Homework 2
Solution

Assigned on February 18, 2024
Due on February 26, 2024

Learning Outcomes:

After this homework, you should be able to:

(a) Use forward kinematics to determine position of any point on manipulator.

Tasks

Let the position of the end-point for the three link planar manipulator shown in Figure 1 be (x, y). Find expressions for x and y in terms of joint coordinates θ_1, d_2, and θ_3, using elementary geometry and trigonometry.

Problem 1
CLO2-C4

10 points

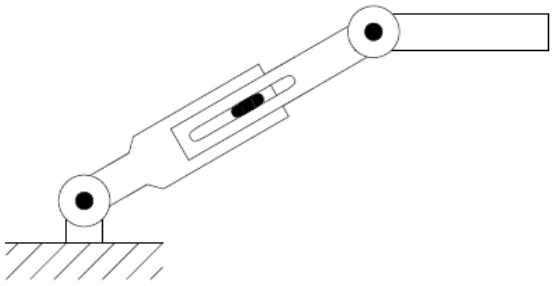

Figure 1: Three link planar robot

We can find the position of the end-effector through geometric means or by following the DH convention. Let's work it out geometrically here to build some intuition.

Solution 1

Let's assume that rotation of the base joint is given by θ_1, measured against the horizontal. For simplification, assume that the length between joint 1 and joint 3 is d_2, which is variable depending on the location of the prismatic joint. This length could also be written as $d = l_1 + l_2$, where l_1 is a fixed length of link 1 and l_2 is variable length of link 2. Finally, the rotation of third joint is θ_3, and $\theta_3 = 0$ when link 3 is extended and aligned with link 2. Length of link 3 is l_3.

If position of end-effector is (x, y), then

$$x = d\cos\theta_1 + l_3\cos(\theta_1 + \theta_3)$$
$$y = d\sin\theta_1 + l_3\sin(\theta_1 + \theta_3)$$

Problem 2
CLO2-C3
30 points

Figure 2 shows an RPRRR open chain in its zero (rest) configuration. Frames $\{0\}$ and $\{b\}$ have already been assigned in the figure. Assign the remaining frames according to standard DH convention, determine all link parameters between valid DH consecutive frames, write the homogeneous transformation matrices between all consecutive frames, and the final homogeneous transformation ${}^0T_b$ for this robot manipulator, illustrated in Figure 2, in terms of the joint variables $\{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5\}$[1]. You may make any reasonable assumptions. Also, determine the values of the joint variables in the present configuration as seen in Figure 2 and find the corresponding end-effector transformation, ${}^0T_b$.

Solution 2

Let's follow the steps of the DH convention to assign frames as illustrated in Figure 3.

1. $\hat{z}_0$ of the provided frame is in the direction of axis of rotation of joint 1, and it has been drawn correctly.
2. $\hat{x}_0$ and $\hat{y}_0$ can be arbitrarily chosen, following right-hand rule.
3. $\hat{z}_1$ is to be chosen in direction of axis of translation of joint 2. **Notice the discrepancy in the index here. $\hat{z}_1$ is along the axis of rotation of joint 2, but frame 1 moves when joint 1 moves since it is rigidly attached to link 1.**
4. $\hat{x}_1$ is chosen perpendicular to the plane formed by $\hat{z}_0$ and $\hat{z}_1$. The origin of frame 1 is at intersection of $\hat{z}_0$ and $\hat{z}_1$.
5. $\hat{y}_1$ is chosen according to right-hand rule in the $-\hat{z}_0$ direction.
6. The DH parameters are:

[1] θ_2 is translation due to prismatic joint.

