EE 366/CE 366/CS 380: Introduction to Robotics Mid-Exam Retake Spring 2023

April 19, 2023

Determining the starting position of the primary end-effector

We can determine p_M, the starting position of the end-effector, using forward kinematics. Let the joint angles for the primary manipulator be $\theta_{1,M}$, $\theta_{2,M}$, and $\theta_{3,M}$. Then, $({}^M x_p, {}^M y_p)$ can be found as:

$$
{}^M x_p = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3)
$$
$$
{}^M y_p = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3)
$$

Substituting the provided joint angles and link lengths:

```
l1 = 0.5;
l2 = 0.5;
l3 = 0.25;
syms theta1 theta2 theta3
% theta1 = 90;
% theta2 = -60;
% theta3 = 0;
x_p = l1*cos(theta1)+l2*cos(theta1+theta2)+l3*cos(theta1+theta2+theta3);
y_p = l1*sin(theta1)+l2*sin(theta1+theta2)+l3*sin(theta1+theta2+theta3);
xp=double(subs(x_p,[theta1, theta2, theta3],[deg2rad(90),deg2rad(-60),deg2rad(0)]))
```

```
xp = 0.6495
```

```
yp=double(subs(y_p,[theta1, theta2, theta3],[deg2rad(90),deg2rad(-60),deg2rad(0)]))
```

```
yp = 0.8750
```

Note that the position is expressed in the frame M.

Determining the initial velocity of the primary end-effector

We can do this by computing the linear Jacobian of the primary manipulator, ${}^M J$. This can be accomplished by differentiating the position of the primary manipulator:

```
J = [diff(x_p,theta1), diff(x_p,theta2), diff(x_p,theta3);
    diff(y_p,theta1), diff(y_p,theta2), diff(y_p,theta3)]
```

```
J =
```

$$\begin{pmatrix} -\sigma_1 - \frac{\sin(\theta_1+\theta_2)}{2} - \frac{\sin(\theta_1)}{2} & -\sigma_1 - \frac{\sin(\theta_1+\theta_2)}{2} & -\sigma_1 \\ \sigma_2 + \frac{\cos(\theta_1+\theta_2)}{2} + \frac{\cos(\theta_1)}{2} & \sigma_2 + \frac{\cos(\theta_1+\theta_2)}{2} & \sigma_2 \end{pmatrix}$$

where

$$\sigma_1 = \frac{\sin(\theta_1+\theta_2+\theta_3)}{4}$$

$$\sigma_2 = \frac{\cos(\theta_1+\theta_2+\theta_3)}{4}$$

The desired velocity is now determined using $J\dot{q}$, where the joint velocities are as provided.

```
v = double(subs(J,[theta1, theta2, theta3],[deg2rad(90),deg2rad(-60),deg2rad(0)])*[deg2rad(-30)
```

```
v = 2×1
    0.6545
   -0.6802
```

Determining the desired position and orientation for the secondary manipulator

We have found the desired position, p_M, and the desired orientation, ϕ, for our secondary manipulator. But, both of these are expressed with respect to the Mframe and we'll have to transform them to the S frame. So, let's find the homogeneous transformation, ${}^{S}T_M$, between the two frames:

```
Tsm = [-1,0,1.6;
        0,-1,0.9;
        0,0,1]
```

```
Tsm = 3×3
   -1.0000         0    1.6000
         0   -1.0000    0.9000
         0         0    1.0000
```

Note that z has not been included in this homogeneous transformation as we're working with planar robots only.

The initial position in the secondary frame, ${}^{S}p_M$, is:

```
p_s = double(Tsm*[xp;yp;1])
```

```
p_s = 3×1
    0.9505
    0.0250
    1.0000
```

To find the orientation, we have already determined the initial velocity vector. We'll transform it to S coordinates first:

```
vs = double(Tsm(1:2,1:2)*v)
```

```
vs = 2×1
   -0.6545
    0.6802
```

The direction of this vector can be determined using $\arctan 2(v_y, v_x)$:

```
psi = double(atan2d(vs(2),vs(1)))
```

```
psi = 133.8979
```

As indicated in the question, we want the orientation of the secondary end-effector to be orthogonal to this line. So, we want our orientation of the secondary manipulator to be:

```
phi = psi-90
```

```
phi = 43.8979
```

