This assignment is due on Friday February, 15 to Gradescope by Noon. You are expected to write or type up your solutions neatly. Remember that you are encouraged to discuss problems with your classmates, but you must work and write your solutions on your own.

Important: Make sure to clearly write your full name, the lecture section you belong to (001 or 002), and your student ID number at the top of your assignment. You may **neatly** type your solutions for +1 extra credit on the assignment.

1. Use rules of inference to show that if $\forall x(P(x) \vee Q(x)), \forall x(\neg Q(x) \vee S(x)), \forall x(R(x) \longrightarrow \neg S(x))$, and $\exists x \neg P(x)$ are true, then $\exists x \neg R(x)$ is true.

2. Prove or disprove the following claims. Be sure to indicate whether you are using a Direct Proof, a Contrapositive Proof, a Proof by Cases, or a Proof by Contradiction, or a Counterexample.

   (a) If the average of $a_1, a_2, ..., a_n$ is some number $\overline{a}$, then at least one of the real numbers $a_1, a_2, ..., a_n$ must be greater than or equal to $\overline{a}$.

   (b) If n is an integer and $3n + 2$ is even, then n is even. [Note: Do NOT use a contrapositive proof.]

   (c) If the lengths of two sides of a triangle are irrational, then the third side must be irrational also.

3. The divisibility rule by 3 is a rule that a number N is divisible by 3 if the sum of the digits is divisible by 3. For example, is 132 divisible by 3? $1 + 3 + 2 = 6$ and 6 is a number divisible by 3. So 132 must also be divisible by 3.

   Please prove this "rule" for three digit numbers:
   Let the positive integer N have the form $N = 100a + 10b + c$ where $a + b + c = 3n$ and a, b, and c are digits with $a \neq 0$. Prove that N is divisible by 3.

4. Prove the following:

   (a) Prove that n is even if and only if $n^2 - 6n + 5$ is odd.

   (b) Prove that if $2n^2 + 3n + 1$ is even, then n is odd.

5. Use proof by cases to prove that $x + |x - 8| \geq 8$ for all real numbers x. [Hint: $|x| = x$ when $x \geq 0$ and $|x| = -x$ when $x < 0$.]