Table of Contents

Lab 03
Simulating Dynamic Models using Simulink

3.1 Objective
To simulate dynamic systems using Simulink in open-loop and closed-loop configurations

3.2 Introduction
Block diagram representation is a simple yet useful technique to represent complicated systems. It helps us visualize the function of an individual element in a complex system. Moreover, individual and overall performance can be studied, and the overall transfer function of a system can be obtained easily. Because of their simplicity and versatility, block diagrams are often used by control engineers to describe different types of systems. A block diagram can be used simply to represent the composition and interconnection of a control system. Also, it can be used, together with transfer functions, to represent the cause-and-effect relationships throughout the system. Simulink, a companion software to MATLAB, allows us to simulate models using blocks available in its different libraries.

3.3 Simulating Dynamic Models Using Simulink
To simulate the mathematical model of a dynamic system in Simulink that is represented by ordinary differential equations, the first step is to ***express the differential equation for the highest derivative of the output.*** Then represent the equation in Simulink model with the help of the blocks available in libraries for different mathematical operations. This is explained through the following example.

Example: Representing a differential equation in Simulink
Consider the differential equation:

$$\frac{d}{dt}y(t) = \frac{2}{t}y \qquad (1)$$

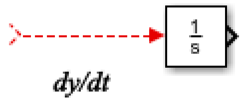
Where $y(1) = 1$.

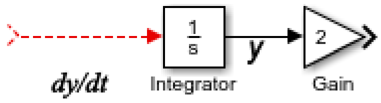
The analytical solution to the differential equation given above is

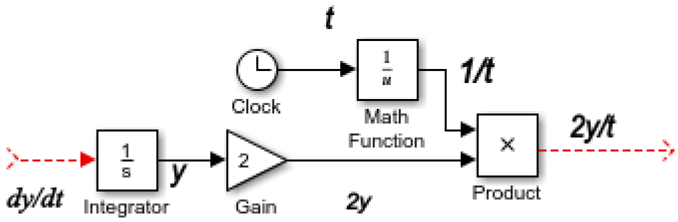
$$y = t^2 \qquad (2)$$

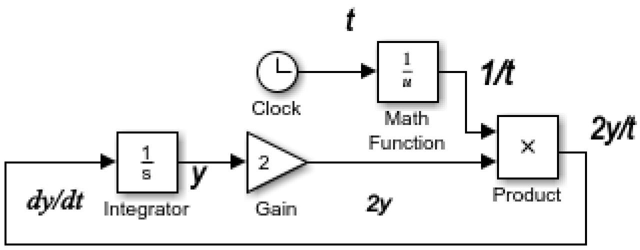
The approach to represent and solve this ODE is explained in step-wise manner. Follow the steps and verify that the obtained numerical solution represents the given analytical solution.

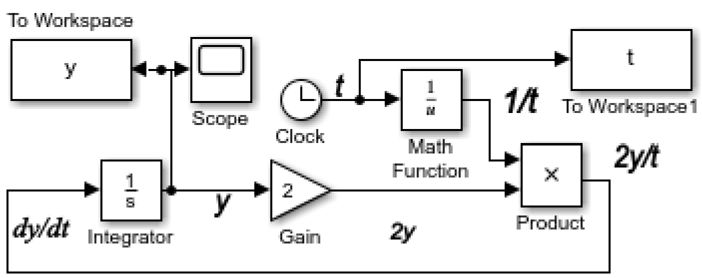
1	To build the Simulink model for this equation, we first re-arrange the equation such that the highest derivative term is at left-hand side. Here, the equation is already in the standard form.	$\frac{dy}{dt} = \frac{2}{t}y$
2	Initially, assume that in Simulink model, we have a signal (or connecting line) for this highest order derivative i.e. $\frac{dy}{dt}$ here.	

<table>
<tr><td>3</td><td>Now, to represent the right-hand side of equation, we need □. It can be obtained by integrating $\frac{□□}{□□}$ once.</td><td>

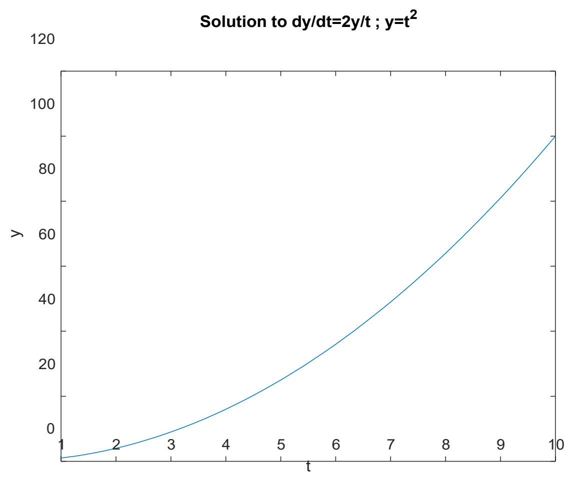
Library path for Integrator: Simulink > Continuous</td></tr>
<tr><td>4</td><td>As per the equation, □ is to be multiplied by a constant 2. In Simulink, this can be easily done by using the Gain block.</td><td>

Library path for Gain: Simulink > Math Operations</td></tr>
<tr><td>5</td><td>The next step is to obtain $\frac{1}{□}$ to multiply it with 2□. In Simulink,the Clock block returns simulation time. Through Math Function its reciprocal can be obtained.</td><td>

Library path for Clock: Simulink > Sources
Library path for Math Function: Simulink>Math Operations (Select Reciprocal in Function)</td></tr>
<tr><td>6</td><td>The obtained signal $\frac{2□}{□}$ completes the right-hand side of the given equation which is equal to the left-hand side $\frac{□□}{□□}$ that was initially assumed. Therefore, the input terminal of Integrator can be now connected to the output terminal of Product.</td><td>
</td></tr>
<tr><td>7</td><td colspan="2">Specifying Initial Conditions: The given initial condition where □ is 1 at □ equal to 1 sec i.e. (1) = 1 , needs to be entered in the model for obtaining its solution.

Enter the initial condition for □ in the Integrator block whose output is □. For simulation time, goto Simulation > Model Configuration Parameters and enter start time (1sec).</td></tr>
<tr><td>8</td><td>Since the required solution is □, aScope can be connected to this signal to view the graphical representation of the function □ for the given values of time. Additionally, the values of any signal can be exported to workspace using To Workspace block.</td><td>

Library path for Scope and To Workspace: Simulink > Sinks
In To Workspace, specify variable name and set Save Format to Array.</td></tr>
</table>

9	Run the simulation model. You will notice the 2 new variables named as ***y*** and ***t*** in Workspace window. Using plot(t,y) command, you can obtain plot for the obtained solution. Verify that it represents given analytical solution $y = t^2$. Open the Scope and observe the same plot for ***y***. Through Configuration Propertiesand Style options in plot window,you can format the image. If you see Offset=1 at the bottom right window of plot, select *Autoscale*.	

Task 1: To simulate the dynamic model of mass-spring-damper system in Simulink

Equation (3) represents the mass-spring-damper system shown in Figure 2. This is the same system simulated in Lab 02 through *ode45* solver.

