Lab 07
Transient Response Characteristics and Proportional Controller

7.1 Objective

To determine the transient response characteristics, design a proportional controller to meet the given transient response requirements for a system and investigate the effect of poles on the second-order system response

7.2 Pre-Lab Task

Task 1: Answer the following

1. Write a mathematical expression as well as sketch the unit-step response of a first order system with transfer function $\frac{K}{\tau s + 1}$.
2. Are these two parameters K and τ enough to describe the step response of any first-order system?
3. What is meant by 'Zeros' and 'Poles' of a transfer function?
4. The transfer function of a general second-order system can be written as $\frac{b}{s^2 + as + b}$.

   Draw a table relating location of poles of transfer function to the different possible shapes of the unit-step response. Are there any names given to these different shapes?
5. Define the terms 'Natural Frequency,ω_n' and 'Damping Ratio,ζ' for a second-order system.

TASK 1

1) we have transfer function

$G(s) = \frac{K}{\tau s+1}$

If we give step input to this transfer function unit step function in frequency domain can be

$Y(s) = \frac{1}{s} G(s) = \frac{K}{s(\tau s+1)}$ {Considering there is no gain and delay}

Now taking inverse laplace

$Y(s) = K\left(\frac{1}{s(\tau s+1)}\right)$

$\mathcal{L}^{-1} Y(s) = K\left\{\mathcal{L}^{-1}\left(\frac{1}{s} - \frac{1}{s+\frac{1}{\tau}}\right)\right\}$

$y(t) = K\left(1 - e^{-t/\tau}\right)$

If there is some initial value at t_0

$y(t) = K\left(1 - e^{-t/\tau}\right) + y(t_0)$

$\frac{K}{s(\tau s+1)} = \frac{A}{s} + \frac{B}{\tau s+1}$

$K = A(\tau s+1) + Bs$

$K = A\tau s + A + Bs$

$A = K$

$A\tau s + Bs = 0$

$K\tau s + Bs = 0$

$B = -K\tau$

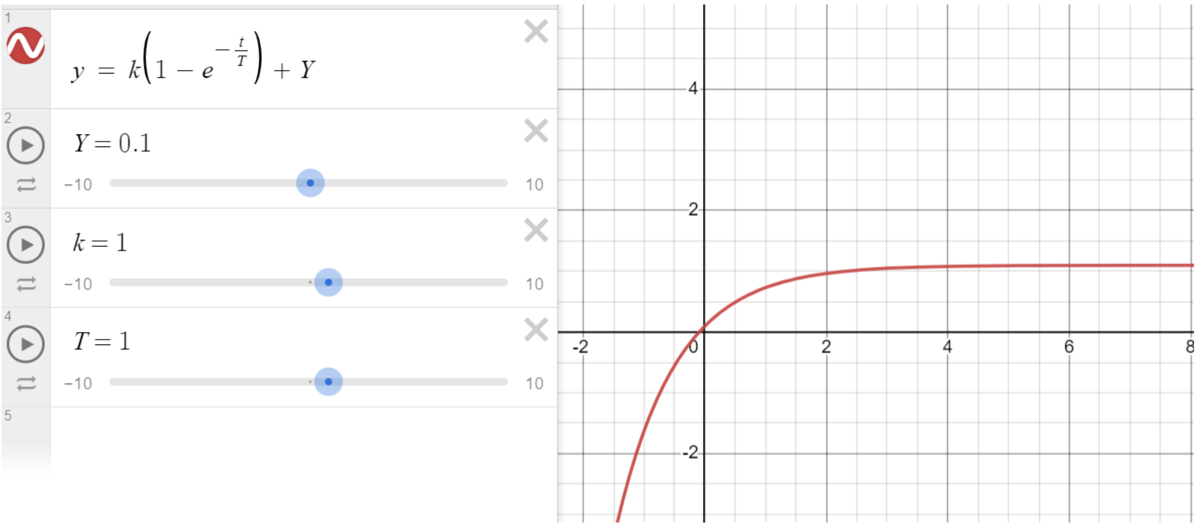
$= \frac{K}{s} + \left(\frac{-K\tau}{\tau s+1}\right)$

$= K\left(\cancel{\frac{1}{s}} + \cancel{\frac{\tau}{1-\tau s}}\right)$

$= K\left(\cancel{\frac{1}{s}} + \cancel{\frac{1}{\frac{1}{\tau}-s}}\right)$

$= \cancel{\frac{K}{s}}\, K\left(\cancel{\frac{1}{s}} - \cancel{\frac{1}{s+\frac{1}{\tau}}}\right)$

$= \frac{K}{s} - \left(\frac{K\tau}{s-(-1/\tau)}\right)$



Unit step response of first order system of given transfer function

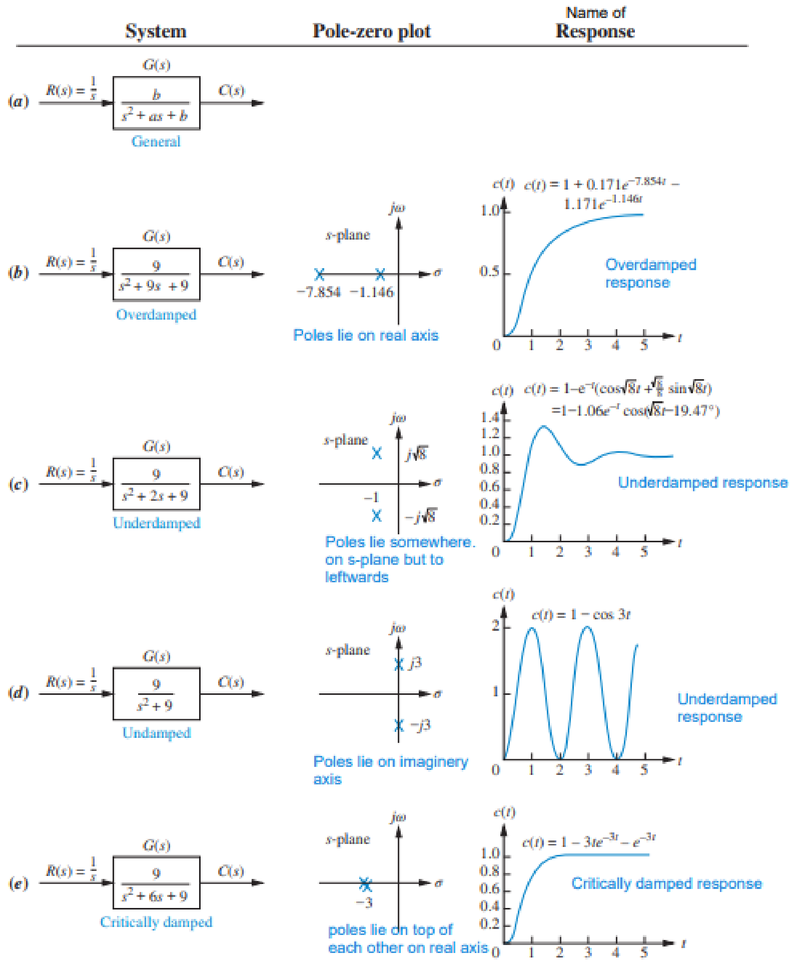
(2) If we know values of K and T, that means we know the transfer function, and response of system can be found by taking inverse Laplace of transfer function. So knowing these two parameters are enough.

