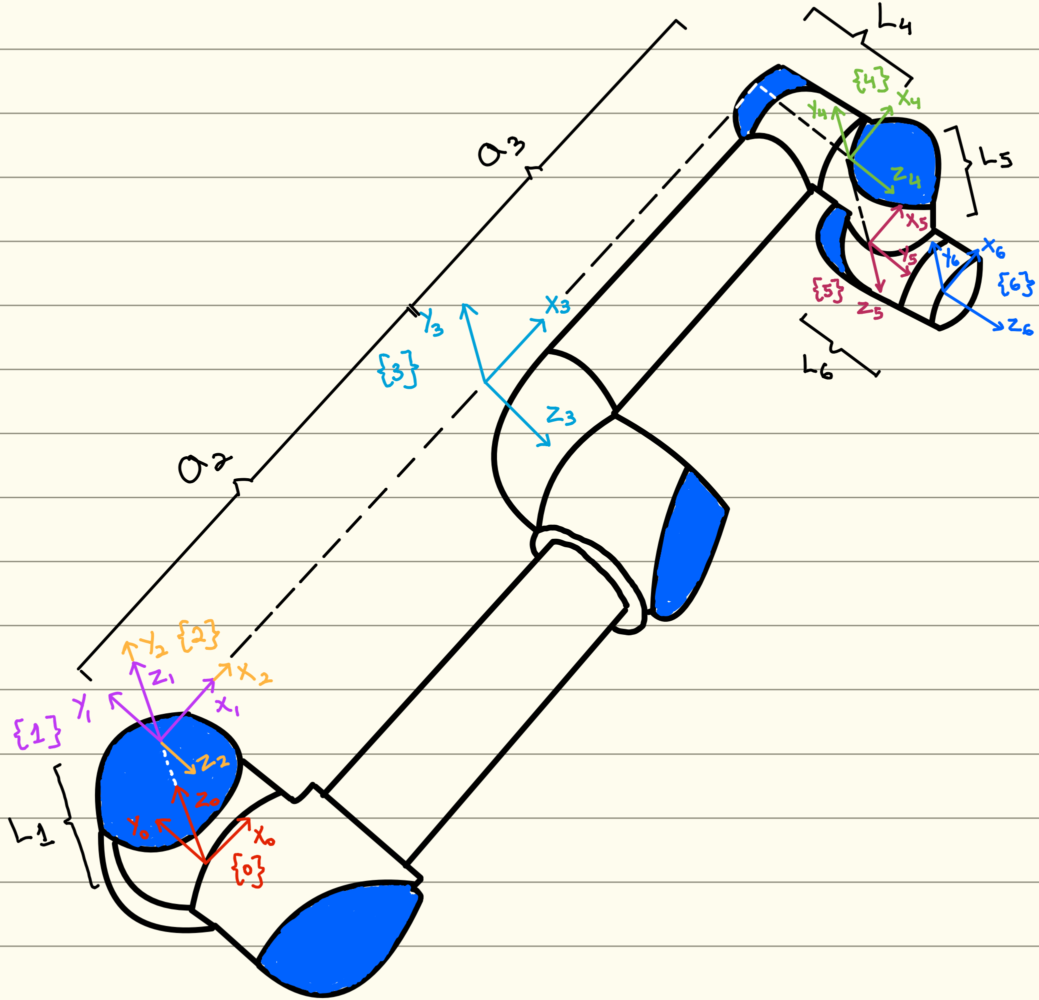
Q.3:)



L4
{4}
X4
Y4
Z4
L5
X5
Y5
{5}
Z5
X6
Y6
{6}
Z6
L6
a3
Y3
X3
{3}
Z3
a2
Y2
{2}
X2
Z1
Y1
X1
{1}
Z2
Z0
Y0
X0
{0}
L1

DH Parameters:

Link	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	L_1	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3
4	0	$\pi/2$	$L4$	θ_4
5	0	$-\pi/2$	$L5$	θ_5
6	0	0	$L6$	θ_6

$$
{}^{i-1}T_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cdot \cos\alpha_i & \sin\theta_i \cdot \sin\alpha_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\theta_i \cdot \cos\alpha_i & \cos\theta_i \cdot \sin\alpha_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Question 3

```
syms theta1 theta2 theta3 theta4 theta5 theta6;
syms r11 r12 r13 r21 r22 r23 r31 r32 r33 px py pz;
syms L1 a2 a3 L4 L5 L6

% Given transformation matrix of the end effector
T_given = [r11, r12, r13, px; r21, r22, r23, py; r31, r32, r33, pz; 0, 0,
0, 1] % Replace ... with the given transformation matrix
```

T_given =

$$\begin{pmatrix} r_{11} & r_{12} & r_{13} & \mathrm{px} \\ r_{21} & r_{22} & r_{23} & \mathrm{py} \\ r_{31} & r_{32} & r_{33} & \mathrm{pz} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T01 = [cos(theta1), -sin(theta1)*cos(pi/2), sin(theta1)*sin(pi/2),
0*cos(theta1);
       sin(theta1), cos(theta1)*cos(pi/2), cos(theta1)*sin(pi/2),
0*sin(theta1);
       0, sin(pi/2), cos(pi/2), L1;
       0, 0, 0, 1]
```

T01 =

$$\begin{pmatrix} \cos(\theta_1) & -\frac{4967757600021511\,\sin(\theta_1)}{81129638414606681695789005144064} & \sin(\theta_1) & 0 \\ \sin(\theta_1) & \frac{4967757600021511\,\cos(\theta_1)}{81129638414606681695789005144064} & \cos(\theta_1) & 0 \\ 0 & 1 & \frac{4967757600021511}{81129638414606681695789005144064} & L_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T12 = [cos(theta2), -sin(theta2)*cos(0), sin(theta2)*sin(0),
a2*cos(theta2);
       sin(theta2), cos(theta2)*cos(0), cos(theta2)*sin(0), a2*sin(theta2);
       0, sin(0), cos(0), 0;
       0, 0, 0, 1]
```

T12 =

$$\begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & a_2\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & a_2\sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T23 = [cos(theta3), -sin(theta3)*cos(0), sin(theta3)*sin(0),
a3*cos(theta3);
       sin(theta3), cos(theta3)*cos(0), cos(theta3)*sin(0), a3*sin(theta3);
       0, sin(0), cos(0), 0;
```

```
       0, 0, 0, 1]
```

T23 =

$$\begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & a_3\cos(\theta_3) \\ \sin(\theta_3) & \cos(\theta_3) & 0 & a_3\sin(\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T34 = [cos(theta4), -sin(theta4)*cos(pi/2), sin(theta4)*sin(pi/2),
0*cos(theta4);
       sin(theta4), cos(theta4)*cos(pi/2), cos(theta4)*sin(pi/2),
0*sin(theta4);
       0, sin(pi/2), cos(pi/2), L4;
       0, 0, 0, 1]
```

T34 =

$$\begin{pmatrix} \cos(\theta_4) & -\frac{4967757600021511\sin(\theta_4)}{81129638414606681695789005144064} & \sin(\theta_4) & 0 \\ \sin(\theta_4) & \frac{4967757600021511\cos(\theta_4)}{81129638414606681695789005144064} & \cos(\theta_4) & 0 \\ 0 & 1 & \frac{4967757600021511}{81129638414606681695789005144064} & L_4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T45 = [cos(theta5), -sin(theta5)*cos(-pi/2), sin(theta5)*sin(-pi/2),
0*cos(theta5);
       sin(theta5), cos(theta5)*cos(-pi/2), cos(theta5)*sin(-pi/2),
0*sin(theta5);
       0, sin(-pi/2), cos(-pi/2), L5;
       0, 0, 0, 1]
```

T45 =

$$\begin{pmatrix} \cos(\theta_5) & -\frac{4967757600021511\sin(\theta_5)}{81129638414606681695789005144064} & -\sin(\theta_5) & 0 \\ \sin(\theta_5) & \frac{4967757600021511\cos(\theta_5)}{81129638414606681695789005144064} & -\cos(\theta_5) & 0 \\ 0 & -1 & \frac{4967757600021511}{81129638414606681695789005144064} & L_5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T56 = [cos(theta6), -sin(theta6)*cos(0), sin(theta6)*sin(0), 0*cos(theta6);
       sin(theta6), cos(theta6)*cos(0), cos(theta6)*sin(0), 0*sin(theta6);
       0, sin(0), cos(0), L6;
       0, 0, 0, 1]
```

