Stokes Theorem

Line Integral of a vector ~~and~~ round a closed path is equal to the surface integral of the normal component of its curl over any closed surface

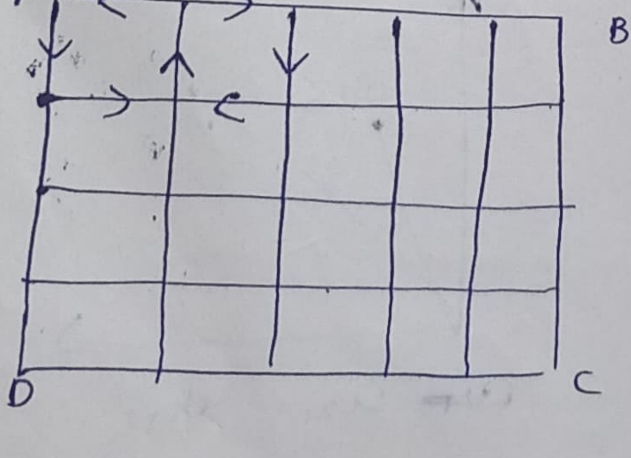
$$\oint H \cdot dl = \iint_S \nabla \times A \cdot ds$$

Application

- Used to convert surface integral into line integral & vise versa.
- To find the line integral of some particular curve.
- To determine curl of bounded surface.

Proof

Consider a closed loop



Stokes Theorem

Line Integral of a vector ~~and~~ round a closed path is equal to the surface integral of the normal component of its curl over any closed surface

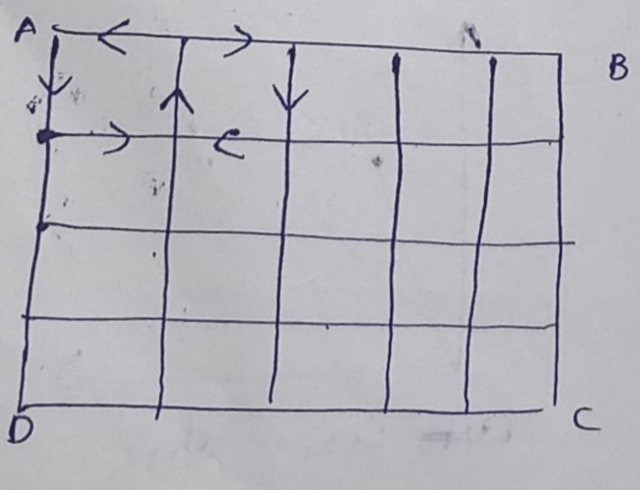
$$\oint H \cdot dl = \iint_s \nabla \times A \cdot ds$$

Application

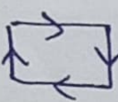
- Used to convert surface integral into line integral & vise versa.
- To find the line integral of some particular curve.
- To determine curl of bounded surface.

Proof

Consider a closed loop



K no. of cells, its divided.
Take one cell



Surface of this cell is ΔS_K
Arrow $\longrightarrow$ flow of charges
Stick is placed on cell, stick rotate due to presence of charges
Circulation of path is taken as
$\oint H \cdot dl$

$\oint H \cdot dl \simeq$ Circulation of $\vec{A}$ & L_K

To find overall closed path

$$\oint H \cdot \vec{dl} = \sum_K \oint_{L_K} \bar{H} \cdot dl$$

Multiply & divide by ΔS_K

$$= \sum_K \oint \frac{H \cdot dl}{\Delta S_K} \times \Delta S_K$$

We know circulation per area is curl.

$$= \sum_K (\nabla \times \vec{A}) \cdot \nabla S_K$$

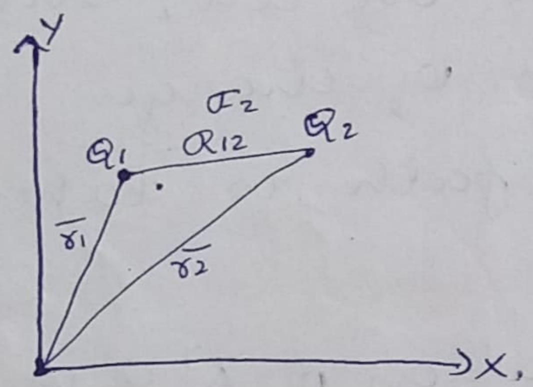
$$= \oiint \nabla \times \vec{A} \cdot ds$$