(3) Zeros can be described as roots of polynomial of numerator of a transfer function and poles are defined as roots of the denominator of a transfer function.

(5) Damping which can be describe as a measure of how rapidly the oscillations decay from one bounce to next. Also, it is a dimensionless measure, so when any disturbance occur which cause oscillation within system to decompose can be given by this damping ratio. Given by $\zeta = \frac{1}{2\pi}\left(\frac{\text{natural period}}{\text{Exponential time constant}}\right)$

Natural frequency

It is the frequency of oscillation of the system without damping. It can be represented as ω_n



4)

7.3 Required Files

LabVIEW with Control Design & Simulation	
Transient-Characteristics.vi	
QUBE Servo	**QNET DC Motor**
NI myRIO 15.0	NI DAQmx
QUBE-Servo-2.lvproj	NI ELVISmx
QUBE-Servo 2 Second Order.vi with sub-VIs	QNET-Motor-Second-Order.vi with sub-VIs

7.4 Characterization of transient response of second-order systems

The standard second-order transfer function has the form

$$\frac{Y(s)}{R(s)} = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2} \quad \text{...(1)}$$

Where, ω_n is the natural frequency and ζ is the damping ratio of the system. As seen in the prelab, the properties of response depend on poles of the system. Looking at the transfer function above, this means that response correspondingly depends on the values of the parameters ω_n and ζ.

If a second-order system is underdamped, then the system response obtained is similar to the one illustrated in Figure 1 when a step input, $R(s) = \frac{R_0}{s}$ is provided.

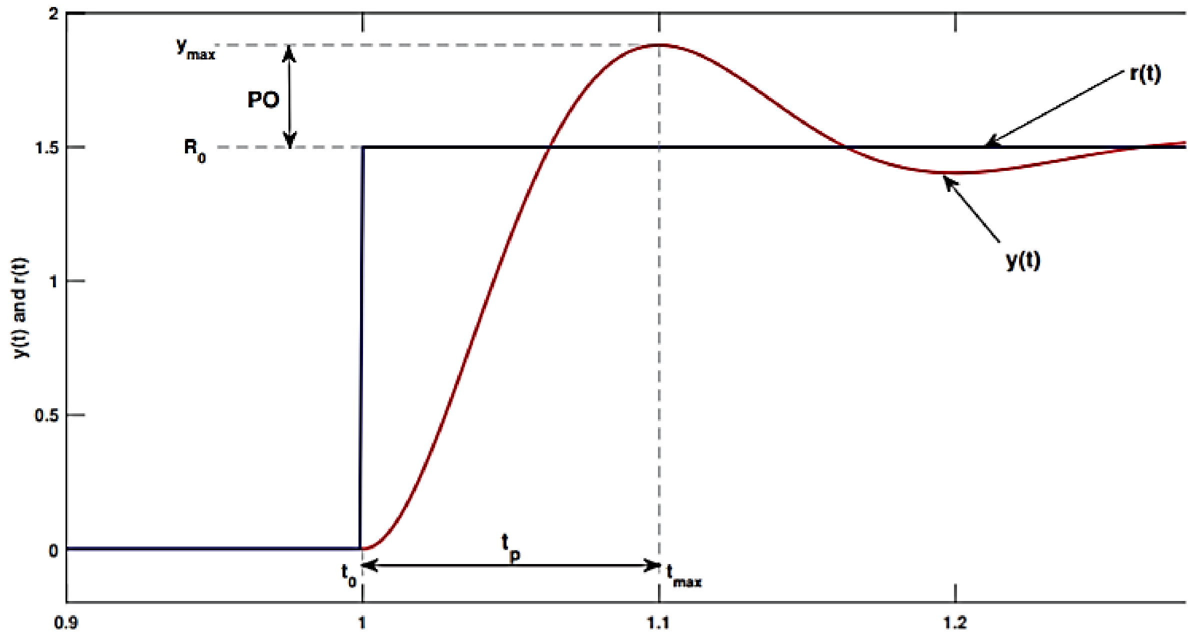


Figure 1: Response of 2nd order system to step input

This step response of an underdamped second order system can be quantified using the following measures:

- **Peak Time (t_p)** is the time required to reach the first, or maximum peak. In Figure 1, the peak time is

$$t_p = t_{max} - t_0 \quad \text{(2)}$$

- **Percent Overshoot (PO)** is the amount that the response overshoots the steady state or final value at the peak time. This is expressed as a percentage of the steady-state value. In Figure 1, the response achieves maximum value, y_{max} at t_{max}, and the percent overshoot is

$$PO = \frac{y_{max} - R_0}{R_0} \times 100 \quad \text{(3)}$$

- **10% Rise Time (t_r)** is the time required for the response waveform to go from 0.1 of the steady-state value to 0.9 of the steady-state value. In Figure 2, the rise time is

$$t_r = t_2 - t_1 \quad \text{(4)}$$

- **2% Settling time (t_s)** is the time required for the transient's damped oscillations to reach and stay within ±2% of the steady-state value. In Figure 2, the settling time is

$$t_s = t_d - t_0 \quad \text{(5)}$$

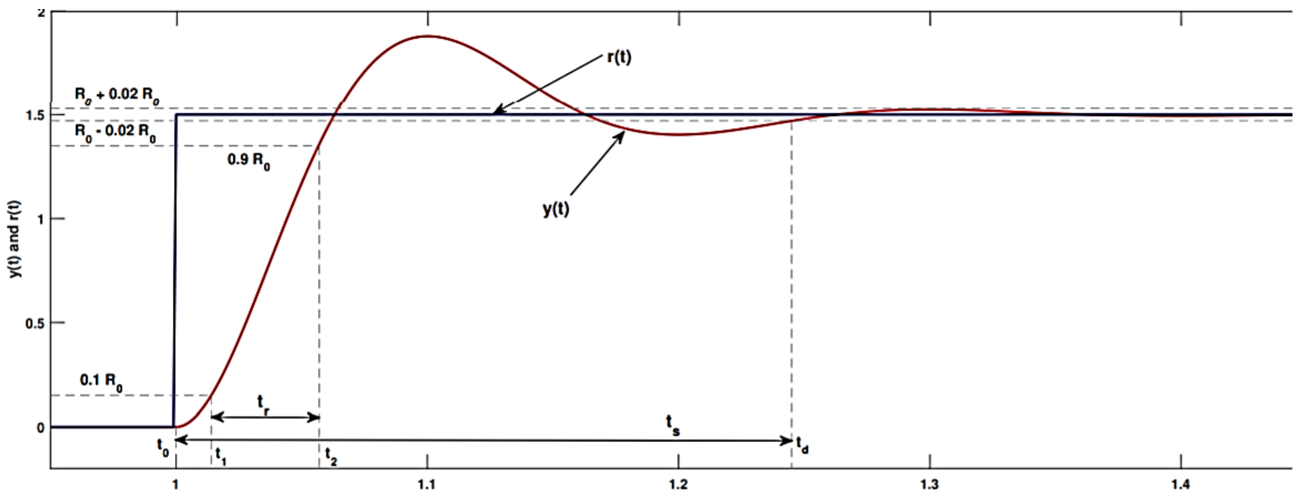


Figure 2: Response of second order system to step input showing rise time and settling time

These definitions are valid for systems of order higher than 2 as well. For a second-order system, the above response specifications can be expressed in terms of the natural frequency and the damping ratio of the system as follows:

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \quad \text{(6)}$$

$$OS = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100 \quad \text{(7)}$$

$$t_s \approx \frac{4}{\zeta\omega_n} \quad \text{(8)}$$

$$t_r \approx \frac{1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1}{\omega_n} \quad \text{(9)}$$

The last two expressions for settling time and rise time are not exact and are just an approximation. Generally speaking, the damping ratio affects the shape of the response while the natural frequency affects the speed of the response.

