Lab 11

Design of Analog Controller

11.1 Objective

To design, tune and implement analog controllers for modification of dynamic response of an RC circuit and investigate the control effort*.

11.2 Pre-Lab Task

Task 1: To find the transfer function of the given circuits

Derive the transfer function of the circuits given in figure 1, 2 and 3.

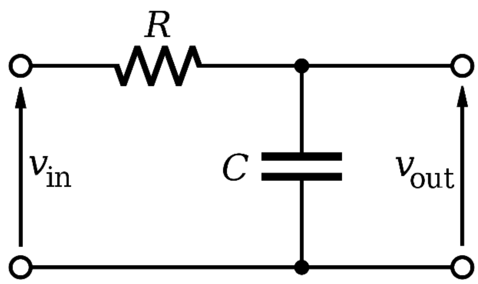


Figure 1: Series RC Circuit

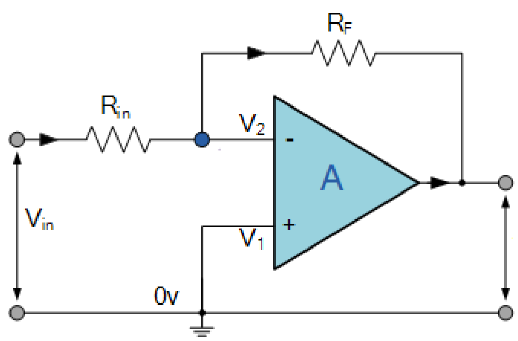


Figure 2: Op-amp with resistor in feedback path

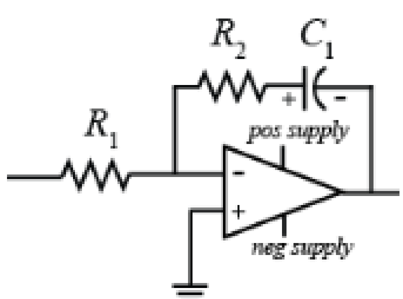


Figure 3: Op-amp with a resistor and capacitor in feedback path

11.3 Introduction

In previous labs, you have observed that how the response of a system can be altered according to the principles of feedback control by implementing controllers using software. In this lab, you will be implementing controller on hardware level using analog electronic components to control the response of a simple first order RC circuit given in figure 1. The output variable is the voltage across capacitor C. To modify the system's dynamic response in order to achieve the desired time-domain specifications, you will design and implement controller utilizing op-amps in different configurations.

The goal of this experiment is to obtain a control system with which the system's step response meets the following specifications:

1. *Settling time (2%) less than 1 second*
2. *Steady State Error = 0 (for step-input)*
3. *Peak time less than 0.350 seconds*
4. *Maximum overshoot less than 25 %*
5. *Maximum control effort less than 8 Volts*

11.4 Response of a First Order RC Circuit

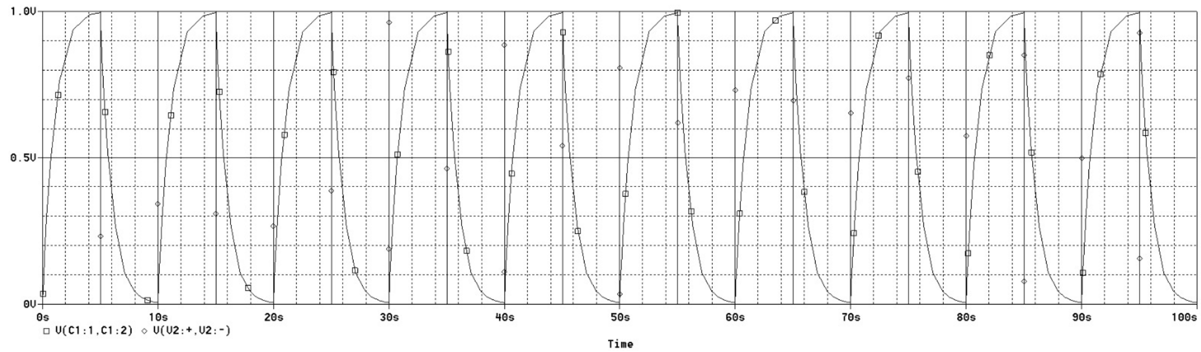
For the series RC circuit shown in figure 1 with input voltage Vi and output voltage Vo, the transfer function can be given as:

$$P(s) = \frac{1}{RCs + 1}$$

This is a first order circuit whose transient response can be characterized by only single parameter i.e. time constant 'τ'. Here time constant determines the speed of response and is given by product of resistance and capacitance (RC). The 2 % settling time of this circuit is four times the time constant (4τ).

Task 2: To implement the first order RC circuit and obtain its time-domain response

1. For the RC series circuit shown in figure 1 where R=2.2k Ω and C=470uF, estimate the time constant, 2% settling time, overshoot, peak time and steady-state error. In your estimations use the measured the value of resistor (i.e. 2.2 k Ω) with DMM.
2. Implement the circuit on breadboard, connect it with function generator to give 1 Volt peak-peak square pulse of frequency 0.1 Hz. Observe the input and output waveforms using oscilloscope and measure the response characteristics to complete table 1.