Coulomb's Law

$$F \propto \frac{Q_1 Q_2}{R^2}$$

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{R^2}$$

Force is a vector quantify.



$$\bar{F}_2 = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{R_{12}^2} \cdot \hat{a}_{12}$$

Force represent by Q_2 is f_1

$$\bar{F} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{R_{12}^2} \hat{a}_{12}$$

$$F_1 = \frac{-1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{R_{12}^2} \cdot \hat{a}_{12}$$

$$F_1 = -F_2$$

$$F_2 = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{R_{12}^2} \frac{\bar{R}_{12}}{|R_{12}|} \quad \left[a_{12} = \frac{\bar{R}_{12}}{|R_{12}|} \right]$$

$$F_2 = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{R_{12}^3} \bar{R}_{12}$$

8(A) Find force exerted on Q_2 where will be $Q_1(1,2,3)$ & $Q_2(2,0,5)$

Ans

$$Q_1 = (1,2,3)$$

$$Q_2 = (2,0,5)$$

$$r_1 = \hat{x} + 2\hat{y} + 3z$$

$$r_2 = 2\hat{x} + 0\hat{y} + 5\hat{z}$$

$$\overline{R_{12}} = r_2 - r_1$$

$$\cancel{\hat{x} + 2\hat{y}}$$

$$2\hat{x} + 0\hat{y} + 5\hat{z} - (\hat{x} + 2\hat{y} + 3z)$$

$$= \hat{x} - 2\hat{y} + 2\hat{z}$$

$$|R_{12}| = \sqrt{1^2 + 2^2 + 2^2}$$

$$= \sqrt{9} = 3$$

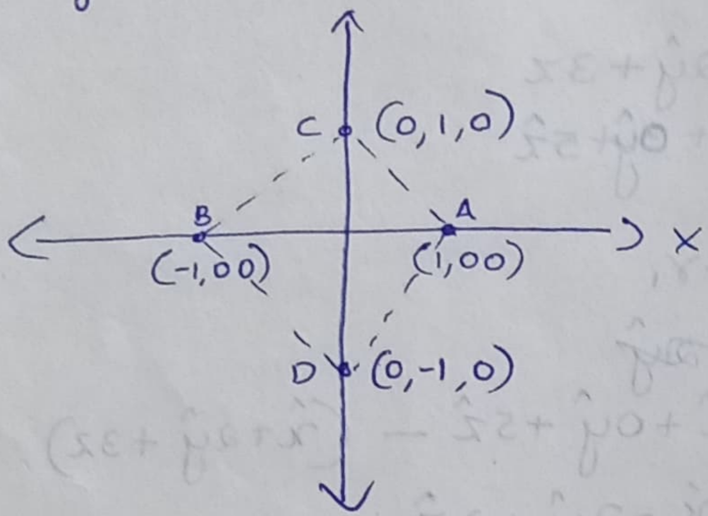
$$F_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^3} \cdot \overline{R_{12}}$$

$$F_2 = \frac{F \cdot \hat{a}_{12}}{R_{12}^2}$$

$$= \frac{F}{R_{12}^2} \times \frac{R_{12}}{|\overline{R_{12}}|}$$

$$= \frac{F \times \overline{R_{12}}}{3^2 \quad 3} \Rightarrow \frac{F}{9} \times \hat{a}_{12}$$

Q. Point charges of 50nC are located at A(1,0,0), B(-1,0,0,) C(0,1,0) D(0,-1,0) in free space. Find total force on charge at A.