7.5 Unity Feedback Control Loop with Proportional Controller

In the last lab, we obtained the transfer function between the speed and voltage of the DC motor setups which was:

$$\frac{\omega_m(s)}{V_m(s)} = \frac{K}{\tau s + 1}$$

It's easy to obtain the transfer function between the position and voltage of the DC motor as well. Speed being the derivative of the position, the transfer function relating position and voltage is:

$$P(s) = \frac{\theta_m(s)}{V_m(s)} = \frac{K}{s(\tau s + 1)}.$$

We're going to use a simple control loop to control the position of a DC motor in this lab. This loop is shown in Figure 3.

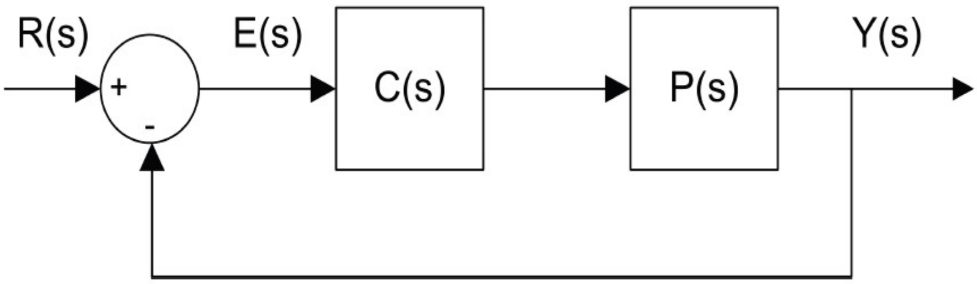


Figure 3: Unity feedback loop

The transfer function of motor setup (plant) is *P(s)*. *Y(s)* represents the output which is position of the motor and *R(s)* is the reference signal or the desired position in radians. Notice that the position of the motor is being fed back and compared. Consequently, the voltage generated *E(s)*corresponds to the error between the reference signal and the measured position.

The controller $C(s)$ then produces a control signal to control the motor. This controller is simply an amplifier here with gain K_P at the moment. It is called a ***proportional controller*** since it produces an output that is proportional to (multiple of) the error signal. In other words, it responds to the magnitude of error only.

Task 2: To find the closed-loop transfer function for motor position control

Find the closed-loop transfer function of this loop from the reference input to the output, i.e. from the desired position $R(s) = \theta_d(s)$ to the actual position $Y(s) = \theta_m(s)$. Compare the system's transfer function to the general second order transfer function, and obtain the values of ω_n and ζ .

TASK 2

$$\frac{\theta_m}{\theta_d} = \frac{Y(s)}{R(s)} = \frac{KK_p/\tau s^2+s}{1+\frac{KK_p}{\tau s^2+s}}$$

$$= \frac{KK_p}{\tau s^2+s+KK_p}$$

$$= \frac{KK_p/\tau}{s^2+\frac{s}{\tau}+\frac{KK_p}{\tau}} \equiv \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$\therefore \omega_n^2 = KK_p/\tau \qquad 2\zeta\omega_n = \frac{1}{\tau}$$

$$\omega_n = \sqrt{\frac{KK_p}{\tau}} \qquad \zeta = \frac{1}{2\sqrt{KK_p\tau}}$$

Task 3: To compute the response specifications of the second order closed-loop transfer function for motor position control

Based on your obtained ω_n and ζ , and the values of K and τ obtained in the last lab through grey-box modeling, calculate the expected values of peak time, percent overshoot, rise time, and settling time for the given value of K_P and fill in the first column of Table 1.

Table 1: Transient response characteristics of motor position control system

Parameters	$K_P = 1$		$K_P = 2$
	Calculated Response	**Measured Response**	**Measured Response**
Peak Time	0.296	0.285	0.172
% Overshoot	47.75	46.82	57.1
Rise Time	0.1138	0.097	0.087
Settling Time	1.6	1.12	0.783

TASK 3 :

From Grey box modelling, $K = 23.65$ & $tau = 0.2$. $K_p = 1$

$$\omega_n = \sqrt{\frac{23.65}{0.2}} = 3.44 \approx 10.87$$

$$\zeta = \frac{1}{2\sqrt{23.65(2)}} = 0.0727 \approx 0.229$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \cancel{0.916} \approx 0.296 \text{ sec}$$

$$OS = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100 = \cancel{79.53\%} \quad 47.75$$

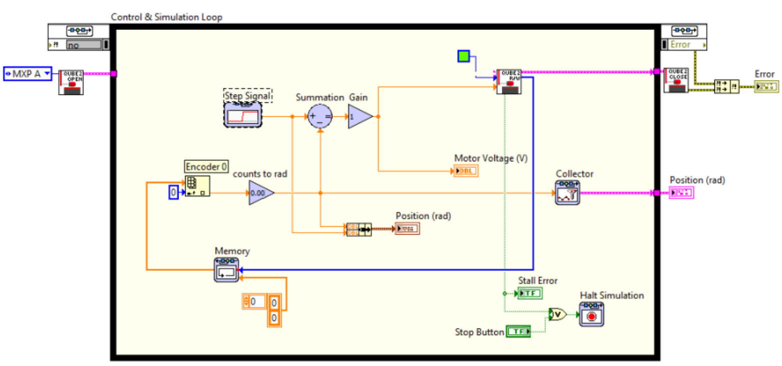
$$t_s \approx \frac{4}{\zeta\omega_n} = \cancel{15.99} \quad 1.6$$

$$t_r \approx \frac{1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1}{\omega_n} = \cancel{0.3122} \quad 0.1138$$

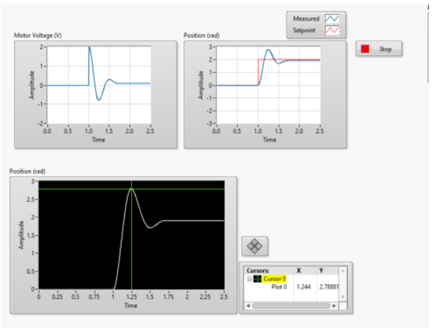
Task 4: To verify the second order specifications by measured response of motor position control and observe the impact of proportional controller gain

1. Modify the VI from previous lab to implement unity feedback system to control the position of motor or use the provided files. Apply a step reference of 2 rad and run the simulation for 2.5 seconds. To apply the step for a 2.5-second time span, set the Final Time of the Simulation Loop to 2.5 (instead of **inf**).

