Industrial Automation

Project #3: Sampling and Reconstruction Demonstration

Due (Report PDF): by 2:00 pm on Tuesday March 12th (via D2L Dropbox folder).
Due (Latex Files): by 2:00 pm on Tuesday March 12th (via D2L Dropbox folder).
Due (Report Hardcopy): at the beginning of class Tuesday March 12th.

For this assignment you will be writing a report (using Latex) to explain the Shannon Sampling Theorem (also attributed to Nyquist (see wiki page)) using an example signal as demonstration.

Your report should include, but not limited to, the following aspects:

- Pretend that you are writing this to an engineer who is not familiar with the nuances of the Shannon Sampling Thm. Therefore, your text should clearing explain what's going on and you should use equations and figures (that you make yourself) to explain. Examples demonstrating what you're describing in words are going to be a crucial way of clearing presenting the topic in a palatable way.
- Reference any equations you don't derive.
- All figures and plots should be created by you. So no figures or images from google.
- You will demonstrate the reconstruction, $x_R(t)$, of a continuous-time signal ,$x(t)$, from the sampled signal $x^*(t) = x(kT_s) = x[k]$. You will show and describe, by using text and figures, that aliasing will occur if you don't sample fast enough.
- The signal you will use for your demonstration is $x(t) = sin(2\pi t) + 0.2cos(12\pi t)$.
- The sampling rates you will use will be
  1. $T_s = 0.17$ seconds. Make sure to explain (text and figures, so think about the figures I drew on the board to describe this process) why this is too slow and how it produces aliasing. Verify that your estimate of the aliasing frequency is approximately what results from the demo.
  2. $T_s = 0.017$ seconds. Make sure to explain (text and figures) why this is too slow and how it produces aliasing.
- One issue that we need to overcome is that there is no such thing as "continuous" in Matlab. So we will use a very small time increment, $\Delta T = .0001$ seconds, to represent a "continuous" signal and then we will "sample" using a multiple of that time increment ($T_s = N_s\Delta T$, where N_s is an integer greater than 1). The following code will plot the continuous signal and start your matlab program:

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%        My_Reconstruction
%              by
%         Shalom Ruben
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
clear all
clc
close all

DT = .0001; %DT is not the sampling rate! It is the "continuous" rate
t = -20:DT:20; %but you can feel free to zoom in later,
               %so the reader can better visualize.
               %But do keep t from -20 to 20 even if you only
               %view the date from -5 to 5 (for example)

%when you plot this, it should look "continuous"
plot(t,x,'linewidth',2)
```

- The next step is to build the sampled signal $x[k]$. This is done by deciding T_s is, and therefore what N_s is also, and then making everything, but those samples, equal to zero. Depending how how you're doing the convolution, it may be useful that the new vector $x[k]$ includes all the zeros in between the non-zero samples for the convolution process later.
- The next step is the Reconstruction of the original signal. You will explain and demonstrate that $x_R(t) = x(t)$ under certain conditions and otherwise they will not be equal.
- As you will explain, using text, figures, and equations, you must pass the sampled data through a low-pass filter to produce $x_R(t)$. One way to do this is to do it in the frequency-domain and the other is to do it in the time-domain via convolution of $x^*(t)$ and $h(t)$ where $h(t)$ is the impulse response of the low-pass filter (actually pulse-response since the input to the low-pass filter was discrete) that you are passing the signal through. In this report you will show the results using the following four low-pass filters:
    1. Ideal Low-Pass Filter (ILP)
    2. Zero-Order-Hold (ZOH)
    3. Non-causal First-Order-Hold (FOH)
    4. causal Predictive FOH (PFOH)
- In class we derived the pulse-response ($h(t)$) for the ZOH, ILP, and FOH, but you show the derivation of the impulse response of the PFOH.
- You must create (build the vector) for each impulse response (corresponding to each low-pass filter) and take care to make sure that you are using the time-increment ΔT (so think continuous) to produce them or the convolution will not be correct.
- Only use Matlab's |`conv()`|function to get the reconstructed signal after passing $x*(t)$ through the low-pass filter.
- I derived in class the Frequency-Response-Function (FRF) (which is also the Fourier Transform of the pulse-response $h(t)$) of the ZOH, so you should use that to plot the the FRF (magnitude only) of that and compare it to the FRF of an ILP and discuss.

Bonus would be to derive the FRF (magnitude only) of the FOH and also plot that on the same plot and discuss why FOH is better than ZOH but not better than ILP.

- Remember that you are not limited to just these bullet points and you should tell a coherent and interesting story.