T56 =

$$\begin{pmatrix} \cos(\theta_6) & -\sin(\theta_6) & 0 & 0 \\ \sin(\theta_6) & \cos(\theta_6) & 0 & 0 \\ 0 & 0 & 1 & L_6 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
% Calculate the transformation matrix based on the symbolic joint angles
T06 = T01*T12*T23*T34*T45*T56
```

```
T06 =
```

$$\left(\begin{array}{cccc}
\cos(\theta_6)\,\sigma_5 - \sin(\theta_6)\,\sigma_2 & -\sin(\theta_6)\,\sigma_5 - \cos(\theta_6)\,\sigma_2 & \sigma_1 & L_5\,(\sigma_{10} - \cos(\theta_4 \\
\cos(\theta_6)\,\sigma_6 - \sin(\theta_6)\,\sigma_4 & -\sin(\theta_6)\,\sigma_6 - \cos(\theta_6)\,\sigma_4 & \sigma_3 & L_4\cos(\theta_1) + L_6\,\sigma \\
\cos(\theta_6)\,\sigma_9 - \sin(\theta_6)\,\sigma_8 & -\cos(\theta_6)\,\sigma_8 - \sin(\theta_6)\,\sigma_9 & \sigma_7 & L_1 + \frac{4967757600021511\,L_4}{81129638414606681695789005144064} + L_5\,(\\
0 & 0 & 0 &
\end{array}\right.$$

where

$$\sigma_1 = \frac{24678615572571482867467662723121\,\sin(\theta_1)}{6582018229284824168619876730229402019930943462534319453394436096} - \sin(\theta_5)\,\sigma_{13} - \sigma_{19}$$

$$\sigma_2 = \sigma_{10} + \frac{4967757600021511\,\sin(\theta_5)\,\sigma_{13}}{81129638414606681695789005144064} - \cos(\theta_4)\,\sigma_{25} + \frac{4967757600021511\,\cos(\theta_5)\,\sigma_{12}}{8112963841460668169578900514406}$$

$$\sigma_3 = \frac{24678615572571482867467662723121\,\cos(\theta_1)}{6582018229284824168619876730229402019930943462534319453394436096} - \sin(\theta_5)\,\sigma_{15} - \cos($$

$$\sigma_4 = \sigma_{11} + \frac{4967757600021511\,\sin(\theta_5)\,\sigma_{15}}{81129638414606681695789005144064} - \frac{4967757600021511\,\cos(\theta_5)\,\sigma_{14}}{81129638414606681695789005144064} + \cos(\theta_4)\,\sigma$$

$$\sigma_5 = \cos(\theta_5)\,\sigma_{13} - \sin(\theta_5)\,\sigma_{12}$$

$$\sigma_6 = \cos(\theta_5)\,\sigma_{15} + \sin(\theta_5)\,\sigma_{14}$$

$$\sigma_7 = \sigma_{23} - \sin(\theta_5)\,\sigma_{17} + \sigma_{22} - \cos(\theta_5)\,\sigma_{16} + \frac{122597380068651197257713 8}{533996758980227520598755426542388028650676130589 16}$$

$$\sigma_8 = \frac{4967757600021511\,\sin(\theta_5)\,\sigma_{17}}{81129638414606681695789005144064} + \cos(\theta_4)\,\sigma_{29} + \sin(\theta_4)\,\sigma_{28} - \frac{4967757600021511\,\cos(\theta}{811296384146066816957890}$$