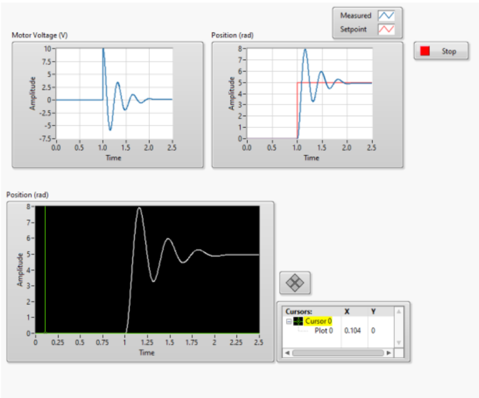
2. Measure the peak time, percent overshoot, rise time, and settling time from the response and compare them with your computed results from Task-3. Use the Cursor palette in the XY Graph to measure points off the plot as done in the previous lab. Fill in Table 2. Now, change the controller gain to 2 and measure the response characteristics to complete Table 2. Add captures of the response.



For gain=1:



For gain=2:



3. Comment on impact of the proportional controller gain on transient response characteristics. You can relate the measured values and dependency of each parameter on ω_n and ζ which in turn depend on $\boldsymbol{K_P}$.

As can be observed when value of Kp increases there is an increase in the natural frequency value but decrease in the damping ratio.
By considering the formula of peak time, due to value of damping ratio, the square of it will be even less and as peak time has already inverse relation with natural frequency so peak time value will decrease as Kp will increase.
By considering the formula of %overshoot, the damping ratio is smaller as Kp increases so square of it will be even samller , so value of %overshoot will increase.
By considering the formula of rise time which only depends on value of damping ratio which is low as Kp increases so rise time value will decrease
By considering the formula of settling time as both natural freq and damping ratio are inversely proportional to it, but damping ratio is very small as compare to natural frequency so, overall denominator will be smaller, giving higher value of settling time.

Task 5: To design a controller for motor-position control system with given specifications

In the previous tasks, you have verified that at given controller gains, the closed-loop system response shows larger percentage overshoot.

1. Design a controller for motor-position control system where allowable overshoot is 25 % only. Mention the formulae being used and calculations to show your approach for designing.

Task 5:

$$OS = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100\%$$

we know ζ is $= \frac{1}{2\sqrt{KK_p\tau}}$

∵ $K = 23.65$, $\tau = 0.2$, we can manage K_p, controller gain according to our desired overshoot.

for an overshoot of 25%.

$$e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} = \frac{25}{100} = 0.25$$

$$\frac{\pi\zeta}{\sqrt{1-\zeta^2}} = 1.3863$$

$$\pi^2\zeta^2 = (1.386)^2(1-\zeta^2)$$

$$\pi^2\zeta^2 = 1.922 - 1.922\zeta^2$$

$$11.79\zeta^2 = 1.922$$

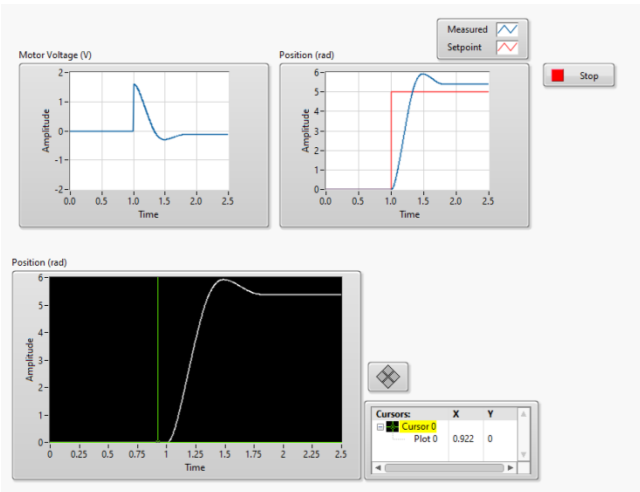
$$\boxed{\zeta = 0.4037}$$

$$0.4037 = \frac{1}{2\sqrt{K_p(0.2)(23.65)}}$$

$$K_p = \frac{1.53399}{0.2 \times 23.65} = 0.32$$

2. Test the designed controller with desired reference angle of 5 rad and show response. Does the system meet required response specification criteria?

For gain=0.32



As per the output, the overshoot % is not 25 but somewhere around 20%, so it is not really as per requirement.

3. If we had to control more than 1 response characteristics e.g. overshoot and peak time, we would compute corresponding ω_n and ζ. Can we tweak these two parameters using single value of K_P alone for the given system?

$$\omega_n = \sqrt{\frac{KK_p}{\tau}} \quad \Bigg\| \quad \zeta = \frac{1}{2\sqrt{KK_p\tau}}$$

As per the expression we found for damping ratio and natural frequency, which is inversely and directly proportional to root of Kp respectively. So, yes by using single value of Kp we can tweak both of these parameters

7.6 Relating Poles-Zeros to step response characteristics

We're now going to try to relate the response characteristics to both poles-zeros and ω_n-ζ of the system.

Task 6: To construct transfer function in LabVIEW from ω_n - ζ and poles

1. Open ***Transient-Characteristics.vi*** You'll notice that VI requires you to enter the real and imaginary parts of two poles and then constructs an array of two complex numbers, which are the poles. To construct a transfer function from these poles, connect the poles array to the *Create polynomial from roots* block from the *Polynomials* palette. This will create a denominator polynomial, which is then wired to *CD Construct Transfer Function Model* block from *Control Design & Simulation|Model Construction* palette. Make sure that *SISO* is selected from drop down menu.
2. The previous task successfully created a transfer function, but you were unable to see it. To display the transfer function, wire the output of the *CD Construct Transfer Function Model* to a *CD Draw Transfer Function Equation* block. Right-click at the *Equation* terminal of the last block and click *Create| Indicator*. You will now be able to see the constructed TF.
3. What is the relationship between poles and ω_n-ζ of a second-order system? Express the poles of the characteristic equation (denominator of the standard second order transfer function) in terms of ω_n and ζ.

$s^2+2\omega_n\zeta+\omega_n^2$

$$= \frac{-(2\omega_n\zeta) \pm \sqrt{(2\omega_n\zeta)^2 - (\omega_n)^2(4)(1)}}{2}$$

$$= \frac{-2\omega_n\zeta \pm \sqrt{4\omega_n^2\zeta^2 - 4\omega_n^2}}{2}$$

$$= \frac{-2\omega_n\zeta \pm \sqrt{4\omega_n^2(\zeta^2-1)}}{2}$$

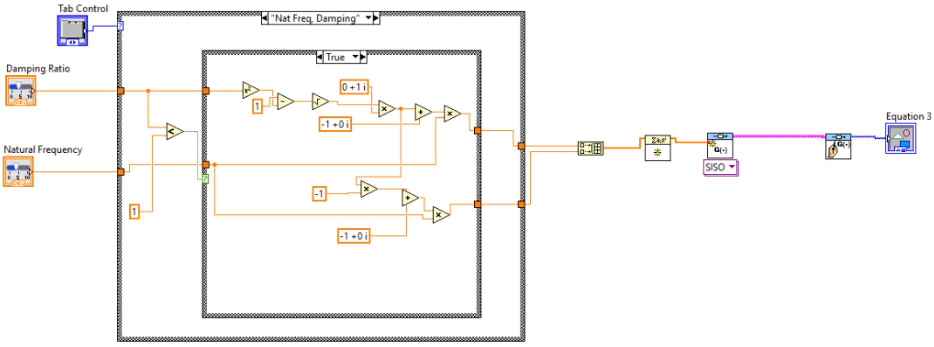
$$= \frac{-2\omega_n\zeta \pm 2\omega_n\sqrt{\zeta^2-1}}{2} \qquad \zeta \geq 1$$

