

Robust, Rapid, and Simple Gravitational-wave Parameter Estimation

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Rapid and robust parameter estimation of gravitational-wave sources is a key component of modern multi-messenger astronomy. We present a novel and straightforward method for rapid parameter estimation of gravitational-wave sources that uses metric-based importance sampling. The method enables robust parameter estimation of binary neutron star and binary black hole binaries and is trivially parallelized, enabling full parameter estimation in seconds with modest resources. The algorithm achieves an average 35% effective sampling efficiency for the majority of aligned-spin neutron star binaries. Surprisingly, this approach is also highly efficient for analyzing the full 15-dimensional parameter space of typical binary black holes, with 20% efficiency achieved for a source detected primarily by the twin LIGO observatories and 9% for a network of three comparable sensitivity observatories. This method can serve immediate use to improve the low-latency data products of the gravitational-wave observatory network and may be a key component of how the millions of sources observed by next-generation observatories could be analyzed. The approach can also be broadly applied for problems where an approximate likelihood metric-space can be constructed.

I. INTRODUCTION

Gravitational waves have become a routine tool for studying the Universe and to date there are over a hundred observations of various compact-binary mergers [1–8]; the vast majority of these are black hole binaries. The growing catalog of observations is beginning to provide insights into binary formation pathways [9–14], nuclear equation of state [15, 16], and the search for theories beyond general relativity [17, 18].

One of the most well-known observations, GW170817, was a well-localized binary neutron star merger [19], accompanied by a gamma-ray burst [20, 21] and electromagnetic emission from a kilonova across the electromagnetic spectrum observed by dozens of observatories [22]. Gravitational-wave observatories are vital to enabling multi-messenger astronomy [23]. To ensure reliable detection of potential neutron star binaries, several analyses have been developed, namely PyCBC Live [24, 25], GstLAL [26, 27], MBTA [28, 29], and SPIIR [30]. These are generally able to identify a source in tens of seconds [31]. However, follow-up observatories need an accurate estimate of source location to know where to point and an estimate of the binary properties (e.g. component masses and spins) to determine if a given candidate is likely to have an electromagnetic counterpart [32]. An initial localization estimate is given by the Bayestar algorithm [33] and source classification can be provided by various approaches [29, 34–37], both within a few seconds. However, the most detailed and accurate approaches rely on using Bayesian parameter estimation analyses that can take hours or days to complete [38–41]. Robust estimates just seconds or minutes after a detection would enable telescopes to best utilize their resources and minimize the risk of missing early emission. Furthermore, as the sensitivity of current observatories improves and new observatories are added

to the network [23], the number of observations could increase by several orders of magnitude [42], necessitating a more efficient approach.

A number of techniques have been proposed to produce rapid parameter estimates for gravitational-wave sources. These include new methods of sampling [43], speeding up the calculation of likelihoods by using reduced representations [44–46], or parameter reduction by marginalization [33, 47, 48]. Neural network enhanced sampling has recently come to the forefront as a potential method to increase the performance by orders of magnitude relative to naive MCMC or nested sampling approaches [49–54]. These include methods that are fully likelihood-free [52], or use neural networks as a basis for proposals within a more standard algorithm [53, 54].

There are also methods that train an algorithm to directly produce a posterior estimate which is then used as the proposal for importance sampling [51]; the success of these algorithms has demonstrated the potential for amortized approaches. A disadvantage of pure neural network based approaches is that the nature of the internal model representation is not always comprehensible. This can make it difficult to robustly extend into new regimes and can result in unpredictable behavior when new and unexpected inputs are observed.

We demonstrate a simple technique for rapidly producing gravitational wave posteriors that is based on the ability for an approximate metric of the likelihood surface to be constructed. We show that using a naive brute-force tiling and exploration of this metric-space can be used as the basis for importance sampling. This approach achieves on average 35% sampling efficiency for a fiducial analysis of binary neutron star mergers defined by their component masses, tidal deformabilities, and the spin components aligned with the orbital angular momentum; this means that an example analysis could achieve 5000 independent samples from the posterior with $\sim 14,000$ likelihood calls. Furthermore, efficiencies