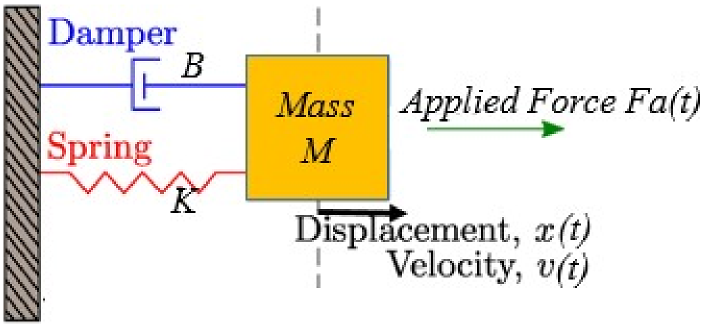


Figure 1: Mass-spring-damper system

$$M\frac{d^2}{dt^2}x(t) + B\frac{d}{dt}x(t) + Kx(t) = F_a(t) \tag{3}$$

To simulate the response of this system through Simulink, when the input force increases from **0 to 300N at t = 1 sec** and the parameter values are **M = 50 kg**, **K= 15 N/m**, and **B = 30 N sec/m**, create a blank Simulink model and follow the steps given below:

1. Express the equation for the highest order derivative i.e. acceleration.

d^x(t)/dt^2=(Fa(t) - B dx(t)/dt - Kx(t))/M

2. From *sources* library, add *step* block for applied force *Fa*. Enter the parameters as given above. You can enter the final value as a number or as a variable. Here, we will add the parameters B, M, K and Fa as variables and then initialize them in workspace before running the simulation in Simulink.
3. To multiply or divide the signals by constants use *gain* block from *Math Operations* and enter the value of gain like B, 1/M etc. You can use variables for parameters here instead of numeric values.

4. If the equation is expressed in the form of highest derivative i.e. acceleration, use *Integrator* where required. For example, to get velocity, integrate acceleration once. To get displacement, integrate twice.
5. Use *sum* block to add the signals. Add *Clock* and *To Workspace* blocks to export simulation time to MATLAB workspace. Similarly, displacement and velocity can also be exported. Don't forget to change **Save Format** to **Array** in Block Parameters of the *To Workspace*.
6. Set up the MATLAB workspace by establishing the values of the parameters needed for the Simulink simulation of the given model. Create an .m file with the given script.

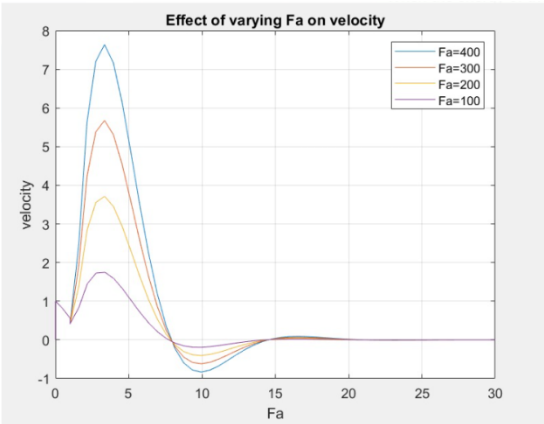
```
Fa=300; %N
B=30;   %Nsec/m
K=15;   %N/m
M=50;   %Kg
```

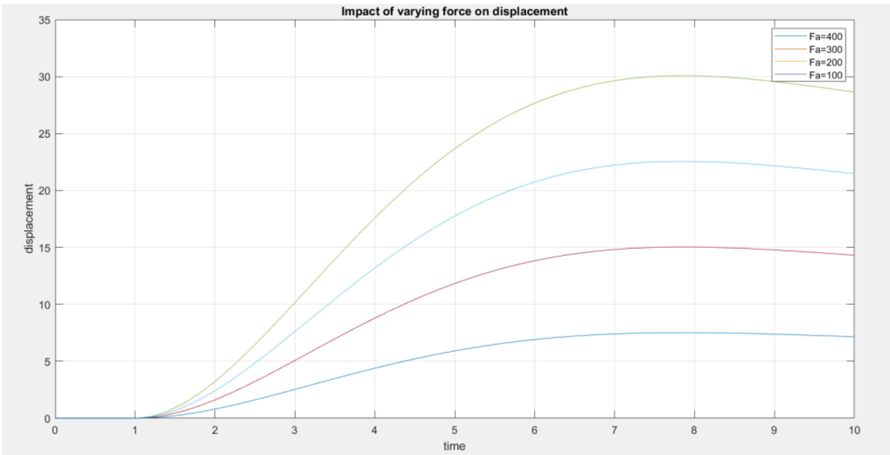
7. First run the above .m file and then run the Simulink model. You can run the Simulink model from MATLAB command window by using **sim('*File_Name*')** command. After running the Simulink model, you will notice that few variables are added in MATLAB workspace. These are the variables exported from Simulink model by using the ***To Workspace*** block. Using these variables, you can obtain plots for speed and displacement w.r.t. time using the **plot** function in MATLAB command window. Alternatively,connect *Scope* block to view results of signals in interest.
   Don't forget to add labels and titles in plots. Remember: Without suitable labels and title, plots are just random lines.
8. The **sim** function in .m file is useful in running the model multiple times with different parameters and plotting their results. Understand the code below and produce plots for different values of B.

```
Fa=300; %N
B=30;   %Nsec/m
K=15;   %N/m
M=50;   %Kg
sim('mass_spring_sys');
% runs the Simulink model, the variables for time and displacement
% will be exported by Simulink model through To Workspace block into
% workspace
plot(t,x); hold on; % for B=30 Nsec/m
B=20;
sim('mass_spring_sys'); % for B=20 Nsec/m
plot(t,x); hold on;
B=10;
sim('mass_spring_sys'); % for B=10 Nsec/m
plot(t,x); hold on;
B=5;
sim('mass_spring_sys'); % for B=5 Nsec/m
plot(t,x); hold off;
```

9. Similarly, by changing the values of F, B, M and K observe and analyze the impact of each on speed and displacement of the mass. Use **stepinfo(x,t)** and note down the obtained results for comparison.

Effect of the applied force: As you can see, the greater the force applied to the spring, the larger the displacement. This is because a greater force will stretch the spring more, and the mass moves with greater velocity. However, the displacement will also decrease over time due to friction which also causes a decrease in velocity. The displacement graph also shows that therate at which the displacement decreases is greater for larger forces and the increasing velocity. This is because a greater force will cause the spring to accelerate more, which will result in more friction. In other words, the force applied to the spring affects both the maximum displacement and the rate at which the displacement decreases. A greater force will result in a larger maximum displacement and afaster rate of decrease in displacement. It also means that greater force results in a greater velocity and a faster rate of decrease in velocity due to more friction.

