Lab 09
PID Controller, Tracking and Disturbance Rejection

8.1 Objective
To investigate the performance of motor speed and position control systems with a PID controller for tracking and disturbance rejection

8.2 Required Files

LabVIEW with Control Design & Simulation	
QUBE Servo	**QNET DC Motor**
NI myRIO 15.0	NI DAQmx
QUBE-Servo-2.Ivproj	NI ELVISmx
QUBE-Servo 2 Speed Control.vi with sub-VIs	QNET-Motor-Speed Control.vi with sub-VIs
QUBE-Servo 2PositionControl.vi with sub-VIs	QNET-Motor-PostionControl.vi with sub-VIs

8.3 System Type and Steady-State Error
Any physical control system inherently suffers steady-state error in response to certain types of inputs. A system may have no steady-state error to a step input, but the same system may exhibit nonzero steady-state error to a ramp input. Whether a given system will exhibit steady-state error for a given type of input depends on the type of open-loop transfer function of the system. Control systems may be classified according to their ability to follow step inputs, ramp inputs, parabolic inputs, and so on. The steady-state error will depend on the type of input (step, ramp, etc.) as well as the system type (0, I, or II).

The system type is defined as ***the number of pure integrators in the forward path of a unity-feedback system***. That is, the system type is equal to the value of ***'n'*** when the system is represented as in the Figure 1. A system is classified as type 0 system when 'n' is 0, as type 1 system when 'n' is 1 and as type 2 system when 'n' is 2.

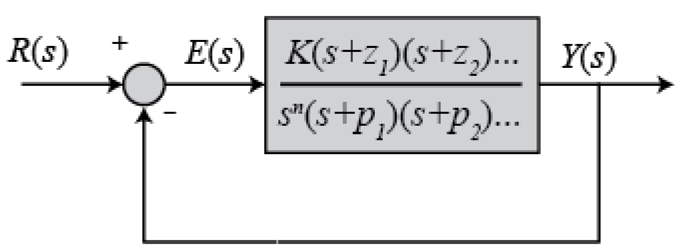


Figure 1: Unity feedback system with representation of system type in forward path gain

According to the final value theorem, steady-state error is given as;

$$E(s) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\frac{R(s)}{1+G(s)*C(s)}$$

For a unity feedback system with controller *C(s)* and plant *G(s)*, the forward path gain will be product of the controller and plant transfer functions as shown in expression above. The system type will be identified using this forward path gain. It does not matter if the integrators in this are part of the controller or the plant.

Task 1: To identify the system type for DC motor speed and position control systems

1. For DC motor position control system with PID controller, identify system type.

System type II

1- System Type for DC motor position control system with PID controller.

$$G(s) = \frac{K}{s(\tau s+1)} \quad ; \quad C(s) = K_p + \frac{K_i}{s} + K_D s = \frac{sK_p + K_i + K_D s^2}{s}$$

$$G(s)C(s) = \frac{K(sK_p + K_i + K_D s^2)}{s^2(\tau s + 1)}$$

2. For DC motor speed controller system with PI controller, identify system type.

System type I

2- System Type for DC motor speed controller system with PI controller.

$$G(s) = \frac{K}{\tau s+1} \quad ; \quad C(s) = K_p + \frac{K_i}{s} = \frac{sK_p + K_i}{s}$$

$$G(s)C(s) = \frac{K(sK_p + K_i)}{s(\tau s + 1)}$$

8.4 Tracking with PID Controller

Tracking is the ability of system to change its output in order to follow a changing reference signal. If the steady-state error becomes zero, we can say that the system is able to perfectly track the input.

Task 2: To investigate the tracking ability of system for triangular reference signals with different controllers

1. For the two systems of Task 1, find the steady-state error with ramp reference input.

Date:

Task 2:

PID controller.

1- $E(s) = \lim_{s\to 0} s \dfrac{R(s)}{1+G(s)*C(s)}$ $\quad \therefore R(s) = \dfrac{1}{s^2}$ = ramp input

$$= \lim_{s\to 0} s \frac{1}{s^2\left(1 + \left[\frac{K(sK_p + K_i + K_D s^2)}{s^2(\tau s+1)}\right]\right)}$$

$$= s \frac{\tau s + 1}{s^2(\tau s+1) + K(sK_p + K_i + K_D s^2)}$$

$$\lim_{s\to 0} = \frac{\tau s^2 + s}{s^2(\tau s+1) + K(sK_p + K_i + K_D s^2)} = 0$$

PI controller

2- $E(s) = \lim_{s\to 0} s \dfrac{R(s)}{1+G(s)\times C(s)}$ $\quad R(s) = \dfrac{1}{s^2}$

$$\lim_{s\to 0} s \left(\frac{1}{s^2\left(1 + \frac{K(sK_p + K_i)}{s(\tau s+1)}\right)}\right)$$

$$\lim_{s\to 0} \frac{\tau s + 1}{s(\tau s+1) + K(sK_p + K_i)} = \frac{1}{KK_i}$$

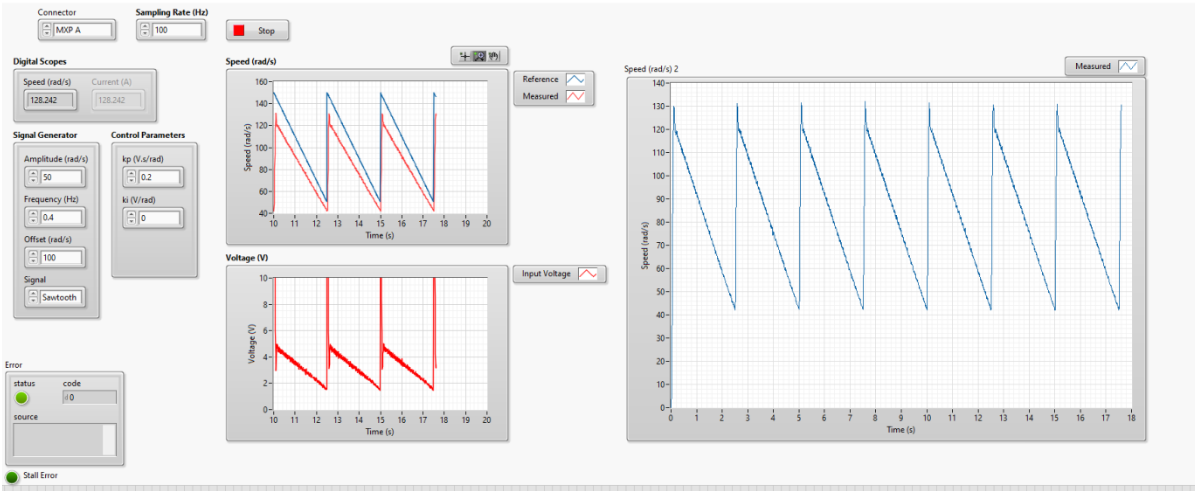
2. Based on your results, predict which one will track the ramp reference with some error and which of these will track perfectly without any error.