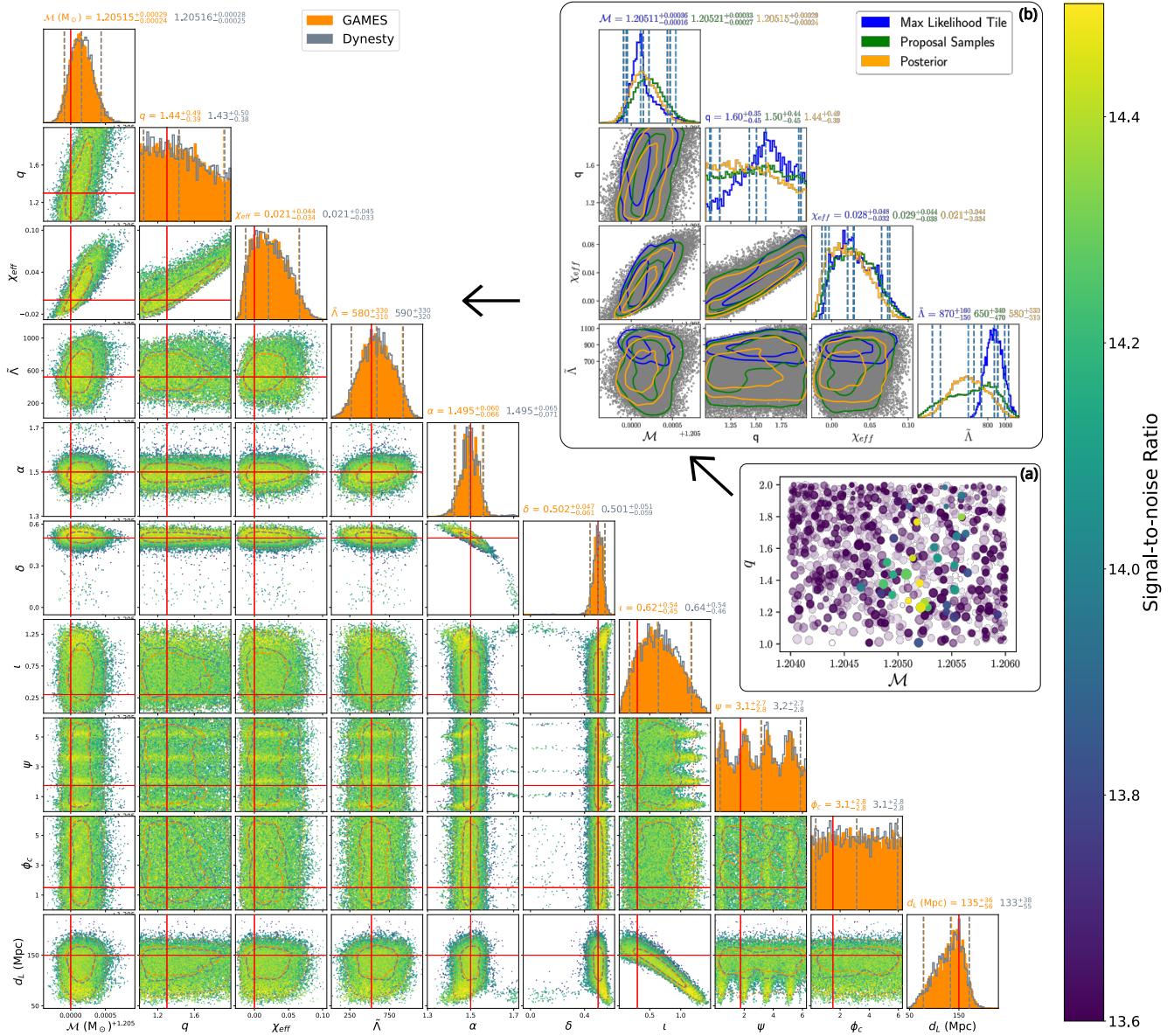


FIG. 1. Key intrinsic (chirp mass \mathcal{M} , mass ratio q , effective spin χ_{eff} [55], effective tidal deformability $\tilde{\Lambda}$ [56]) and extrinsic parameter distributions (right ascension α , declination δ , inclination ι , polarization angle ψ , coalescence phase ϕ_c , luminosity distance d_L) for a representative simulated BNS source added to simulated noise from a three-detector network. The true parameters are shown with red lines. We compare the posterior distribution obtained from our our importance-sampling procedure (GAMES, orange) to Dynesty (gray) [57], an implementation of nested sampling. The sub-panels schematically show how the importance sampling procedure works. Sub-panel A shows the initial stage where the likelihood is calculated for a precalculated set of metric points. Each point is colored by the signal-to-noise; only a handful of metric points contribute to informing the final posterior. Sub-panel B shows the distribution of the prior volume contained by the largest likelihood metric tile (blue), the likelihood-weighted average of the prior used as the proposal distribution (green), and the final posterior after importance sampling (orange). Importance sampling is highly efficient because the proposal distribution is already quite close to the posterior distribution.

of $> 9 - 20\%$ can be achieved for typical precessing binary black hole binaries. This performance is superior for typical sources to state-of-the art neural posterior methods [51] and avoids the need to train a potentially complicated latent space. Because this approach has

no trained parameters and works directly with existing implementations of the gravitational-wave likelihood, it should be similarly robust to expected variations in detector noise as traditional sampling techniques. Given these advantages, we expect significant impacts

on both low-latency and high-throughput gravitational wave science. This approach may also become a model for similar scientific problems where the signal model has a well defined metric.

