2/8/23

MODULE-3

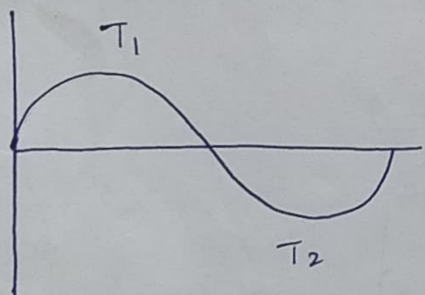
Uniform plane wave

If ϕ & amplitude of wave is same on all points.

Properties

1) Electric field ~~is~~ ~~two~~ & magnetic field are perpendicular to each other.
2) Field varies harmonically with time.
3) No electric & magnetic field is in direction of propagation.

Electromagnetic wave Eq



Consider a sine wave with time period.

Consider 2 maxwell eq.

$$\nabla \times H = J + \frac{\partial D}{\partial t} \quad \text{—①}$$

$J = \sigma E$ — Electric field intensity

σ → Surface charge density

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad \text{—②}$$

$$\nabla \times H = \sigma E + \frac{\partial D}{\partial t} \quad \text{—③}$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} \quad \text{—— (4)}$$

$$\nabla \times (\nabla \times E) = -\mu \frac{d}{\partial t} (\nabla \times H) \quad \text{(5)}$$

$$\cancel{\nabla \times (\nabla \times E) = -\mu\, d\left(\frac{\partial}{\partial t}\left(\sigma E + \frac{\partial D}{\partial t}\right)\right)}$$

$$\cancel{\nabla(\nabla \cdot E) - \nabla^2 E = \mu \frac{d}{\partial t}(\sigma E + \varepsilon}$$

Taking curl of eq (4) $\boxed{B = \mu H}$

$$\nabla \times (\nabla \times E) = -\mu \frac{\partial}{\partial t} (\nabla \times H) \quad \text{—— (5)}$$

$$\nabla \times \nabla \times E = -\mu \frac{\partial}{\partial t}\left(\sigma E + \frac{\partial B}{\partial t}\right)$$

$$\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A - \nabla^2 A)$$

$$\nabla(\nabla \cdot E) - \nabla^2 E = -\mu \frac{\partial}{\partial t}\left(\sigma E + \varepsilon \frac{\partial E}{\partial t}\right) \quad \boxed{D = \varepsilon_0 E}$$

$\nabla \cdot E = 0$ (charge free medium)

$$-\nabla^2 E = -\mu \frac{\partial}{\partial t}\left(\sigma E + \varepsilon \frac{\partial E}{\partial t}\right)$$

$$\boxed{\nabla^2 E = \sigma \mu \frac{\partial E}{\partial t} + \mu \varepsilon \frac{\partial^2 E}{\partial t^2}}$$

Wave eq. for electric field

$$\boxed{\nabla^2 H = \sigma H \cdot \frac{\partial H}{\partial t} + \mu \varepsilon \frac{\partial^2 H}{\partial t^2}}$$

- For magnetic field.

In free space

$\sigma = 0$

$$\nabla^2 E = \mu \varepsilon \frac{\partial^2 E}{\partial t^2}$$
$$\nabla^2 H = \mu \varepsilon \frac{\partial^2 H}{\partial t^2}$$
$\Big\}$ General wave eq. for EF & MF.

In phasor form

$$\nabla^2 E = j\omega\mu(\sigma + j\omega\varepsilon) E_s$$

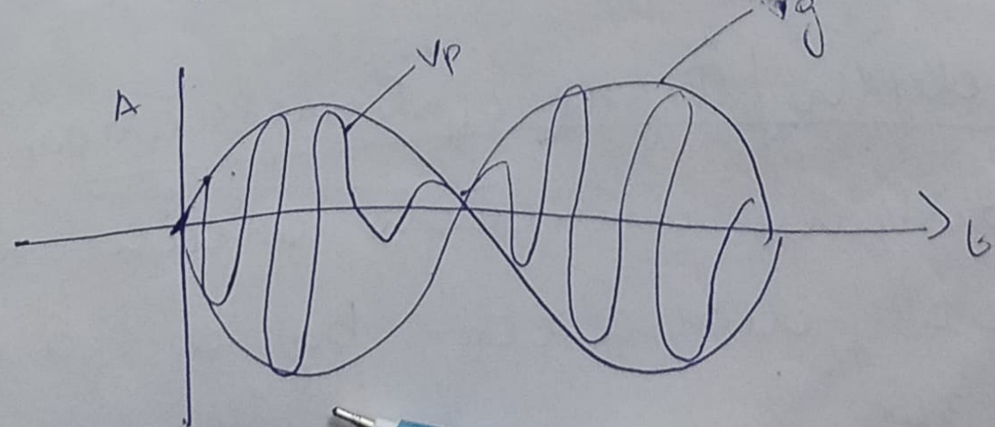
$$-\nabla^2 E = -\mu \frac{\partial}{\partial t}\left(\sigma E + \frac{\partial E}{\partial t}\right)$$