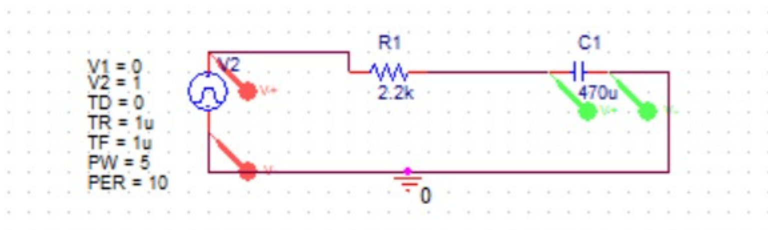


Table 1: Time-domain response of the plant in open loop

Parameters	Measured	Estimated
Time Constant	1.2s	1.043s
Settling Time (2%)	4.8s	4.136s
% Overshoot	0	0
Peak Time	4.8s	4.136s
Steady-state error	0	0

11.5 Design of Analog Controller

To achieve the response of our plant faster to meet the desired response specifications, one solution would be to modify the circuit components; the capacitor and resistor. The other solution is to modify the system's behavior without modifying the physical parameters of the plant is to add a feedback control to the system so that the performance becomes robust to the conditions under which the circuit operates and the noise or disturbances added to the system.

Figure 4 shows the plant P(s) in unity feedback control loop with a controller C(s). Here, E(s) is the error signal, U(s) is the control signal, R(s) is the input and Y(s) the output. The amplitude of this control signal represents the *control effort*.

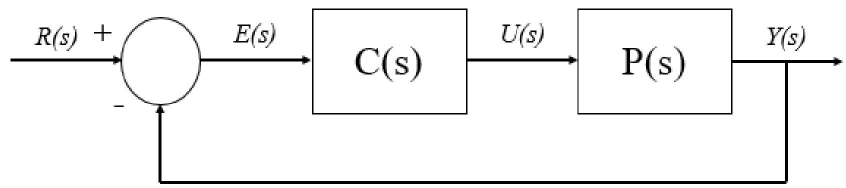


Figure 4: Unity feedback control

The response of open loop plant has been observed. A controller is now needed to modify the response. We will be implementing a proportional controller and a proportional-integral controller to investigate the effect of controller gains on system's response.

Task 3: To implement a proportional controller using operational amplifiers

1. Obtain the closed loop transfer function of system (Figure 04) with a P-controller i.e. C(s)= Kp. where Kp is gain of proportional controller.

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$= \frac{C(s)\cdot P(s)}{1+C(s)P(s)H(s)}$$

unity feedback means $H(s) = 1$

$C(s) = K_p$ for proportional controller

$$P(s) = \frac{1}{RCs+1}$$

Therefore $$T(s) = \frac{K_p/(RCs+1)}{1+K_p/(RCs+1)} = \frac{K_p/(RCs+1)}{1+\frac{K_p}{RCs+1}}$$

$$= \frac{K_p}{RCs+1+K_p}$$

$$= \frac{K_p/RC}{s+1/RC+K_p/RC}$$

To operate the plant using negative feedback and a P-controller, we need a voltage subtractor to obtain the error signal from the difference of input and output and a gain block to implement the P-controller. This can be done using op-amps in different configurations. Figure 5 shows an op-amp as subtractor whose output is the difference of the two voltages V_1 and V_2 with unity gain i.e. $V_{out} = V_2 - V_1$

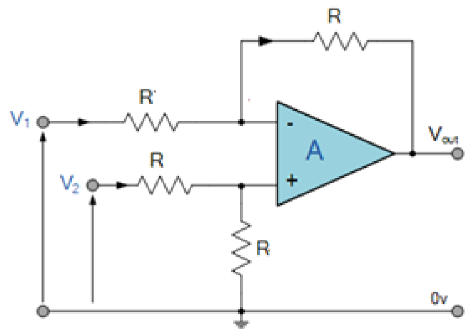


Figure 5: Op-amp as voltage subtractor

To implement a P-controller we can use another op-amp in inverting configuration as given in figure 2 whose gain has been obtained in the pre-lab task.

$$\frac{V_{out}}{V_{in}} = -\frac{R_f}{R_{in}}$$

We will be implementing the circuit of figure 6 to operate the plant with unity-feedback and P controller. The difference amplifier gives the error signal as output; $e = r - y$. The P-controller is implemented by an inverting amplifier with a gain of K_p.

Where, $K_p = \frac{R_2}{R_1}$

The output v_o of this amplifier will be given as; $vo = -e * \frac{R2}{R1}$

The inverting amplifier following the P-controller is implemented to cancel the negative gain obtained by the P-controller. The gain of this inverting amplifier is -1.

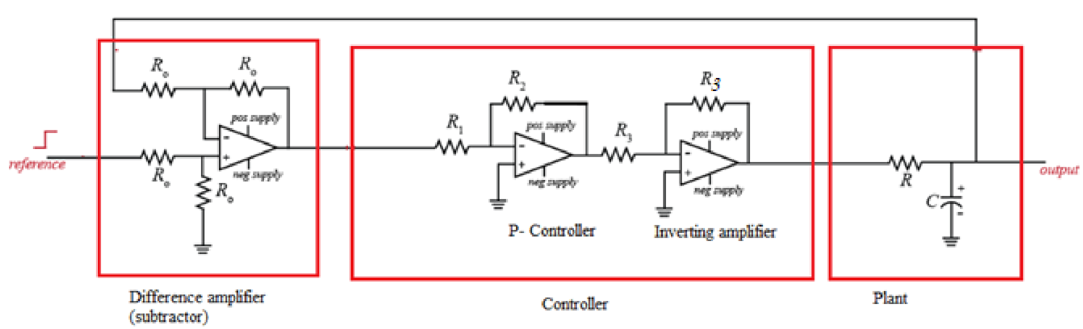


Figure 6: Plant with P controller

2. Implement the circuit shown above by using the schematic given in figure 7. Here, two ICs are used. LM741 is an operational-amplifier and LM-358 is a dual operational-amplifier IC. Two amplifiers of LM-358 are used to implement the controller and the amplifier of LM-741 is used to implement the subtractor. Refer the data sheets of the two ICs to explore further. The component values are listed below.
   - Ro = 5.6 k Ω
   - R1= 10 k Ω
   - R2= 100 k Ω (variable resistor)
   - R3= 1k Ω