II. METRIC-BASED POSTERIOR IMPORTANCE SAMPLING

We propose a method of importance sampling to directly obtain accurate posterior samples for gravitational-wave problems. Importance sampling can allow for efficient estimation of integrals and expectations, especially in cases where direct sampling is difficult or computationally expensive. However, it is strongly affected by the choice of proposal distribution. An optimal proposal distribution is proportional to the target posterior distribution, however as that is clearly not known a priori, the aim is to construct an accurate proxy for this with a straightforward, computationally inexpensive procedure. It is possible to efficiently create an accurate proposal distribution for gravitational-wave problems because the likelihood can be defined in terms of an inner product whose salient features can be exhaustively mapped independent of any specific observation. This allows for an approximate metric-space to be constructed and for the prior volume to be efficiently tiled. A proposal distribution can be constructed by simply drawing from the prior volume associated with each tile in proportion to the approximate posterior probability contained within each tile. A similar approach would likely apply to any problem of this nature.

A. Gravitational-wave Likelihood

We'll start by motivating why a metric space can be constructed. The likelihood for gravitational-wave data analysis can be expressed in terms of the inner product

$$(a|b) = 4 \operatorname{Re} \int \frac{a(f)^* b(f)}{S_n(f)} df \quad (1)$$

in the case of frequency-domain functions a , b and the noise power spectral density S_n . The full likelihood can be written as

$$\ln \mathcal{L} = -\frac{1}{2}(d - h|d - h) = (d|h) - \frac{1}{2}(h|h) + (d|d) \quad (2)$$

where d is the data and h is the signal proposed to be in the data. This form naturally results from the assumption of wide-sense stationary colored Gaussian noise at the time of a potential signal. This is the standard assumption in gravitational-wave astronomy, though deviations from this have been explored [38]. Since the data is the sum of the noise n and an actual

source signal s , the ratio of the additive signal to noise-only hypothesis can be written as

$$\ln(\mathcal{L}/\mathcal{L}_n) = (n|h) + (s|h) - \frac{1}{2}(h|h). \quad (3)$$

If we focus on factors related to the phase evolution of a signal and are independent of its amplitude, we can maximize the likelihood to give

$$\ln(\mathcal{L}/\mathcal{L}_n)_{maxA} = (n|h) + \frac{(s|h)^2}{2(h|h)} = (n|h) + \frac{1}{2}\mathcal{O}(h|s)^2(s|s). \quad (4)$$

where \mathcal{O} is the normalized overlap between the source and proposed signal. The non-trivial structure of this likelihood as a function of the proposed template $h(\theta)$ arises from the behavior of $\mathcal{O}(h|s)$. This can be viewed as the inner product of the space defined on the gravitational waveform manifold and can be fully explored independently of specific observational data. Our algorithm recasts parameter estimation in terms of proposals defined naturally on the metric space defined by this inner product.

Constructing a set of points within waveform the manifold defined by this metric that provides an efficient packing is a well-known problem that has been explored as ‘template bank generation’ in the context of searches for gravitational-wave sources [58]. There exist several algorithms for tiling this space such that coverage is achieved with a minimum metric distance maintained between points in the space [59–61]. Because the likelihood can be expressed entirely in terms of this inner product, a tiling of the metric space is also an approximate tiling of the likelihood space. Notably, we don't require the metric to exactly match that of the likelihood itself, but only be consistent for small patches. For a small region, terms in the likelihood such as $(n|h)$ can be treated as nearly constant with small changes bounded by a triangle inequality.

Once the metric space is tiled, we determine the portion of the prior volume closest to each tiling point. To generate the posterior, we first calculate the posterior probability for each tiling point; there are typically few tiling points that need to be calculated $\mathcal{O}(10^{3-4})$ in practice when using an information from an initial search identification. The proposal distribution is then the portion of prior volume associated with each tile weighted by the posterior probability of its associated tiling point. In Fig 1, we show schematic picture of this process starting from the template bank and likelihood with prior regions already encapsulated.