The PID controller for the DC motor position controller will track the ramp reference perfectly without any error since its steady-state error is 0.
The PI controller for the DC motor speed controller system will track the ramp reference with some error of 1/(K*Ki)

In this lab, we're going to make DC motor's speed to follow a triangular signal.

3. Open the VI with PI controller implementation for DC motor speed control.
4. Provide a saw-tooth signal as reference speed to the motor. The parameters of saw-tooth signal are:
   - Amplitude: 50 rad/s
   - Frequency: 0.4 Hz
   - Offset: 100 rad/s

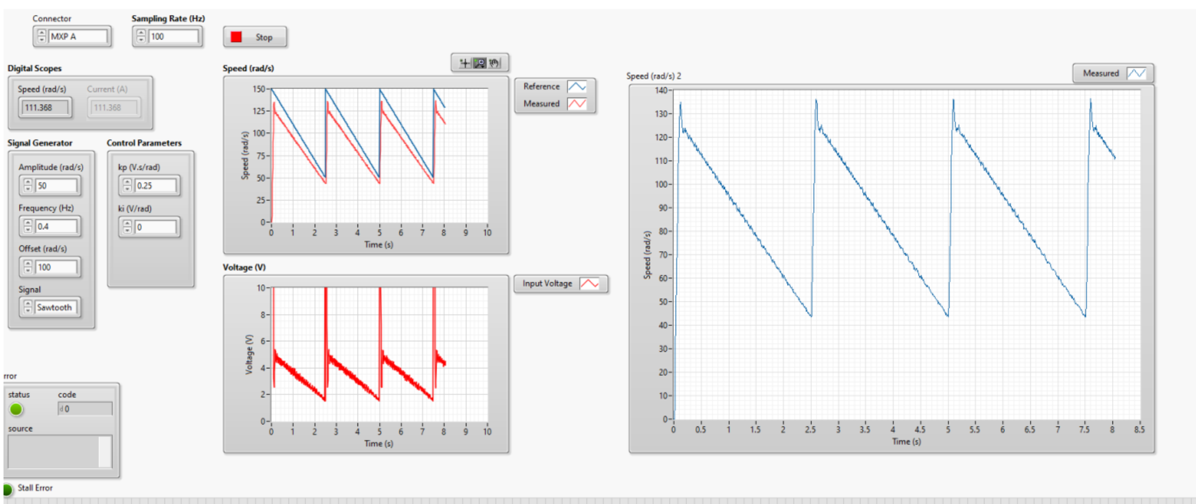
5. Set $K_I = 0$ and $K_P = 0.2$. Compare the measured speed and reference speed. Increase K_P gradually. Justify the presence or absence of tracking error.



As can be seen there is quite a difference between measured and reference speed. Here reference speed is reaching at about 150 rad/sec and measured speed is reaching at about 128.242 rad/sec

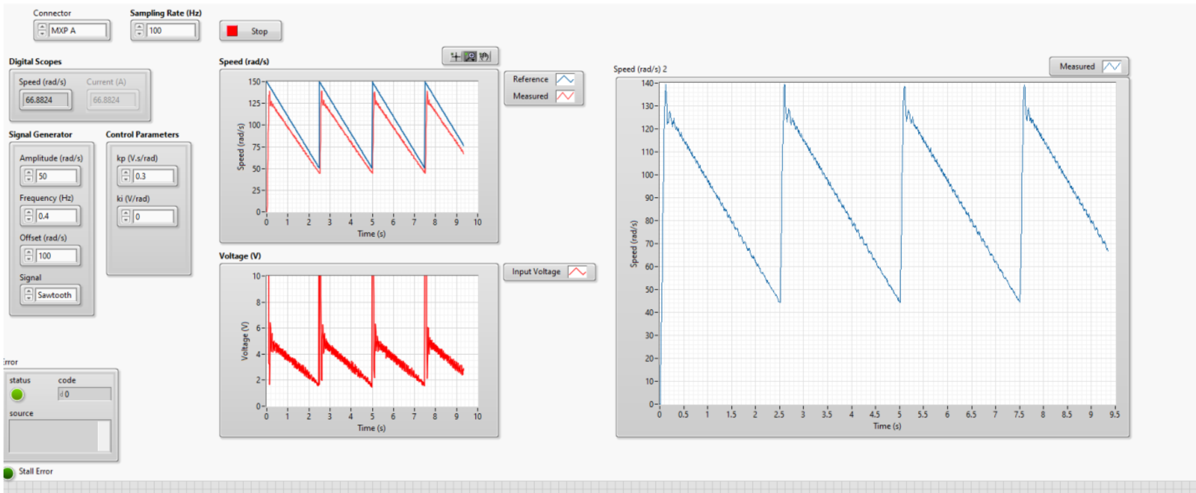
However, to reduce this unwanted error we can vary the value of Kp to observe the error either its going to be reduced or increased.