$q = 50nC$

$\bar{r_A} = \hat{x}$

$\bar{r_B} = -\hat{x}$

$\bar{r_C} = \hat{y}$

$\bar{r_D} = -\hat{y}$

$R_{CA} = \bar{r_A} - \bar{r_C} \Rightarrow \hat{x} - \hat{y}$

$R_{DA} = \bar{r_D} - \bar{r_A} \Rightarrow \hat{x} + \hat{y}$

$R_{BA} = \bar{r_B} - \bar{r_A} \Rightarrow 2\hat{x}$

$|R_{CA}| = \sqrt{2}$

$|R_{DA}| = \sqrt{2}$

$|R_{BA}| = 2$

$$F_A = \frac{q^2}{4\pi\varepsilon_0}\left[\frac{R_{BA}}{|R_{BA}^3|} + \frac{R_{DA}}{|R_{DA}^3|} + \frac{R_{CA}}{|R_{CA}^3|}\right]$$

$$= \cancel{50\times10^{-9}}$$

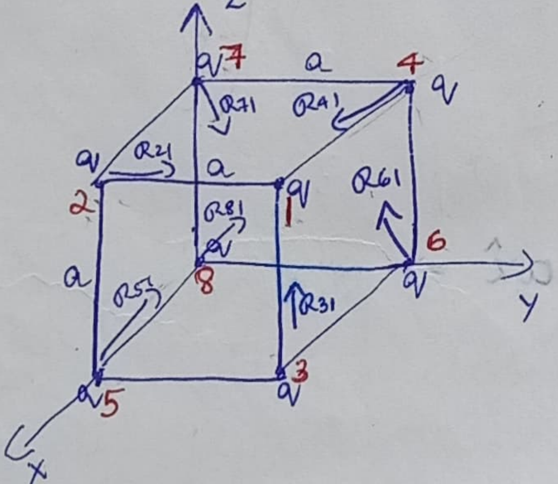
$$\frac{\cancel{(50\times10^{-9})^2}}{4\times3.14\times8.85\times10^{-12}}\Big[$$

$$(9\times10^{9})\frac{(50\times10^{-9})^2}{\cancel{a\times10^{-9}}}\left[\frac{2\hat{x}}{8}+\frac{\hat{x}}{2\sqrt{2}}-\frac{\hat{y}}{2\sqrt{2}}+\frac{\hat{x}}{2\sqrt{2}}+\frac{\hat{y}}{2\sqrt{2}}\right]$$

$$\Rightarrow (50\times10^{-9})^2(9\times10^{9})\times0.957\hat{x}$$

$$21.5\times10^{-6}\hat{x}N$$

Q 8 identical charges q each are placed on the corners of a cube of side 'a'. Find resultant force on a charge.



Position vectors of 8 corners will be

$\bar{r}_1 = a\hat{x}+a\hat{y}+a\hat{z}$ [Length = $\bar{a}$]

$\bar{r}_2 = a\hat{x}+0\hat{y}+a\hat{z}$

$\bar{r}_3 = a\hat{x}+a\hat{y}+0\hat{z}$

$\bar{r}_4 = 0\hat{x}+a\hat{y}+a\hat{z}$

$r_5 = a\hat{x} + 0\hat{y} + 0\hat{z}$

$r_6 = a\hat{x} + a\hat{y} + 0\hat{z}$

$r_7 = a\hat{x} + 0\hat{y} + a\hat{z}$

$r_8 = 0$ (origin)

$\cancel{R_{12} = r_1 - r_2}$

$\cancel{a\hat{x} + a\hat{y} + a\hat{z} - a\hat{x} + 0\hat{y} + a\hat{z}}$

$\cancel{0\hat{x} + a\hat{y} + 0\hat{z}}$

$\cancel{R_{13} = r_1}$

$R_{21} = r_2 - r_1$

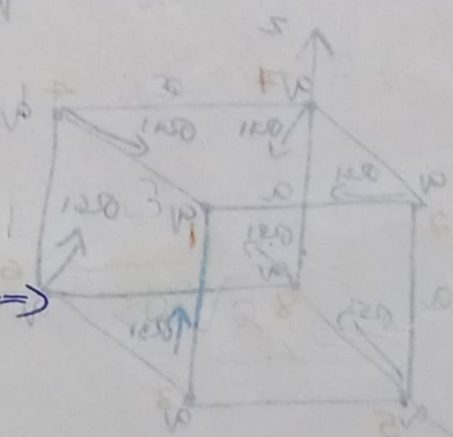
$0\hat{x} + a\hat{y} + 0\hat{z} \quad |1| = \sqrt{\sqrt{a}}$

