Introduction to Robotics

Lab Instructor: Miss Sadaf Sheikh
Group Members: Afsah Hyder, Ailiya Fatima, Maleeha Hussain
Course Instructor: Dr Basit Memon

LAB 06-INVERSE KINEMATICS

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1 Task 6.1 Inverse Kinematics Solutions

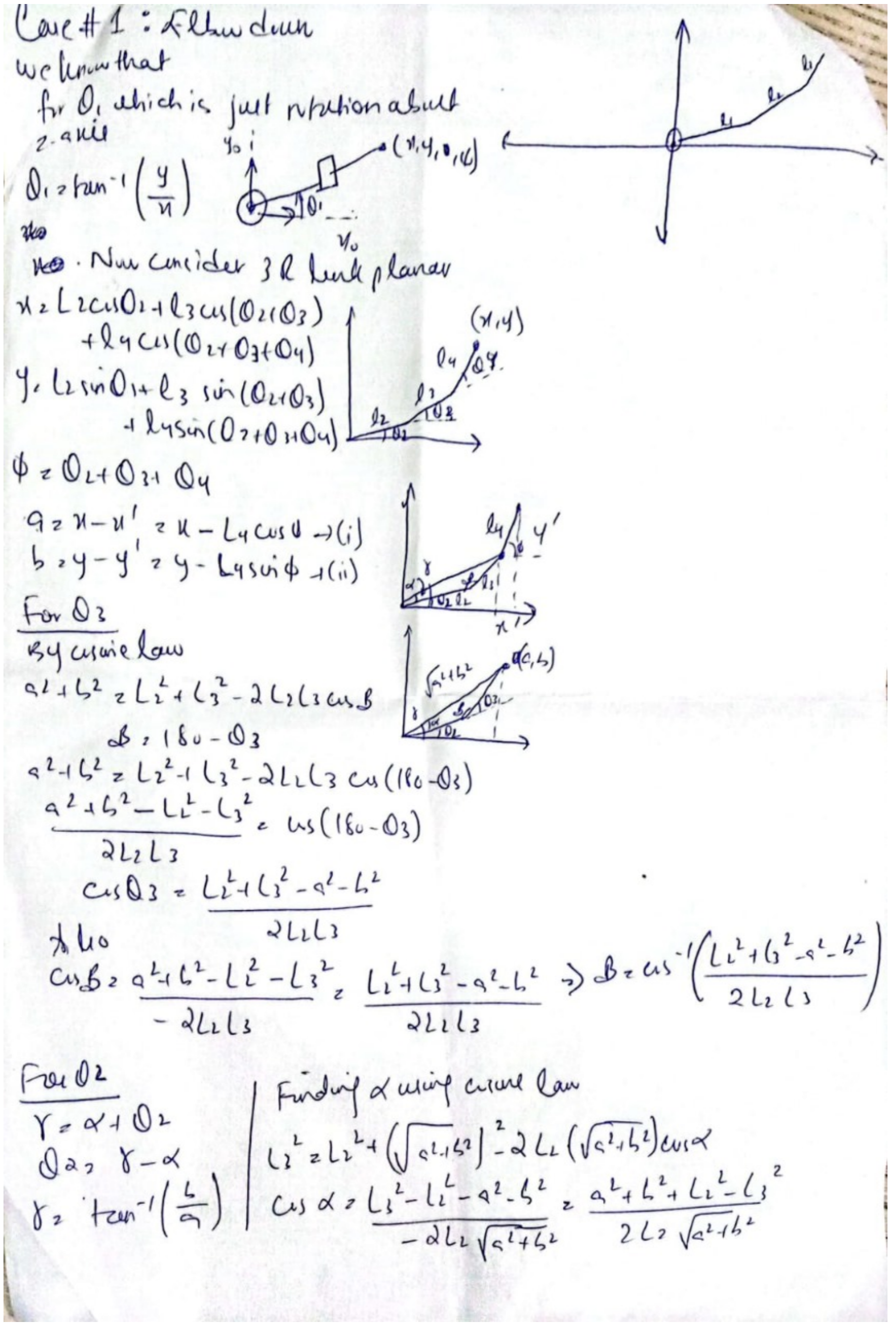
Task 6.1 Inverse Kinematics Solutions (60 points)

Given a desired position, (x, y, z), of the end-effector and orientation, ϕ, find mathematical expressions for all solutions to this inverse kinematics problem. Show all steps and specifically state how many solutions exist? Assuming that direction of $\hat{x}$ of the last frame or the end-effector frame is along the length of the last link, ϕ is the angle it makes with the x-axis of frame 1, i.e. $\phi = \theta_2 + \theta_3 + \theta_4$. When the gripper is parallel to the base board, then $\phi = 0^{\circ}$[a].