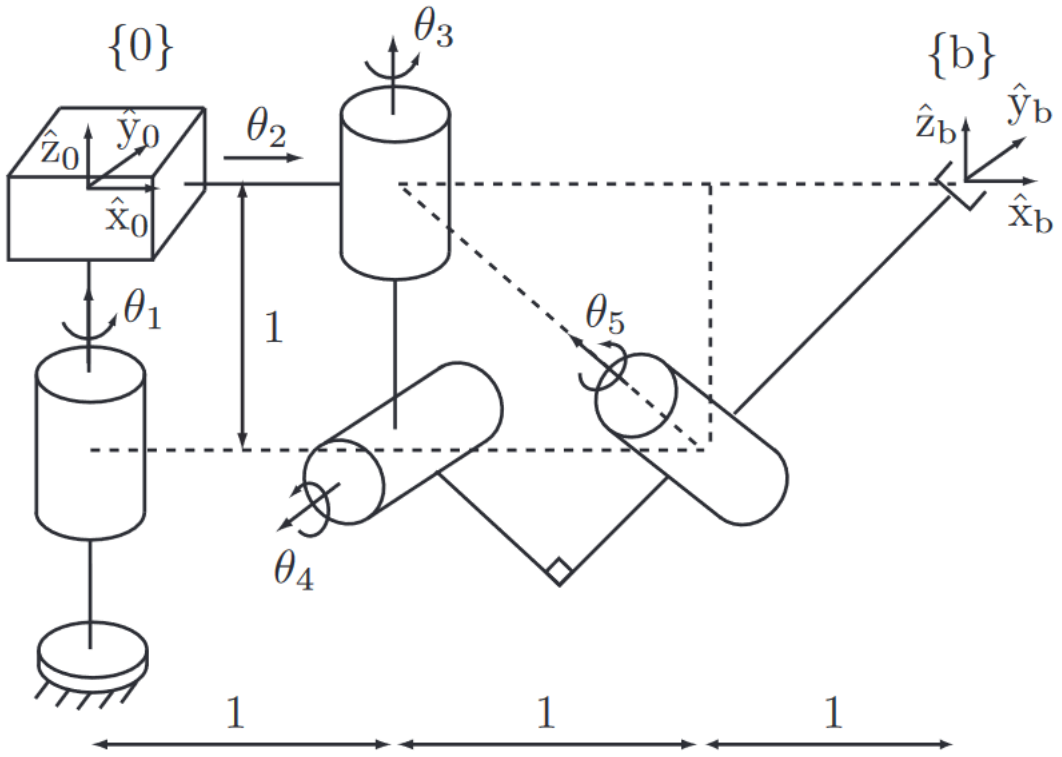

Figure 2: Manipulator for Problem 2

Link	a_i	α_i	d_i	θ_i
1	0	-90°	0	θ_1

Notice that zeroth configuration is when $\hat{x}_1$ is aligned with $\hat{x}_0$, or the arm is going into the page. For configuration drawn in the picture, $\theta_1 = -90^\circ$.

7. $\hat{z}_2$ is along axis of rotation of joint 3. $\hat{x}_2$ is normal to the plane formed by $\hat{z}_1$ and $\hat{z}_2$.

Link	a_i	α_i	d_i	θ_i
1	0	-90°	0	θ_1
2	0	90°	θ_2	0

The origin of frame 2 is at intersection of $\hat{z}_1$ and $\hat{z}_2$. The current value of θ_2 as seen in the figure is 1.

8. $\hat{z}_3$ is along axis of rotation of joint 4. $\hat{x}_3$ is normal to the plane formed by $\hat{z}_2$ and $\hat{z}_3$. The origin O_3 is at their intersection.

Link	a_i	α_i	d_i	θ_i
1	0	-90°	0	θ_1
2	0	90°	θ_2	0
3	0	90°	-1	θ_3

Note that d_3 is negative as the link offset is along the direction of $\hat{z}_2$ by definition, but $\hat{x}_3$ lies along $-\hat{z}_2$ direction from O_2. The zero configuration is when $\hat{x}_3$ is aligned with $\hat{x}_2$, or the arm is coming out of the page. For configuration drawn in the picture, $\theta_3 = 90^\circ$.

9. $\hat{z}_4$ is chosen along the axis of rotation of joint 5. $\hat{x}_4$ is chosen along the common normal between $\hat{z}_3$ and $\hat{z}_4$, pointing towards $\hat{z}_4$. The origin O_4 is at the intersection of $\hat{z}_4$ and $\hat{x}_4$. The DH parameters are:

Link	a_i	α_i	d_i	θ_i
1	0	-90°	0	θ_1
2	0	90°	θ_2	0
3	0	90°	-1	θ_3
4	$\frac{1}{\sqrt{2}}$	-90°	0	θ_4

In determining the last row, it is assumed that the right-angled bars between joint 4 and joint 5 are of the same length. The zero configuration is when $\hat{x}_4$ is aligned with $\hat{x}_3$, or when right-angled bars lines up with the previous link. For configuration drawn in the picture, $\theta_4 = 45^\circ$.