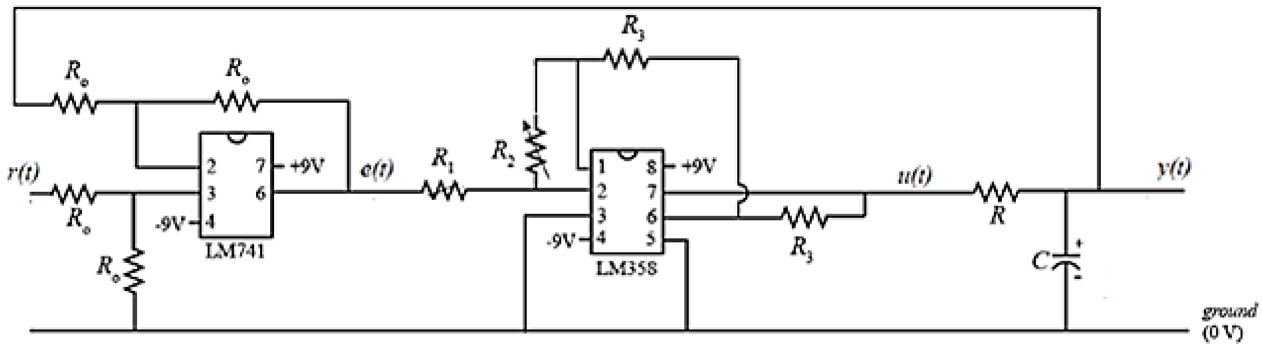


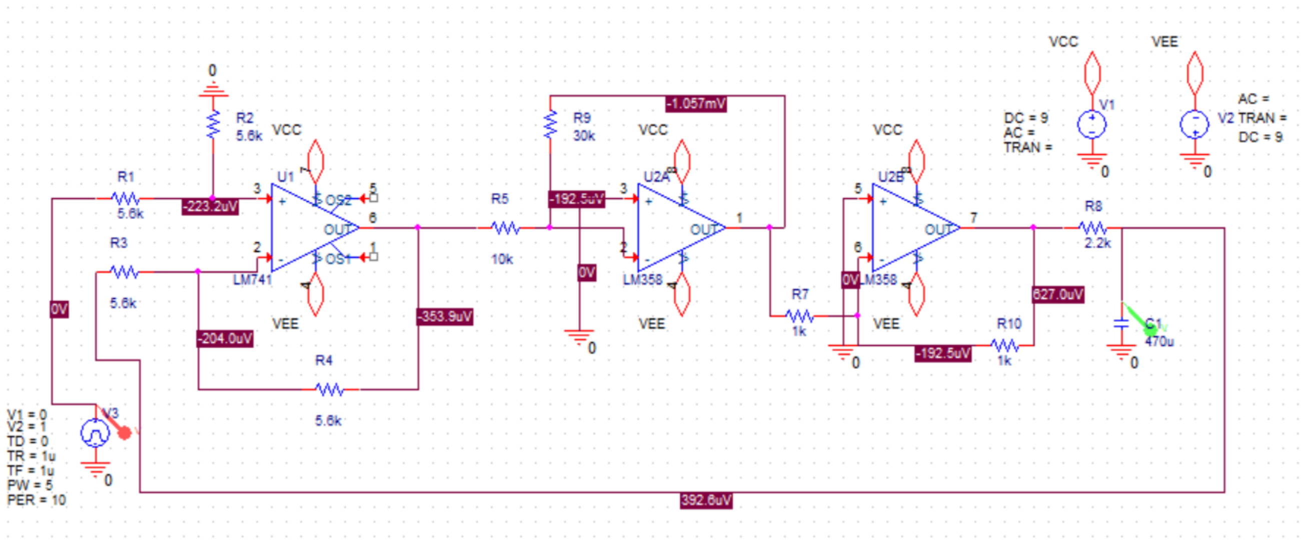
Figure 7: Schematic for the system with P controller

To give +9V and –9V supply voltages to the ICs, operate the DC power supply in series by selecting the second option out of the given three on the equipment (Independent, Series and Parallel). The common terminal of the 2 supplies in series is the common ground of the circuit. The input is supplied to the circuit from function generator. Set the function generator to give 1 Vpp at 0.1 Hz frequency.