[a] See the remarks below for further explanation

Figure 1: Question

Four solutions exist, the working is shown below.



Case # 1 : Elbow down

we know that

for θ_1 which is just rotation about z-axis

$$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right)$$

Now consider 3R link planar

$$x = L_2\cos\theta_2 + l_3\cos(\theta_2+\theta_3) + l_4\cos(\theta_2+\theta_3+\theta_4)$$

$$y = L_2\sin\theta_2 + l_3\sin(\theta_2+\theta_3) + l_4\sin(\theta_2+\theta_3+\theta_4)$$

$$\phi = \theta_2+\theta_3+\theta_4$$

$$a = x - x' = x - L_4\cos\phi \rightarrow (i)$$

$$b = y - y' = y - L_4\sin\phi \rightarrow (ii)$$

For θ_3

By cosine law

$$a^2+b^2 = L_2^2+L_3^2-2L_2L_3\cos\beta$$

$$\beta = 180-\theta_3$$

$$a^2+b^2 = L_2^2+L_3^2-2L_2L_3\cos(180-\theta_3)$$

$$\frac{a^2+b^2-L_2^2-L_3^2}{2L_2L_3} = \cos(180-\theta_3)$$

$$\cos\theta_3 = \frac{L_2^2+L_3^2-a^2-b^2}{2L_2L_3}$$

Also

$$\cos\beta = \frac{a^2+b^2-L_2^2-L_3^2}{-2L_2L_3} = \frac{L_2^2+L_3^2-a^2-b^2}{2L_2L_3} \Rightarrow \beta = \cos^{-1}\left(\frac{L_2^2+L_3^2-a^2-b^2}{2L_2L_3}\right)$$

For θ_2

$$\gamma = \alpha+\theta_2$$

$$\theta_2 = \gamma-\alpha$$

$$\gamma = \tan^{-1}\left(\frac{b}{a}\right)$$

Finding α using cosine law

$$L_3^2 = L_2^2+\left(\sqrt{a^2+b^2}\right)^2-2L_2\left(\sqrt{a^2+b^2}\right)\cos\alpha$$

$$\cos\alpha = \frac{L_3^2-L_2^2-a^2-b^2}{-2L_2\sqrt{a^2+b^2}} = \frac{a^2+b^2+L_2^2-L_3^2}{2L_2\sqrt{a^2+b^2}}$$