When Kp was set = 0.25



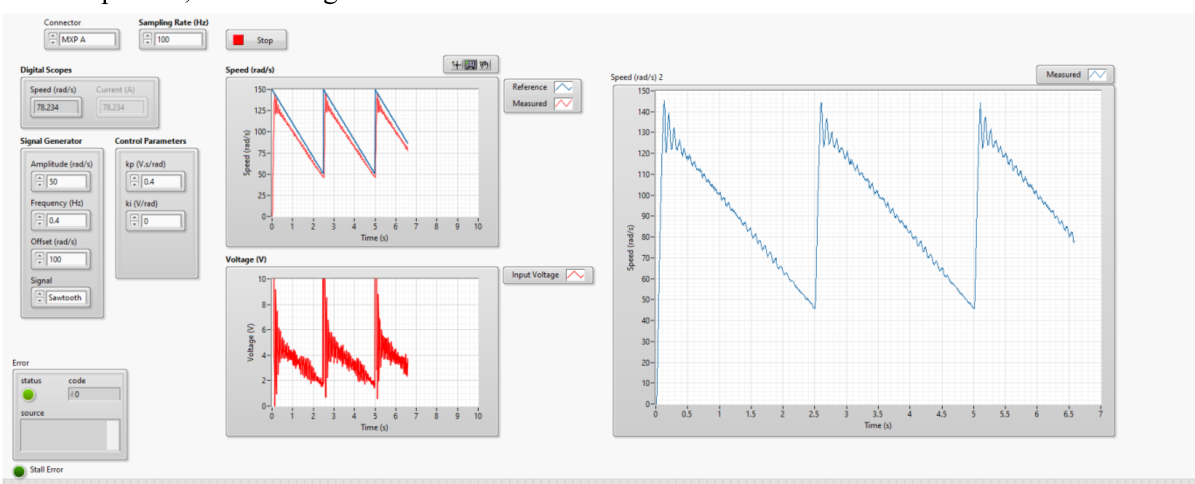
The measured value increases and become closer to reference output. Since we are using purely proportional system, the system output is always lesser than desired set point value, this phenomenon is known as system droop. This system droop can be lowered by increasing Kp but should not be increased to much as it could lead to instability.

When Kp = 0.3



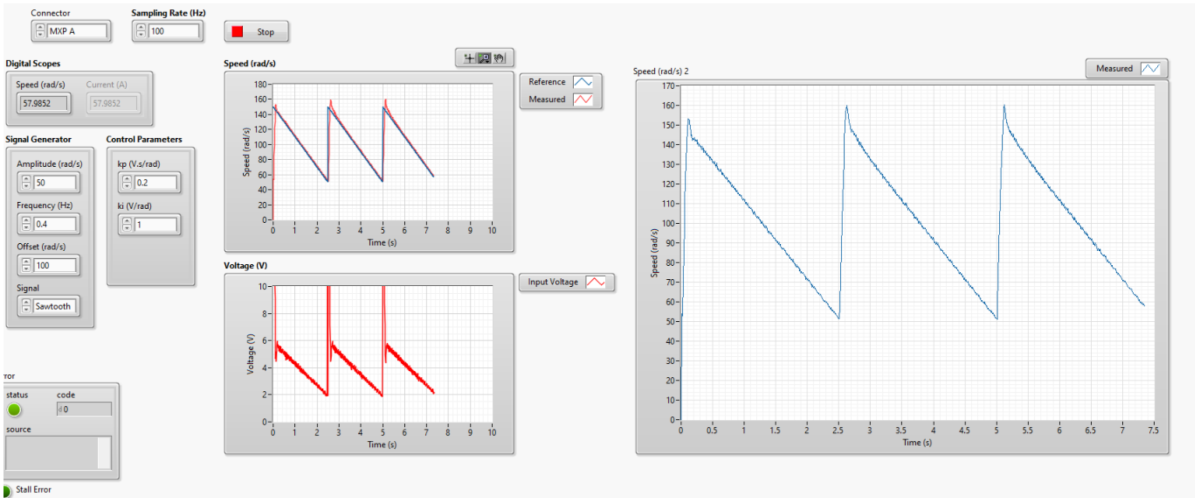
The tracking error decreases further and measured response output is getting closer to reference response output

When Kp = 0.4, the tracking error further decreases

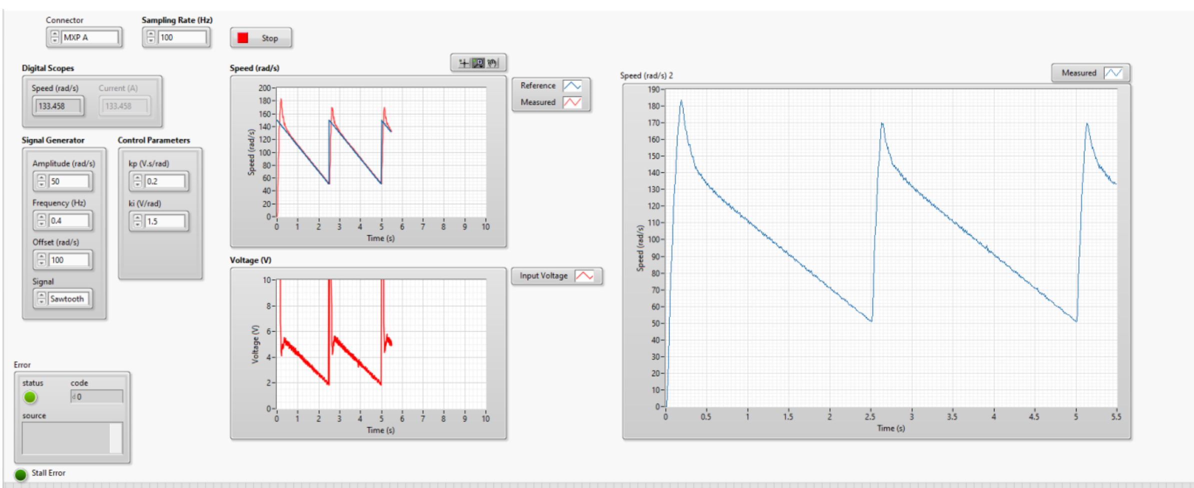


6. Set $K_P = 0.2$ and $K_I = 1.0$. Increase K_I gradually and examine the response. What effect does increasing K_I have on the tracking ability?