$$= \omega_n\left(-\zeta \pm j\sqrt{1-\zeta^2}\right) \qquad \zeta < 1$$

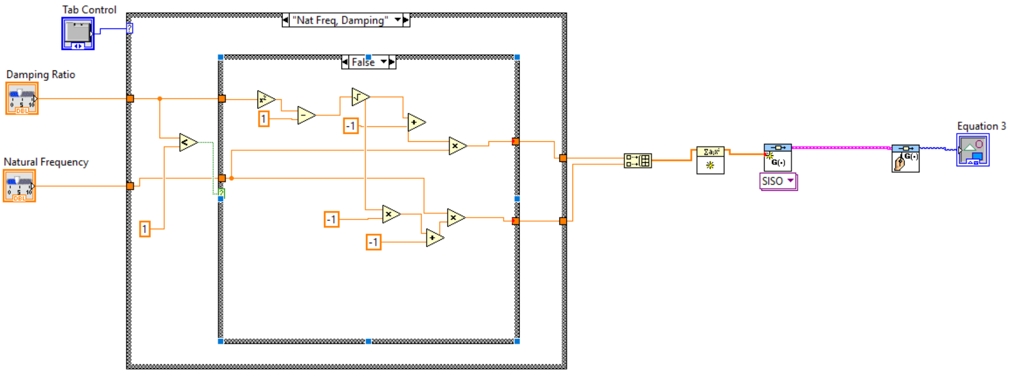
As per the expression if the values of natural frequency and damping ratio increases, the poles will radially away from each other at particular value of ω_n-ζ and if decreases, the poles will move radially closer to each other.

4. Add another case in the big case structure. In this new case, use the damping ratio and natural frequency inputs and convert them to two poles, which are then to be wired to the array. This will enable you to construct a transfer function using ω_n and ζ now

For damping Ratio < 1:



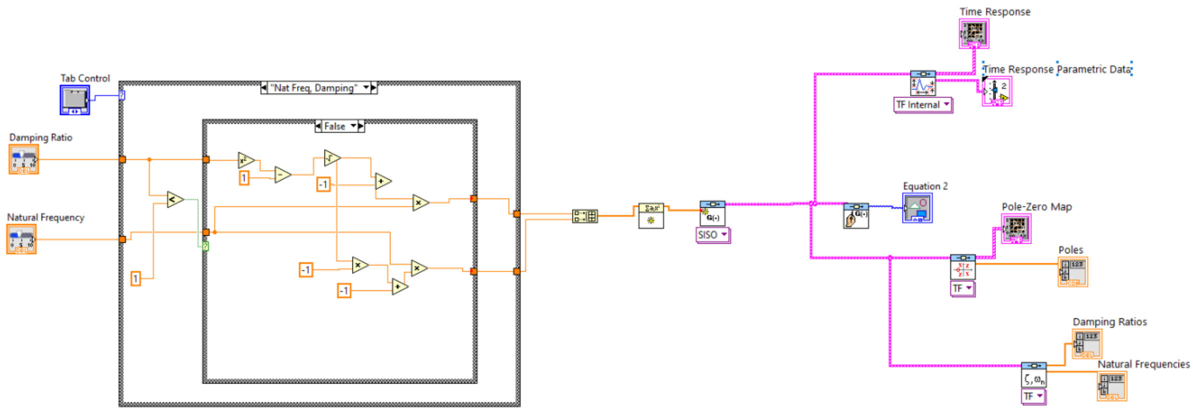
For damping > 1:



Task 7: To display the pole-zero map, step response and its characteristics

Whether you provide poles or ω_n - ζ values, to see the other set of values and step response of the transfer function, follow the give steps:

1. To display the poles-zeros of a TF, use the *CD Pole-Zero Map* block from *Control Design & Simulation|Dynamic Characteristics* palette. Make sure that **TF** is selected from the dropdown menu. Create two indicators from both the *Pole-Zero Map* and *Poles* terminals.
2. To see the ω_n-ζ values of a TF, use the *CD Damping Ratio* and *Natural Frequency* block from *Control Design & Simulation|Dynamic Characteristics* palette. Make sure that **TF** is selected from the dropdown menu. Create two indicators from both the *Natural Frequencies* and *Damping Ratios* terminals.
3. To be able to see the step response and its characteristics, use *CD Parametric Time Response* block from *Control Design & Simulation|Time Response* palette. Make sure that *TF Internal* is selected from the dropdown menu. Create two indicators from both the Time Response Graph and *Time Response Parametric Data* terminals.

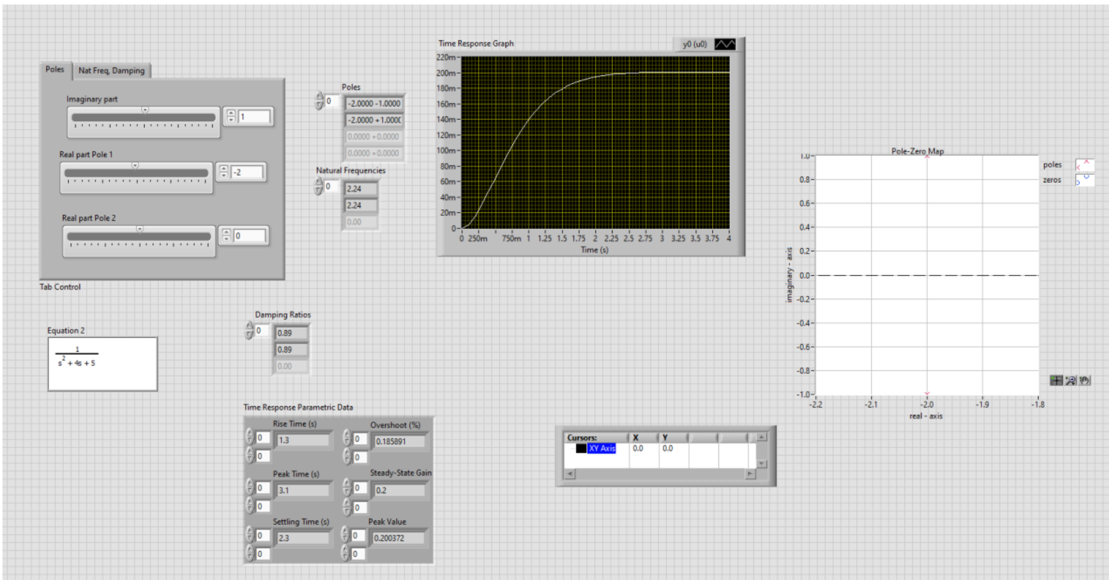


Task 8: To investigate the impact of system parameters on response

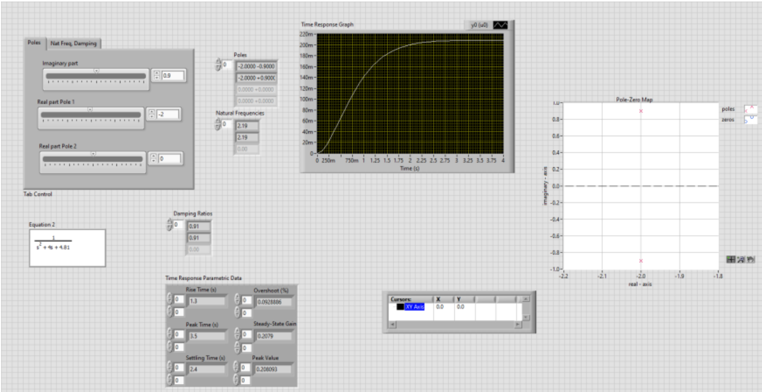
We're now going to see the effect of varying the parameters of system on response characteristics. For each of the following cases, comment on which response characteristic stays the same throughout the variation and which response characteristic is the most affected. Save plots for a few values in each case as supporting data for your argument. Note down the response characteristics and the different values of parameters for which the response is being observed.

