Lab 12
Black-Box System Identification

12.1 Objective
To carry out black-box system identification for DC motor using frequency response

12.2 Mathematical Modelling using Frequency Response
A mathematical model for a system can be obtained by observing the internal structure of the system. For the DC motor, we used physical principles to obtain such a mathematical model. At times, a designer may not have access to the internal structure of a system or modeling the system using physical principles may be too complicated. In situations like this, the system can be characterized by observing how it responds to different frequencies or in other words constructing a frequency response of the system. The frequency response has two components: ***magnitude response* | *G*(*jω*)** |, which tells us how the system amplifies or attenuates different frequencies, and ***phase response* ∠*G*(*jω*)**, which tells us how the system delays certain frequencies.

We already know that if the input to a linear time invariant system is a sinusoid, then the steady state output is also a sinusoid of the same frequency but possibly with different amplitude and phase. Based on this fact, we can experimentally determine the frequency response. We provide a sinusoidal signal of a certain frequency (plus a DC offset to obtain equilibrium solution) and a fixed amplitude as input to a system and observe the steady state output which will also be a sinusoidal signal (plus a DC offset) of the same frequency. Measuring the ratio of magnitudes of this output and input will yield magnitude response and measuring the phase difference between input and output tells us about the phase response of the system. Repeating this procedure for frequencies of interest will result in desired frequency response.

Bode plot and cut-off frequency
Bode plot is graph of frequency response of a system. It is magnitude response (in dB) and phase response plotted against frequencies. Cut-off frequency is a boundary in system's frequency response where energy flowing through the system begins to be reduced. It is the point where gain reduces to -3dB in magnitude response.

12.3 Required Files

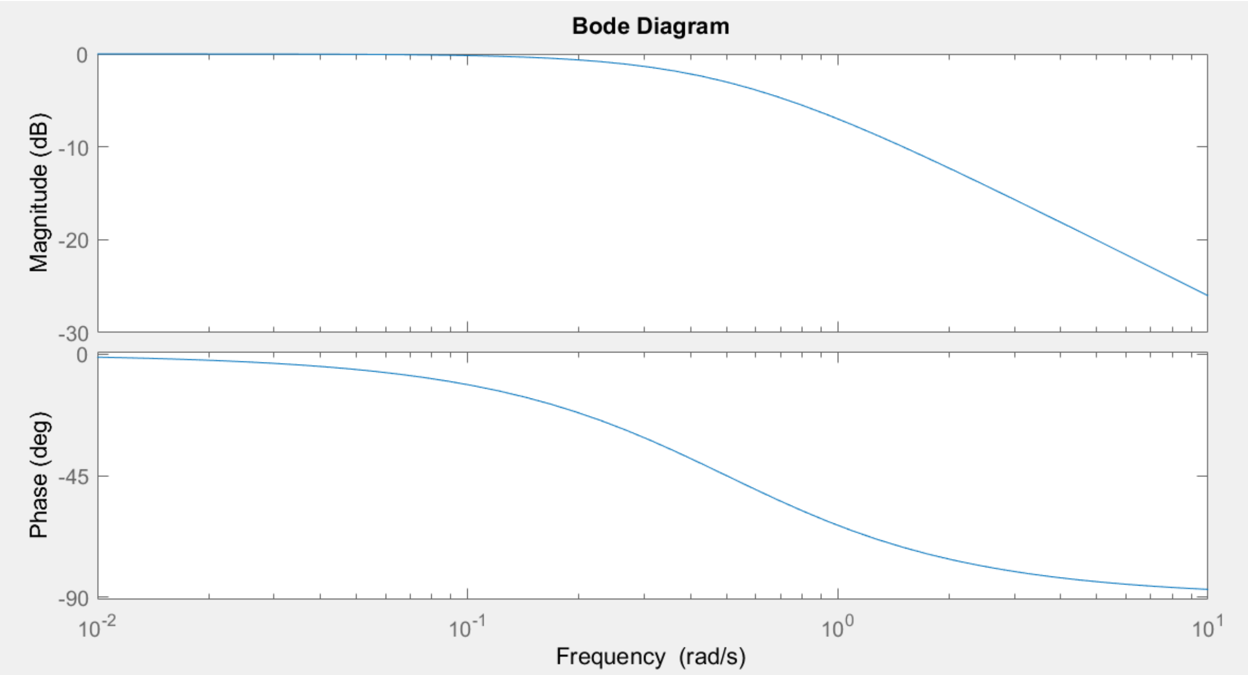
1.	LabVIEW with Control Design and Simulation Module
2.	DC Motor Modelling.vi with associated sub-VIs
3.	Frequency Response.vi with associated sub-VIs

Task 1: To obtain bode plot of the given system using MATLAB
The transfer function of a first order low-pass filter is of the form: $G(s) = \frac{\omega_c}{s + \omega_c}$

Where, $\omega_c$is called the cut-off frequency. A low-pass filter is implemented using RC circuit whose transfer function is found to be

$$G(s) = \frac{1}{RCs + 1} = \frac{1}{2s + 1}$$

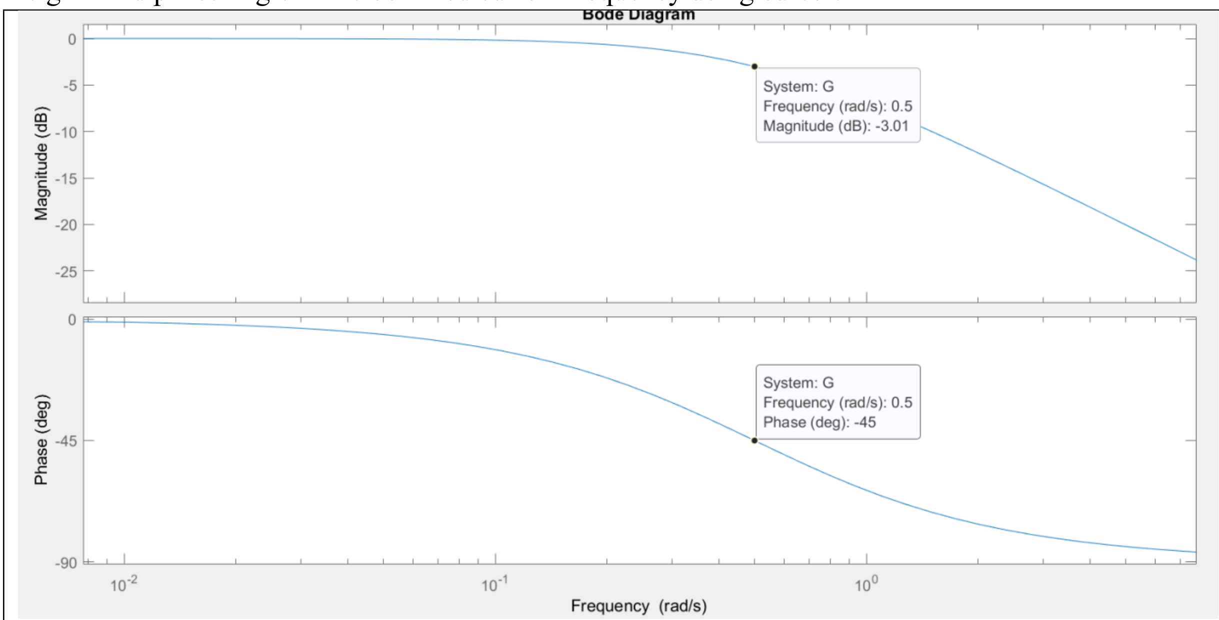
1. Obtain the bode plot (phase and magnitude frequency response) of the low-pass filter using MATLAB. Hint: search for ***bode( )*** command in MATLAB.



2. What is meant by cut-off frequency? Determine the cut-off frequency from the obtained bode-plot.

A cutoff frequency, also known as a corner frequency or break frequency, is a boundary in a system's frequency response at which energy flowing through the system begins to be reduced (attenuated or reflected) rather than passing through. In simpler terms, it is the frequency at which a filter starts to block or reduce the amplitude of certain frequencies. The cut-off frequency is 0.5 rad/s at the magnitude -3.

3. Determine phase at the cut-off frequency from the obtained phase response. Add the plots and highlight the gain and phase angle at the obtained cut-off frequency using cursor.



The phase is 45 degree at the cut-off frequency.