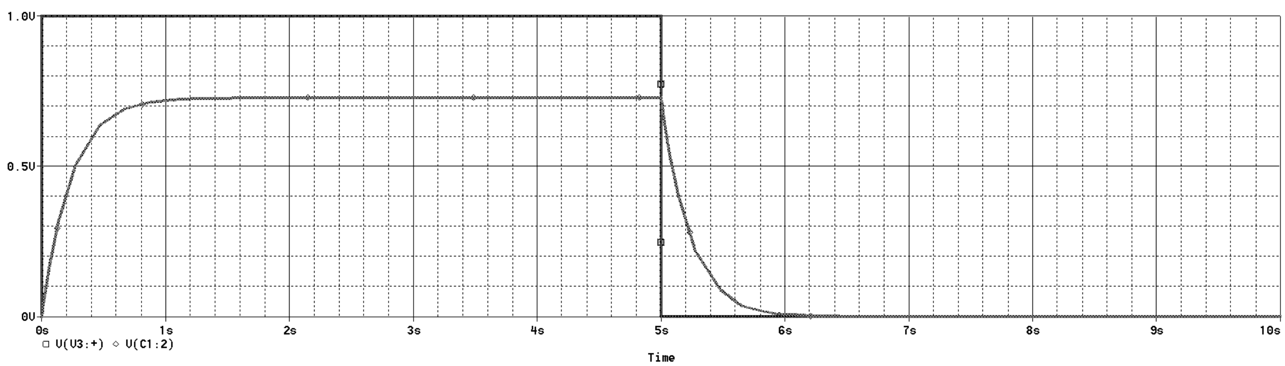
3. Observe the output and input waveforms. Vary the controller gain Kp using the variable resistor R_2. Also observe the control signal u(t). Complete table 2 for two different values of Kp.

Table 2: Time-domain response for different values of P-controller gain

Parameters	**Kp = ___3__________** **R1= __10K_________** **R2= 30K**	**Kp = _1____________** **R1= _10k____________** **R2= 10k**
Settling Time (2%)	1.2881s	1.994s
% Overshoot	0%	0%
Peak Time	1.689s	2.327
Steady-state error	27.5%	50%
Maximum Control Effort	3V	1V



For Kp = 3



4. Analyze the results you have obtained and discuss the impact of *Kp* on the time-domain response parameters.
5.

Ans: As Kp increases, the following changes can be observed:
> The steady-state error decreases. This is because the controller is more aggressive in correcting the error between the desired and actual output.
> The peak time decreases. This is because the controller applies a larger control signal, which causes the system to respond more quickly.
> The settling time decreases. This is because the system reaches its steady-state value more quickly.

However, it is important to note that increasing Kp too much can also have negative consequences. For example, it can cause the system to become unstable and oscillate.

Task 4: To design a proportional-integral controller to obtain the desired time-domain response

1. The response of circuit has been observed in open loop and in closed loop with a P-controller. To modify the response in order to meet the specifications given in section 11.3, we will be designing and implementing a PI controller now. The formulae for peak time, settling time, overshoot and damping ratio are given below.

$$t_s = \frac{4}{\sigma} \qquad t_p = \frac{\pi}{\omega_d}$$

$$\%OS = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100 \qquad \zeta = \cos(\beta)$$

2. Use the given formulae to find the range of σ, ζ, ω_d and β that satisfy the required specifications.

Task 4:

$$t_s = \frac{4}{\sigma} \qquad t_p = \frac{\pi}{\omega_d}$$

$$\%OS = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100 \qquad \zeta = \cos(\beta)$$