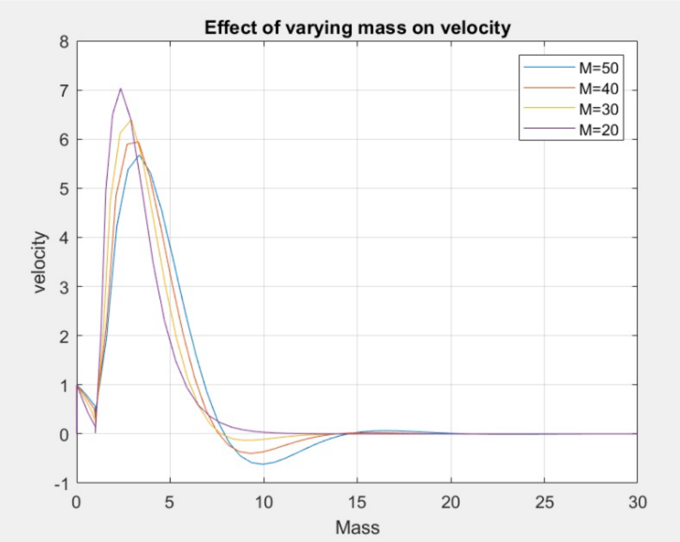


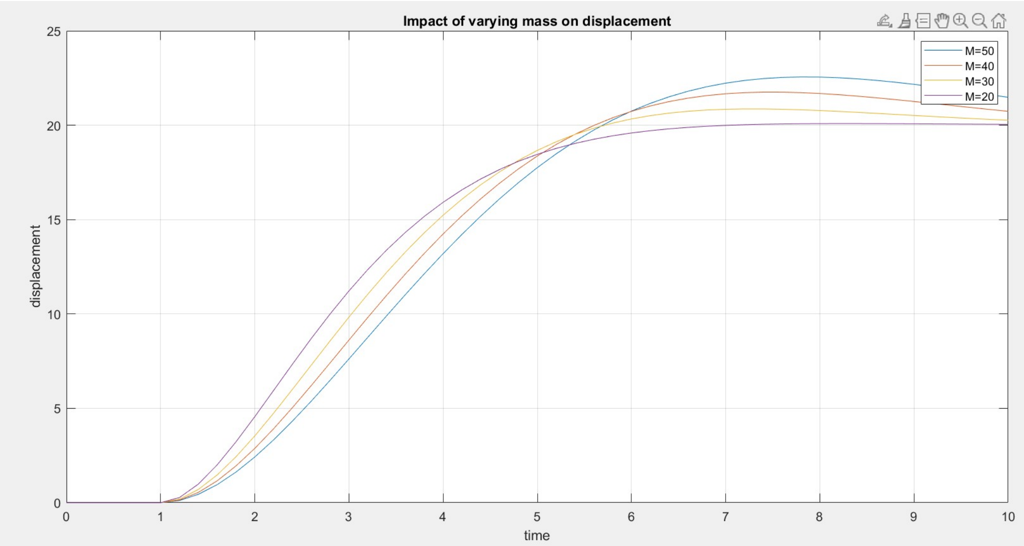


- Effect of mass:

The graph shows how the displacement of a mass-spring system decreases over time fordifferent masses. The greater the mass, the slower the displacement decreases. This is because a more massive object has more inertia, so it takes more force to move it. Friction also causes the displacement to decrease over time.
In other words, the mass of the object affects how quickly the displacement of the mass-spring system decreases over time. A more massive object will have a slower rate of decrease in displacement.

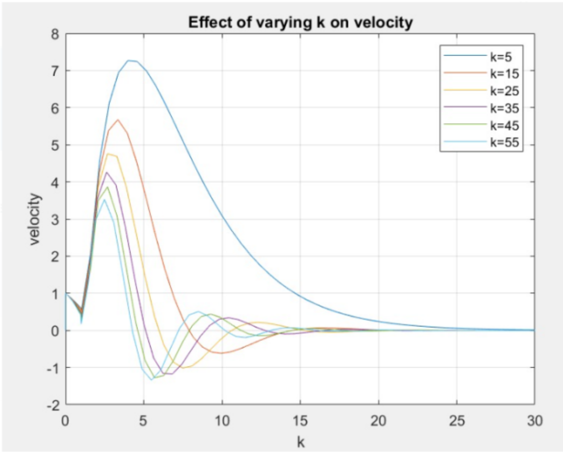
We can observe that the higher the mass, the lower the velocity. This is because the mass has more inertia, which resists changes in its motion. The spring and the damper work to oppose the motion of the mass, but the greater the mass, the more force is needed to overcome the inertia and move the mass

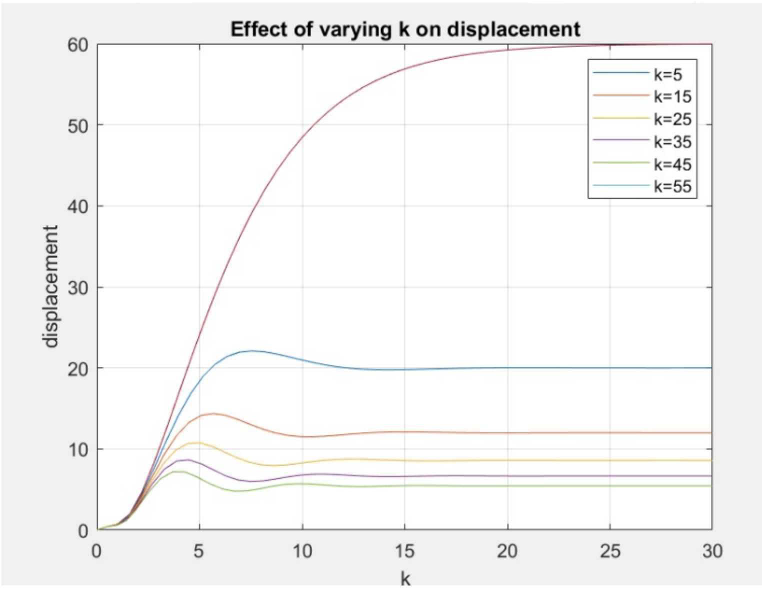




- Effect of spring constant:

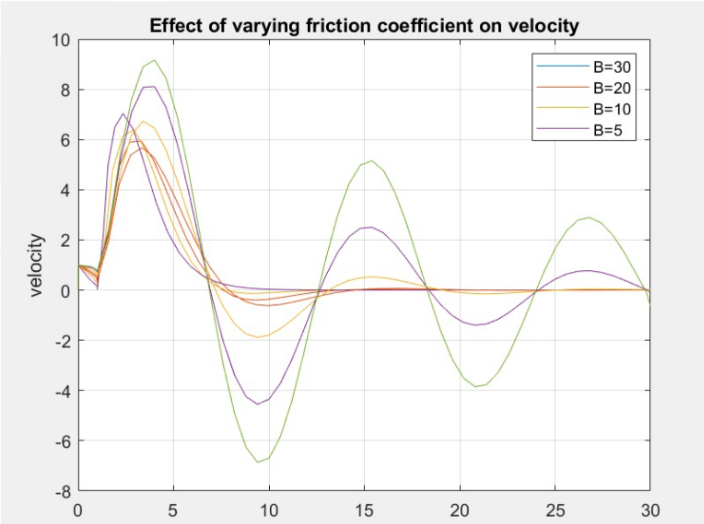
The spring constant is a measure of how stiff a spring is. A stiffer spring will require more force to stretch or compress it.The greater the spring constant, the less the displacement for the same applied force. The displacement of the spring will also decrease with time due to energy loss. The rate at which the displacement decreases will depend on the spring constant, the mass of the object attached to the spring, and the amount of friction in the system. In other words, a spring with a higher spring constant will have less displacement for the same applied force and will lose energy more slowly, so the displacement will decrease more slowly. In the velocity graph, we can observe that the higher the spring constant, the lesser the velocity. The lesser the spring constant, the higher the velocity. This is because at high spring constant, the mass will accelerate more slowly when it is displaced from its equilibrium. In other words, the velocity of the mass will be lower. The greater the spring constant, the stronger the restoring force, hence the mass will accelerate more slowly

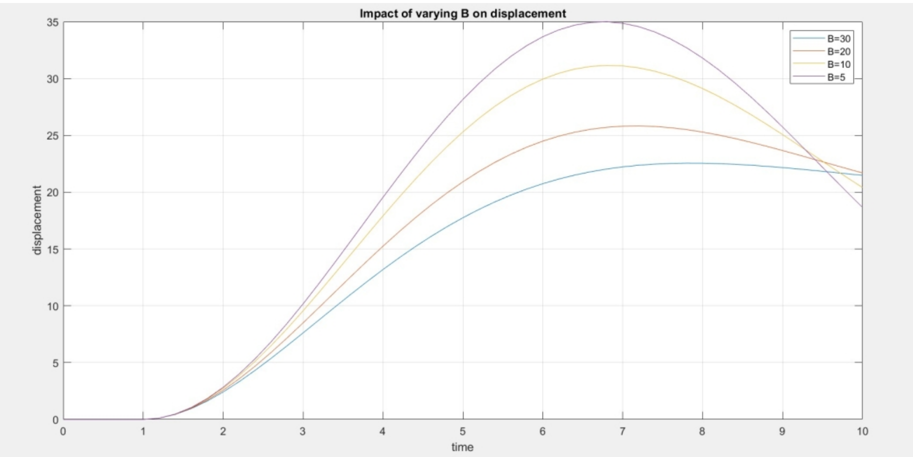




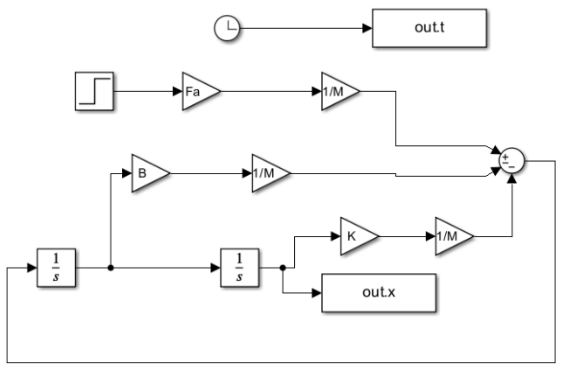
- Effect of friction coefficient:

The friction coefficient, B, in a mass spring system is a measure of the amount of energy that is lost to friction each time the mass moves. A higher friction coefficient means that more energy is lost, which will cause the mass to slow down and eventually stop oscillating. A lower friction coefficient means that less energy is lost, which will allow the mass to oscillate for a longer period of time and with a larger displacement. Whereas in the velocity graph, we see that the lower the friction coefficient, the higher the velocity is. The higher the friction coefficient the lower the velocity is. This is because a lower friction coefficient means less resistance to motion, which means mass can move more easily resulting in a higher velocity.





Add the snippet of Simulink model.



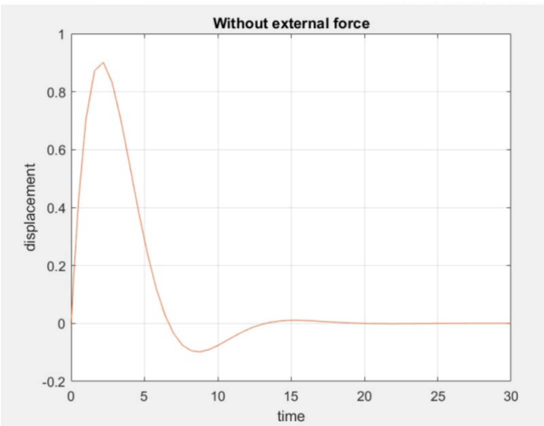
Task 2: To obtain mass-spring system response from the stored energy with zero input

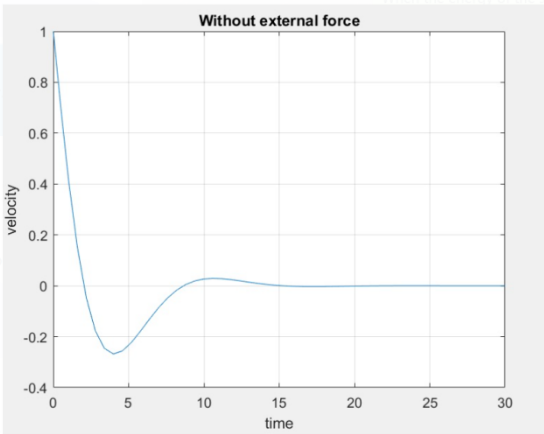
Find the response of the same mass-spring-damper system when there is no input for t ≥ 0 and the system responds to the stored energy such that the initial value of the displacement (0) is zero and the initial velocity (0) is 1 m/s.

In the previous program, make the following changes:

1. Set the final value of the step input to zero
2. Set the initial condition in parameters of the ***Integrator*** block used for obtaining velocity to 1.0 m/sec.
3. Obtain and analyze the plots for displacement and velocity. How do you interpret the movement of mass from its initial position to final position in the absence of external input?

The mass starts away from its equilibrium point, and the spring force tries to return it there, while the damping force resists its motion. The net force is the difference between these two. The mass oscillates, with the amplitude decreasing due to damping, until it eventually stops at equilibrium. The initial velocity gives the system energy, which transforms from kinetic to potential energy in the spring and dissipates as heat via damping. This process continues until all energy becomes heat. The time it takes to stop depends on the damping coefficient; higher values lead toquicker settling.





Task 3: To simulate the response of a series RLC Circuit

Consider the series RLC circuit shown in Figure 2. Let, $V = 10V$, $R = 10\Omega$, $L = 2.0\ H$ and $C = 0.01F$.

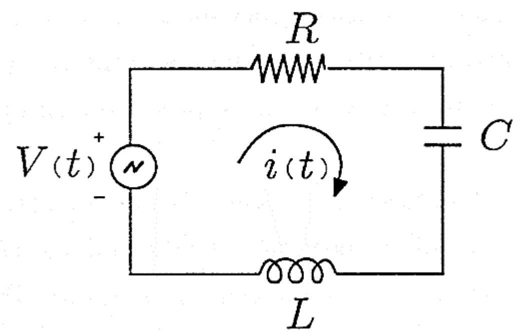


Figure 2: Series RLC circuit

1. If the capacitor is initially discharged i.e. □□(0) = 0, obtain an expression for current through the circuit □(□). Hint: apply KVL and express voltage across R, L and C in terms of current or its derivative or integral.

$$\frac{di}{dt} = \frac{V}{L} - \frac{RI}{L} - \frac{1}{LC}\int_0^t i\,dt$$

2. Create a Simulink model for the RLC series network using the obtained equation. Obtain a plot for current

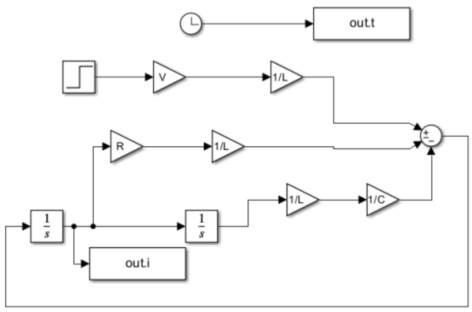
through the circuit (□). Add snippet of your block diagram. Comment on the behavior of circuit through your obtained plot.

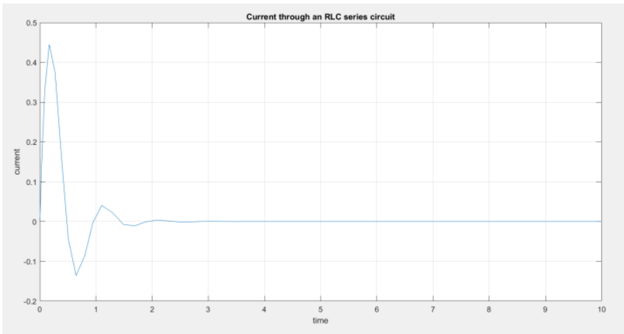
```
Vt = 10;        %voltage in volts
C = 0.01;       %capacitance in farads
R=100;          %resistance in ohms
L =2.0;         %inductance in Henry

res = sim('task3');

 plot(res.t, res.i);
xlabel("Time (s)");
ylabel("Current (A)");
title("Graph of Circuit current against time");
```

The graph shows that as the voltage increases and reaches a steady state, the capacitor acts as an open circuit and at that point there is zero current passing through the circuit given above



