

(3 Hours)

[Total Marks : 80]

- N. B. : (1) Question no. one is **compulsory**.
 (2) Answer any three questions from Q.2 to Q.6.

1. (a) If $A = \begin{bmatrix} x & 4x \\ 2 & y \end{bmatrix}$ has eigen values 5 and -1 then find values of x and y. 5

(b) Evaluate $\int_C (\bar{z} + 2z) dz$ along the circle $C: x^2 + y^2 = 1$. 5

(c) State true or false with justification: If the two lines of regression are $x + 3y - 5 = 0$ and $4x + 3y - 8 = 0$ then the correlation coefficient is + 0.5. 5

(d) Find dual of following LP model 5

$$\max z = 2x_1 + 3x_2 + 5x_3$$

subject to

$$x_1 + x_2 - x_3 \geq -5$$

$$x_1 + x_2 + 4x_3 = 10$$

$$-6x_1 + 7x_2 - 9x_3 \leq 4$$

& $x_1, x_2 \geq 0$ and x_3 is unrestricted.

2. (a) Using Cauchy's integral formula, evaluate $\int_C \frac{(12z - 7) dz}{(z - 1)^2 (2z + 3)}$ where 6

$$C: |z + i| = \sqrt{3}.$$

(b) Determine whether matrix A is derogatory $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$. 6

(c) In a competitive examination, the top 15% of the students appeared will get grade 'A', while the bottom 20% will be declared fail. If the grades are normally distributed with mean % of marks 75 and S.D. 10, determine the lowest % of marks to receive grade A and the lowest % of marks that passes. 8

3. (a) The daily consumption of electric power (in millions of kwh) is r.v. X with PDF $f(x) = k x e^{-x/3}$, $x > 0$. Find k and the probability that on a given day the electricity consumption is more than expected electricity consumption. 6

[TURN OVER]

- (b) Using Simplex method, solve the following LPP

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$$\max z = 15x_1 + 6x_2 + 9x_3 + 2x_4$$

$$\text{s.t. } 2x_1 + x_2 + 5x_3 + 6x_4 \leq 20$$

$$3x_1 + x_2 + 3x_3 + 25x_4 \leq 24$$

$$7x_1 + x_4 \leq 70$$

$$\& \quad x_1, x_2, x_3, x_4 \geq 0$$

- (c) Obtain ALL Taylor's and Laurent's series expansions of function

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$$\frac{(z-2)(z+2)}{(z+1)(z+4)} \text{ about } z=0.$$

4. (a) Find the moment generating function of Poisson distribution and hence find mean and variance.

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- (b) Obtain the equation of the line of regression of cost on age from the following table giving the age of a car of certain make and the annual maintenance cost. Also find maintenance cost if age of the car is 9 years.

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Age of car (in years) : x	2	4	6	8
Maintenance cost : y (in thousands)	5	7	8.5	11

- (c) Show that the matrix A is diagonalizable, find its diagonal form and

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transforming matrix, if $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$.

5. (a) A sample of 8 students of 16 years each shown up a mean systolic blood pressure of 118.4 mm of Hg with S.D. of 12.17 mm. While a sample of 10 students of 17 years each showed the mean systolic BP of 121.0 mm with S.D. of 12.88 mm during investigation. The investigator feels that the systolic BP is related to age. Do you think that the data provides enough reasons to support investigator's feeling at 5% LoS? Assume the distribution of systolic BP to be normal.

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- (b) Using Cauchy's residue theorem, show that
- $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4 \cos \theta} d\theta = \frac{\pi}{6}$
- .

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(c) Using dual simplex method, solve

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$$\begin{aligned} \text{max } z &= -2x_1 - x_3 \\ \text{s.t. } x_1 + x_2 - x_3 &\geq 5 \\ x_1 - 2x_2 + 4x_3 &\geq 8 \\ \& x_1, x_2, x_3 \geq 0 \end{aligned}$$

6. (a) A total of 3759 individuals were interviewed in a public opinion survey on a political proposal. Of them, 1872 were men and the rest were women. A total of 2257 individuals were in favour of the proposal and 917 were opposed to it. A total of 243 men were undecided and 442 women were opposed to the proposal. Do you justify on the hypothesis that there is no association between sex and attitude, at 5% LoS.

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(b) