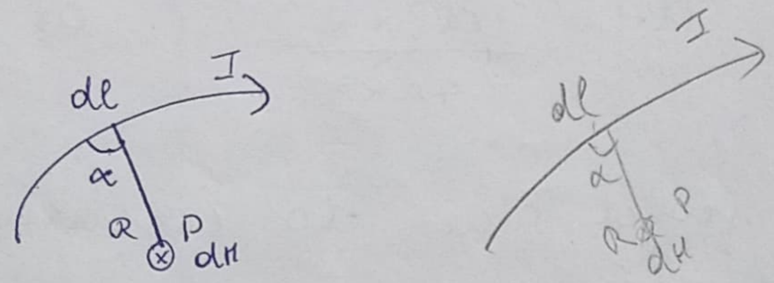
MODULE – 2

Bio Savart's Law



The differential magnetic field intensity 'dH' produced at a point 'P' ~~tary~~ by the differential current element 'Idl' is proportional to the product of current element & ~~one~~ the ~~line joining~~ sine of angle 'α' b/w the elements & line joining 'P' and is inversely proportional to square of the distance

$$dH \propto \frac{I\,dl \sin\alpha}{R^2}$$

$$dH = \frac{K I\,dl \sin\alpha}{R^2} \qquad K = \frac{1}{4\pi}$$

$$dH = \frac{I\,dl \sin\alpha}{4\pi R^2}$$

$$dH = \frac{I\,dl \times \hat{a}_r}{4\pi R^2}$$

$$dH = \frac{I\,dl \times \vec{R}}{4\pi R^3} \qquad \hat{a}_r = \frac{\vec{R}}{|R|}$$

Magnetic field due to continuous current distributions

i) Line distribution

$$H = \int_L \frac{I \cdot dl \times \hat{a}_r}{4\pi R^2}$$

ii) Surface Current

$$H = \int_S \frac{K\,ds \times \hat{a}_r}{4\pi R^2}$$

iii) Volume

$$H = \int_V \frac{J \cdot dv \times \hat{a}_r}{4\pi R^2}$$

Ampere's Circuit Law

It states that the line integral of the tangential componential of 'H' around a closed path is same as the current enclosed by path.

We are considering magnetic field

$$\oint_L H \cdot dl = I_{enclosed}$$

We know

$$I_{enclosed} = \int_S I \cdot ds$$

where J is the volume current density.

Apply stokes theorem

$$\oint_S A \cdot dl = \int_S (\nabla \times A)\, ds$$

After equaling

$$\int_S (\nabla \times H)\, ds = \int J \cdot ds$$

ie $\boxed{\nabla \times H = J}$

This is 3rd maxwell eqn.

19/7/23

Magnetic Scalar & Vector potential

Consider V_m as scalar, A as vector.
Considering 2 properties of curl

$$\nabla \times (\nabla \cdot A) = 0 \quad \text{--- (1)}$$

$$\nabla \cdot (\nabla \times A) = 0 \quad \text{--- (2)}$$

We know

$$E = -\nabla V$$

In the case of magnetic field

$$H = -\nabla V_m \quad \text{--- (A)}$$

This is scalar magnetic potential.

This expression is valid only if $J = 0$

$$J = \nabla \times H \quad \text{--- (3)}$$

Ampere circuit law

Sub (A) in (3)

$$J = \nabla \times -\nabla V_m$$

$$= \nabla^2 V_m = 0 \quad \text{--- (4)}$$

From 4th maxwells eq.

$$\nabla \cdot B = 0 \quad \text{--- (5)}$$

From (5) & (2)

$$B = \nabla \times A$$

If flux is considered.

$$\Psi = \int_S B \cdot ds$$

$$\Psi = \int_S \nabla \times A \, ds$$

If we apply divergence theorem

$$\oint A \, ds = \int_S \nabla \times A \, ds$$

$$\Psi = \oint_L A \, dl$$

These are magnetic scalar & vector potential.

Q: <u>Derivation of Maxwell's eqn from Faraday's law (Dynamic)</u>

Faraday figured out that changing magnetic flux within a circuit produce an induced emf or voltage within circuit.

$$\text{Emf} = \frac{-d\phi}{dt}$$

$$\phi = \int B \cdot ds$$

According to KVL, total emf around the circuit is equal to suming of small contribution at each emf at point