III. APPLICATION TO BINARY NEUTRON STAR AND BINARY BLACK HOLES

We examine two specific cases of interest. The first case is that of binary neutron stars, which we'll

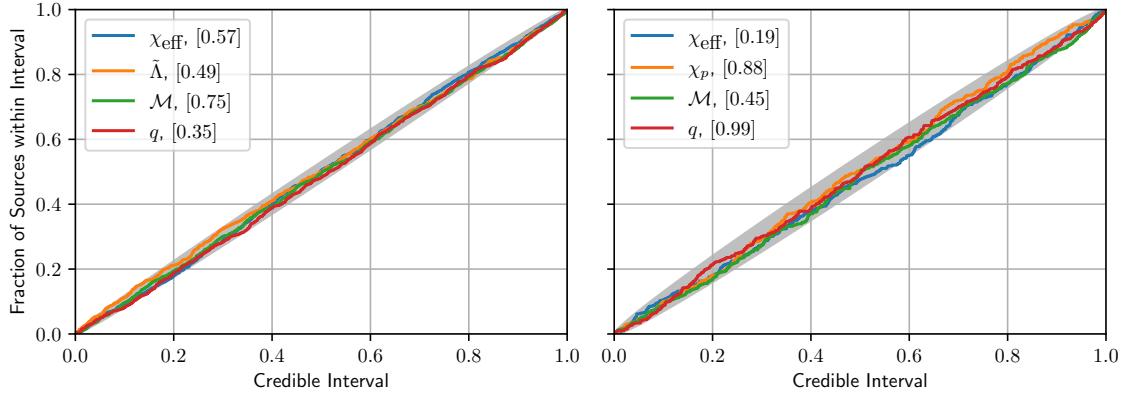


FIG. 2. The fraction of sources with true values within a credible interval for simulated sources from both our BNS population (left) and BBH population (right) along with 2σ confidence bands (gray). For each parameter we perform a KS test (shown in legend); the p-value in each case is consistent with the algorithm producing self-consistent posteriors. Key parameter combinations are shown (\mathcal{M} , q , χ_{eff} , χ_p [62], $\tilde{\Lambda}$ [56]), the remainder, including extrinsic parameters are similarly consistent.

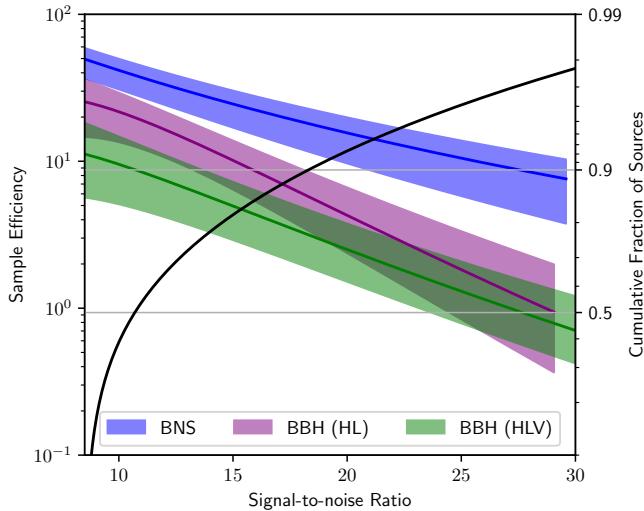


FIG. 3. The sample efficiency as a function of signal-to-noise ratio for aligned spin BNS (blue), three-detector BBH (green), and two-detector BBH sources (purple). In each case, the efficiency is calculated over the average of 80,000 likelihood calls. The cumulative number of detectable sources is shown in black.

characterize by their masses, aligned spin components, and tidal deformabilities. Second, we'll examine binary black holes of typical mass ($\mathcal{M} \sim 33M_\odot$) with the full parameter space of spin vectors, allowing for precession and higher order modes. This covers the most typical cases for the current observed population of sources. We assume natural priors for extrinsic parameters (e.g. sky location, orientation) and allow for spin magnitudes up to 0.4 and 0.85 for the BNS and BBH cases, respectively. To model the gravitational-wave signal, we use the IMRPhenomD_NRTidal (BNS) [63, 64] and IMRPhenomXPHM (BBH) [65] waveform models.

We will use our algorithm to estimate the intrinsic parameters of the source in both cases, while the other

parameters are marginalized over using SNR-informed importance sampling [48] or analytic formulas [33, 47]. In the case of precessing BBH signals, we also choose to sample over inclination explicitly rather than marginalizing over the parameter; we also numerically marginalize the orbital phase in this case. We will use this marginalized likelihood for the weights of the metric-space importance sampling.