Figure 2: Solution

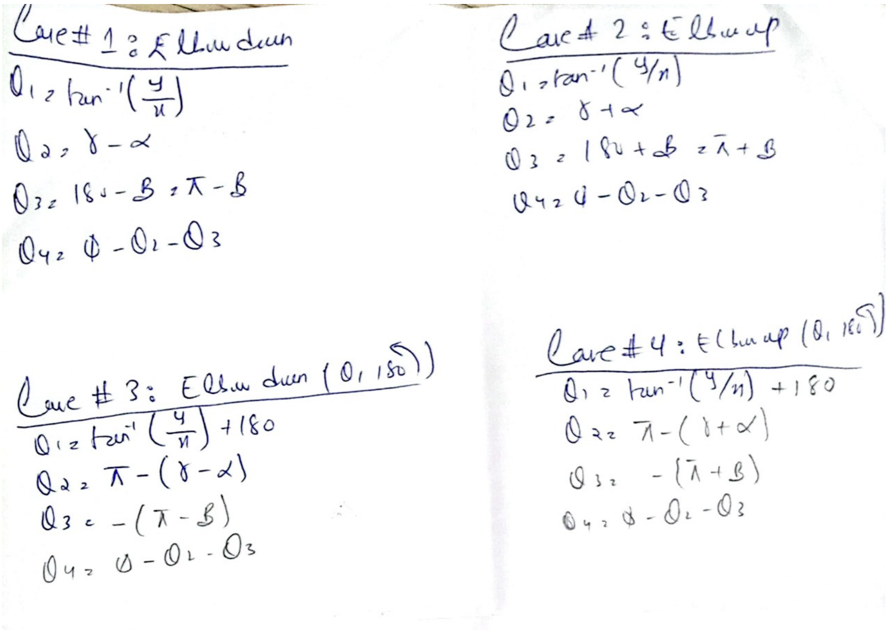


Figure 3: Solution

2 Task 6.2 Inverse Kinematics MATLAB function

```
function q=findJointAngles(endeff)
    q=zeros(4,4);
    x=endeff(1);y=endeff(2);z=endeff(3);phi=endeff(4);
    L2=108; L3=108; L4=76; L1=54;
    r=sqrt(x*x+y*y);
    s=z-L1;
    position=[r-L4*cos(phi),s-L4*sin(phi)];
    xl=position(1); yl=position(2);

    theta1=atan(y,x);
    gamma=atan(yl,xl);
    alpha=acos((L2^2+xl^2+yl^2-L3^2)/(2*L2*sqrt(xl^2+yl^2)));
    beta=acos((L2^2+L3^2-xl^2-yl^2)/(2*L2*L3));

    % 1
    form1=double([theta1 ; gamma-alpha; sym(pi)-beta; phi-(gamma-alpha
       )-(sym(pi)-beta)])
    % 2
    form2=[theta1 ; gamma+alpha; -sym(pi)+beta; phi-(gamma+alpha)-(-
       sym(pi)+beta)]
    % 3
    form3=[theta1+sym(pi) ; sym(pi)-(gamma-alpha); -sym(pi)+beta; phi
       -(sym(pi)-(gamma-alpha))-(-sym(pi)+beta)]
    % 4
    form4=[theta1+sym(pi) ; sym(pi)-(gamma+alpha); sym(pi)-beta; phi-(
       sym(pi)-(gamma+alpha))-(sym(pi)-beta)]
    q=[form1, form2, form3, form4]
```

```
    q=vpa(q,2)
end
```

3 Task 6.3 Optimal Solution

```

function findOptimalSolution_lab6(desiredendeff , currentPos)
    %currentPos= arb.getpos() % this gives us the current joint angles
%      currentPos=currentPos(1:4)
%      size(currentPos)
    desx=desiredendeff(1);desy=desiredendeff(2);desz=desiredendeff(3);desphi=desiredendeff(4);

    %now running inverse on desried post and orient to get joint angles

    joint_angles_final= findJointAngles([desx, desy,desz,desphi]) % this gives multiple possible joint angle sols
    % each column corresponds to one solution

    %now chosing best sol
    difference = zeros(4,4)
    for i = 1: 4
    difference(:,i)=double(abs( transpose(currentPos(1:4))-joint_angles_final(:,i)))

    end
    % each column of diff corresponds to difference of each sol with the current configuration
    % we need to sum each column to note the cumulative diff and make a
    % decision
    cum_diff = zeros(1,4)
    for i = 1:4
    cum_diff(i) = sum( difference(:,i));%sums each col

    end

    cum_diff
    min_val= min(cum_diff)

    % the min col of difference corresponds to the most optimal column of
    % solution matrix
    index = find(cum_diff== min_val)

    optimalSol = joint_angles_final(index) %gives column of the nearest sol

    return


```

```
end
```

4 Task 6.4 error code

```

function errorCode(jointAngles_deg)

% acccepts joint angles of Phantom X Pincher as argument, and sets
    them as goal positions for
% the respective motors in the arm.

% accepts angle in degrees for easy visualization
% the input is wrt to the DH so we convert it to servo angles
  jointAngles_deg(2) = jointAngles (2) - 90;

  % now check if any of the angles is outside [-150, 150]
  errorval = 0
  if any(jointAngles_deg > 150) || any(jointAngles_deg < -150)
        error('Vector contains values outside the range [-150, 150].')
            ;
        errorval = 1
  end

  if errorval==0

     jointAngles_rad = jointAngles_deg* pi/180
     % return joint angles radian position to input into arb.setpos
  end


end
```