Habib University
Course Code: EE366/CE366/CS380
Course Title: Introduction to Robotics
Instructor name: Dr. Basit Memon
Examination: Final Exam Exam Date: May 8, 2023
Total Marks: 100 Duration: 180 minutes

Instructions

1. You can refer to course slides, your notes, homework solutions, or any of the books included in the course syllabus. You're not permitted to specifically search for responses to any of the exam questions online. Where appropriate, you can cite the slides and don't have to redo what has already been done.
2. You are welcome to utilize MATLAB in whichever way you find useful. You can submit the response to a question in the form of a MATLAB live script as well, but you cannot use numerical methods where question explicitly asks you to employ analytical methods. In case, you're utilizing make sure that your answers are organized.
3. The exam will be administered under HU student code of conduct (see Chapter 3 of `https://habibuniversity.sharepoint.com/sites/Student/code-of-conduct`). You may not talk, discuss, compare, copy from, or consult with any other human on this planet (except me) on questions or your responses during the exam.
4. Make sure that you show your work (intermediate steps). Most of the points for any question will be given based on the followed process.
5. Kindly use a black/blue pen to write your hand-written solutions, so that they are legible.
6. The questions or their associated points are not arranged by complexity or time consumption.

Questions

Consider the 4-dof RRRP robot in Figure 1. Assign a set of DH frames and derive the associated table of parameters. Problem 1 CLO2-C3

10 points Solution 1

- z_0 in direction of z_w, and x_0 in direction of x_w.

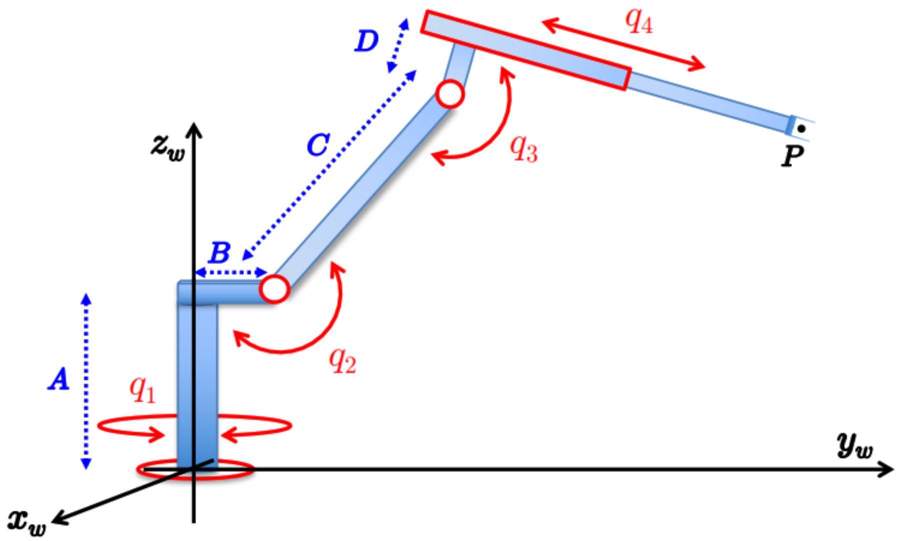

Figure 1: Spatial RRRP Robot

- z_1 coming out of page. x_1 from left to right.
- z_2 coming out. x_3 along C from bottom to top.
- z_3 along q_4 towards P. x_3 along D from bottom to top.
- z_4 the same as z_3 at P. x_4 parallel to x_3.

	a	**d**	α	θ
1	B	A	90°	θ_1
2	C	0	0	θ_2
3	D	0	90°	θ_3
4	0	q_4	0	0

Set $B = D = 0$ in the previous problem. Now, (i) determine the Jacobian; (ii) find at least one singular configuration; (iii) the unreachable instantaneous velocity direction in this configuration.

Problem 2
CLO2-C4

10 points

Solution 2

Solution is provided in the accompanying MATLAB-based PDF.

Problem 3
CLO1-C3

20 points

For the planar RPR robotic arm of Figure 2, position sensors at the joints are not available. Instead, two cameras have been installed, as shown in Figure 2, where the displayed y-axis is along the optical axes of the camera. The two cameras are mounted parallel to each other and the arm is centered between them. The cameras have focal length f meters, and pixel size ρ meters/pixel. The origins of the image planes are at center of the plane. Given distance a and b, in pixels, what is the position (x, y) of the end-effector dot in the figure?