$$\text{Emf} = \oint E \cdot dl$$

Considering 2 eq^n

$$\oint_L E \cdot dl = -\frac{d}{dt}\int_S B \cdot ds$$

$$= -\int_S \frac{\partial B}{\partial t} \cdot ds$$

Apply stokes theorem

$$\oint_L A \cdot dl = \int_S (\nabla \times A)\, ds$$

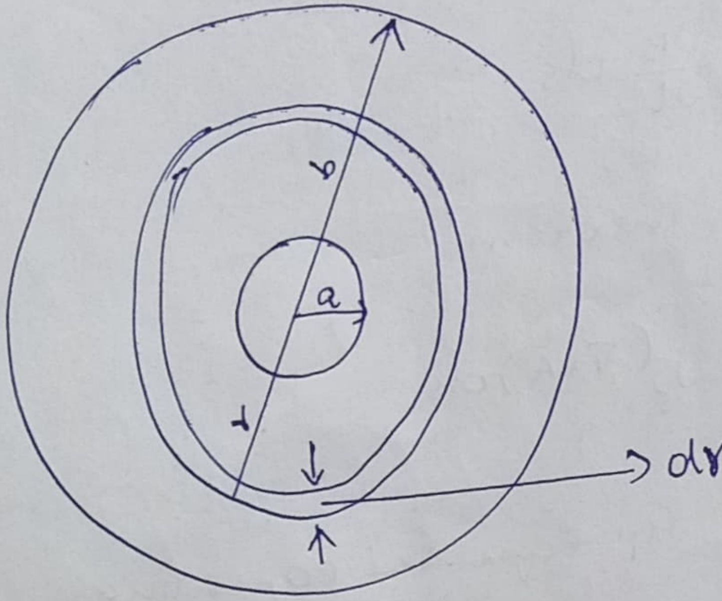
Rearranging & ~~equili~~ equating

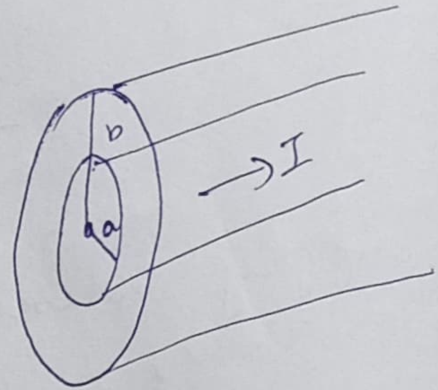
$$\int_S (\nabla \times E)\, ds = -\int_S \frac{\partial B}{\partial t} \cdot ds$$

$$\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$$

1) Ampere Circuital Law
2) Bio savart
3) Inductance of Coaxiable Cable
4) Maxwell derivation.

Inductance of Coaxial





$$L = \frac{N\phi}{I}$$

For a coaxial cable there are only one ring so $N = 1$

$$L = \frac{Q}{I} \quad \text{——— (1)}$$

Total magnetic flux linkage

$$\phi = \int B \cdot ds$$

Magnetic flux density

$$B = \frac{\mu_0 N I}{l}$$ Here $N = 1$

$l = 2\pi r$; it is a coaxial cable

$$B = \frac{\mu_0 I}{2\pi r} \text{ ——— } ②$$

Sub the above eqn in $\phi = \int B\,ds$

$$\phi = \int_a^b \frac{\mu_0 I}{2\pi r} \cdot dr$$

$$= \frac{\mu_0 I}{2\pi} \int_a^b \left(\frac{1}{r}\right) dr$$

$$= \frac{\mu_0 I}{2\pi} \left[\ln r\right]_a^b$$

$$= \frac{\mu_0 I}{2\pi} \left[\ln b - \ln a\right]$$

$$\phi = \frac{\mu_0 I}{2\pi} \ln(b/a) \text{ ——— } ③$$

Sub ③ in ①

$$L = \frac{\mu_0 I}{2\pi} \ln(b/a)$$

$$L = \frac{\mu_0}{2\pi} \ln b/a \text{ H/m}$$

21/7/23 Inductance of Solenoid

We know that

$$emf = L \cdot \frac{di}{dt}$$

According to faraday's law

$$emf = -N \frac{d\phi_B}{dt}$$

From above 2 eq^n

$$-L\frac{di}{dt} = -N\frac{d\phi_B}{dt}$$

$$N\phi_B = Li \qquad \therefore \phi = BA.$$

$$L = \frac{N\phi_B}{i}$$

$$L = \frac{N(BA)}{i}$$

Magnetic flux

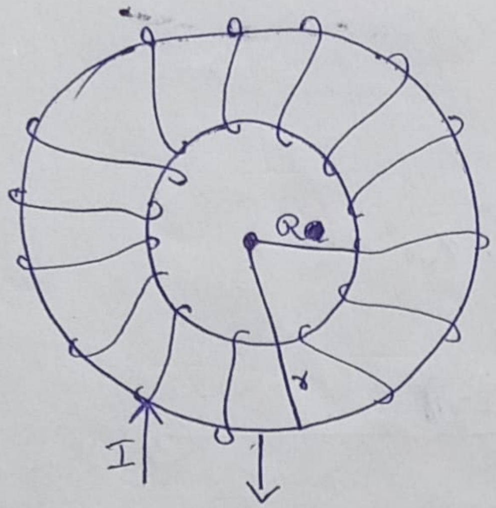
$$B = \frac{\mu_0 N i}{l}$$

Self Inductance only depends on geometric factors.

$$L = \frac{N \mu_0 N i/l \, A}{i}$$

$$\boxed{L = \frac{\mu_0 N^2 A}{l}}$$

Inductance of toroid



Consider a toroid of 'N' No of turns current I with Radius R.

$$L = \frac{N\phi}{I} \quad \text{--- (1)}$$

Magnetic flux

$$\phi = B \cdot A \quad \text{--- (2)}$$

flux density of toroid is

$$B = \frac{\mu_0 N I}{l} \qquad \text{here } l = 2\pi R$$

~~If the~~

$$B = \frac{\mu_0 N I}{2\pi R} \quad \text{--- (3)}$$

If the radius of coil is 'r' then the cross sectional area is given as

$$A = \pi r^2 \quad \text{--- (4)}$$

Sub (3) & (4) in (2)

$$\phi = B \cdot A$$

$$\phi = \frac{\mu_0 N I}{2\pi R} \cdot \pi r^2$$

$$= \frac{\mu_0 N I r^2}{2R} \quad \text{--- (5)}$$

Sub (5) in (1)

$$L = \frac{N\phi}{I} = \frac{N}{I} \frac{\mu_0 N I r^2}{2R}$$

$$\boxed{L = \frac{\mu_0 N^2 r^2}{2R} \cdot H}$$

27/03/23