Habib University
Course Code: EE366/CE366/CS380
Course Title: Introduction to Robotics
Instructor name: Dr. Basit Memon
Examination: Mid-term Exam Exam Date: March 12, 2022
Total Marks: 100 Duration: 120 minutes

Instructions

1. You can refer to course slides, your notes, homework solutions, or any of the books included in the course syllabus. You're not permitted to specifically search for responses to any of the exam questions online. Where appropriate, you can cite the slides and don't have to redo what has already been done.

2. You are welcome to utilize MATLAB in whichever way you find useful. You can submit the response to a question in the form of a MATLAB live script as well, but you cannot use numerical methods where question explicitly asks you to employ analytical methods.

3. The exam will be administered under HU student code of conduct (see Chapter 3 of `https://habibuniversity.sharepoint.com/sites/Student/code-of-conduct`). You may not talk, discuss, compare, copy from, or consult with any other human on this planet (except me) on questions or your responses during the exam.

4. Make sure that you show your work (intermediate steps). Most of the points for any question will be given based on the followed process.

5. Kindly use a black/blue pen to write your hand-written solutions, so that they are legible.

6. The questions or their associated points are not arranged by complexity or time consumption.

Questions

Problem 1
50 points

Figure 1 is a modification of the ride illustrated in homework assignment 2 (modulo my illustration abilities). Assign a frame $\{B\}$ to the rider in Figure 1, such that its origin is at the point where rider is seated. Find an expression for the homogeneous transformation ${}^{W}T_{B}$, using DH frames procedure, in terms of transformations between consecutive frames. **What is different from the figure in homework 2?**

- The frame illustrated in blue in Figure 1 is the world frame (fixed). Axis $\hat{z}_W$ is along the vertical side of the page and $\hat{y}_W$ is along the horizontal side of page. Correspondingly, $\hat{x}_W$ is coming out of the page.
- The bar of this ride is attached between points O_W, origin of the world frame, and point B. The coordinates of points O_W and B, in frame $\{W\}$, are $(0, 0, 0)$ and $(0, D, -h)$ respectively. This is one point where the problem differs from HW 2 as the bar is slanted downwards.
- Another point of difference is that we have a prismatic joint, which allows the point C, where vertical bar of ride connects, to slide along the length of the wall-connecting rod. The distance of C from O_W along this wall-connecting rod is d.
- The vertical length L is along $-\hat{z}_W$ direction, i.e. the angle between L and d is not a right-angle.
- The ride is able to rotate about the wall-connecting rod in the indicated direction of θ_1, in the same way as the homework question.
- Similar to the homework, the ride is able to rotate about L in the indicated positive direction of θ_2.

Solution 1 DH frame assignment is given in Figure 2. Parameters are:

Link	a_i	α_i	d_i	θ_i
W-0	0	$(90^\circ + \beta)$	0	180°
1	0	0	d	0
2	0	$-(90^\circ + \beta)$	0	θ_1
3	R	0	$-L$	θ_2

where $\beta = \arctan 2(h, D)$.

Problem 2
50 points

Consider a 2R planar robot having link lengths $l_1 = 0.8$ and $l_2 = 0.4$ m. The robot should execute a motion along the straight path from the initial point $A = \begin{bmatrix} 1.42 & 0.6 \end{bmatrix}^T$ to the final point $B = \begin{bmatrix} 1.42 & -1.6 \end{bmatrix}^T$, both expressed in the world reference frame $\{W\}$. Answer the following questions for this robot with justifications.

(a) Determine a position $P_0 = \begin{bmatrix} x_0 & y_0 \end{bmatrix}^T$, expressed in coordinates of frame $\{W\}$, where the robot base can be placed so that its end-effector is capable of moving along the entire given path.