10. The last frame, the end-effector frame can be assigned arbitrarily. So, let's choose $\hat{z}_5$ perpendicular to the last link.

11. Since $\hat{z}_5$ is parallel to $\hat{z}_4$, $\hat{x}_5$ will be along the common normal, i.e. it will always be along the direction of the last link and pointing towards $\hat{z}_5$. The origin O_5 is chosen such that $\hat{x}_5$ passes through O_4.

12. The final DH parameters are:

Link	a_i	α_i	d_i	θ_i
1	0	-90°	0	θ_1
2	0	90°	θ_2	0
3	0	90°	-1	θ_3
4	$\frac{1}{\sqrt{2}}$	-90°	0	θ_4
5	$\sqrt{2}$	0°	0	θ_5

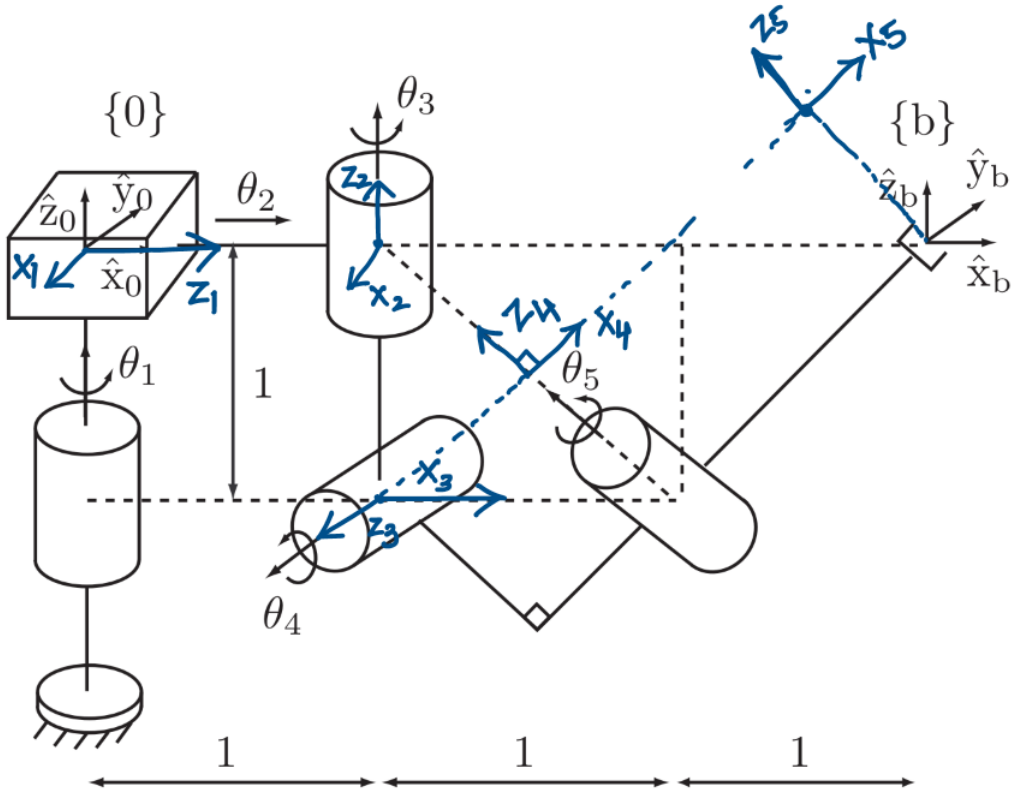

Figure 3: RPRRR Robot Frame Assignment

The current value of θ_5 is $0°$.

