈ R
  x ∈ P and (x,y) ∈ PxR
  (x,y) ∈ P ∩ PxR
  Thus Px(Q ∩ R) = (PxQ) ∩ (PxR)

- Suppose that P, Q, and R are non-empty sets. Prove that Px(Q ∩ R) = (PxQ) ∩ (PxR) by using set builder notation and set identities and definitions.
  let (x,y) ∈ Px(Q ∩ R), this is equivalent to
  Px(Q ∩ R) = (x,y) : x ∈ P, (x,y) (Q ∩ R)
  Px(Q ∩ R) = (x,y) : x ∈ P, x ∈ Q and y ∈ R
  Px(Q ∩ R) = (x,y) : x ∈ P, x ∈ Q and x ∈ P, y ∈ R
  Px(Q ∩ R) = (x,y) : (x,y) ∈ PxQ and (x,y) ∈ PxR
  Px(Q ∩ R) = (x,y) : (x,y) ∈ PxQ and (x,y) ∩ PxR
  Thus Px(Q ∩ R) = (PxQ) ∩ (PxR)

- Let U be the set of all integers. Let E be the set of all even integers, D the set of all odd integers, P the set of positive integers, and N the set of all negative integers. Find the following sets.
  i. Set of all even Negative integers ii. Set of all integers (positives and negative integers) iii. Set of all odd integers iv. Set all non integers

Problem 2

- Give an example of two uncountable sets A and B with a nonempty intersection, such that A B is

i. A set with size 0 such as A and B are equal
ii. A is set of all real number and B is set of all real number except negative integer. so A-B is set of all positive integers.
iii. A is set of all real number, and B is set all positive real number. Thus A-B is set of all real negative number.

- Use the Cantor diagonalization argument to prove that the number of real numbers in the interval [3, 4] is uncountable.
  Suppose that we can list all the number in the [3,4]
  3.100110111
  3.100101011
  3.100011011
  3.01110101
  ....
  ....
  let m=b1b2b3b4 and
  bi=0 if aii different than 1, 1 if aii=0
  m=0110....

- Use a proof by contradiction to show that the set of irrational numbers that lie in the interval [3, 4] is uncountable. (You can use the fact that the set of rational numbers (Q) is countable and the set of reals (R) is uncountable). Show all work.
  Assume that the set all irrational in [3,4] is countable
  We know that the set of all rational in [3,4] is countable and the set of reals (R) is uncountable.
  The subtraction of (Q) and (R) must then be uncountable which is the set of all irrational number in [3,4]. This is a contradiction.

Problem 3

- Find a closed form for the recurrence relation: a_n = $2a_{n-1}$ - 2, a_0 = -1
  a_0 = -1
  a_1 = -4 =-4 -2 +2
  a_2 = -10= -8 -4 +2
  a_3 = -22 = -16 -8 +2
  a_n = -2^{n+1} - 2^n + 2

- Find a closed form for the recurrence relation: a_n = (n + 2)a_{n-1}, a_0 = 3
  a_0 = 3
  a_1 = (1 + 2)a_0 =9
  a_2 = (2 + 2)a_1 =36

a_3 = (3 + 2)a_2 =180
a_4 = (4 + 2)a_3 =1080
a_n =$\frac{3}{2}$ (n+2)!

- Show that $a_n = 5(-1)^n$ -n + 2 is a solution of the recurrence relation $a_n = a_{n-1} +$ $2a_{n-2}$ + 2n - 9.
  let $a_n = a_{n-1} + 2a_{n-2}$ + 2n - 9
  if $a_n = 5(-1)^n$ -n + 2, then $a_{n-1} = 5(-1)^{n-1}$ -n + 3 and $a_{n-2} = 5(-1)^{n-2}$ -n + 4
  $a_n = a_{n-1} + 2a_{n-2}$ + 2n - 9 become
  $a_n = 5(-1)^{n-1}$ -n + 3 + 2*5$(-1)^{n-2}$ -2n + 8 + 2n - 9
  $a_n = 5(-1)^n * (-1)^{-1}$ + 2*5$(-1)^n * (-1)^{-2}$ + 2 - n
  $a_n = 5(-1)^n$ -n + 2
  a_n=a_n

Problem 4

- Consider the function f : Z x Z → Z where f(m, n) = 2m - n. Is this function onto?

  The function is onto because for every 2m-n there is an integers m and n for which 2m-n is integers

- Consider the function f : Z x Z → Z where f(m, n) = m^2 - n^2. Is this function onto?

  The function is not onto because m^2 or n^2 can not be satisfied when negative that is there is not inter for which m^2 or n^2 are negatives.

- Define the set C = the set of all residents of Colorado. Define in words a function f : C → Z. Is your function one-to-one? Is it onto? Be sure that the f you defined is indeed a function. Be creative and have fun!

  let x be number of job an individual has
  let y how many hour an individual works per week
  The function f(x,y)= xy is not onto or one to one because for every total hour of work xy we can not find a resident with x and y satisfied. For example no individual work 1000 hours per week.

- Again, define the set C = the set of all residents of Colorado. Define in words a function f : C → Z. However this time, make sure that your function is one-to-one. (Make sure to give a different example from part (c)).

  Let f(x)= m with m be how much each resident spend at grocery store per month. f is one-to-one because the amount the individual x spend in one week will have a unique value in z