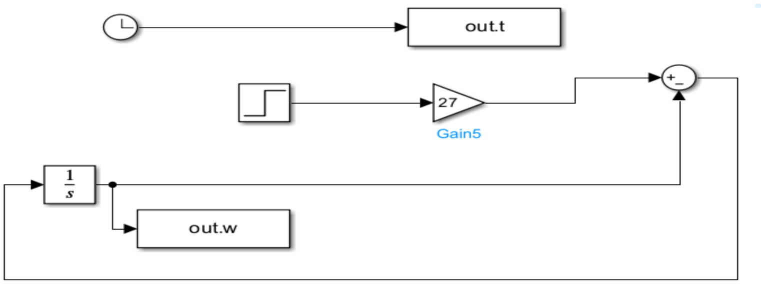


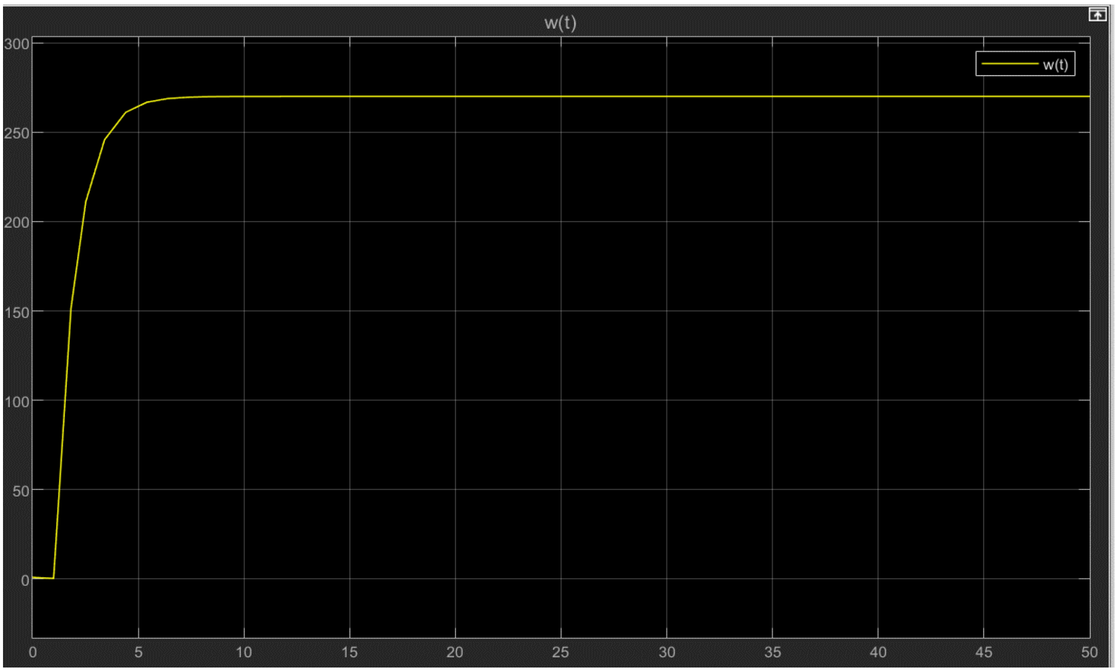
Task 4: To simulate speed control of a DC motor in open-loop and closed-loop strategies using its differential model

Consider a DC motor whose speed (□) and the applied armature voltage □□(□) are related by the following simplified differential equation (4);

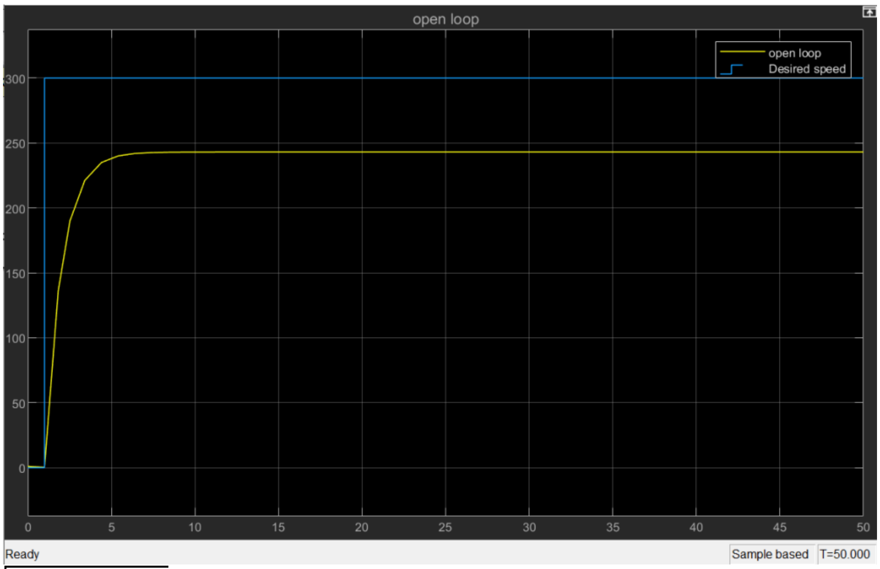
$$\frac{□}{□□}(□) + □(□) = 27\ □_{□}(□) \quad (4)$$

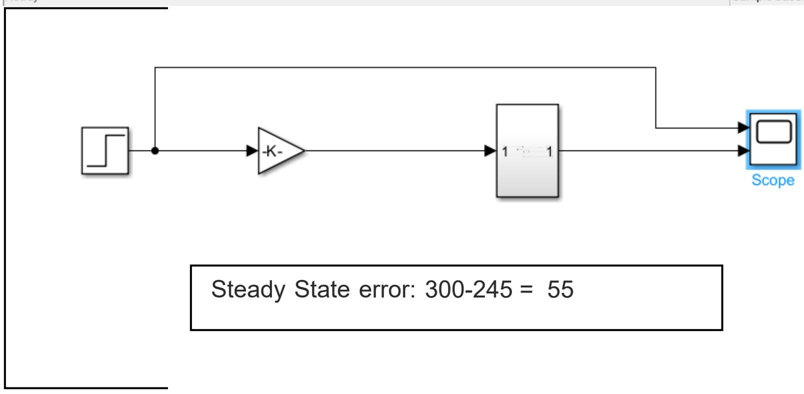
1. Using the given differential equation, model the DC motor and obtain its open-loop speed response for the applied input voltage of **10V.** Add capture of the Simulink model and response.





2. The DC motor is to be operated at a constant desired speed. For simplicity we assume that the transducer that converts the desired input speed in rad/sec to the voltage applied across the motor armature in Volts can be modelled with a gain of **0.03 V.sec/rad**.
3. Modify the Simulink diagram to enclose the motor model in a sub-system. You can do this by selecting all blocks except the Step block for input and the Scope block for output, and then by right-clicking and selecting Create Sub-System from Selection option.
4. Simulate the motor in open-loop configuration, with a desired speed of 300 rad/sec. To do this connect the transducer and motor in series. Obtain plot for motor speed and find the steady-state speed error.





5. Now, modify the Simulink model to operate it in closed-loop configuration such that the measured output speed is fed back through another transducer of gain **0.03 V.sec/rad**. The computed error is fed to an error-amplifier of gain **K**. This is represented in Figure 3.

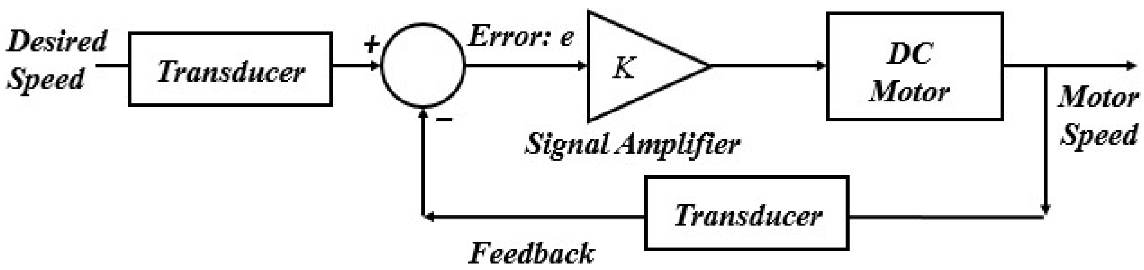
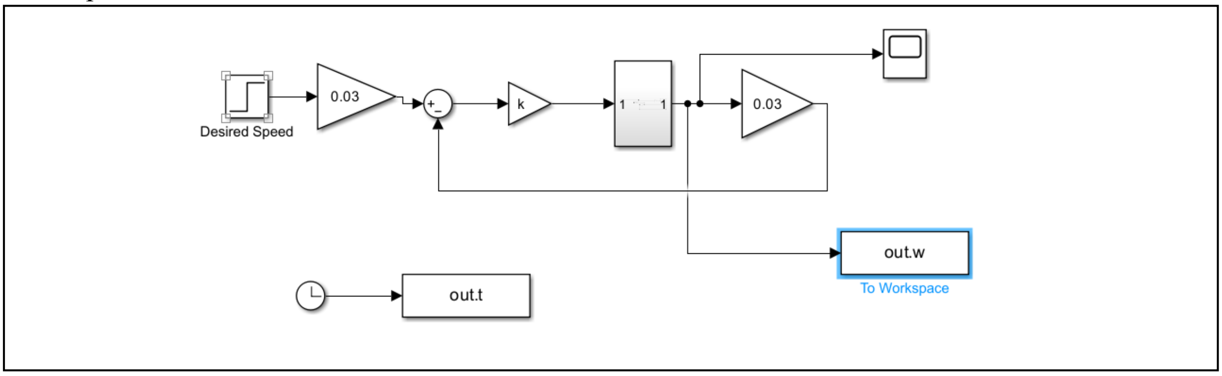