1. Vary the imaginary part of poles keeping real part constant

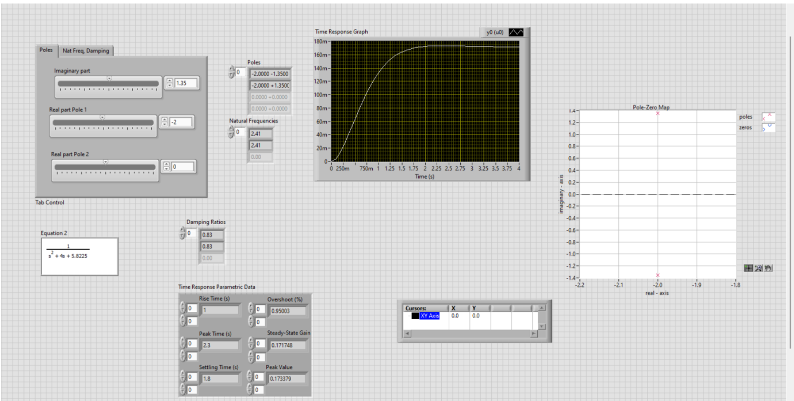
a) For damping ratio= 0.894, and wn= 2.23



If changing imaginary part a bit



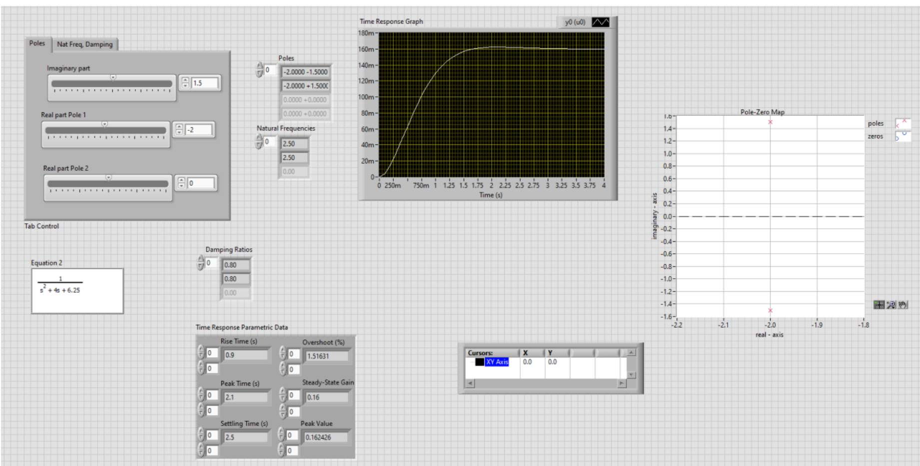
When imaginary part taken 0.9, the damping ratio increased but natural frequency decrease a bit, so overall poles value become smaller and they become radially closer to each other.



When imaginary value increases, the natural frequency increases, so overall poles value increases, so poles move radially away from each other.

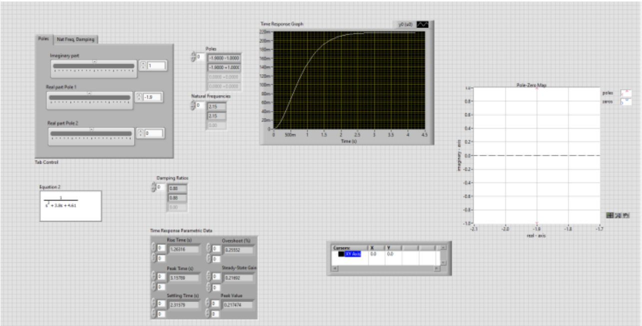
However changing imaginary part there is no rightwards or leftwards shift

When imaginary part taken 1.35

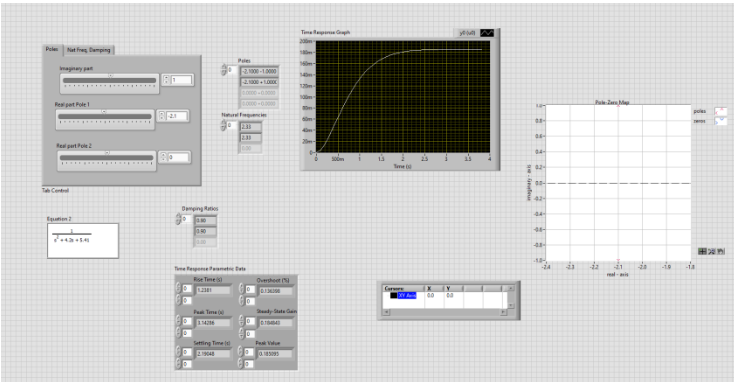


When imaginary part taken as 1.5

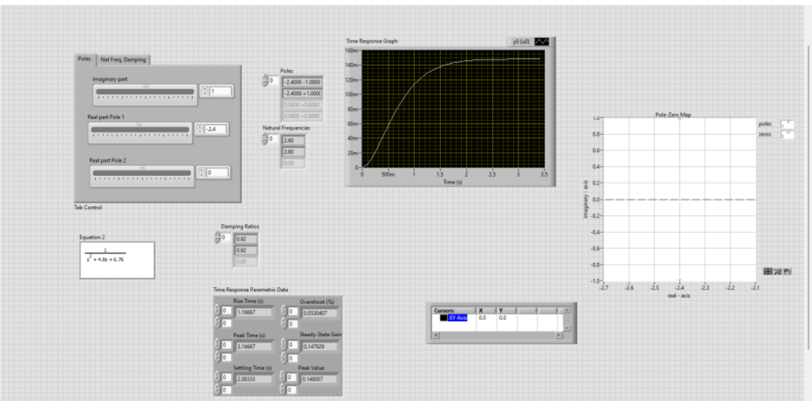
Real part increases



When real part is -1.9, Both damping ratio and natural frequencies decreases, but the poles position remain same as before and no change was observed.