$R_{31} = r_3 - r_1$

$= a\hat{z}$

$R_{41} = a\hat{x}$

$R_{51} = +a\hat{y} + a\hat{z} \Rightarrow$



$R_{61} = +a\hat{z}$

$R_{71} = +a\hat{y}$

$R_{81} = +a\hat{x} + a\hat{y} + a\hat{z}$

$F_1 = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{}$

$$F = \frac{q^2}{4\pi\varepsilon_0}\left[\frac{\bar{R_{21}}}{R_{21}^3} + \frac{\bar{R_{31}}}{R_{31}^3} + \frac{\bar{R_{41}}}{R_{41}^3} + \frac{\bar{R_{51}}}{R_{51}^3} + \frac{\bar{R_{61}}}{R_{61}^3} + \frac{\bar{R_{71}}}{R_{71}^3} + \frac{\bar{R_{81}}}{R_{81}^3}\right]$$

$$F = (9\times10^9)\, q^2\left[\frac{a\hat{y}}{a} + \frac{a\hat{z}}{a} + \frac{a\hat{x}}{a} + \frac{a\hat{y}+a\hat{z}}{2a\sqrt{2}} + \frac{a\hat{x}+a\hat{y}+\ldots}{3a\sqrt{3}}\right.$$

$\cancel{F = \hat{x}(1+}$

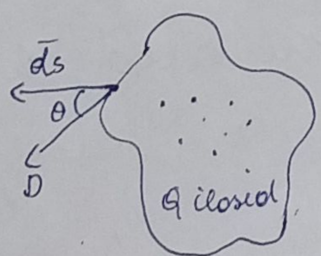
$$F = \frac{(9\times10^9)q^2}{a^2}\left[\hat{x}\left(1+\frac{1}{\sqrt{2}}+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}\right] + \hat{y}\left[1+\frac{1}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}\right] + \hat{z}\left[1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}\right]$$

$$F = \frac{q^2\times1.89\times9\times10^9}{a^2}(\hat{x}+\hat{y}+\hat{z}]$$

$$\frac{17.01q^2(\hat{x}+\hat{y}+\hat{z})}{a^2} \Rightarrow \frac{17q^2}{a^2}(\hat{x}+\hat{y}+\hat{z})$$

10/23 **Gauss Law**

Electric flux leaving a closed surface is equal to the total electric charge enclosed by closed surface.



$D \rightarrow$ Electric flux density

$d\psi \rightarrow D_N ds$ Normal

$= |D|\cos\theta|\bar{ds}|$ unit Coulomb

$= D\cdot ds$. (C) Differential surface

Here $d\bar{s} \rightarrow$ the "direction normal to the surface area

According to Gauss Law

Ψ leaving = Q enclosed

Ψ crossing = $\int_s d\psi = \int_s \bar{D} \cdot \bar{ds}$ C

If it is closed surface

Ψ leaving = $\oint_s \bar{D} \, \bar{ds}$ C

If charge is in the volume form

Q enclosed = $\int_v \rho_v \, dv$

Ψ leaving = Q enclosed

$\oint D \cdot ds = \int_v \rho_v \, dv$

This is integral form of gauss law.

Trust wala after figure

If a closed surface is present, having some charge Qenclosed. We know that if there is a positive charge, flux lines are going outwards towards -ve charge. 1

In faraday's exp, ~~inde~~ despite of any ~~di~~ perfect dielectric, the electric flux line move from inner sphere to outer sphere. 2

So electric flux line generated because the charges are crossing the surface wall. 3

Flux density can be split into normal & tangential components.

Here the normal component will be more considered as it is perpendicular to enclosed surface.

Differential amount of electric flux = $d\phi$

Equipotential Surface

It is the contact in space having same potential ~~at~~ on the surface.