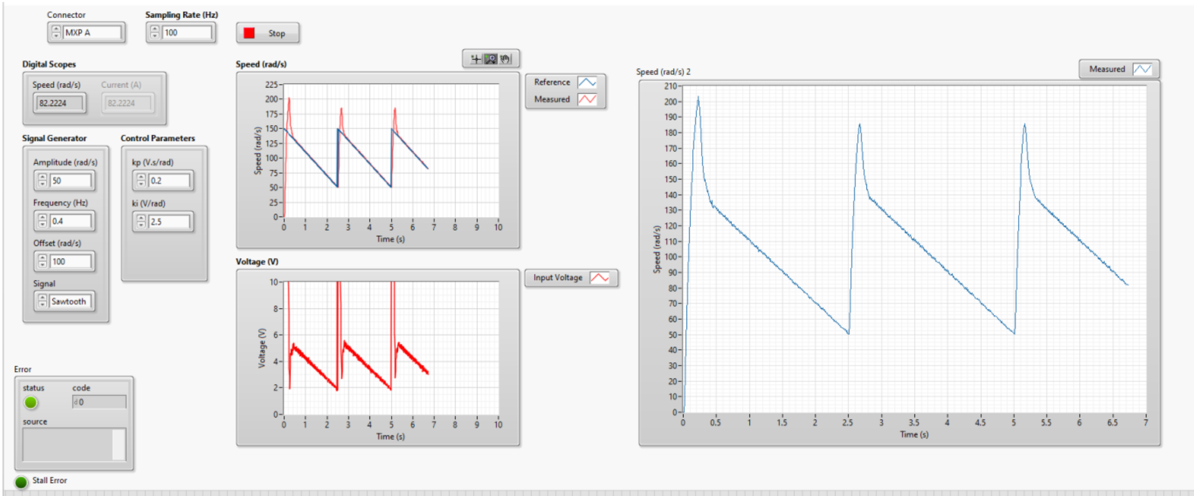
When Ki was set equal to 1, we will get following result



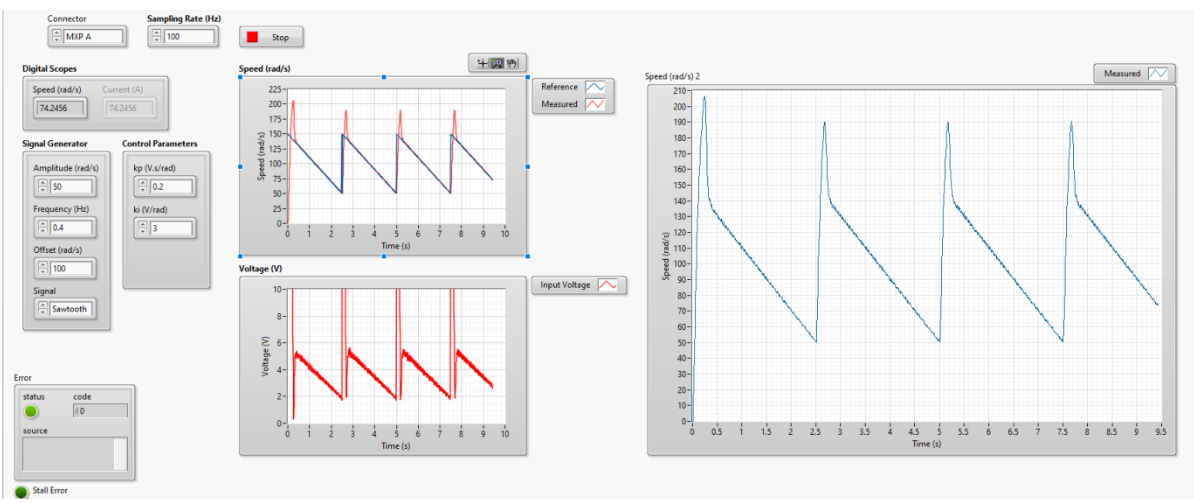
As Ki increased to 1.5, the measured response output gets an higher value than reference response output as compare to when Ki = 1.



When Ki increased to 2.5



There is further increase in measured response then input and error still persist. Similarly, for 0.3 it still persist and error is keep on increasing as ki increases



8.5 Disturbance Rejection

We'll now explore the advantage of the PID controller in reducing the effects of disturbances or unwanted signals on our output. A disturbance is typically modeled as a signal added to the control input as shown in Figure 2.

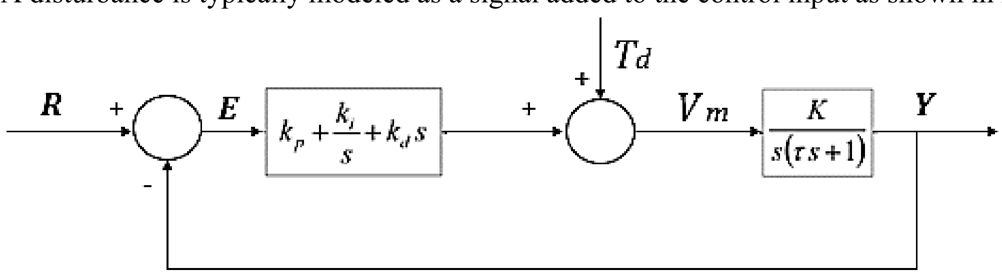
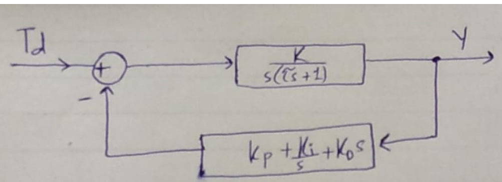


Figure 2: Closed loop motor position control with PID controller in presence of disturbance

Task 3: To find the steady-state motor angle with step-disturbance with PD and PID controllers

1. Write down the transfer function of the closed-loop system from the disturbance torque T_d to motor position θ_m, when a PID controller is used. The reference input 'R' is set to zero.



for R=0

$$\frac{\theta_m}{T_d} = \frac{K/s(\tau s+1)}{1 + \frac{K}{s(\tau s+1)}\left(K_p + \frac{K_i}{s} + K_D s\right)}$$

$$= \frac{K}{s(\tau s+1) + K\left(K_p + \frac{K_i}{s} + K_D s\right)}$$

$$= \frac{Ks}{s^2(\tau s+1) + K\left(K_p s + K_i + K_D s^2\right)}$$

$$\frac{\theta_m}{T_d} = \frac{Ks}{\tau s^3 + (K_D K+1)s^2 + KK_p s + KK_i}$$

2. In the absence of reference input R(s), find the steady-state motor angle due to step disturbance $T_d(s)$ of magnitude V_o where the motor position is being controlled by a PID controller.

b)

$$\theta_m = \frac{Ks}{\tau s^3 + (KK_D+1)s^2 + KK_p s + KK_i} \times T_d \qquad \therefore T_d = \frac{V_o}{s}$$

$$\theta_m = \frac{KV_o}{\tau s^3 + (KK_D+1)s^2 + KK_p s + KK_i}$$

$$\lim_{s\to 0} s\,\frac{KV_o}{\tau s^3 + (KK_D+1)s^2 + KK_p s + KK_i} = \frac{0}{\tau(0)^3 + 0^2 + 0 + K_i K} = 0$$