Determining the joint angles for the secondary manipulator

Now, we can use inverse kinematics to find the joint angles of the secondary manipulator that achieve the desired position and orientation. Since it's a three-link manipulator, we can find the coordinates of the end-point of link 2 and then use IK solution for a 2-link manipulator to find θ_1 and θ_2. Let the coordinates of the end-point of link 2 be (x_2, y_2). Then,

$$x_2 = x - l_3 \cos \phi$$
$$y_2 = y - l_3 \sin \phi$$

```
x2 = p_s(1)-l3*cosd(phi)
```

```
x2 = 0.7703
```

```
y2 = p_s(2)-l3*sind(phi)
```

```
y2 = -0.1483
```

According to the class slides, there are two possible IK solutions for a 2R manipulator. Let's say that we'll always use the elbow up solution in this task. Then,

$$\theta_1 = \arctan 2(y_2, x_2) - \arccos\left(\frac{l_1^2 + x_2^2 + y_2^2 - l_2^2}{2l_1\sqrt{x_2^2 + y_2^2}}\right)$$

$$\theta_2 = 180^\circ - \arccos\left(\frac{l_1^2 + l_2^2 - x_2^2 - y_2^2}{2l_1 l_2}\right)$$

```
theta1_s = atan2d(y2,x2)+acosd((l1^2+x2^2+y2^2-l2^2)/(2*l1*sqrt(x2^2+y2^2)))
```

```
theta1_s = 27.4264
```

```
theta2_s = -(180-acosd((l1^2+l2^2-x2^2-y2^2)/(2*l1*l2)))
```

```
theta2_s = -76.6529
```

Finally,

$\theta_3 = \phi - \theta_1 - \theta_2$

```
theta3_s = phi-theta1_s-theta2_s
```

```
theta3_s = 93.1244
```

Determining the initial velocity of the secondary manipulator

We can use the Jacobian of the secondary manipulator to find the initial end-effector velocity. Since the manipulators are identical, the expressions for the linear Jacobian of the secondary end-effector, ${}^{S}J_v$, will be exactly the same as the primary, i.e.

```
sJv = J
```

sJv =

$$\begin{pmatrix} -\sigma_1 - \frac{\sin(\theta_1+\theta_2)}{2} - \frac{\sin(\theta_1)}{2} & -\sigma_1 - \frac{\sin(\theta_1+\theta_2)}{2} & -\sigma_1 \\ \sigma_2 + \frac{\cos(\theta_1+\theta_2)}{2} + \frac{\cos(\theta_1)}{2} & \sigma_2 + \frac{\cos(\theta_1+\theta_2)}{2} & \sigma_2 \end{pmatrix}$$

where

$$\sigma_1 = \frac{\sin(\theta_1+\theta_2+\theta_3)}{4}$$

$$\sigma_2 = \frac{\cos(\theta_1+\theta_2+\theta_3)}{4}$$

What about the angular Jacobian? Note that the angular velocity is always in the z direction in this case, and

${}^{S}\omega_{0e} = \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3$

So, the angular Jacobian, ${}^{S}J_\omega$, can be represented as:

```
sJw = [1 1 1];
```

The complete Jacobian is:

```
sJ = [sJv; sJw]
```

```
sJ =
```

$$\begin{pmatrix} -\sigma_1 - \frac{\sin(\theta_1+\theta_2)}{2} - \frac{\sin(\theta_1)}{2} & -\sigma_1 - \frac{\sin(\theta_1+\theta_2)}{2} & -\sigma_1 \\ \sigma_2 + \frac{\cos(\theta_1+\theta_2)}{2} + \frac{\cos(\theta_1)}{2} & \sigma_2 + \frac{\cos(\theta_1+\theta_2)}{2} & \sigma_2 \\ 1 & 1 & 1 \end{pmatrix}$$

where

$$\sigma_1 = \frac{\sin(\theta_1+\theta_2+\theta_3)}{4}$$

$$\sigma_2 = \frac{\cos(\theta_1+\theta_2+\theta_3)}{4}$$

Now, we can easily find the initial joint velocities using $\dot{q} = J^{-1}\begin{pmatrix} v \\ \omega \end{pmatrix}$. Here $\omega = 0$, as we know that no change in the orinetation of secondary end-effector should occur as it traverses along the desired trajectory:

```
qs = double(subs(sJ,[theta1, theta2, theta3],[deg2rad(theta1_s),deg2rad(theta2_s),deg2rad(theta
```

```
qs = 3×1
    1.9374
   -2.4875
    0.5501
```

These joint speeds are in rad/s and are equivalent to

```
rad2deg(qs)
```

```
ans = 3×1
  111.0040
 -142.5247
   31.5207
```

degrees/s.