It means charge will have same potential energy at every point

Properties

1 Equipotential & Electric field are at right angle to each other.

2 The net work done on a charge for carrying it between any 2 point of an equipotential surface is 0

3 Direction of electric field is from high to low potential.

Electric Displacement

- The induced ~~d~~ charge density and polarisation vector is replaced by

$$\sigma_P = \vec{P}\cdot\hat{n} \qquad P \rightarrow \text{Polarization vector.}$$

Electric field in dielectric in vector form

$$E = \frac{\sigma - \sigma_P}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

In the vector form

$$\vec{E}\,\hat{n} = \frac{\sigma - \vec{P}\cdot\hat{n}}{\varepsilon_0} \qquad \left[\because \sigma_P = \vec{P}\,\hat{n}\right]$$

$$\sigma = \left[\varepsilon_0 \vec{E} + \vec{P}\right]\hat{n}$$

$$\varepsilon_0\vec{E} + \vec{P} = \vec{D} \quad \text{Electric Displacement}$$

$$\boxed{\vec{D}\cdot\hat{n} = \sigma}$$

Coloumb's Law

The force of attraction or repulsion between any two charge is direly proportional to product of charges and inversely proportional to the square of the distance between them.

$$F = K \cdot \frac{q_1 q_2}{r^2}$$

5/6/2 **Divergence, Curl & Gradient**

There are various mathematical operation that can be used in various physical problems.

Here $\vec{\nabla}$ operator is a vector differential operator.

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

Gradient

$\vec{\nabla}$ operates on scalar function.

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}.$$

Let ϕ be a scalar function.

$$\text{grad}\,\phi = \nabla\phi = \hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}$$

[illegible]

Here ϕ is a scalar quantity. When ∇ operates on scalar quantity, we get vector quantity, since it has a direction, so we call gradient as directional derivative.

① Divergence

∇ operates on vector function by dot product

Consider vector as ~~$a\hat{i} + b\hat{j} + c\hat{k}$~~

$$\bar{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

Then

$$\boxed{\begin{aligned} i\cdot i = j\cdot j = k\cdot k = 1 \\ i\cdot j = j\cdot k = k\cdot i = 0 \end{aligned}}$$

$$\nabla\cdot A = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

$$\vec{\nabla}\cdot A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Here it is scalar quantity.

Curl

∇ operates on vector function by cross product.

Consider

$$\bar{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\nabla\times\bar{A} = \left(i\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\times\left(A_x i + A_y i + A_z\hat{k}\right)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) - \hat{j}\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right) + \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

Resultant is a vector Quantity.

Poisson's Equation

It relates the electric potential to charge density ie, Potential of a conductor is directly proportional to charge it contains

$$\nabla^2 V = \frac{-\rho_v}{\varepsilon}$$

where $V \longrightarrow$ Electrical potential.

$\rho_v \longrightarrow$ Volume charge density

$\varepsilon \longrightarrow$ Dielectric constant.

Proof

The relation b/w electric potential & field intensity is

$$E = -\nabla V \quad \text{--- (1)}$$

Gauss Law in point form

$$\nabla \cdot D = \rho_v \quad \text{--- (2)}$$

The divergence of electric flux density = volume charge density

$$\nabla \cdot (\varepsilon E) = \rho_v \qquad D = \varepsilon E$$

$$\varepsilon (\nabla \cdot E) = \rho_v$$

$$\nabla \cdot E = \frac{\rho_v}{\varepsilon} \quad \text{——— ③}$$

Sub ① in ③

$$\nabla(-\nabla V) = \frac{\rho_v}{\varepsilon}$$

$$\boxed{\nabla^2 V = \frac{-\rho_v}{\varepsilon}}$$

For cartesian coordinate system (x, y, z)

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{-\rho_v}{\varepsilon}$$

In cylindrical coordinate system (ρ, ϕ, z)

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho_v}{\varepsilon}$$

For spherical coordinate (r, θ, ϕ)

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \cdot \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \cdot \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial^2 V}{\partial \phi^2}$$

$$= -\frac{\rho_v}{\varepsilon}$$

Laplace Equation

If an enclosed surface does not have any volume charge (ρ_v). Then poisson's eqn becomes

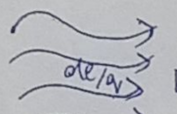
$$\nabla^2 V = 0$$

This is called Laplace Equation.

& ∇^2 = Laplacian operator.

- Bonding value problem - Application.