3. What is the steady-state motor angle due to a disturbance step of magnitude V_o for a PD controller (K_I=0)?

c) $T_d = V_0/s$

$$\frac{\theta_m}{T_d} = \frac{K}{s(\tau s+1) + K(K_p + K_D s)}$$

$$\theta_m = \frac{K}{\tau s^2 + (KK_D+1)s + KK_p} \times \frac{V_0}{s}$$

$$\lim_{s\to 0} \cancel{s}\, \frac{K V_0/\cancel{s}}{\tau s^2 + (KK_D+1)s + KK_p} = \frac{K V_0}{K K_p} = \frac{V_0}{K_p}$$

Task 4: To investigate the rejection of step disturbance by DC motor position control system with PD and PID controllers

1. Open the VI that implements a PID controller to control the position of the DC motor.
2. Modify the VI to simulate a disturbance by adding a step signal to the control input. Make sure to add a switch to the signal path so that you're able to turn the disturbance off. Add a switch with the signal generator as well. Implementation help: Add a Boolean Control (Slide switch / Push button). Connect it with ***Select*** Boolean input and with other two inputs of select, connect the disturbance signal and a constant 0 as shown in figure 3.

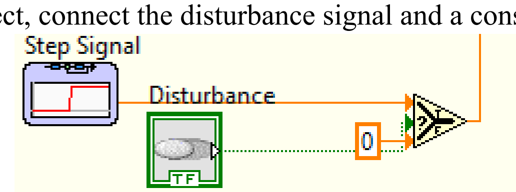
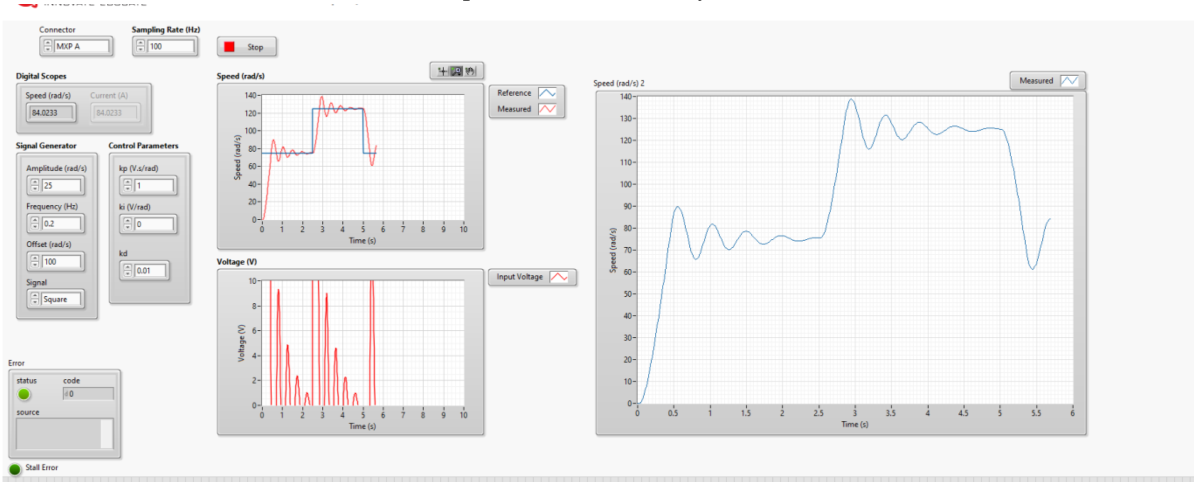


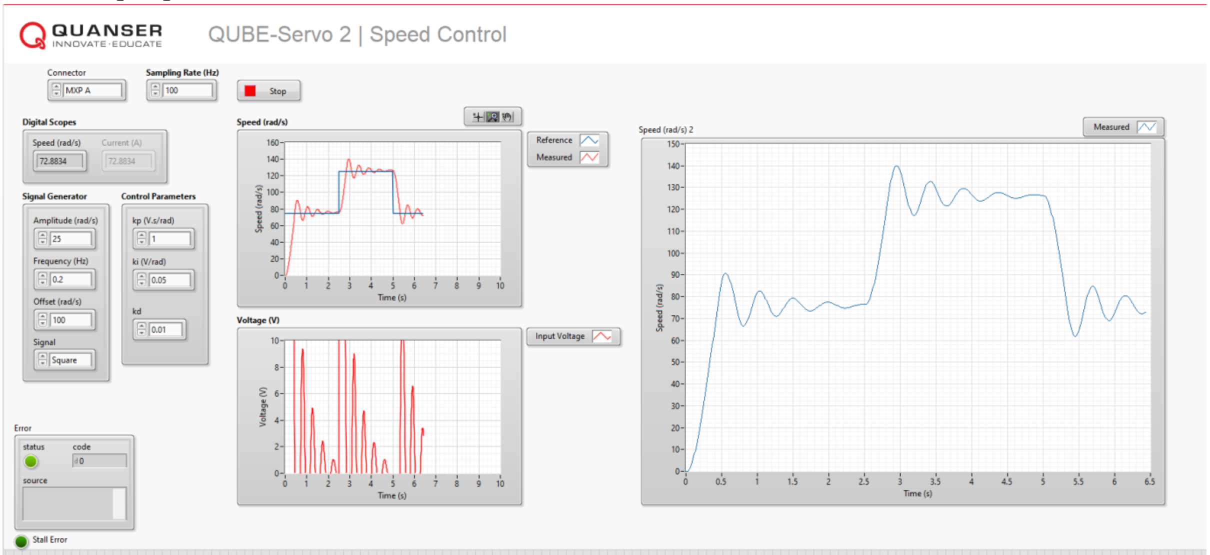
Figure 3: Adding a Boolean control

1. Set the disturbance signal to zero. Set the input from signal generator (Square wave, 0.2Hz frequency, offset 100 and amplitude 25) and observe the output at K_D =0.01 and K_P=*1*. Is there any steady-state error when a PD controller is used i.e. Ki=0 or, when a PID controller is used with K_I = 0.05?

