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▣ Multiplicative inverse of -7 modulo 20 :

⇒ We need to find x such that $-7x \equiv 1 \pmod{20}$

→ if $ax \equiv 1 \pmod{m}$, then x is the multiplicative inverse of a modulo m .

$$\rightarrow -7 \equiv 13 \pmod{20}$$

Since $-7 \equiv 13 \pmod{20}$, we can rewrite the congruence as $13x \equiv 1 \pmod{20}$.

→ We are looking for an integer x such that $13x = 20k + 1$ for some integer k .
We can test values of x :

if $x=1$, $13(1) = 13 \not\equiv 1 \pmod{20}$

if $x=2$, $13(2) = 26 \not\equiv 1 \pmod{20}$

if $x=3$, $13(3) = 39 \not\equiv 1 \pmod{20}$

if $x=4$, $13(4) = 52 \not\equiv 1 \pmod{20}$

if $x=5$, $13(5) = 65 \not\equiv 1 \equiv 5 \pmod{20}$

if $x=6$, $13(6) = 78 \not\equiv 1 \pmod{20}$

if $x=7$, $13(7) = 91 \equiv 11 \pmod{20}$

if $x=8$, $13(8) = 104 \equiv 4 \pmod{20}$

" $x=9$, $13(9) = 117 \equiv 17 \pmod{20}$

" $x=10$, $13(10) = 130 \equiv 10 \pmod{20}$

" $x=11$, $13(11) = 143 \equiv 3 \pmod{20}$

" $x=12$, $13(12) = 156 \equiv 16 \pmod{20}$

" $x=13$, $13(13) = 169 \equiv 9 \pmod{20}$

$$\text{if } x = 14, 13(14) = 182 \equiv 2 \pmod{20}$$

$$11 \quad x = 15, 13(15) = 195 \equiv 15 \pmod{20}$$

$$11 \quad x = 16, 13(16) = 208 \equiv 8 \pmod{20}$$

$$11 \quad x = 17, 13(17) = 221 \equiv 1 \pmod{20}$$

Thus, $x = 17$ is multiplicative inverse of 13 modulo 20.

\therefore The multiplicative inverse of -7 modulo 20 is 17.

$$* \quad -17 \pmod{23} :$$

$$\begin{array}{r} 23 \overline{) -17} \quad (-1) \\ \underline{-23} \\ 6 \end{array}$$

$$\Rightarrow -17 = (-1 \times 23) + 6$$

$$-17 \pmod{23} = 6 \quad A;$$

* Multiplicative Inverse of $-13 \bmod 23$:

\Rightarrow The multiplicative inverse of a number $a \bmod m$ is a number x such that: $ax \equiv 1 \bmod m$

In our case, we are looking for a number x such that:

$$-13x \equiv 1 \bmod 23$$

To simplify, we first convert -13 into a positive number equivalent modulo 23:

$$-13 \bmod 23 = -13 + 23 = 10$$

So, the equation becomes:

$$10x \equiv 1 \bmod 23$$

Now, we find the integer x such that

$$10x \equiv 1 \bmod 23$$

if $x = 1$, $10 \times 1 = 10 \not\equiv 1 \bmod 23$

if $x = 2$, $10 \times 2 = 20 \not\equiv 1 \bmod 23$

" $x = 3$, $10 \times 3 = 30 \equiv 7 \bmod 23$

" $x = 4$, $10 \times 4 = 40 \equiv 17 \bmod 23$

" $x = 5$, $10 \times 5 = 50 \equiv 4 \bmod 23$

" $x = 6$, $10 \times 6 = 60 \equiv 14 \bmod 23$

" $x = 7$, $10 \times 7 = 70 \equiv 1 \bmod 23$

We found it : $10 \cdot 7 = 70 \equiv 1 \pmod{23}$
since $-13 \equiv 10 \pmod{23}$ and $10^{-1} \pmod{23} = 7$
we conclude —

The multiplicative inverse of $-13 \pmod{23}$ is 7.
A.