We'll construct a tiling stochastically using the algorithm introduced in [61] and construct the mapping between the prior volume and the tiled metric space by explicit numerical waveform comparison. For specific parameter cases, a known analytic metric could potentially be used instead to map the prior volume, which may be able to reduce or remove the costs associated with mapping out the metric space. We simplify the tiling process by taking advantage of the fact that the intrinsic and extrinsic parameters partially decouple [33]; this is a weaker approximation for highly precessing signals which does reduce the overall efficiency. In the case of aligned spin signals, the association between metric point and prior volume is done using the simple waveform overlap, whereas for precessing systems we approximate this as the minimum of the overlap between the metric point and the individual plus and cross gravitational waveform polarizations; for aligned spin systems the polarizations differ only be an overall phase shift.

The tiling is only weakly sensitive to expected changes in the power spectral density $s_n(f)$ as has been studied in the context of searches [66]. We would expect that the space would only need to be remapped when there are significant changes such as after between observing runs. The method can also be straightforwardly extended to incorporate calibration and waveform uncertainties either directly through the metric space of the waveform model or by using a calibration-marginalized likelihood [67].

A. Validity and Efficiency

To test the self-consistency of our method, we simulate a population of sources added to simulated observatory noise. We consider a network of three observatories with sensitivity consistent with Advanced LIGO’s third observing run. For each source, we follow our method and estimate the parameters of each source. In Fig. 2 we demonstrate that the resulting estimates are consistent with unbiased probability distributions. A handful of individual sources are also directly compared against the standard Dynesty sampler. In each case, the results are indistinguishable. One such example is shown in Fig. 1. To test the robustness of the approach in cases where the noise curve may have deviated from what it was constructed for, we apply the same pre-computed metric tiling used for simulated O3 data to O4 simulated data; this also produces unbiased posterior estimates.

To assess the efficiency of the algorithm, we calculate the effective sample size for each simulated source relative to the number of likelihood evaluations. Fig. 3 shows how the sample efficiency scales as a function of signal SNR. For the BNS case, high efficiency $> 10\%$ is achieved for signals up to SNR of 30 and for the majority of sources the efficiency is $> 35\%$. While it is perhaps not surprising that for a simple enough parameter space this approach may be efficient, we find that it can also handle the full 15 dimensional parameter space of typical BBH mergers including precession effects and the inclusion of higher order modes. Because the metric mapping however is further approximated in comparison to the aligned spin BNS case, we find a reduction in the sampling efficiency, however, we note that for the majority of sources it is still $> 10\%$ which is competitive with the state-of-art neural importance sampling methods [51].

Given the naive implementation of this method tested here, we would expect the efficiency to drop with increasing SNR as is observed. The drop in efficiency occurs for two reasons: (1) the fixed metric tiling distance means that as the SNR of a signal increases, the coarseness of the tiling is increasingly apparent in the proposal distribution and (2) the intrinsic dimensional of the metric space increases as more subtle parameter effects become measurable. A more sophisticated treatment of the metric space in this case, would very likely result in further efficiency improvements by recording the details of the local metric around each of the prior points and incorporating that into an iterative procedure to update the proposal distribution.

IV. DISCUSSION

In this work, we have demonstrated a novel approach to rapid and robust gravitational-wave parameter estimation using metric-based importance sampling. This method is effective for the majority of typical gravitational-wave sources, including binary neutron

star (BNS) and typical binary black hole (BBH) systems. Our approach, which tiles the parameter space using an approximate metric, achieves high sampling efficiencies—over 35% for BNS and 9–20% for BBH sources.

While the method handles most sources efficiently, it does not eliminate the need for other parameter estimation algorithms. It may prove more difficult to extend to highly precessing NSBH or lower mass BBH mergers where the intrinsic parameter space size is significantly higher. Furthermore, the current implementation is not optimal for extremely loud signals, where precision is crucial, or for cases where the waveform model is not well-understood in advance. The method relies on pre-computation steps, which assume the signal model is well-defined. In situations where this is not the case or where a full tiling of the parameter space is impractical, more general algorithms will still be necessary. These edge cases will likely be of high scientific interest and will require further development of complementary techniques. However, our method offers a clear advantage for the majority of sources, significantly reducing computational costs and time.

We have not yet integrated this method into a full low-latency pipeline for real-time gravitational-wave parameter estimation, but the potential for such an implementation is clear. After non-trivial optimization of data ingestion, the main time constraint would be likelihood evaluations. For example, achieving 10,000 effective samples for BNS sources could be completed in $O(10)$ seconds on a machine with $O(10)$ cores. This would drastically improve the ability to deliver fast, reliable source estimates to multi-messenger partners, enhancing the chances of capturing early electromagnetic counterparts.

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The algorithm is available through the PyCBC toolkit [68] project and a corresponding data release provides a working example [69].

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