$$t_s = \frac{4}{\sigma} < 1 \qquad t_p = \frac{\pi}{\omega_d} < 0.350$$

$$\sigma > 4 \qquad \omega_d > \pi/0.35$$

$$\omega_d > 8.98$$

$$\%OS = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100 < 25$$

$$e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} < 0.25$$

$$\frac{-\pi\zeta}{\sqrt{1-\zeta^2}} < \ln(0.25)$$

$$\frac{\zeta}{\sqrt{1-\zeta^2}} < \frac{\ln 0.25}{-\pi}$$

$$\left(\frac{\zeta}{\sqrt{1-\zeta^2}} > 0.44\right)^2$$

$$\zeta^2 > 0.195(1-\zeta^2)$$

$$1.195\,\zeta^2 > 0.195$$

$$\zeta^2 > 0.16$$

$$\zeta > \sqrt{0.16} > 0.4$$

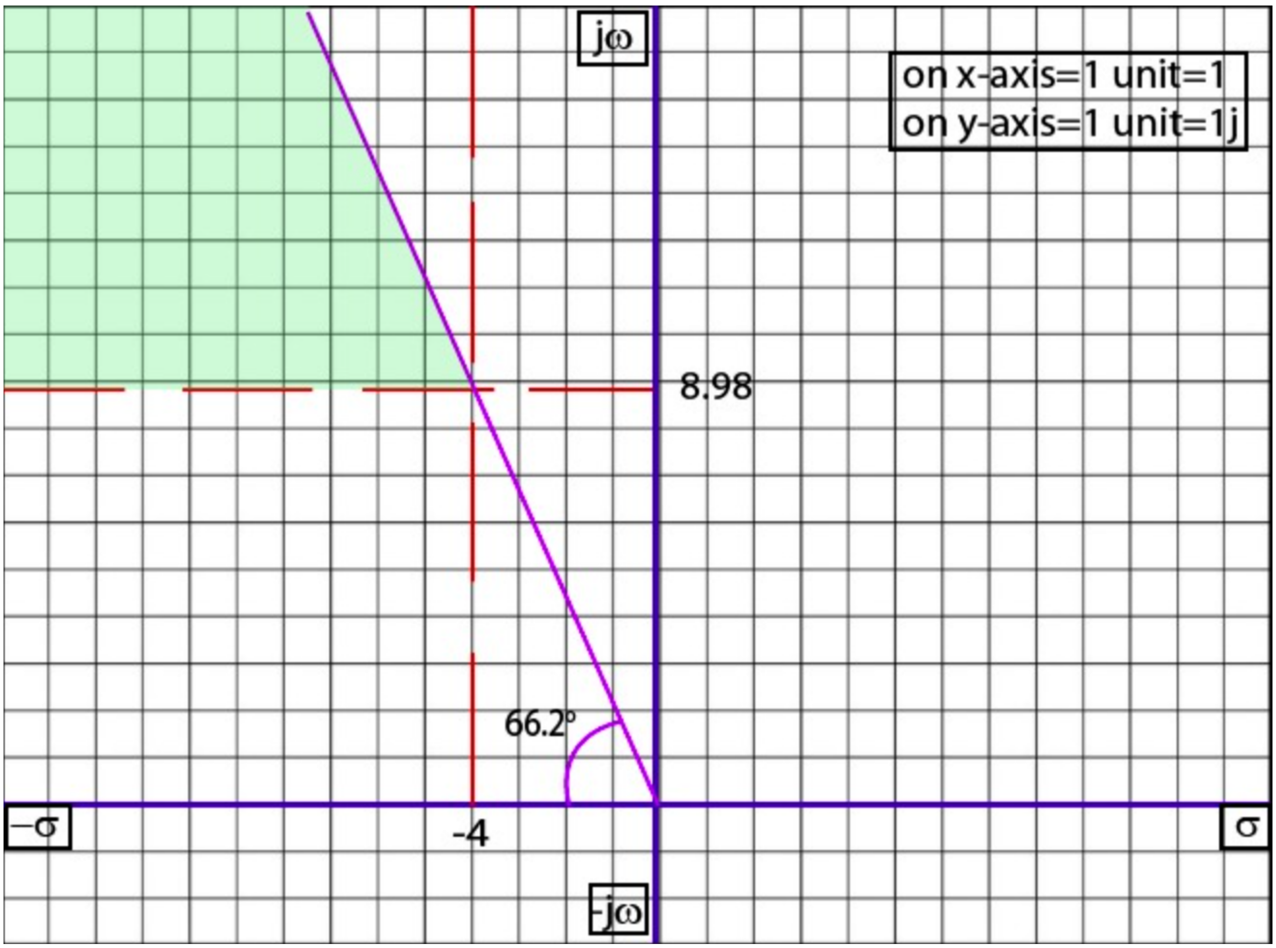
$$\zeta = \cos(\beta) < 0.4$$

$$\beta = \cos^{-1}(\zeta)$$

$$\beta < \cos^{-1}(0.4)$$

$$\beta < 66.2^\circ$$

3. In order to meet above constraints (i.e. on σ_d, ω_d *and* β), draw the region on graph paper below and highlight the area in the s-plane where closed loop poles should be to achieve the desired performance. Label properly the axes and scale of the graph.



4. From this region, choose the poles to meet the requirements with greater margin (choosing greater value of σ_d and ω_d to shift poles more towards the left side), the only drawback is that making the system "faster" (smaller settling time and smaller peak time) tends to come at the cost of increased control effort. Considering this, what poles would you select that lie in the obtained range and doesn't require much control effort?

Selected closed loop transfer function poles = <u>-5±10j</u>

5. Obtain the transfer function of the closed loop plant (see Figure 4) with a PI controller given below.

For PI controller: $C(s) = \frac{K_p s + K_i}{s}$

Where *Kp'* is proportional gain and *'Ki' is* the integral controller gain. Compare it with the standard second order characteristic equation to express *Kp* and *Ki*.

6. Calculate the value of Kp and Ki for the poles selected to meet the desired specifications.
   Hint: $\sigma_d = \zeta\omega_n$ and $\omega_n^2 = \sigma_d^2 + \omega_d^2$

R=2.2k Ω and C=470uF (taken from task 2)

Ki = ωn^2 * R * C = 125 * 2.2k * 470μ = 129.25

Kp = (R*C*2*ζ*ωn) -1 = (R*C*2*σd)-1 = (2.2k*470μ*2*5)-1 = 9.34

Task 5: To implement the designed PI controller

The PI controller can be implemented using the op-amp in the configuration given in figure 8.

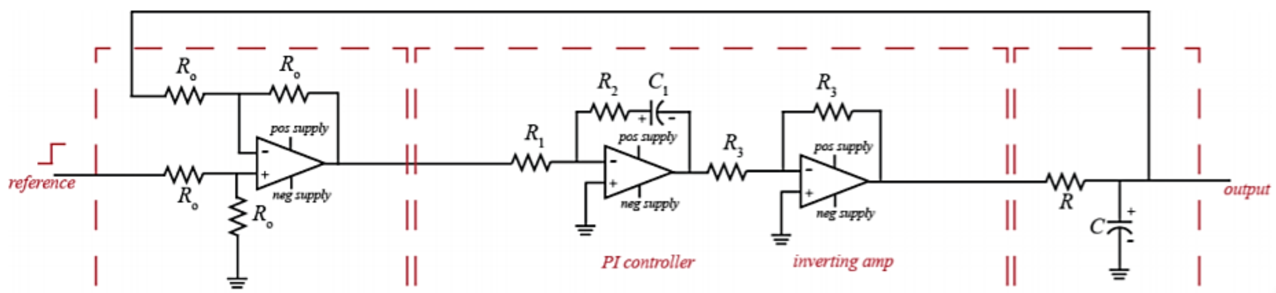


Figure 8: Implementation of analog PI controller

1. The controller gains in terms of the circuit components are given below. These can be verified from the transfer function obtained in the pre-lab task.

$$Ki = \frac{R_3}{C_1 R_1 R_3} = \frac{1}{C_1 R_1}$$

$$Kp = \frac{R_2 R_3}{R_1 R_3} = \frac{R_2}{R_1}$$

   Only R_1 and R_2 are variable resistors here. R_3 is equal to 1kΩ. Let, C_1= 1uF. Then, calculate the value of R_1 and R_2 for the values of *Kp* and *Ki* calculated above.