$$\sigma_9 = \cos(\theta_5)\,\sigma_{17} + \sin(\theta_5)\,\sigma_{16}$$

$$\sigma_{10} = \frac{4967757600021511\,\sin(\theta_1)}{81129638414606681695789005144064}$$

$$\sigma_{11} = \frac{4967757600021511\,\cos(\theta_1)}{81129638414606681695789005144064}$$

$$\sigma_{12} = \sigma_{19} - \sin(\theta_1) + \sigma_{18}$$

$$\sigma_{13} = \cos(\theta_4)\,\sigma_{24} - \sin(\theta_4)\,\sigma_{25}$$

$$\sigma_{14} = \cos(\theta_1) + \sigma_{21} - \sigma_{20}$$

$$\sigma_{15} = \cos(\theta_4)\,\sigma_{26} + \sin(\theta_4)\,\sigma_{27}$$

$$ {}^{0}T_6 = {}^{0}T_1 \cdot {}^{1}T_2 \cdot {}^{2}T_3 \cdot {}^{3}T_4 \cdot {}^{4}T_5 \cdot {}^{5}T_6 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

The set of equations we get by simplifying are;

$$ r_{11} = C_1 C_{234} C_5 C_6 + C_6 S_1 S_5 - C_1 S_{234} S_6 $$

$$ r_{12} = -C_1 C_{234} C_5 C_6 - S_1 S_5 S_6 - C_1 C_6 S_{234} $$

$$ r_{13} = -C_1 C_{234} S_5 + C_5 S_1 $$

$$ r_{21} = C_{234} C_5 C_6 S_1 - C_1 C_6 S_5 - S_1 S_{234} S_6 $$

$$ r_{22} = -C_{234} C_5 S_1 C_6 + C_5 S_5 S_6 - C_6 S_1 S_{234} $$

$$ r_{23} = -C_{234} S_1 S_5 - C_1 C_5 $$

$$r_{31} = C_5 C_6 S_{234} + C_{234} S_6$$

$$r_{32} = -C_5 C_6 S_{234} + C_{234} C_6$$

$$r_{33} = -S_{234} S_5$$

$$P_x = -C_1 C_{234} S_5 L_6 + C_5 S_1 L_6 + C_1 S_{234} L_5$$
$$+ S_1 L_4 + C_1 C_{23} a_3 + C_1 C_2 a_2$$

$$P_y = -C_{234} S_1 S_5 L_6 - C_1 C_5 L_6 + S_1 S_{234} L_5$$
$$- C_1 L_4 + C_{23} S_1 a_3 + C_2 S_1 a_2$$

$$P_z = -S_{234} S_5 L_6 - C_{234} L_5 + S_{23} a_3 + S_2 a_2 + L_1$$

Considering we know the LHS of all of the above equations, solving them for $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ and θ_6;

$$j = P_x - L_6 r_{13}$$

$$S = L_6 r_{23} - P_y$$

$$K = P_x C_1 + P_y S_1 + C_{234} S_5 L_6 - S_{234} L_5$$

$$u = P_z - L_1 + C_{234} L_5 + S_{234} S_5 L_6$$

These are the error terms calculated.

Now using these we get the following;

$$\theta_1 = \arctan2(j, s) \pm \arctan2\left(\sqrt{j^2 + s^2 - L_4^2},\, L_4\right)$$

$$\theta_2 = \arctan2(u, k) - \arctan2(a_3 s_3,\, a_3 c_3 + a_2)$$

$$\theta_3 = \pm \arctan2(s_3, c_3)$$

$$\theta_4 = \theta_{234} - \theta_2 - \theta_3$$

$$\theta_5 = \pm \arctan2(s_5, c_5)$$

$$\theta_6 = \arctan2(s_6, c_6)$$

Here $\pm$ indicates the existence of multiple solutions.