   When a PD controller is used, there is quite small but steady state error do exist

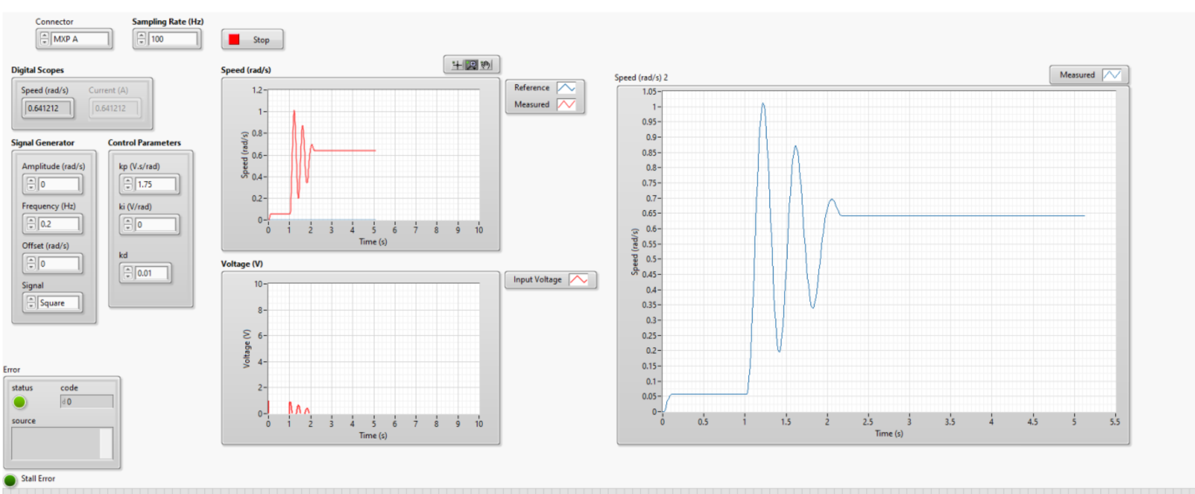


And when PID controller is used, the small error still exists and these small errors can be eliminated by varying the value pf kp,kd, and ki

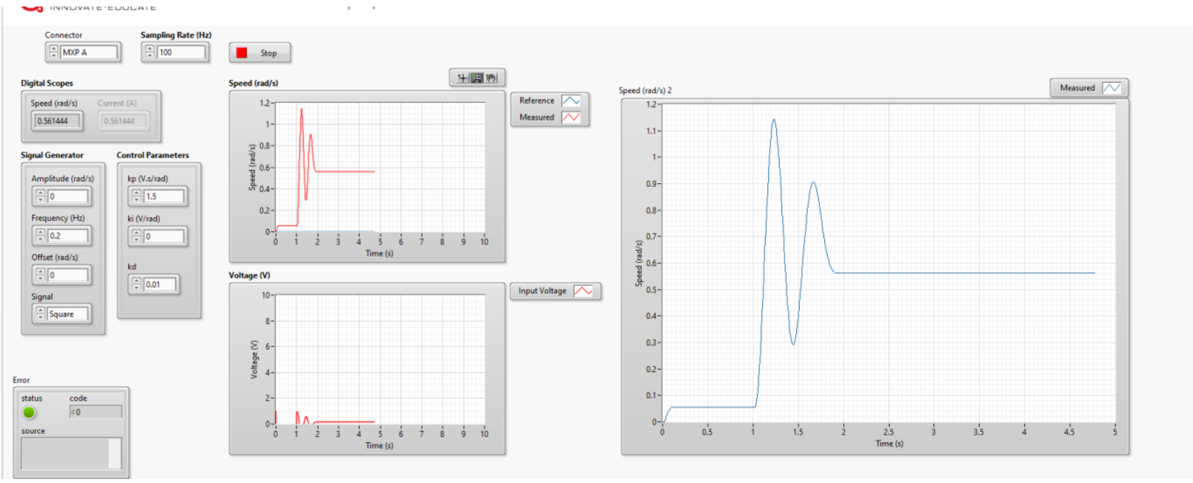


2. Now, set the reference signal to zero and switch on disturbance. Set $K_I = 0$. Vary K_P and K_D, and try to completely reject the effect of disturbance on the measured position, i.e. the output position should be zero since the reference is zero. Comment on your investigation.

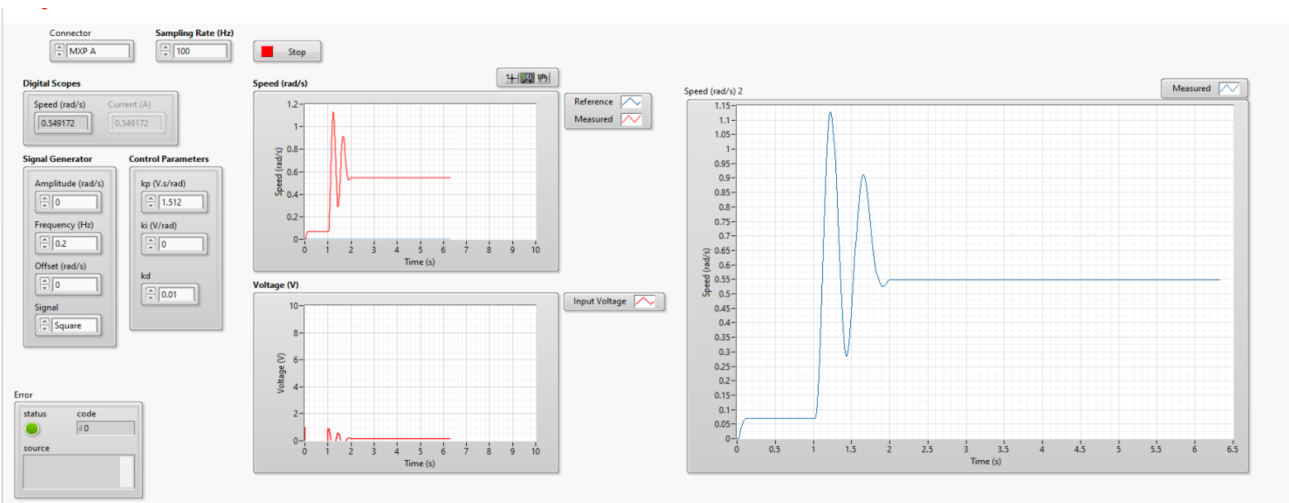
As when Kp and Kd is varied we were able to remove the error to a certain extent but was not completely able to make it to zero as it should as Vo is zero, the error should be zero as well.