Relation b/w Electric field intensity & scalar potential function



Here E is vector

V is scalar

Workdone in moving a charge Q by dl ~~dl~~

$$dw = -Q\vec{E} \cdot dl \quad \text{——} \; ①$$

We know

$$\partial V = \frac{\partial V}{\partial x} \cdot \hat{a_x} + \frac{\partial V}{\partial y} \cdot \hat{a_y} + \frac{\partial V}{\partial z} \cdot \hat{a_z}$$

$$= \underbrace{\left(\frac{\partial V}{\partial x} \hat{a_x} + \frac{\partial V}{\partial y} \hat{a_y} + \frac{\partial V}{\partial z} \hat{a_z}\right)}_{\nabla V} \; \underbrace{a_x \hat{a_x} + a_y \hat{a_y} + a_z \hat{a_z}}_{dl}$$

$dv = \nabla V \cdot dl$ —— ③

By ② & ③

$E = -\nabla V$ i.e, E is − gradient of V

Thus the relation b/w electric field & potential is obtained.

Q) $V = 2x + 4y$

Determine E

$E = -\nabla V$

$= -\left(\frac{\partial V}{\partial x}\hat{a_x} + \frac{\partial V}{\partial y}\hat{a_y} + \frac{\partial V}{\partial z}\hat{a_z}\right)$

$-\left(2\hat{a_x} + 4\hat{a_y}\right)$

9/6/23 Curl

Q Determine the curl of each of vector field.

1) $\vec{P} = x^2yz\hat{a_x} + xz\hat{a_z}$

Ans: $\nabla \times \vec{P} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2yz & 0 & xz \end{vmatrix}$

$\nabla \times P =$

$\hat{a_x}\left(\frac{\partial}{\partial y}xz - 0\right) - \hat{a_y}\left(\frac{\partial}{\partial x}xz - \frac{\partial}{\partial z}x^2yz\right) + \hat{a_z}\left(0 - \frac{\partial}{\partial y}x^2yz\right.$

$$a\hat{x}\times 0 - a\hat{y}\left(\cancel{2}z - x^2 y^{\cancel{2}}\right) + a\hat{z}\left(0 - x^2 z\right)$$

$$\cancel{\text{or}}\ \cancel{2}(z - x^2 y)a\hat{y} \cancel{+} - x^2 z\, a\hat{z}$$

$$-(z - x^2 y)a\hat{y} - x^2 z\, a\hat{z}$$

Properties of Curl

- The curl of a vector field should be an another vector field.
- The curl of a scalar field $\nabla\times V$ makes no sense.
- $\nabla\times(A+B) = (\nabla\times A) + (\nabla\times B)$
- $\nabla\times(A\times B) = A(\nabla\cdot B) - B(\nabla\cdot A) + \left(B(\nabla\cdot A) - A(\nabla\cdot B)\right)B$
- The divergence of the curl of a vector field vanishes ie $\nabla\cdot(\nabla\times A) = 0$
- The curl of a gradient of scalar field vanishes, ie $\nabla\times(\nabla V) = 0$

Prog of vector field

1) Divergence of curl of any vector field is zero. $\bar{\nabla}\cdot(\vec{\nabla}\times A) = 0$

$$\cancel{\nabla\times\partial}\ \nabla = \frac{\partial}{\partial x}a\hat{x} + \frac{\partial}{\partial y}a\hat{y} + \frac{\partial}{\partial z}a\hat{z}$$

$$\Rightarrow \left(\frac{\partial}{\partial x}\hat{a_x} + \frac{\partial}{\partial y}\hat{a_y} + \frac{\partial}{\partial z}\hat{a_z}\right) \cdot \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\hat{a_x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) - \hat{a_y}\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right) + \hat{a_z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$= 0$$

$$\left(\frac{\partial}{\partial x}\hat{a_x} + \frac{\partial}{\partial y}\hat{a_y} + \frac{\partial}{\partial z}\hat{a_z}\right) \cdot (0+0+0)$$

$$\Rightarrow \underline{\underline{0}}$$

<u>Proof for a scalar field</u>

$\nabla \times (\nabla V) = 0$ S.T

A $\nabla V = \frac{\partial}{\partial x}\hat{a_x} + \frac{\partial}{\partial y}\hat{a_y} + \frac{\partial}{\partial z}\hat{a_z}$

$$\nabla \times \left(\frac{\partial}{\partial x}\hat{a_x} + \frac{\partial}{\partial y}\hat{a_y} + \frac{\partial}{\partial z}\hat{a_z}\right)$$