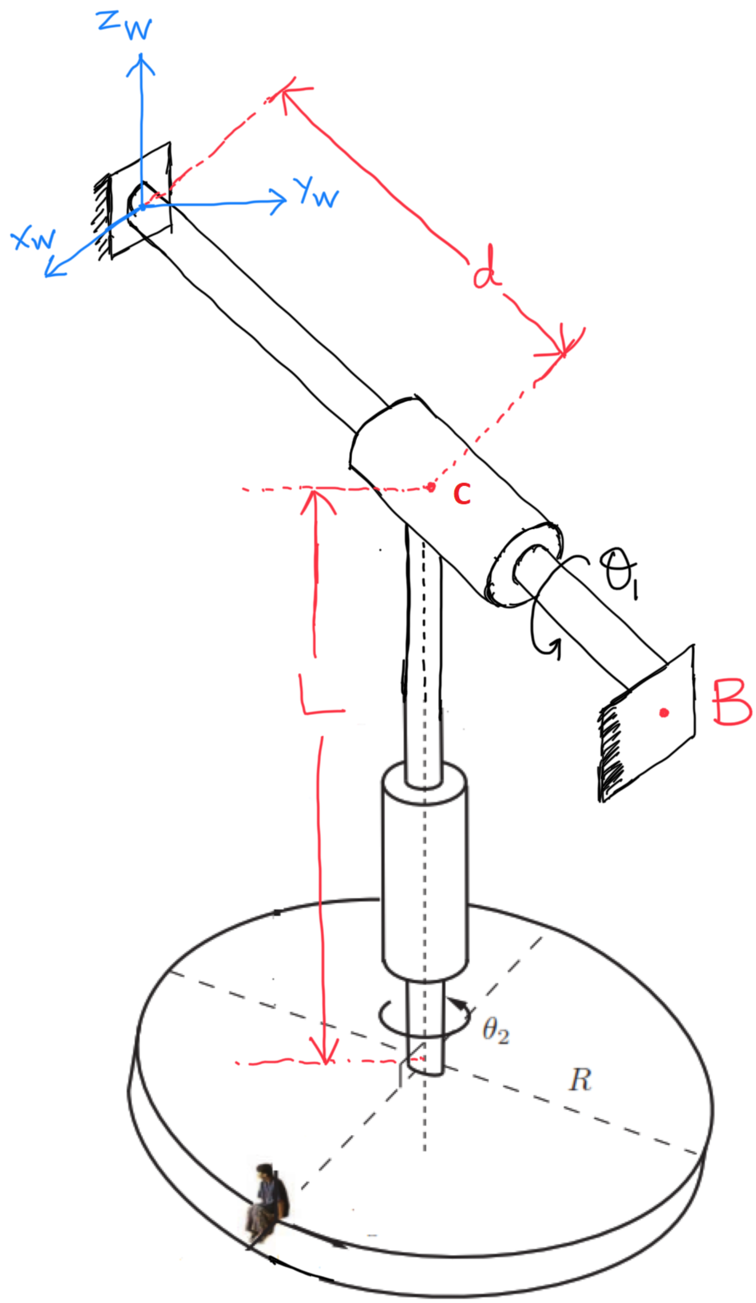

Figure 1: Amusement ride of HW 2 modified for mid-term exam

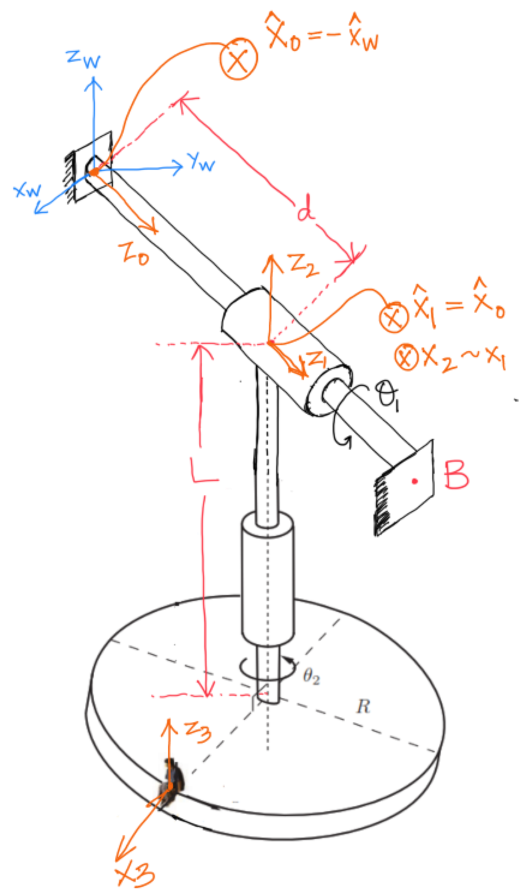

Figure 2: Caption here

(b) Are there any kinematic singularities encountered along this path?

(c) Find a robot configuration, q^* such that the end-effector is at the midpoint of the given path.

(d) At $q = q^*$, compute instantaneous joint velocity $\dot{q} \in \mathbb{R}^2$ that realizes the desired Cartesian motion on the line with a speed $V = 1.5$ m/s.