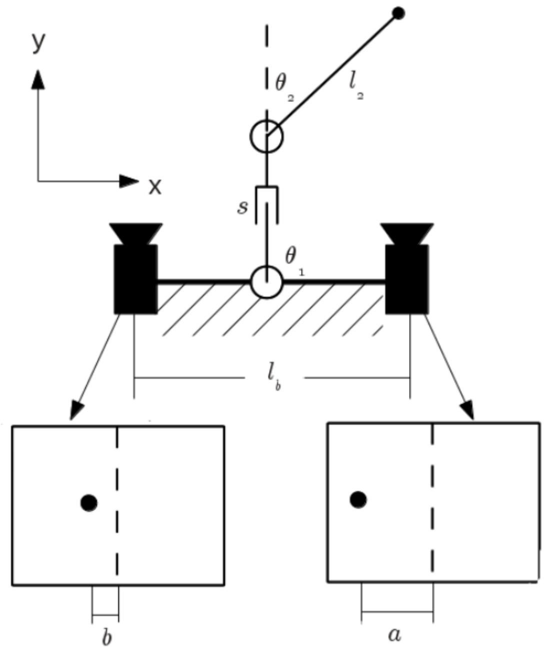

Figure 2: Camera with planar arm

Solution 3

This is the same stereo camera setup as Problem 3 of Homework Assignment 4. Since the optical axes of the cameras are along y-axis in this setup, depth measurements from the camera, i.e. Z, will correspond to y-coordinates. Using the homework solution,

$$y = Z_2 = \frac{f^2 l_B}{f\rho\,(b-a)} = \frac{f\, l_B}{\rho\,(b-a)},$$

where the left camera and right cameras are assumed as cameras 1 and 2 respectively. The origin of the x-y frame has been placed at the camera center of the left camera. Having obtained the depth value,

$$x = X_1 = \frac{\rho\, b\, Z_1}{f} = \frac{\rho\, b\, Z_2}{f} = \frac{b\, l_b}{b-a}.$$

Problem 4
CLO3-C3

15 points

Given a two camera setup, suppose we know that one of the camera always moves on the XY plane relative to the other.

(a) Show that the corresponding essential matrix has the special form:

$$\begin{pmatrix} 0 & 0 & a \\ 0 & 0 & b \\ c & d & 0 \end{pmatrix}.$$

(b) Find the epipoles e and e'.

(c) What is the use of epipolar lines?

Solution 4

(a) If the camera only moves in the XY plane then $\mathbf{t}$ is of the form $\begin{pmatrix} x & y & 0 \end{pmatrix}^T$ and the rotation is about the z-axis, i.e.

$$R = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Correspondingly, the essential matrix is:

$$\begin{aligned} \mathcal{E} &= \begin{pmatrix} 0 & 0 & y \\ 0 & 0 & -x \\ -y & x & 0 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 & y \\ 0 & 0 & -x \\ x\sin\theta - y\cos\theta & x\cos\theta + y\sin\theta & 0 \end{pmatrix}, \end{aligned}$$

which is of the same form as provided in the question.

(b) The epipoles, e and e', lie in the nullspace of $\mathcal{E}^T$ and $\mathcal{E}$ respectively.

$$\begin{pmatrix} 0 & 0 & a \\ 0 & 0 & b \\ c & d & 0 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = 0$$
$$\Rightarrow c_3 = 0$$
$$cc_1 + dc_2 = 0$$

Recall that these are homogeneous coordinates and given that $c_3 = 0$, we have the epipoles at infinity, i.e. out of the image plane.

(c) The epipolar lines reduce the search space for matching a point in the image plane to its corresponding point in stereo view.

Problem 5
CLO4-C3
10 points

A DC motor with inertia J_M drives a link through a gear with toothed wheels, as shown in Figure 3. The gear on the motor shaft has radius $r_M = 2$ cm, while the radius of the gear on the link rotation axis is $r_L = 10$ cm. The link has inertia $J_L = 0.3$ kgm^2 around its rotation axis. Assuming that gear ratio is chosen according to the optimal criteria $\frac{r_L}{r_M} = \sqrt{\frac{J_L}{J_M}}$, determine the torque τ_M that the motor needs to produce around its z_M axis in order to accelerate the link at $\ddot{\theta}_L = -5$ rad/s^2. Neglect dissipative effects as well as the inertia of the transmission components.