When the Kp and Kd was further varied, we were able to reach to about 0.58 but wasn't able to reach to zero



And when varied further, we were able to reach to 0.55 approx but wasn't able to reach to zero



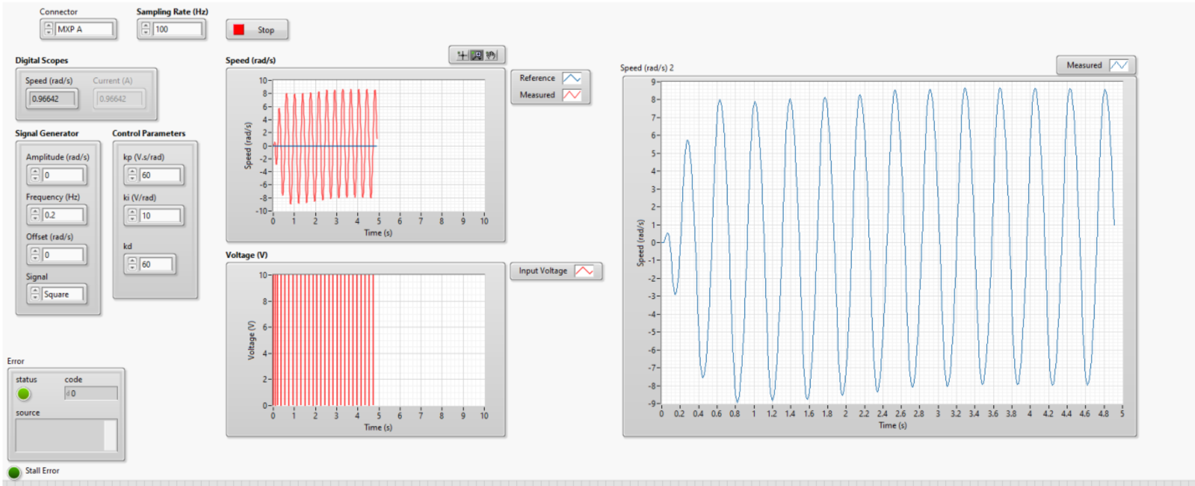
Also, note that changing value of Kd will have no impact error depends on Kp

3. Record the obtained steady-state angle, and compare it to the calculated value using the derived expression.

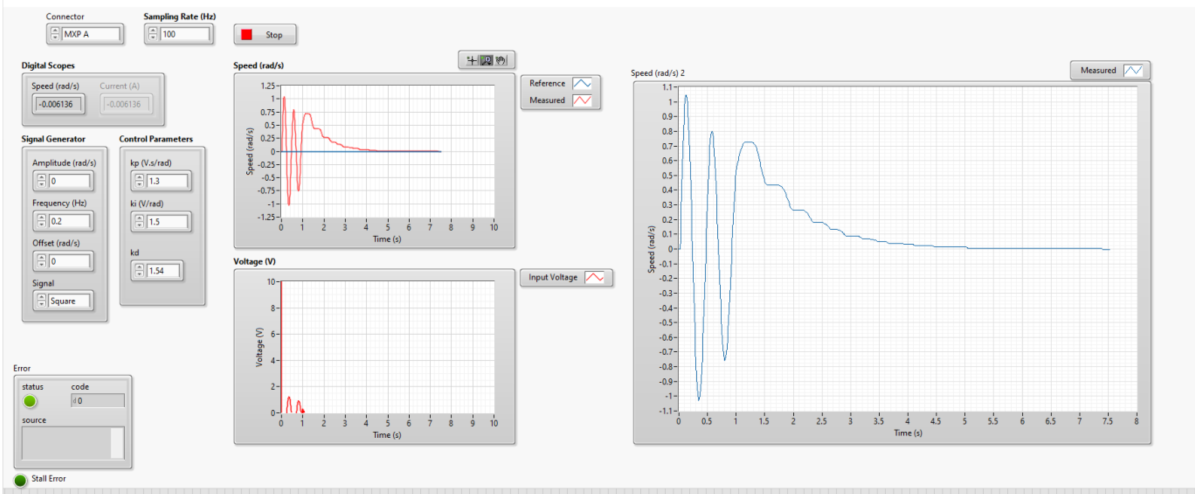
Vo is 0, so according to the steady-state angle calculated above: Vo/Kp ; steady state angle should have been 0 too. The obtained steady-state angle is almost 0.55, which is very close.

4. Now, include K_I as well. Examine the effect of disturbance on the measured position, now in the presence of PID controller. Explain the difference of the disturbance response with the integral action added and compare your result to calculations done in task.

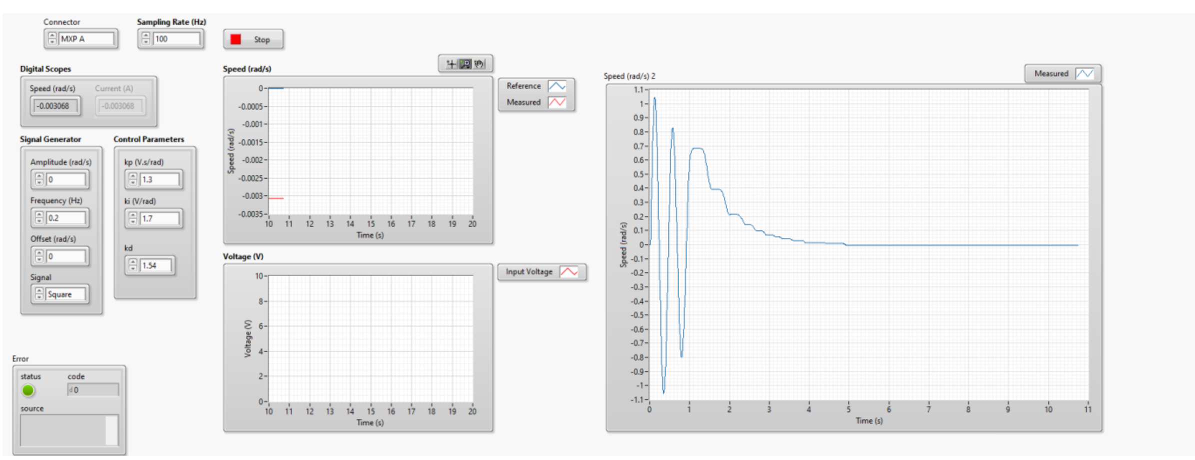
   When the integral action was added, we have observed lot of noise and rise in disturbance signal