   R_1= 7.72k Ω R_2= 72.12k Ω

2. Implement the circuit given in figure 9. Use the same circuit of P-controller just insert capacitor in series with R_2 and replace R_1 with variable resistor. Set R_1 and R_2 as calculated above.

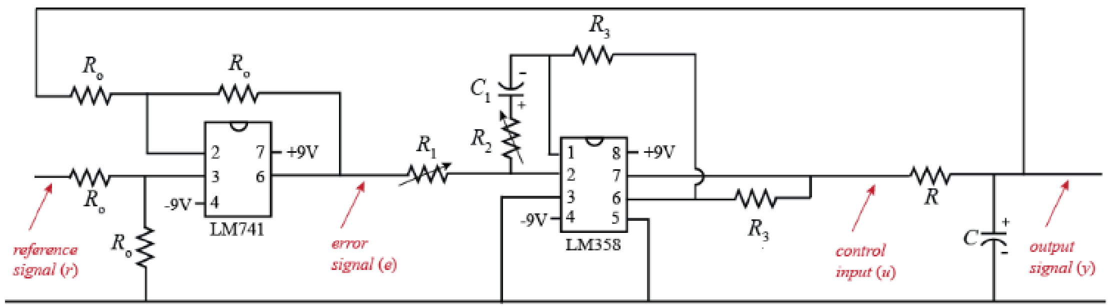
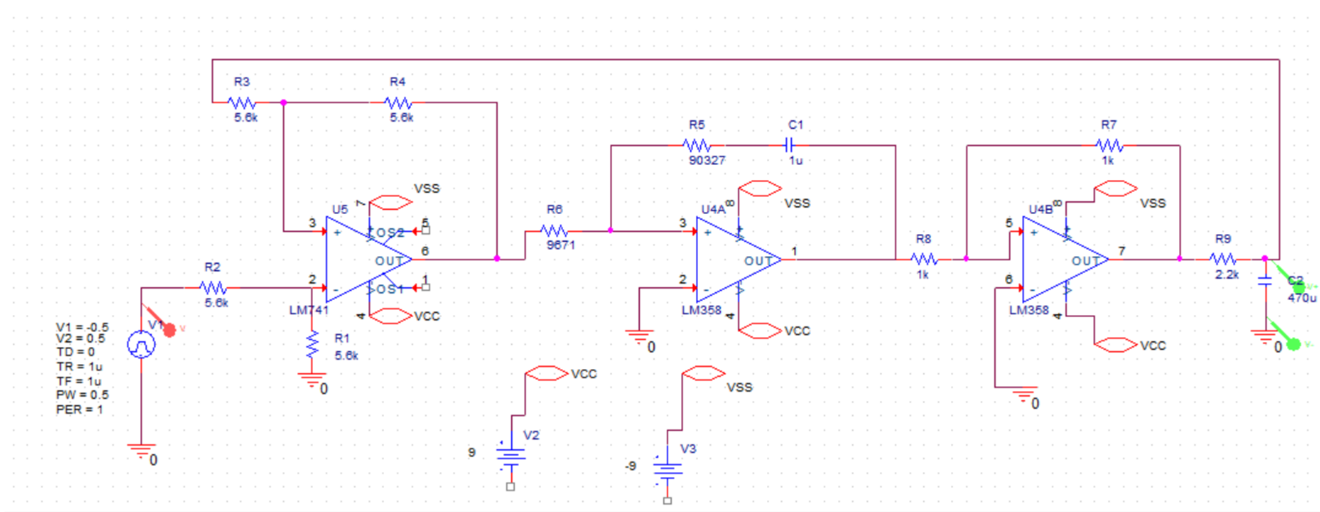
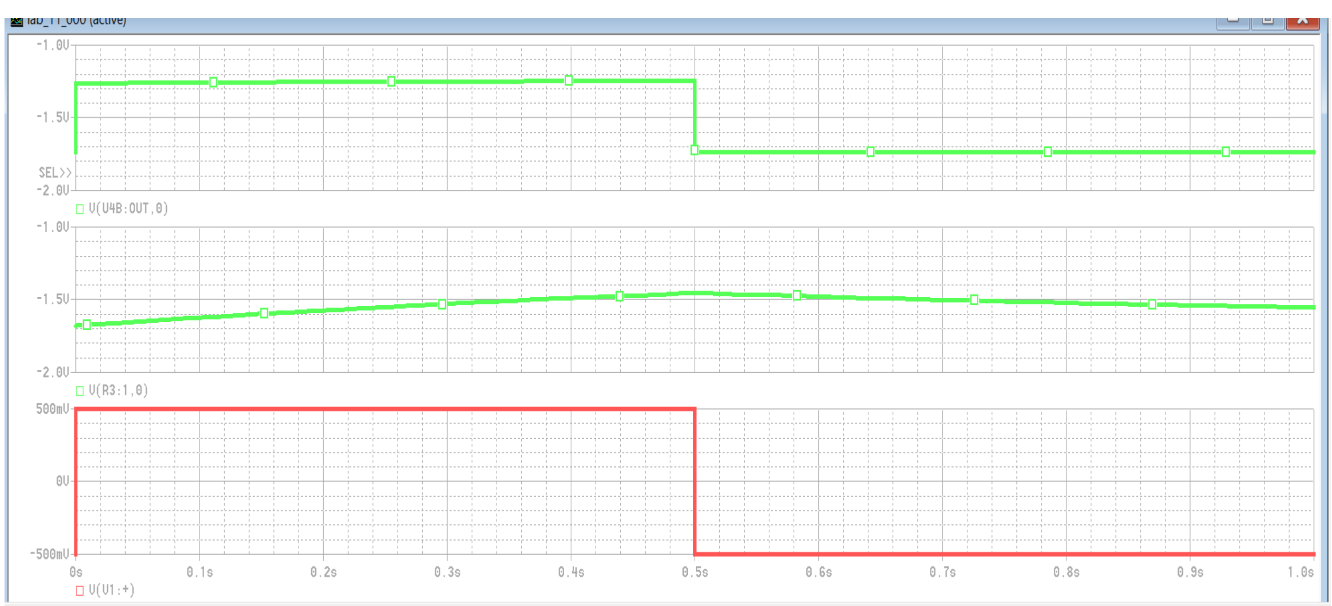