(e) If you were given the instantaneous end-effector acceleration, $\ddot{p}$, and end-effector velocity, $\dot{p}$, at the current configuration q, describe the steps for obtaining $\ddot{q}$.

Solution 2

(a) The problem requires us to ensure that the vertical line AB lies in the workspace of the planar robot. We know from Homework Assignment 1 that the workspace of a 2R planar robot is an annular region centered at the origin of the base, as illustrated in blue in Figure 3. The outer circle is traced when $\theta_2 = 0°$, and so the radius of outer

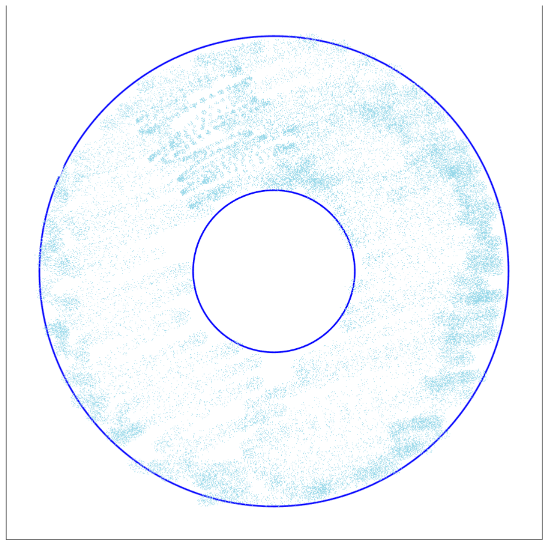

Figure 3: Workspace of 2R Planar Robot

circle is $l_1 + l_2 = 1.2$ m. The inner circle is trace when $\theta_2 = 180°$, and so radius is $l_1 - l_2 = 0.8 - 0.4 = 0.4$ m.

The path AB is a vertical line of length 2.2 m, so we can argue geometrically that there is only a small region in this robot's workspace where a line of this length could fit; this is marked in red in Figure 4. The robot needs to be placed or O needs to be chosen such that AB lies in this red region. The right border of this region or the farthest we can be away from the center is when the length of the chord is the same as AB. Using the green and orange triangles in Figure 4, we can find DE and OG. Specifically,

$$\begin{aligned} DE &= \sqrt{OD^2 - OE^2} \\ &= \sqrt{1.2^2 - 0.4^2} = 1.13 \\ OG &= \sqrt{OF^2 - FG^2} \\ &= \sqrt{1.2^2 - 1.1^2} = 0.48 \end{aligned}$$

So, the red region extends from 0.4m to 0.48m along the horizontal from the center. This figure assumes that y-coordinate of O, i.e. y_0 is at midpoint of AB, i.e. $y_0 = -0.5$.

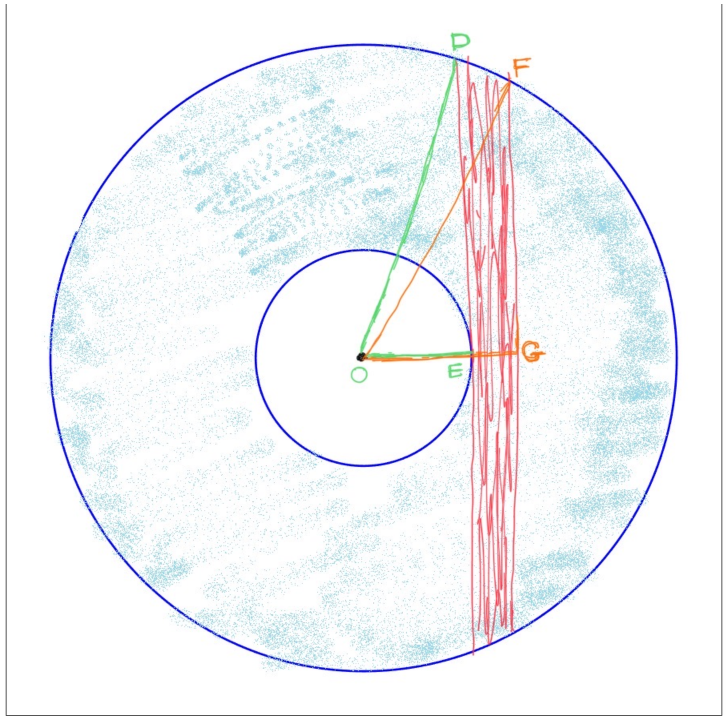

Figure 4: Region of the workspace where AB could be placed

The x-coordinate could be anywhere between $1.42 - 0.48 = 0.94$ and $1.42 - 0.4 = 1.02$. Let's choose $(x_0, y_0) = (1, -0.5)$. There are other possibilities for P_0 too if we allow base to be asymmetrically placed with respect to AB, i.e. robot is closer to one end than the other.