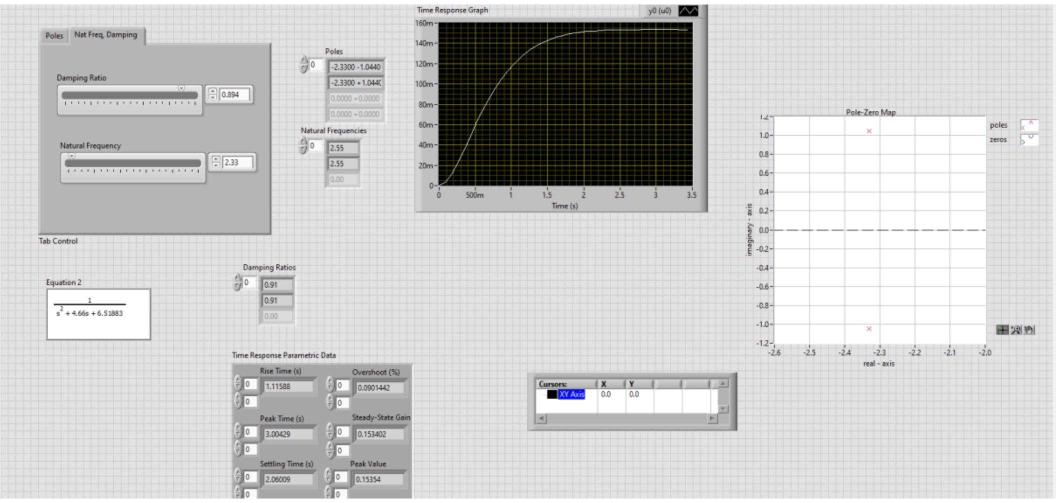
When real part -2.1, still the poles were remain in same position



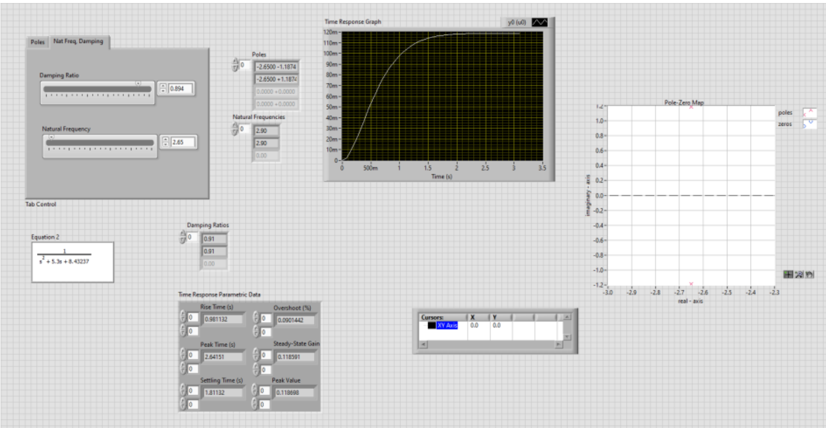
When real part -2.4, still the poles were remain in same position

In conclusion, changing real part has no impact on the position of poles but changing imaginary value change poles position radially.

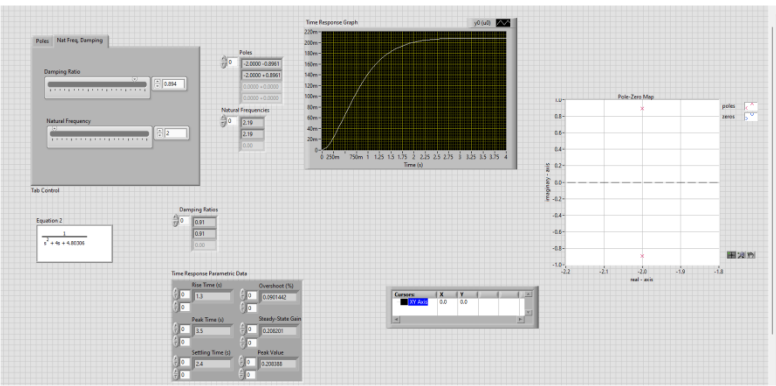
3. Vary ω_n keeping ζ constant



When increasing the natural frequency, the poles position move towards left and move radially away.



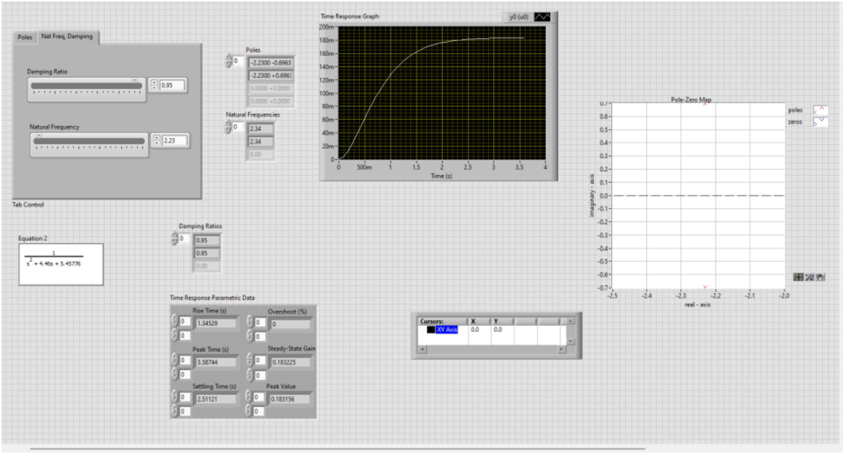
When increasing the natural frequency, the poles position move towards left and move radially away further.



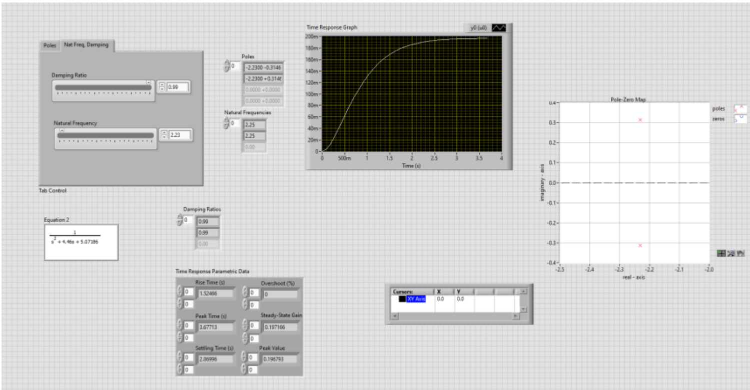
When natural frequency value decreases, the poles move radially closer.

As observed when natural frequency value decreased, the poles didn't shift but when increased the poles move left wards to value (- ω_n), but has less impact on up and down movement of poles

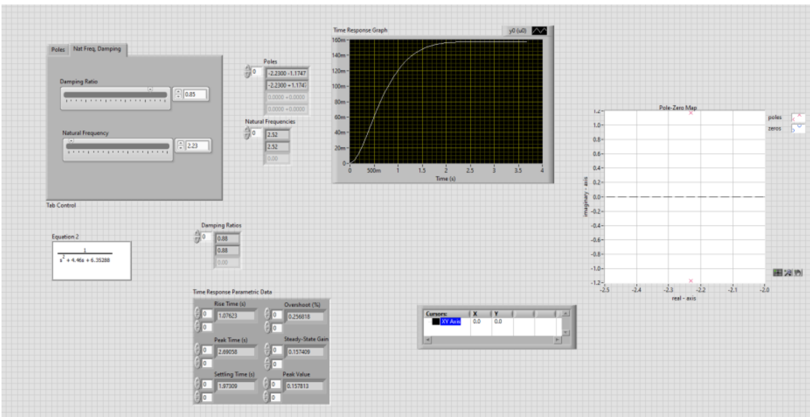
4. Vary ζ keeping ω_n constant



When damping ratio increases, the poles move closer radially as well as move towards left



When increased further, the poles become closer radially further, no significant shift towards left was observed.



When damping ratio decreases, the poles move towards left but move radially away from each other.

When natural frequency remain constant, even if damping ratio changes, the poles is at leftwards (-wn)

But when damping ratio increases the poles move more closer.

5.