The corresponding A matrices have the structure:

$$A_i = \begin{bmatrix} c\theta_i & -s\theta_i\, c\alpha_i & s\theta_i\, s\alpha_i & a_i\, c\theta_i \\ s\theta_i & c\theta_i\, c\alpha_i & -c\theta_i\, s\alpha_i & a_i\, s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and transformations ${}^{i-1}T_i$ can be determined using this matrix for $1 \leq i \leq 5$. The final homogeneous transformation ${}^{5}T_b$ cannot be determined using DH parameters as the transformation requires rotation about y, which is not encapsulated in DH transformations.

$${}^{5}T_b = R_y(\theta_5)$$

Thus,

$$^0T_b = A_1A_2A_3A_4A_5\,^5T_b.$$

Problem 3
CLO2-C4

45 points

RoboCup is an annual robotics competition, with its finals being held in summer every year, to promote robotics and AI research. While the title comes from a soccer competition modeled after FIFA, Robocup has grown to include a variety of leagues and competitions. RoboCup Autonomous Robot Manipulation (ARM) Challenge, sponsored by Mathworks, has been a recent addition to the competition. This year the challenge will utilize Universal Robotics' UR5e arm, and your task is to determine its forward kinematics. You are to submit:

- schematic of kinematic chain using our class's convention for drawing revolute and prismatic joints schematically;
- a proper assignment of DH frames on the bottom right figure in the specifications sheet;
- a table of link parameters (You may indicate and assume reasonable values for missing information);
- an implementation of a workspace approximation algorithm that calculates an approximate end-effector position workspace of this robot. You should loop through all combinations of joint angles within its joint range, up to a resolution of 10°, and for each configuration calculates the end-effector position using forward kinematics. Output and plot these points. What kind of shape do the points approximate? How densely or sparsely is this shape sampled?

Solution 3

The frame assignment is shown in Figure 4. Corresponding to this frame assignment, the DH parameters are:

Link	a_i	α_i	d_i	θ_i
1	0	90°	162.5	θ_1
2	425	0°	0	θ_2
3	392.2	0°	0	θ_3
4	0	90°	133	θ_4
5	0	-90°	99.7	θ_5
6	0	0°	$99.7/2 + 41.7$	θ_6

Problem 4
CLO2-C2

15 points

Answer the following questions individually:

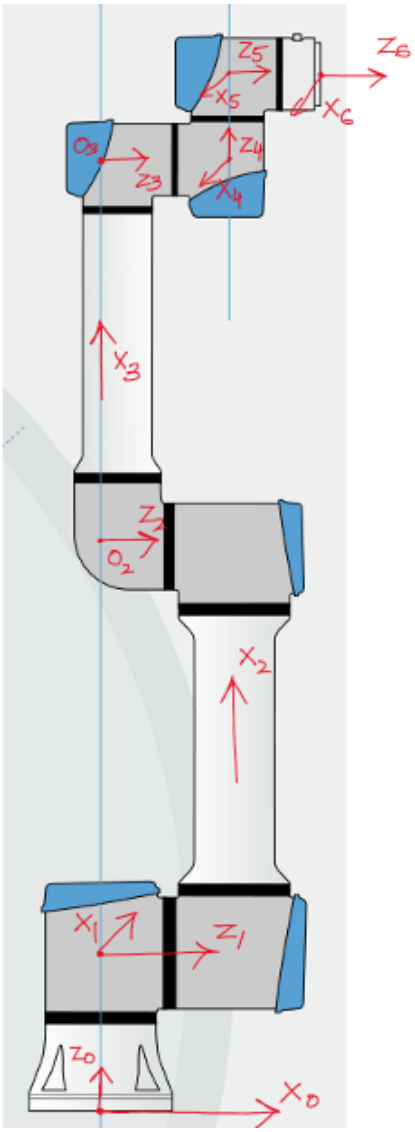

Figure 4: Frame Assignment of UR5e

(a) How many hours did each of you spend on this homework and specifically state your contribution in this homework assignment? Answer as accurately as you can, as this will be used to structure next year's class.

(b) Do you have any specific advice for students attempting this homework next year?

(c) Each group member is to provide a self-reflection in the form of a note or a concept

map. This requires you to reflect on your learning in relation to each of the outcomes stated at the beginning of this document.

Some questions that may help in this regard are: Have I achieved this outcome? What do I currently understand about content related to this outcome? How does it help me understand or build any robot? Do I have unanswered questions? What went wrong? How can I enable myself to achieve this outcome? What could I do to know more or enhance my skills in this context?