Figure 9: Schematic for implementation of PI controller

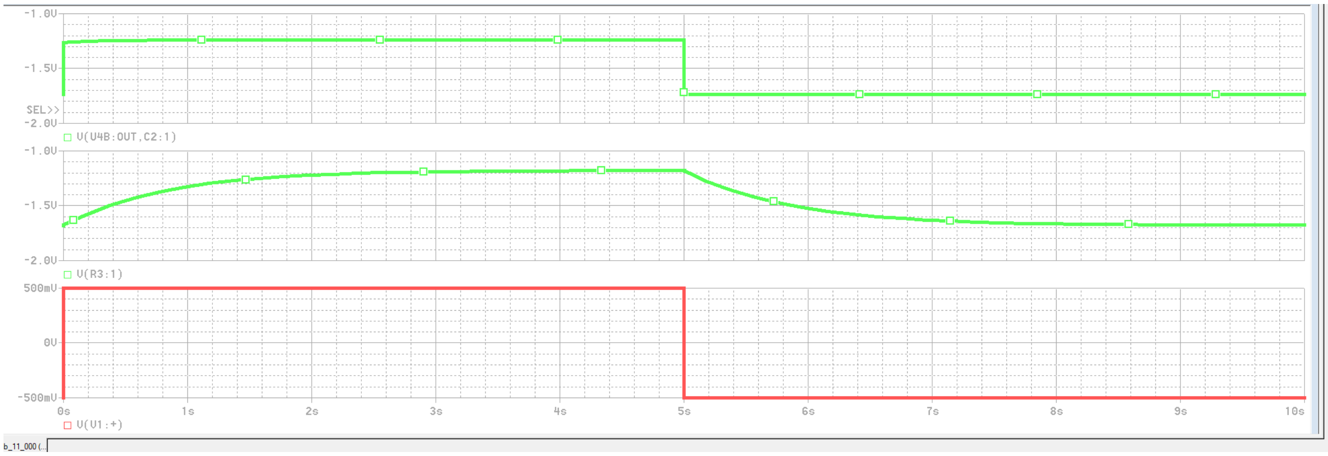




Red one is input, middle one is output and upper one is control signal

It should be noted that we have used frequency of 1Hz

If frequency taken as 0.1 Hz, we will get



but we measured value when considered frequency of 1 Hz because estimated and measured values are then showing some correlation

3. Observe the waveforms of input, output and control signals. Complete table 3 with the measured results and estimated results.
Note: Estimated values are calculated from the location of selected poles.

Table 3: Table 4: Time-domain response specifications of system with PI controller

Parameters	Estimated	Measured
Settling Time (2%)	0.8	0.817
% Overshoot	16.29%	11.11%
Peak Time	0.349	0.488
Steady-state error	0	1.17
Maximum Control Effort		1V

Estimated values

Settling time $= \frac{4}{\sigma} = \frac{4}{5} = 0.8$ sec

%overshoot, zeta = 0.5

$= e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100 = e^{-\pi(0.5)/\sqrt{1-0.5^2}} \times 100 = 16.29\%$

Peak time $= \pi/\omega_d = \pi/9 = 0.349$ sec

Steady state error

$E(s) = R(s)[1 - T(s)]$

$e(\infty) = \lim_{s\to 0} sE(s)$

$= \lim_{s\to 0} s\left(R(s)(1-T(s))\right) \rightarrow (i)$

$sR(s)(1-T(s)) = \frac{1}{s} \times s\left(1 - \frac{k_p s + k_i}{RCs^2 + s + k_p s + k_i}\right)$

$= \frac{RCs^2 + s + k_p s + k_i - k_p s - k_i}{RCs^2 + s + k_p s + k_i}$

$x = \frac{RCs^2 + s}{RCs^2 + s + k_p s + k_i}$

By (i)

$e(\infty) = \lim_{s\to 0} x \Rightarrow 0/k_i = 0$

Estimated control effort = e^(R1/R2) = 1.11V

Measured values

It is not possible to take freq 1 the measuring time period = 1 sec as well

C, Now

settling time = 0.817 sec

peak time = 0.488 sec

$$OS\% = \frac{1.41 - 1.269}{1.269} \times 100$$

$$= 11.11\%$$

Steady state error =

$$E = C(s) - R(s)$$

$$= -1.67 + 0.5$$

$$= \cancel{1.07}\ 1.1$$

4. Compare the estimated and measured results. Analyze why the measured results deviate from the estimated ones. How accurate is this method of **tuning** the PI controller?