Task 2: To obtain the frequency response of DC motor

1. Modify the DC motor modelling VI to generate a square wave with the following input parameters:
   - *Amplitude: 0V*
   - *Frequency: 0Hz*
   - *Offset: 2V*

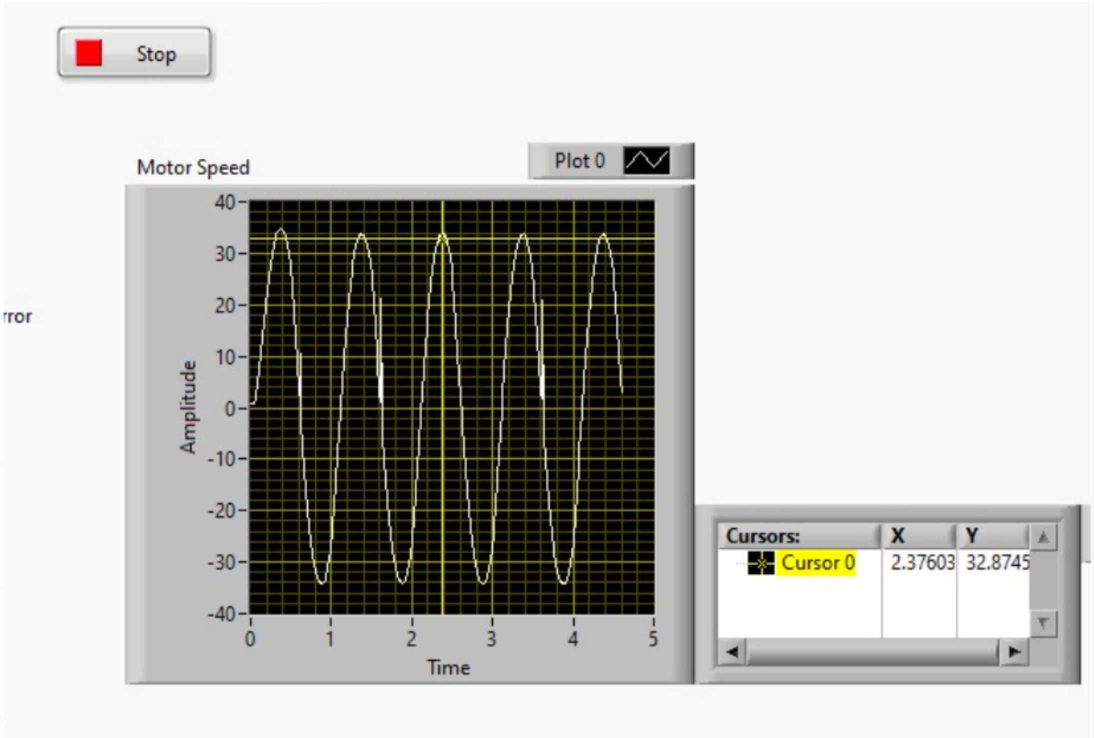
   This implies that a step-signal of 2V is being applied.
2. Excite the DC motor and observe the output till a steady state has been achieved. Find the peak amplitude of output using cursors. Note it in Table 1.
3. Proceed to measure the speed of the DC motor for different input frequencies of a **sinusoidal** signal, starting from 0.5 Hz up to 3 Hz in 0.5 Hz increments. The input parameters of signal generator are as follows:
   - *Amplitude: 2V*
   - *Frequency: 0.5Hz*
   - *Offset: 0V*

Table 1: Measurements for frequency response

Frequency (Hz)	**Output Speed** (rad/s)	**Gain** $\lvert G(j\omega)\rvert = \frac{Output}{Input}$	**Gain** (decibels) $\lvert G\rvert_{dB}$
0	47.57	23.785	27.53
0.5	41.09	20.54	26.25
1	32.87	16.43	24.297
1.5	27.53	13.76	22.77
2	23.583	1.791	21.43
2.5	20.202	10.101	20.087
3	17.125	8.56	18.65

4. Find the gain in dB using the expression $\lvert G\rvert_{dB} = 20log_{10}\,\lvert G\rvert$to complete Table 1.

For frequency =1



12.4 DC Motor Modelling by Frequency Response

The Bode plot can be used to determine the model of DC motor. Remember that the voltage- speed transfer function of DC motor is given by:

$$G(s) = \frac{K_f}{\tau_f s + 1}$$

We can use the obtained Bode plot to determine the values of K_f and τ_f. Substituting $s = j\omega$, we obtain the frequency response of the system as

$$|G(j\omega)| = \left|\frac{K_f}{\tau_f(j\omega) + 1}\right| = \frac{K_f}{\sqrt{1 + (\tau_f \omega)^2}}$$

