# Homework 2

# Problem 1

Finding the nature of the inhabitant of the island.

A: Aramis is a Knight B: Berthand is a Knight C: Charleston is a Knight
They say:
Aramis: ¬ B
Berthand: ¬ C ⊕ B
Charleston: A ∧ ¬ B
lets D:¬ B ⇔ ¬ C ⊕ B
E: ¬ B ⇔ A ∧ ¬ B
G: ¬ C ⊕ B ⇔ A ∧ ¬ B

| A | B | C | ¬ B | ¬ C | ¬ C ⊕ B | A ∧ ¬ B | D | E | G | D∧E∧G |
|---|---|---|---|---|---|---|---|---|---|---|
| T | T | T | F | F | T | F | F | T | F | F |
| T | F | F | T | T | T | T | T | T | T | T |
| T | T | F | F | T | F | F | T | T | T | T |
| T | F | T | T | F | F | T | F | T | F | F |
| F | T | T | F | F | T | F | F | T | F | F |
| F | F | F | T | T | T | F | T | F | F | F |
| F | T | F | F | T | F | F | T | T | T | T |
| F | F | T | T | F | F | F | F | F | T | F |

Bertrand is a Night, Charleston is Knave and Aramis is a Knave

# Problem 2

1. Using the truth table to demonstrate that $((p{\to}q)\wedge(q{\to}r)){\to}(p{\to}r)$ is a tautology

| p | q | r | p→q | q→r | (p→q)∧(q→r) | p→r | ((p→q)∧(q→r)→ (p→r) |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | F | F | F | T | F | F | T |
| T | T | F | T | F | F | F | T |
| T | F | T | F | T | F | T | T |
| F | T | T | T | T | T | T | T |
| F | F | F | T | T | T | T | T |
| F | T | F | T | F | F | T | T |
| F | F | T | T | T | T | T | T |

The statement is a tautology.

2. Using chain of logical equivalent.
$((p{\to}q)\wedge(q{\to}r)){\to}(p{\to}r) \equiv \neg((\neg p\vee q)\wedge(\neg q\vee r))\vee(\neg p\vee r)$
$\equiv\neg((\neg p\vee q)\wedge(\neg q\vee r))\vee(\neg p\vee r)$

$\equiv ((p \wedge \neg q) \wedge (q \wedge \neg r)) \vee (\neg p \vee r)$
$\equiv ((q \wedge \neg q) \wedge (p \wedge \neg r)) \vee (\neg p \vee r)$
$\equiv ((p \wedge \neg r)) \vee (\neg p \vee r)$

3. Showing that (p→q) → r and p→(q → r) are not logical equivalent.

| p | q | r | p→q | q→r | (p→q) → r | p→(q → r) |
|---|---|---|-----|-----|-----------|-----------|
| T | T | T | T | T | T | T |
| T | F | F | F | T | T | T |
| T | T | F | T | F | T | F |
| T | F | T | F | T | F | T |
| F | T | T | T | T | T | T |
| F | F | F | T | T | T | T |
| F | T | F | T | F | T | T |
| F | F | T | T | T | T | T |

The last two columns contain contradictory statements, thus the two logical statements are not equivalent.

# Problem 3

1. $\forall x p(x) \equiv P(5) \wedge P(6) \wedge P(7) \wedge P(8)$

2. $\neg \exists x p(x) \equiv \neg P(5) \vee \neg P(6) \vee \neg P(7) \vee \neg P(8)$

3. $\neg \forall x p(x) \equiv \neg P(5) \wedge \neg P(6) \wedge \neg P(7) \wedge \neg P(8)$

# Problem 4

| 163 | 1 |
|-----|---|
| 54 | 0 |
| 18 | 0 |
| 6 | 0 |
| 2 | 2 |

163 in base 3 is $(20001)_3$

# Problem 5

1. Translating each of the groups travel requirements from English into a proposition.

   (a) Shaggy: $V \vee \neg S$;

   (b) Velma: $P \rightarrow \neg V$

   (c) Daphne: B ↔ (L ∧ P)

   (d) Fred: ¬ P

   (e) Scooby: $\exists x T(x) \equiv$ P ∨ V ∨ S ∨ B

2. Satisfaction of the gangs proposition. Lets Sat be the logical the gangs go on vacation. For Sat to be True: (V ∨ ¬ S) ∧ (P → ¬ V) ∧ (B ↔ (L ∧ P)) ∧ (¬ P) ∧ (P ∨ V ∨ S ∨ B) must be true.

3. The team travel whish is possible. If V is true proposition a) will be true no matter the values of S. For b) to be true, P must be false. The statement c) is true only if B is false no matter the outcome of L. Because P was false, the statement d) must be true.Finally as one of the valuein each is true, statement e) is true.
   -Venice
   -Shanghai

4. Park W in west Africa.