7.7 Post-Lab Task

Task 9: To give a procedure of determining ω_n and ζ from measured response

In the last lab, we devised a way to experimentally obtain the two parameters, K and τ, needed to completely write a first-order transfer function from the step response. This method belonged to grey-box modeling. We now know that a second-order system can in general be described by three parameters: ω_n, ζ and a gain parameter (numerator of TF). Conclude a way to determine the values of these parameters from the step response of a second-order underdamped system obtained experimentally.

To determine the values of the three parameters (ω_n, ζ, and gain) of a second-order underdamped system from the step response of the system obtained experimentally, we can use the following steps:

1. Identify the key features of the step response: This includes the rise time (t_r), peak time (t_p), peak overshoot (PO), and settling time (t_s).
2. Use the key features of the step response to calculate the three parameters:

   ω_n: This can be calculated using the following equation:

   ω_n = 4 / (π * t_r)

   ζ: This can be calculated using the following equation:

   ζ = (ln(PO) / π) / sqrt(√(ln(PO)^2/4) - 1)

   Gain: This can be calculated using the following equation:

   Gain = (y_s - y_0) / y_0

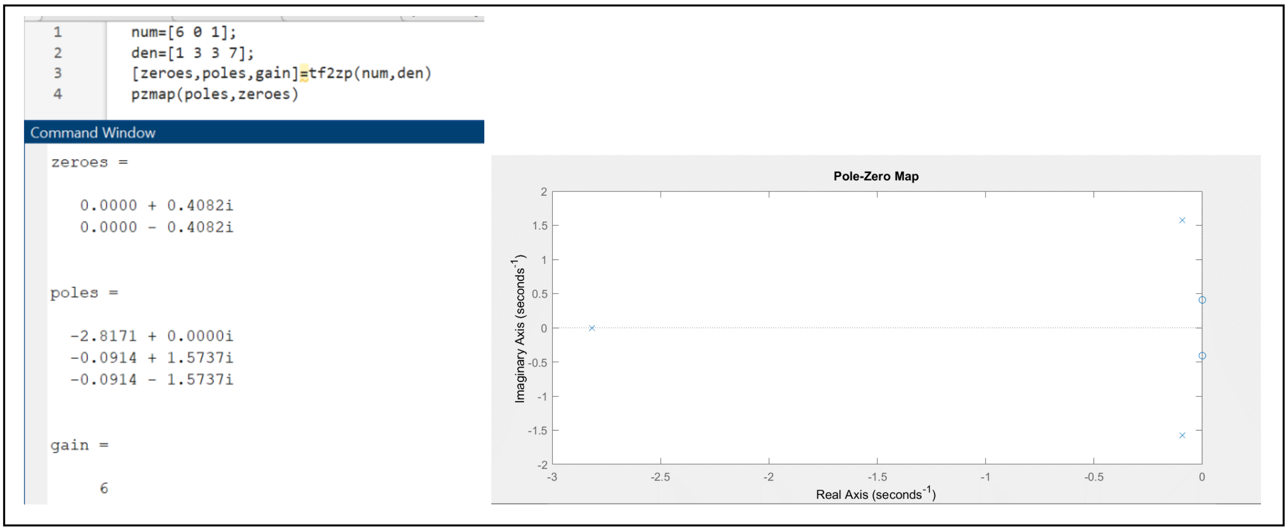
where y_s is the steady-state value of the step response and y_0 is the initial value of the step response.

Task 10: To define transfer functions of systems in MATLAB and obtain system response

1. Use MATLAB to create the given transfer function *G(s)* and obtain its poles, zeros and pole-zero map.

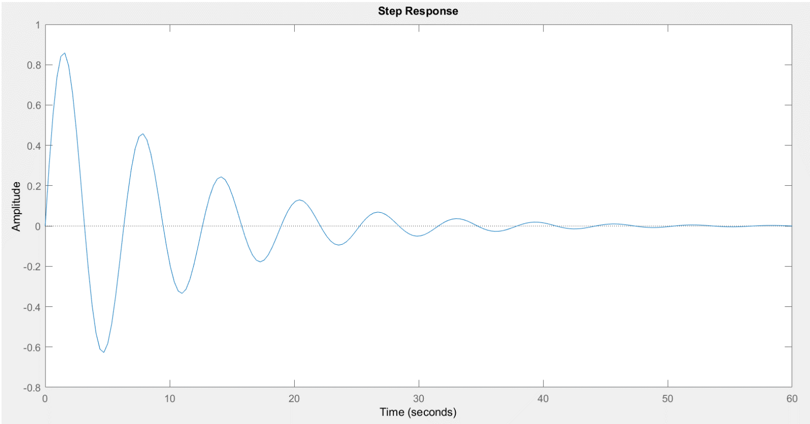
$$G(s) = \frac{6s^2 + 1}{s^3 + 3s^2 + 3s + 7}$$

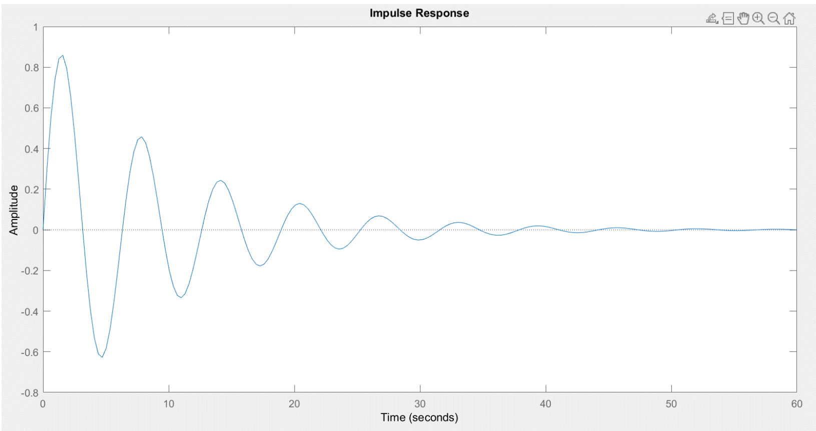
2. Obtain the unit impulse response of *H(s)* and the unit step response of *T(s)*. Explain why both the results are same.



$$H(s) = \frac{1}{s^2 + 0.2s + 1} \qquad T(s) = \frac{s}{s^2 + 0.2s + 1}$$

b)





The outputs for both are same because in case of the step response: C(s)= (1/s) * T(s) = 1/(s^2+0.2s +1). This is the same as the impulse response: C(s) = 1 * H(s).

Assessment Rubric

Lab 07
Transient Response Characteristics & Proportional Controller

Name: Afsah Hyder	**Student ID: 07065**

Points Distribution

Task No.	LR2 Simulation /Code	LR4 Data Collection	LR5 Results	LR 6 Calculations	LR 10 Analysis	AR 6 Class Participation
Task 1	-	-	-	-	12	-
Task 2	-	-	-	4	-	-
Task 3	-	-	-	4	-	-
Task 4	4	8	4	-	4	-
Task 5	-	2	4	4	4	-
Task 6	4	-	-	4	-	-
Task 7	4	-	-	-	-	-
Task 8	-	-	8	-	8	-
Task 9	-	-	-	-	4	
Task 10	6	-	6	-	2	
SEL	-	-	-	-	-	/20
Course Learning Outcomes	CLO 2					CLO 4
Total Points	/100					/20
	/120					

For details on rubrics, please refer to *Lab Evaluation Assessment Rubrics*.