$$\nabla^2 H = j\omega\mu(\sigma + j\omega\varepsilon) H_s$$

Phase Velocity & Group Velocity

V_p - Velocity of individual wave.
(phase velocity)

V_g - Create an envelope
(group velocity)



$$\boxed{V_p = \frac{c}{\sqrt{1-\left(\frac{\omega_p}{\omega}\right)^2}}} = \boxed{\frac{\omega}{\beta}}$$

$\omega_p \rightarrow$ Angular frequency of phase wave

$\omega \rightarrow$ fundemental frequency

$\beta \rightarrow$ phase constant

$$\boxed{V_g = c\sqrt{1-\left(\frac{\omega_p}{\omega}\right)^2}}$$

$$\boxed{V_p = 2V_g}$$

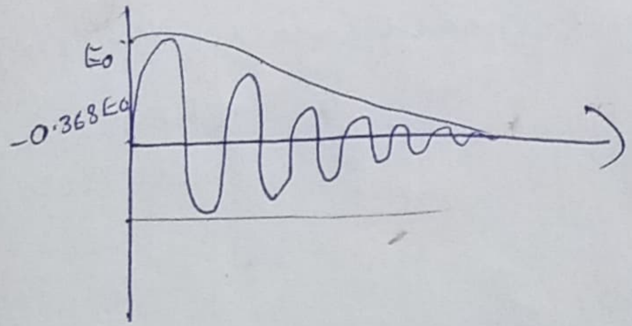
Polarisation

- Time varying electric field intensity with time.

1) Linear
2) Circular
3) Elliptical

* Skin depth / Depth of Penetration

The distance through which amplitude of a wave decreased by a factor of e^{-1} or 0.368



$$E(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, a_x$$

$$H(z,t) = H_0 e^{-\alpha z} \cos(\omega t - \beta z)\, a_z$$

$$\boxed{\delta = \frac{1}{\alpha}}$$

$$\boxed{\alpha = \frac{1}{\sqrt{\pi f \mu \sigma}}}$$

Propogation of EM wave in conducting medium (lossy dielectric)

Propogation constant (γ)

Attenuation constant (α)

Phase constant (β)

2 types of medium

1) Low loss medium (no power loss)

2) Lossy medium (power loss)

Low loss, $\sigma \neq 0$

For linear isotropic homogeneous medium (Lossy dielectric)

$j=0, \rho=0, \sigma=0, \rho_v=0$

$$\nabla^2 E = j\omega\mu(\sigma + j\omega\varepsilon)E_s$$

$$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$$

$$\nabla^2 E_s = \gamma^2 E_s$$

$$\boxed{\begin{matrix}\nabla^2 E_s - \gamma^2 E_s = 0 \\ \nabla^2 H_s - \gamma^2 H_s = 0\end{matrix}}$$

Helmholtz equation / Homogeneous vector wave eqn.

$$\gamma = \alpha + j\beta$$

$$\gamma^2 = (\alpha + j\beta)^2$$

$$\gamma^2 = \alpha^2 - \beta^2 + 2\alpha j\beta \quad \text{—— (1)}$$

$$\gamma = j\omega\sigma\mu - \omega^2\mu\varepsilon \quad \text{—— (2)}$$

$$\alpha^2 - \beta^2 = -\omega^2\mu\varepsilon \quad \text{—— (3)}$$

$$\alpha\beta = \omega\sigma\mu \quad \text{—— (4)}$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(1+\sqrt{1+\frac{\sigma}{\omega\varepsilon}}\right)^2 - 1\Bigg]}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(1+\frac{\sigma}{\omega\varepsilon}\right)^2 + 1\Bigg]}$$

$$\gamma = \pm\sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

$$\nabla^2 E_s - \gamma^2 E_s = 0$$