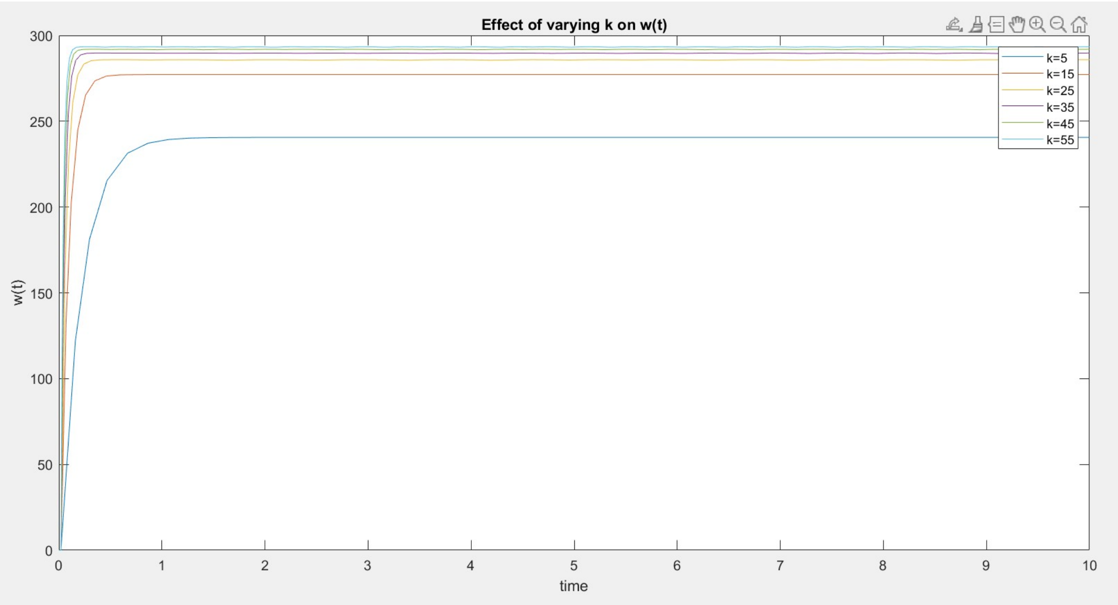
Figure 3: Block diagram representation of DC motor speed control system

Add capture of the Simulated model.



6. Vary the error-amplifier gain K in the range of 5 to 55 in steps of 10. Run the simulation and obtain responses for different values of K in one plot. Add capture of the speed control response. (Add suitable legends to distinguish plots). Compare the response of open-loop speed control with closed-loop speed-control system. Conclude the effect of gain ***K*** on the output.

In an open loop, the system stabilizes at around 260 whereas with the closed loop system, with higher gain values i.e. with K= 55 we can stabilize the system faster and at/around 290 (close to 300).



Task 5: To simulate an elevator position control system with varying gain

Figure 4 shows a feedback position control system for an elevator that is being considered by a system designer Mr. PFC. It's block diagram representation is given in Figure 5.

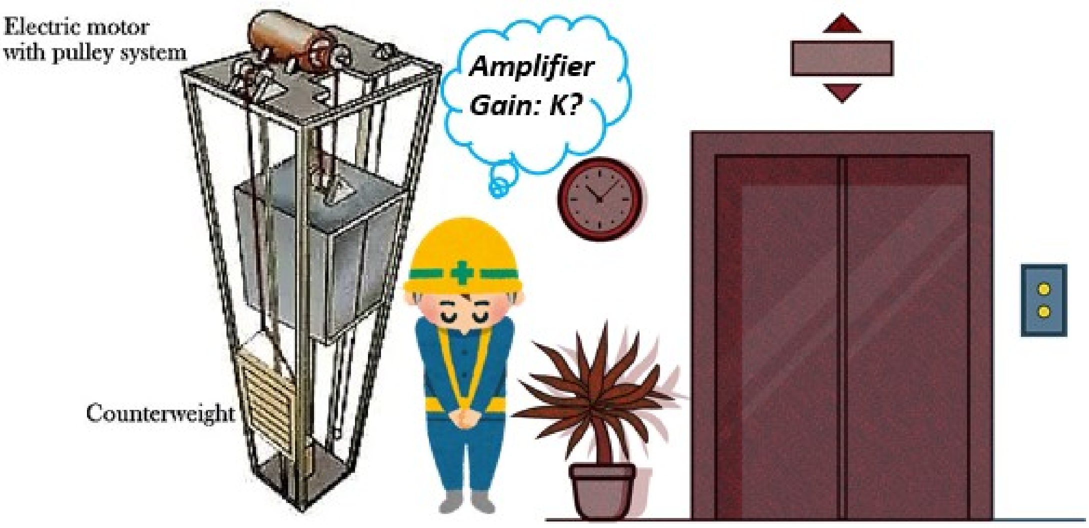


Figure 4: Elevator position control system

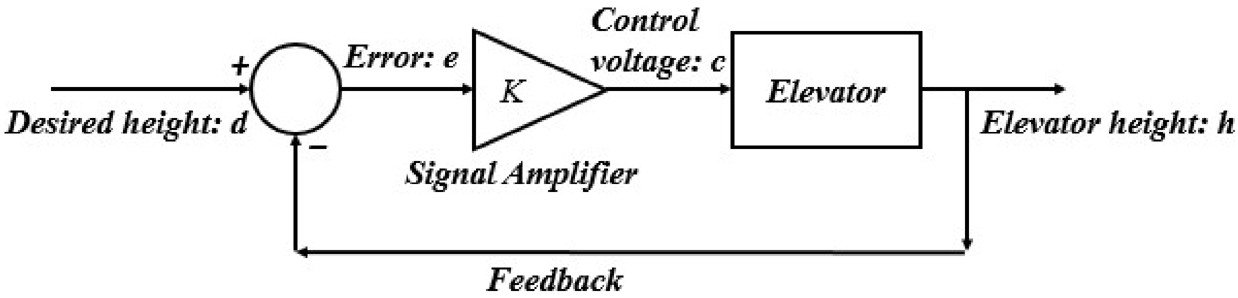


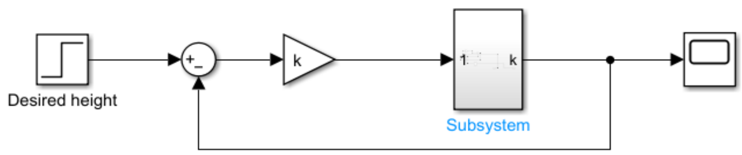
Figure 5: Block diagram representation of feedback system elevator position control

He uses a desired height; ***'d'*** provided by a user, and the actual height of the elevator ***'h'***. The difference between these two is called the error ***'e'***. An amplifier with gain ***'K'*** will use this error 'e' to control the speed of the lift motor by providing appropriate control voltage ***'c'***. The elevator is described with the differential equation (5) in simplified form,