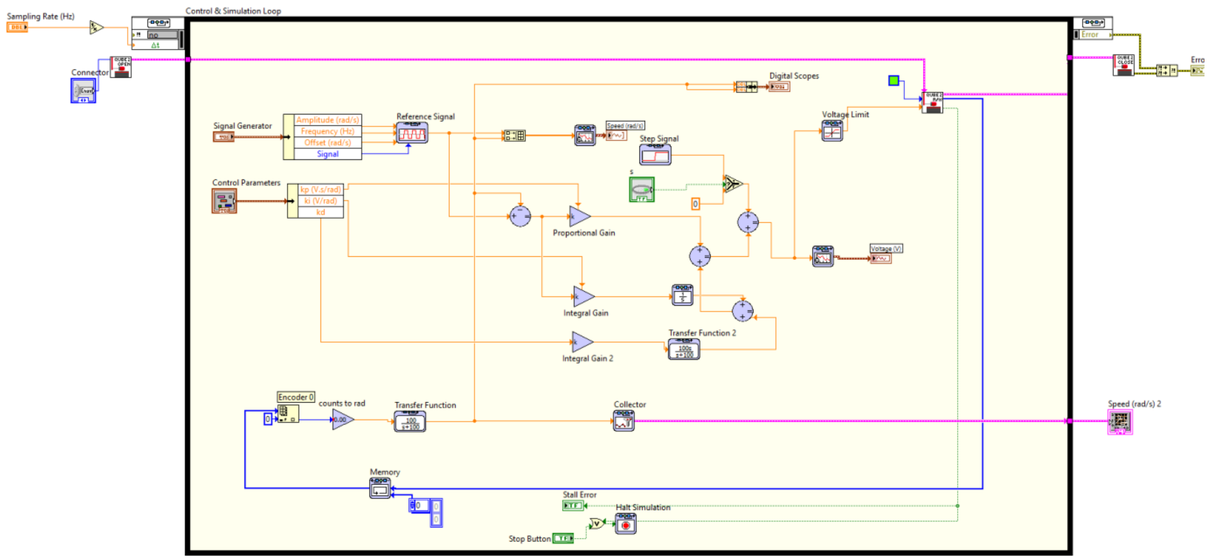


When all three parameters were varied, it can be observed that we were able to eliminate disturbance as done in task 3 of Q2



As these parameters were varied further, it can be seen that we get a steady state response equal to zero, which should precisely be the case.





8.6 Post-Lab Task

Task 5: To analyze disturbance rejection ability of PID for higher order disturbances

1. What will be the steady-state angular position in case of ramp disturbance with zero reference signal? Is the PID controller able to reject this disturbance completely?
2. What is the maximum order of disturbance T_d rejected by the position control system with PID controller?
3. If a system can reject a step disturbance, does this mean that the output will not be affected by any type of disturbance?

1. What will be the steady-state angular position in case of ramp disturbance with zero reference signal? Is the PID controller able to reject this disturbance completely?

The steady-state angular position in case of a ramp disturbance with zero reference signal will be a constant value that is proportional to the slope of the ramp disturbance. The PID controller will be able to reject the disturbance completely if the integral term of the PID controller is non-zero.

The PID controller output is calculated as follows:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d de(t)/dt$$

where:

u(t) is the controller output

e(t) is the error between the desired and actual angular position

K_p is the proportional gain

K_i is the integral gain

K_d is the derivative gain

The proportional term, $K_p e(t)$, responds to the current error. This means that the controller output will increase if the error is positive and decrease if the error is negative.

The integral term, $K_i \int_0^t e(\tau) d\tau$, responds to the accumulated error over time. This means that the controller output will continue to increase until the error is eliminated.

The derivative term, $K_d\, de(t)/dt$, responds to the rate of change of the error. This means that the controller output will increase if the error is increasing and decrease if the error is decreasing.

In the case of a ramp disturbance with zero reference signal, the error will be a linear function of time. The integral term of the PID controller will respond to the linear error and will cause the controller output to increase linearly. This will cause the system to track the ramp disturbance perfectly.

Therefore, the PID controller can reject a ramp disturbance with zero reference signal completely.

2. **What is the maximum order of disturbance Td rejected by the position control system with PID controller?**

The PID controller can reject any order of disturbance, but the performance of the controller will degrade as the order of the disturbance increases.

The PID controller is a linear controller, and linear controllers are not able to perfectly track non-linear disturbances. However, the PID controller can still be used to reduce the impact of non-linear disturbances on the system output.

The higher the order of the disturbance, the more difficult it is for the PID controller to track it. This is because the PID controller only has three parameters, and these parameters must be tuned to achieve a balance between tracking performance and disturbance rejection.

In general, it is good practice to choose a PID controller that has a bandwidth that is higher than the frequency of the most significant disturbance. This will ensure that the controller is able to track the disturbance and reduce its impact on the system output.

3. **If a system can reject a step disturbance, does this mean that the output will not be affected by any type of disturbance?**

No, if a system can reject a step disturbance, this does not mean that the output will not be affected by any type of disturbance.

A step disturbance is a sudden change in the system input. The PID controller is able to reject a step disturbance by using the integral term to eliminate the steady-state error.

However, other types of disturbances, such as ramp disturbances and sinusoidal disturbances, cannot be eliminated by the integral term. The PID controller can still be used to reduce the impact of these disturbances on the system output, but the performance of the controller will degrade as the order of the disturbance increases.

In general, a system that can reject a step disturbance will be more robust to disturbances than a system that cannot reject a step disturbance. However, the system will still be affected by disturbances, and the magnitude of the impact of the disturbance will depend on the order of the disturbance and the tuning of the PID controller.

Assessment Rubric

Lab 09

PID Controller, Tracking and Disturbance Rejection

Name: Afsah Hyder	**Student ID:07065**

Points Distribution

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Analysis</th><th>AR 6
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For details on rubrics, please refer to *Lab Evaluation Assessment Rubrics*.