The steady-state or DC gain of the model is: $K_f = |G(j0)|$

Since the cut-off frequency ω_c is defined as the frequency where the gain drops by 3dB or by $\frac{1}{\sqrt{2}}$ i.e.,

$$|G(\omega_c)| = \frac{1}{\sqrt{2}}|G(0)|$$

$$\frac{K_f}{\sqrt{1 + (\tau_f \omega_c)^2}} = \frac{1}{\sqrt{2}}|G(0)|$$

$$\frac{|G(0)|}{\sqrt{1 + (\tau_f \omega_c)^2}} = \frac{1}{\sqrt{2}}|G(0)|$$

Task 3: To obtain the mathematical model of the DC motor using its Bode plot

1. Use ***BodePlot.vi*** to plot the frequency response of the data collected in Table 1.
2. Determine the cutoff frequency from the Bode plot.
3. Using the above formulae find K_f and τ_f to complete Table 2.

Table 2: Parameters obtained from the Bode plot of DC motor

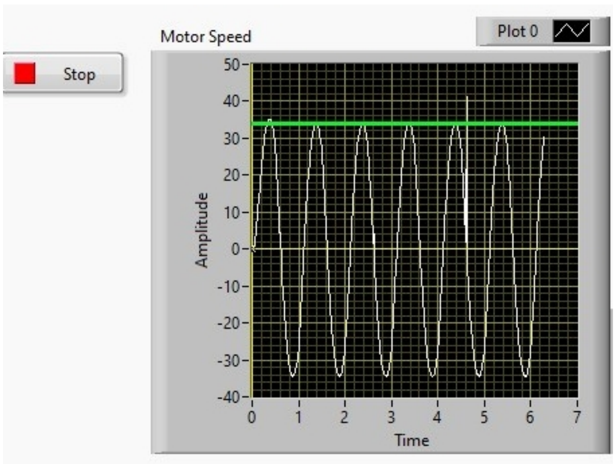
Parameters	Values
Cut-off frequency: • f_c (Hz) • ω_c (rad/sec)	Fc= 1.315Hz Wc=8.26 rad/s
Gain: K_f	23.785 (for Fc=0, wc=0)
Time constant: τ_f	0.121s

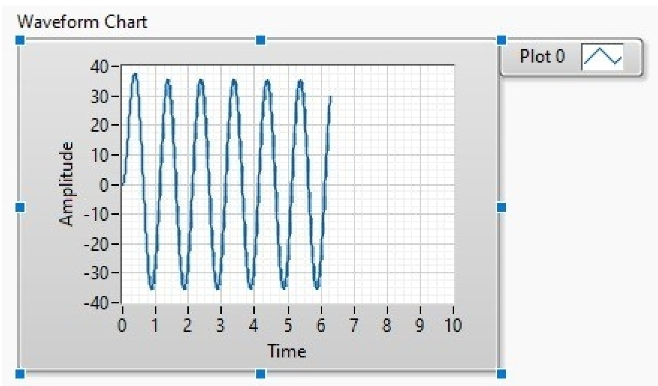
4. Write down the mathematical model of the DC motor obtained using its frequency response.

$$G(s) = \frac{23.785}{0.121s + 1} = \frac{196.57}{s + 8.26}$$

Task 4: To validate the obtained mathematical model for different inputs

1. In the DC motor modelling VI, add blocks to simulate the obtained mathematical model. To validate this model, give same input to the mathematical model and the DC motor and plot their responses on the same graph. Add plots to show validation.
2. Compare the measured and modeled responses.





At frequency = 1 we are getting 32.87 as our motor speed. If compared to our table 1 we are gettong 32.63

Assessment Rubric

Lab 12
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Points Distribution

Task No.	**LR2** Simulation	**LR4** Data Collection	**LR5** Plots	**LR 6** Calculations	**LR 10** Analysis	**AR 6** Class Participation
Task 1	4	4	4	-	-	-
Task 2	8	12	8	12	-	-
Task 3	8	4	8	8	4	-
Task 4	8	-	4	-	4	-
SEL	-	-	-	-	-	20
Course Learning Outcomes	CLO 1					CLO 4
Total Points	100					20

For details on rubrics, please refer to *Lab Evaluation Assessment Rubrics*.