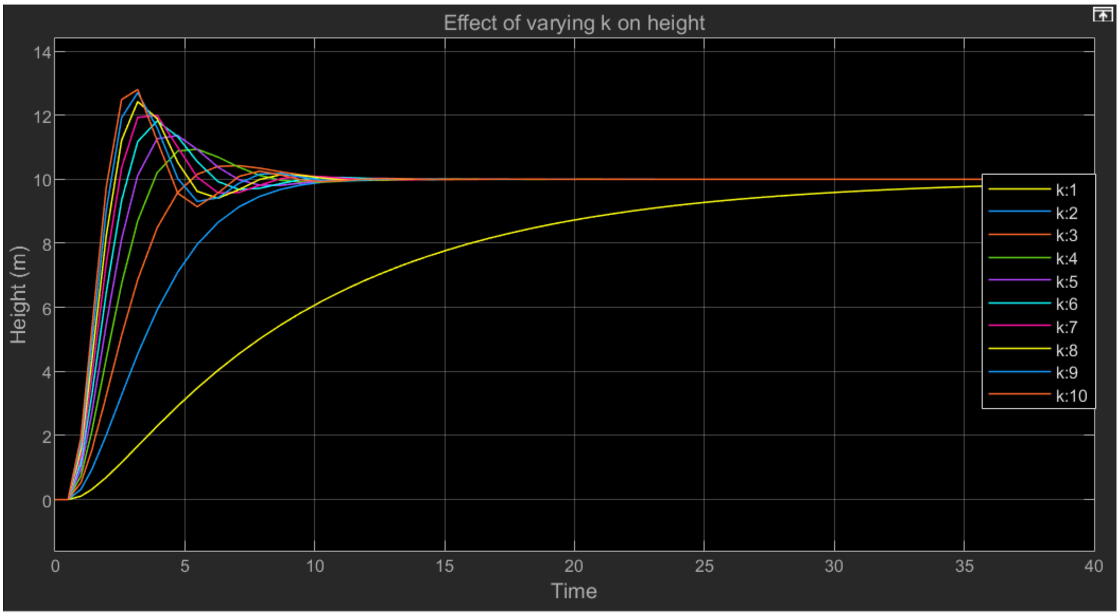
$$\frac{d^2}{dt^2}h(t) + \frac{d}{dt}h(t) = 10\ c(t) \tag{5}$$

Now, Mr. PFC is confused in selection of an appropriate value for the signal amplifier gain ***'K'.***

1. Model the complete closed-loop system and obtain the response for elevator position for different values of gain ranging from 0.01 to 0.2 in steps of 0.02. Run the simulation for 40 secs for a desired height of 10m.



K=[0.01,0.03,0.05,0.07,0.08,0.09,0.11,0.13,0.15,0.17,0.19]



2. Compare the pros and cons of high and low gain values for the system under consideration. In your analysis, don't forget to include mechanical / physical impacts on the system along with the passenger comfort and patience. Your commentary would be helpful for Mr. PFC in designing the system.

The high values of gain results in system stabilizing to the desired value quickly. But high gain value comes at the expense of discomfort for the passengers, since the elevator moves quickly and they amy experience joltsduring the ride. The elevator will also change its height quickly, e.g when k=2, the height varies from 9 to 13 m in a span of almost 5 sec, so this will be uncomfortable for the passengers and test their patience, and the sudden movements will also effect the mechanical parts of the elevator, the gears and the pulley.

The lower gain values means the system takes more time to reach the desired value. But lower gain values resultin a gradual change of the height which means a relaxed, comfortable journey for the passengers, and since there are no sudden movements, it's also better for mechanical parts of the elevator. But it takes alot of time to reach the desired height e.g. for k=0.01, it takes almost 40 sec for the height to reach 10m, even then its an approximate not exactly the desired height

3.4 Post Lab Task: Rotational Motion and System Linearization

Many physical systems are **nonlinear** and must be described by nonlinear differential equations. For instance, the following differential equation (6) that describes the motion of a pendulum of mass $\boldsymbol{m}$ and length $\boldsymbol{L}$ is nonlinear because of the term $\sin\theta$.

$$mL\frac{d^2}{dt^2}\theta + mg\sin\theta = 0 \tag{6}$$

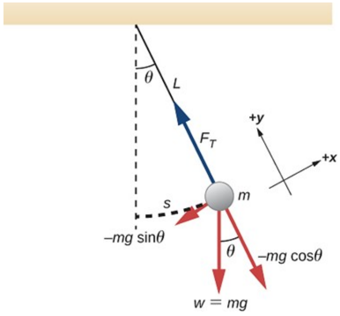


Figure 6: A simple pendulum

This system was solved in given in lab 02 through *ode45* when pendulum was initially displaced by some angle and due to the stored energy, it kept oscillating. There wasn't any external constant force or torque applied to the system.

Now, consider the same system where there is a torque T_c applied at the pivot as shown in Figure 7. The equation of motion for this system is obtained by applying the Newton's law for one-dimensional rotational systems which states that if more than one torque acts on a rigid body about a fixed axis, then the sum of the torques equals the moment of inertia times the angular acceleration.

$$\sum r = I\alpha \tag{7}$$

Where, I is the body's mass moment of inertia, in Kg.m^2, about its center of mass and α is the angular acceleration of the body in rad/sec^2.

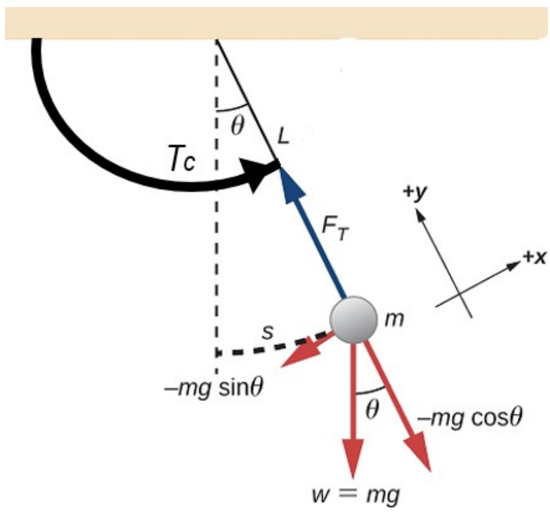


Figure 7: Pendulum system with applied torque T_c

Application of Newton's law on the pendulum system results in the following differential equation;

$$T_c - mgL\sin\theta = I\frac{d^2}{dt^2}\theta \tag{8}$$

Putting $I = mL^2$ in the above, we get

$$\square_{\square} - mgL\sin(\theta) = mL^2 \frac{d^2}{\square\square^2}\theta \quad (9)$$

Such equations are much more difficult to solve than linear ones. It is therefore useful to linearize models in order to gain access to linear analysis methods. It may be that the linear models and linear analysis are used only for the design of the control system (whose function may be to maintain the system in the linear region). Once a control system is synthesized and shown to have desirable performance based on linear analysis, it is then prudent to carry out further analysis or an accurate numerical simulation of the system with the significant nonlinearities in order to validate that performance.

We can linearize this case by assuming the motion is small enough that sin $\theta \cong \theta$ in equation (8). Hence, the linearized equation for pendulum becomes,

$$\square_{\square} - mgL\theta = mL^2 \frac{d^2}{\square\square^2} \quad (10)$$

Task 6: To compare the response of a pendulum's linear and nonlinear model

For the system given in Figure 7, the mass of the pendulum is 1 Kg and its length is 1m. There is a torque applied at the pivot $\square_{\square}$.

1. For estimating pendulum angular displacement, create two Simulink models in same file; one using the linear equation (10) and the other using the non-linear equation (9). For nonlinear model, you can find ***sin*** block in Simulink >> Quick Insert >> Math Operations.
2. To obtain the response by both the models in one plot, use a ***Mux*** before scope.
3. You can add a gain block of 57.3 to convert angular displacement from radians to degrees. To get smoother output, go to Simulation >> Model Configuration Parameters >> Solver Details. Change maximum step size to 0.001sec.
4. Add capture of the simulated model. Test your models and obtain responses for all the cases listed below.