(b) The response to this question depends on the placement of the base. We know from the class notes that the two singularities of this arm occur at $\theta_2 = 0°$ and $\theta_2 = 180°$, i.e. the arm will pass through a kinematic singularity if any point of AB intersects with either the outer or inner boundary of the workspace. With our choice of P_0 from (a), the robot will not encounter any singularity. But if P_0 were chosen such that AB aligns with DE, then arm will encounter $\theta_2 = 180°$ singularity at E, and if AB aligns with FG then it will encounter $\theta_2 = 0°$ singularity at F and its mirror point.

(c) The midpoint of AB is at $(1.42, -0.5)$ in the world coordinates. We will find q^* assuming P_0 from (a). This can be done using IK expressions for 2R planar robot from class notes, but recall that these expressions are in the base frame of the robot. The midpoint in the base frame coordinates is $(1.42-1, -0.5+0.5) = (0.42, 0)$.

$$\theta_2 = 180^\circ - \arccos\left(\frac{0.8^2 + 0.4^2 - 0.42^2 - 0^2}{2 \times 0.4 \times 0.8}\right) = 167^\circ$$
$$\theta_1 = \arctan 2(0, 0.42) - \arccos\left(\frac{0.8^2 + 0.42^2 + 0^2 - 0.4^2}{2 \times 0.8 \times \sqrt{0.42^2 + 0^2}}\right) = -12.4^\circ$$

Note that this is one of the two possible solutions. The other solution is:

$$\theta_2 = -180^\circ + \arccos\left(\frac{0.8^2 + 0.4^2 - 0.42^2 - 0^2}{2 \times 0.4 \times 0.8}\right) = -167^\circ$$
$$\theta_1 = \arctan 2(0, 0.42) + \arccos\left(\frac{0.8^2 + 0.42^2 + 0^2 - 0.4^2}{2 \times 0.8 \times \sqrt{0.42^2 + 0^2}}\right) = 12.4^\circ$$

(d) We need to find the Jacobian. We know that for 2R planar manipulator,

$$x = 0.8\cos\theta_1 + 0.4\cos(\theta_1 + \theta_2)$$
$$y = 0.8\sin\theta_1 + 0.4\sin(\theta_1 + \theta_2)$$

Differentiating both equations,

$$\dot{x} = -0.8\,\dot{\theta}_1\ \sin\theta_1 - 0.4(\dot{\theta}_1 + \dot{\theta}_2)\sin(\theta_1 + \theta_2)$$
$$\dot{y} = 0.8\,\dot{\theta}_1\ \cos\theta_1 + 0.4(\dot{\theta}_1 + \dot{\theta}_2)\cos(\theta_1 + \theta_2)$$
$$\Rightarrow \underbrace{\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix}}_{v} = \underbrace{\begin{bmatrix}-0.8\sin\theta_1 - 0.4\sin(\theta_1+\theta_2) & -0.4\sin(\theta_1+\theta_2)\\ 0.8\cos\theta_1 + 0.4\cos(\theta_1+\theta_2) & 0.4\cos(\theta_1+\theta_2)\end{bmatrix}}_{J(q)}\underbrace{\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}}_{\dot{q}}$$
$$\Rightarrow \dot{q} = J^{-1}(q^*)v$$

where $q^* = (\theta_1^*, \theta_2^*) = (-12.4^\circ, 167^\circ)$ and since direction of end-effector velocity is along negative y-direction going from A to B, we have that $v = \begin{bmatrix}0\\ -1.5\end{bmatrix}$. Thus,

$$\dot{q} = \begin{bmatrix}0 & -0.003\\ 0.0073 & -0.0063\end{bmatrix}^{-1}\begin{bmatrix}0\\ -1.5\end{bmatrix}$$
$$= \begin{bmatrix}-204.8\\ -0.26\end{bmatrix}$$

(e) We know that

$$\dot{p} = J(q)\dot{q}$$

Differentiating this equation on both sides,

$$\begin{aligned} \ddot{p} &= J(q)\ddot{q} + \dot{J}(\dot{q})\dot{q} \\ \Rightarrow \ddot{q} &= J^{-1}\left[\ddot{p} - \dot{J}(\dot{q})\dot{q}\right] \end{aligned}$$

where

$$\dot{q} = J^{-1}\dot{p}$$