$$\begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial V}{\partial x} & \frac{\partial V}{\partial y} & \frac{\partial V}{\partial z} \end{vmatrix}$$

$$\Rightarrow \hat{ax}\left(\frac{\partial}{\partial y}\cdot\frac{\partial V}{\partial z}-\frac{\partial}{\partial z}\frac{\partial V}{\partial y}\right)-\hat{ay}\left(\frac{\partial}{\partial x}\cdot\frac{\partial V}{\partial z}-\frac{\partial}{\partial z}\frac{\partial V}{\partial x}\right)$$
$$+\hat{az}\left(\frac{\partial}{\partial x}\frac{\partial V}{\partial y}-\frac{\partial}{\partial y}\frac{\partial V}{\partial x}\right)$$

$$\cancel{\nabla}\cancel{x} = \hat{ax}(0)-\hat{ay}(0)+\hat{az}(0)$$

$$0 //$$

SOLENOID

- If divergence of field is 0, then it is solenoid. $(\nabla\cdot V)=0$

Q S.T the vector $A = x^2z^2\vec{i} + xyz^2\vec{j} + (-xz^3)\hat{k}$ is solenoid

$$\cancel{\nabla}\ \cancel{\partial}\ \nabla = \frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}$$

$$\nabla\cdot A = \cancel{\{}\frac{\partial}{\partial x}x^2z^2+\frac{\partial}{\partial y}xyz^2+\frac{\partial}{\partial z}(-xz^3)$$

$$= 2xz^2 + xz^2 + -3z^2x$$

$$3xz^2 - 3z^2x$$

$$= 0 //$$

So it is solenoid.

Q S.T vector $r = x\vec{i} + y\vec{j} + z\vec{k}$ is irrotational

A $\nabla \times r = 0$

$$\begin{vmatrix} \hat{ax} & \hat{ay} & \hat{az} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$$

$$\hat{ax}\left(\frac{\partial z}{\partial y} - \frac{\partial y}{\partial z}\right) - \hat{ay}\left(\frac{\partial z}{\partial x} - \frac{\partial x}{\partial z}\right) + \hat{az}\left(\frac{\partial y}{\partial x} - \frac{\partial x}{\partial y}\right)$$

$$0 - 0 + 0$$

$$0 //$$

So it is irrotational.

Q. Verify that vector fied $\hat{A} = yz\hat{a_x} + zx\hat{a_y} + xy\hat{a_z}$ is both irrotational & solenoid.

$(\nabla \cdot V) = 0$ – Solenoid.

$(\nabla \times V) = 0$ – Irrotational

$$\begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & zx & xy \end{vmatrix} = \hat{a_x}\left(\frac{\partial xy}{\partial y} - \frac{\partial zx}{\partial z}\right) - a_y\left(\frac{\partial xy}{\partial x} - \frac{\partial yz}{\partial z}\right) + a_z\left(\frac{\partial zx}{\partial x} - \frac{\partial yz}{\partial y}\right)$$

$$\hat{a_x}(x - x) - \hat{a_y}(y - y) + a_z(z - z)$$

$$0 - 0 + 0 \Rightarrow 0$$

It is irrotational.

$\underline{\nabla \cdot V = 0}$

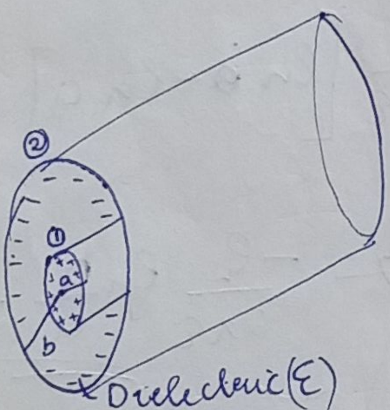
$$i\frac{\partial}{\partial x}yz + j\frac{\partial}{\partial y}zx + \hat{k}\frac{\partial}{\partial z}xy$$

$$0 + 0 + 0 \Rightarrow 0$$

Hence it is solenoid.