The measured results deviate from the estimated ones for the following reasons:

> The model of the system used to estimate the PI controller parameters is not perfect. This means that the model does not capture all of the dynamics of the real system.
> There are disturbances acting on the system that are not accounted for in the model.
> The PI controller is a linear controller, but the real system may be nonlinear. This means that the controller may not be able to perfectly control the system.
In general, the method of tuning the PI controller used in this case is quite accurate. The measured results

are close to the estimated results, and the system is able to settle to its steady-state value within a reasonable amount of time. However, there is still some deviation between the estimated and measured results, which is likely due to the factors mentioned above.

The accuracy of this method of tuning the PI controller can be further improved by using a more accurate model of the system, identifying and compensating for disturbances acting on the system, and using a nonlinear controller.

Overall, the method of tuning the PI controller used in this case is a good and effective method. It is simple to implement and can provide good results for a wide range of systems.

5. What is the maximum value of control effort? Is it less than 8 volts as initially required?

Yes the control effort is at 1V, which is less than 8V.

Task 6: Post-Lab Task

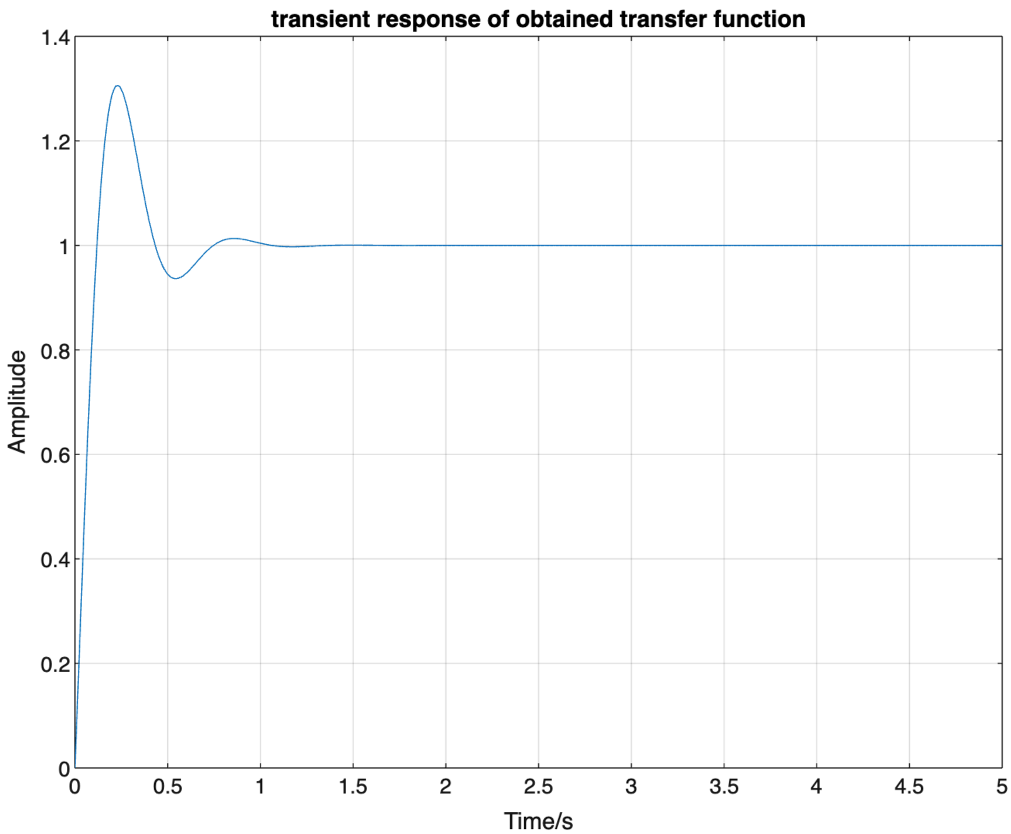
Obtain the step response of the closed-loop transfer function of the system implemented in Task 5 using MATLAB. Compare the MATLAB simulation results with the results obtained experimentally.

```
>> tf1 = tf([9.34 129.25], [2200*0.00047 10.34 129.25])

tf1 =

        9.34 s + 129.2
  ---------------------------
  1.034 s^2 + 10.34 s + 129.2

Continuous-time transfer function.
Model Properties
>>
```



The response obtained via MATLAB by directly inputting the transfer equation, showed a better response than the circuit simulation in task 5, with lesser steady state error.

Reference:

*http://ctms.engin.umich.edu/CTMS/index.php?aux=Activities_RCcircuitC

Assessment Rubric

Lab 11
Design of Analog Controllers

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Points Distribution

Task No.	**LR1** **Circuit Layout**	**LR2** **Simulation**	**LR4** **Data Collection**	**LR5** **Results**	**LR 6** **Calculations**	**LR 10** **Analysis**	**AR 6** **Class Participation**
Task 1	-	-	-	-	-	12	-
Task 2	4	-	4	-	-	2	-
Task 3	12	-	8	4	-	8	-
Task 4	-	-	-	4	8	2	-
Task 5	4	-	8	-	2	8	-
Task 6	-	4	2	2	-	2	-
SEL	-	-	-	-	-	-	/20
Course Learning Outcomes	CLO 3						CLO 4
Total Points	/100						/20
	/120						

For details on rubrics, please refer to *Lab Evaluation Assessment Rubrics*.