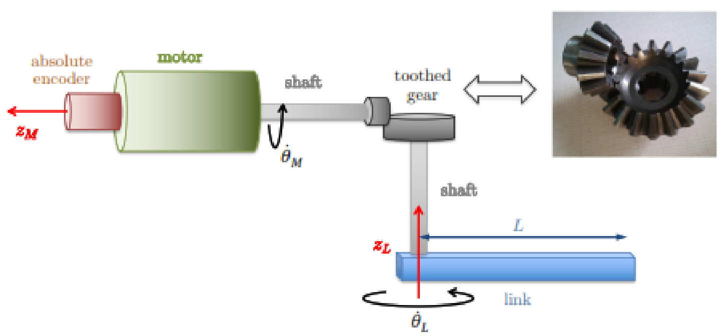

Figure 3: Motor connected to link through gear transmission

Solution 5

The gear ratio, provided the radii, is

$$n_r = \frac{r_L}{r_M} = \frac{10}{2} = 5.$$

This gear ration has been chosen according to the optimality criterion $n_r = \sqrt{\frac{J_L}{J_M}}$, and we can use that to find J_M:

$$J_M = \frac{J_L}{n_r^2} = \frac{0.3}{5^2} = 0.012 \text{ kg.m}^2.$$

From slide 23 of slide deck 19,

$$J_M\ddot{\theta}_M = \tau_M - \left(\frac{K_m^2}{R} + B_m\right)\dot{\theta}_M - \frac{J_L\ddot{\theta}_L}{n_r} - \frac{B_l\dot{\theta}_L + mgl\cos\theta_L}{n_r}$$

Ignoring the red terms and using $\ddot{\theta}_M = n_r\ddot{\theta}_L$,

$$\begin{aligned} J_M\, n_r\ddot{\theta}_L &= \tau_M - \frac{J_L\ddot{\theta}_L}{n_r} \\ \Rightarrow \tau_M &= J_M\, n_r\ddot{\theta}_L + \frac{J_L\ddot{\theta}_L}{n_r} \\ &= (0.012)(5)(-5) + (0.3)(-5)(0.2) = -0.6 \text{ N.m} \end{aligned}$$

Problem 6
CLO4-C3
15 points

A single revolute joint of a robot needs to move between $q_i = \pi/2$ and $q_f = 0$, under the velocity and acceleration bounds

$$|\dot{q}| \leq 2\, rad/s \qquad |\ddot{q}| \leq 4\, rad/s^2.$$

Determine a trajectory of the LSPB kind and the total time.

Solution 6 The solution is provided in the attached MATLAB PDF file.

Problem 7
CLO4-C3
15 points

An RP robot is described by the following dynamic equations in the two variables θ and d:

$$\begin{aligned} (I_1 + m_2 d^2)\ddot{\theta} + 2m_2 d\dot{\theta}\dot{d} + 10(m_1 l_1 + m_2 l_2)\cos\theta &= \tau \\ m_2\ddot{d} - m_2 d\dot{\theta}^2 + 10m_2\sin\theta &= f, \end{aligned}$$

where τ and f are control inputs.

(a) Design a computed torque controller, i.e. find τ and f, such that resulting system is decoupled (one equation completely in θ and other completely in d).

(b) Consider the system $\ddot{\theta} = u$, where u is the controller. Design an appropriate controller, i.e. find u, such that system tracks a given desired trajectory θ_r with zero error, is critically damped, and has natural frequency 50.

Solution 7

(a) By computed torque controller or inverse dynamics approach, choose

$$\tau = (I_1 + m_2 d^2)a_1 + 2m_2 d\dot{\theta}\dot{d} + 10(m_1 l_1 + m_2 l_2)\cos\theta$$
$$f = m_2 a_2 - m_2 d\dot{\theta}^2 + 10 m_2 \sin\theta$$

With this choice of control signals, the system dynamics are reduced to:

$$\ddot{\theta} = a_1$$
$$\ddot{d} = a_2.$$

Notice that θ and d are now uncoupled, and a_1 and a_2 are the new control signals that are completely under our control.

(b) To meet desired characteristics, we can set

$$a_1 = \ddot{\theta}_r + K_P(\theta_r - \theta) + K_D(\dot{\theta}_r - \dot{\theta})$$
$$a_2 = \ddot{d}_r + K_P(d_r - d) + K_D(\dot{d}_r - \dot{d})$$

The closed loop system for θ is now:

$$(\ddot{\theta} - \ddot{\theta}_r) + K_D(\dot{\theta} - \dot{\theta}_r) + K_P(\theta - \theta_r) = 0$$

Interpreting this differential equation in terms of the variable $(\theta - \theta_r)$ and comparing its coefficients to standard second-order system:

$$K_P = \omega_{cl}^2 = 2500$$
$$K_D = 2\zeta\omega_{cl} = 100.$$

In summary, the overall controller is:

$$\tau = (I_1 + m_2 d^2)[\ddot{\theta}_r + 2500(\theta_r - \theta) + 100(\dot{\theta}_r - \dot{\theta})] + 2m_2 d\dot{\theta}\dot{d} + 10(m_1 l_1 + m_2 l_2)\cos\theta$$
$$f = m_2[\ddot{d}_r + 2500(d_r - d) + 100(\dot{d}_r - \dot{d})] - m_2 d\dot{\theta}^2 + 10 m_2 \sin\theta$$