Thus proved

Capacitance of a Coaxial Cable



Consider ~~graph~~ length L of a coaxial cable. (inner radius) & outer ~~...~~ a radius b, ~~a~~. Let the space between conductor be filled with dielectric with permittivity ε, Assuming that conductors resp. carry positive & negative charge uniformly distributed by

$$C = \frac{Q}{V} \quad \text{——— (1)}$$

According to Gauss Law
The electric field intensity $E = \frac{\rho_L}{2\pi\varepsilon r}$

The potential diff. b/w 2 cylinder.

$$V = -\int_a^b E \cdot d\cancel{r}\,r = -\int_a^b \frac{\rho_L}{2\pi\varepsilon r}\,dr$$

$$= \frac{-\rho_L}{2\pi\varepsilon}\int_a^b \frac{1}{r}\,dr$$

$$= \frac{-\rho_L}{2\pi\varepsilon}\left[\ln r\right]_a^b$$

$$= \frac{-\rho_L}{2\pi\varepsilon}\left[\ln b - \ln a\right]$$

$$= \cancel{\frac{-\rho_L}{2}}\ -\frac{\rho_L}{2\pi\varepsilon}\ln\frac{b}{a} \quad \text{——— (2)}$$

$$C = \frac{Q}{V} = \frac{-\rho_L}{\rho_L/2\pi\varepsilon\,(\ln b/a)}$$

$$= \frac{2\pi\varepsilon}{\ln b/a}\ F/m$$

$$V = -\int_b^a E \cdot dr = -\int_b^a \frac{\rho_L}{2\pi\varepsilon_0} dr$$

$$= \frac{-\rho_L}{2\pi\varepsilon} \int_b^a \frac{1}{r} dr$$

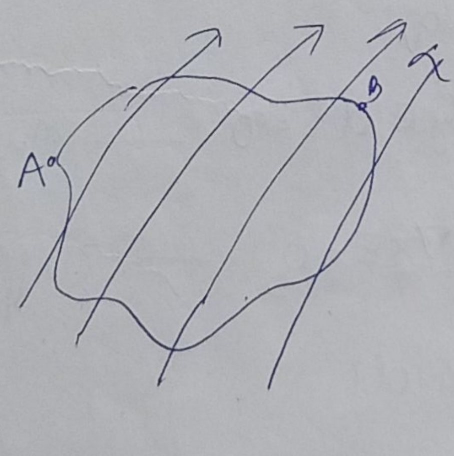
$$= \frac{-\rho_L}{2\pi\varepsilon} \left[\ln r \right]_b^a = \frac{-\rho_L}{2\pi\varepsilon} \left[\ln a - \ln b \right]$$

$$= \frac{\rho_L}{2\pi\varepsilon} \left[\ln b - \ln a \right]$$

$$= + \frac{\rho_L}{2\pi\varepsilon} \ln b/a$$

$$C = \frac{Q}{V} = \frac{\rho_L}{\frac{\rho_L}{2\pi\varepsilon} \ln b/a)} = \frac{2\pi\varepsilon}{\ln b/a} \ f/m$$

Relation b/w E & V



The potential diff. b/w points A & B is independent of path taken

$$V_{BA} = -V_{AB}$$

$$V_{BA} + V_{AB} = \oint E \cdot dl = 0$$

$$\boxed{\oint E \cdot dl = 0}$$

No net work is done in moving charge along closed path in electrostatic field.

Applying Stokes Theorem

$$\oint E \cdot dl = \int (\nabla \times E) \cdot ds = 0$$

$$\boxed{\nabla \times E = 0}$$

Any vector field satisfies eq is said to be conservative or irrotational.

Thus electrostatic static field is conservative

Maxwell's Eq.

$$\oint E \cdot dl = 0 \text{ — Integral form}$$

$$\nabla \times E = 0 \text{ — Differential form}$$

$$V = -\int E \cdot dl.$$

$$dV = -E \cdot dl.$$

$$= -E_x dx - E_y dy - E_z dz$$

$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy + \frac{\partial v}{\partial z}dz$$

comparing 2 expression for dv

$$E_x = -\frac{\partial v}{\partial x}, \quad E_y = -\frac{\partial v}{\partial y}, \quad E_z = -\frac{\partial v}{\partial z}$$

$$\boxed{E = -\nabla V}$$

This is electric field intensity ~~is~~ the gradient of V.

The -ve sign show E is opposite to direction in which V increases,

E is directly from high to low filed.