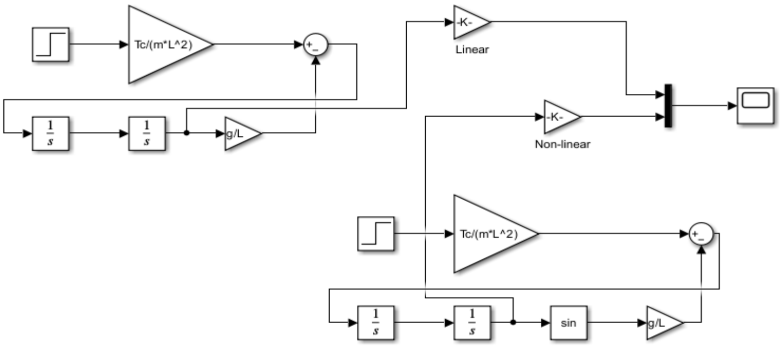
   Case 1: Applied torque $\square_{\square}$ is equal to 1 N.m. and the pendulum is at rest initially

   Case 2: Applied torque $\square_{\square}$ is equal to 4 N.m. and the pendulum is at rest initially

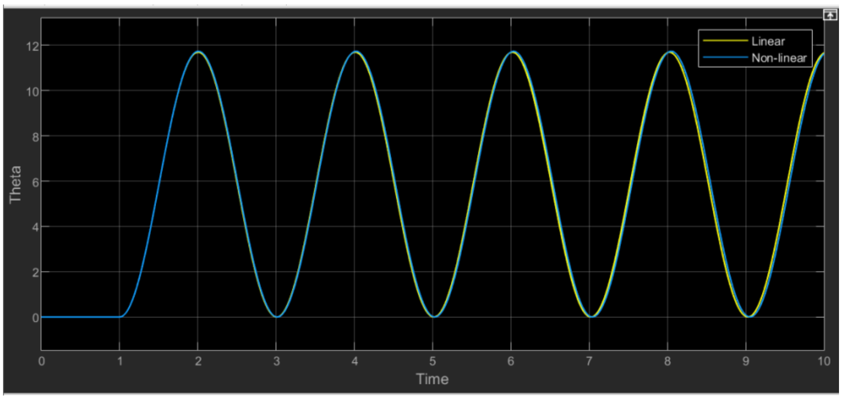
   Case 3: External torque is not applied and the pendulum is at rest but displaced by 10 degrees from its mean position initially

   Case 4: External torque is not applied and the pendulum is at rest but displaced by 80 degrees from its mean position initially

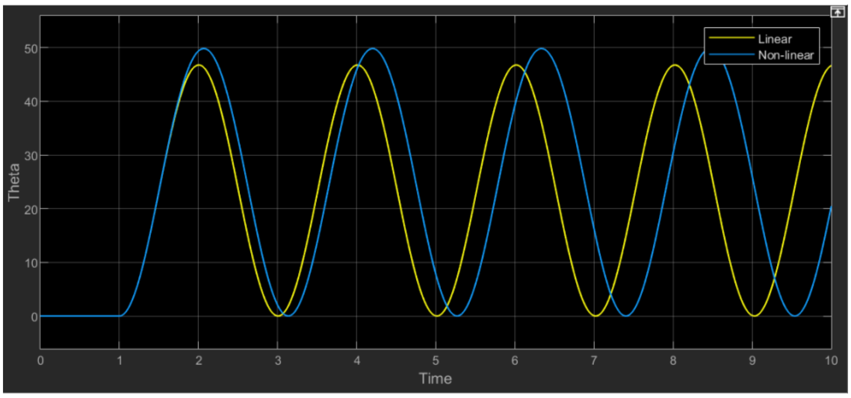
5. Compare the response produced by the linear and nonlinear models for the above cases. Comment about the applied linearization. How well does it work in different cases?



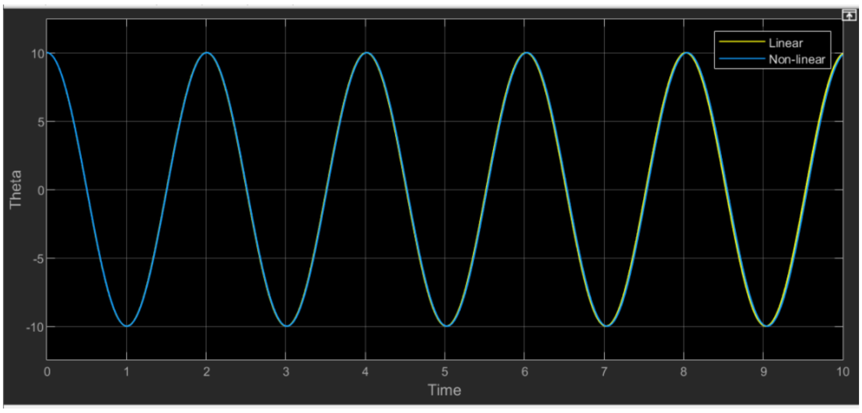
Case 1



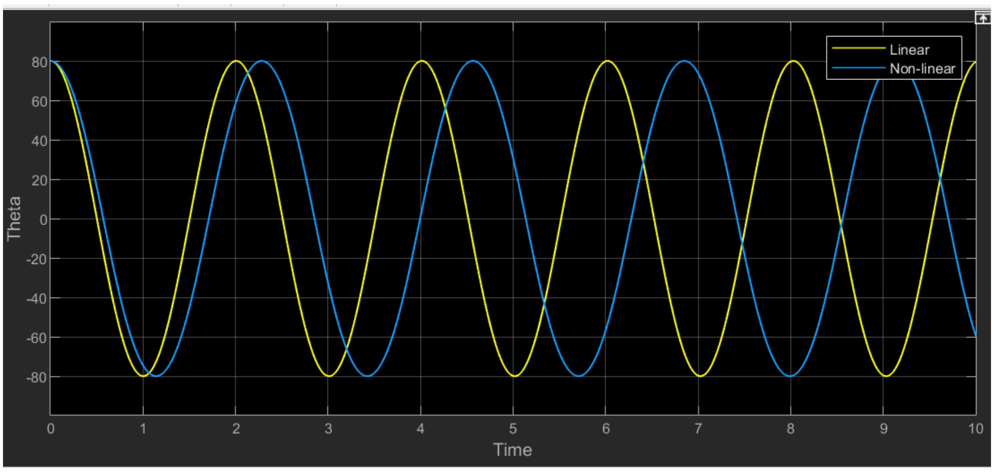
Case 2



Case 3



Case 4



When the energy of the system is low that is either the torque applied is small or the initial angular displacement is small. In both these cases, sin(theta) is approximated as theta is valid since both linear and non-linear systems give almost the same results. However, when the torque or initial angular displacement is greater then system energy is also greater and the approximation is no longer suitable since nonlinear and linear systems give very different results

Assessment Rubric

Lab 03
Simulating Dynamic Models using Simulink

Name:Afsah Hyder	**Ah07065**

Points Distribution

Task No.	**LR2** Simulation/Model/ Code	**LR5** Results/Plots	**LR 10** Analysis	**AR 6** Class Participation
Task 1	/8	/8	/6	-
Task 2	/4	/4	/2	-
Task 3	/6	/4	/4	-
Task 4	/8	/6	/6	-
Task 5	/6	/4	/6	-

Task 6	/8	/6	/4	-
SEL	-	-	-	/20
Course Learning Outcomes	CLO 1		CLO 4	
Total Points	/100		/20	
	/120			

For details on rubrics, please refer to *Lab Evaluation Assessment Rubrics*.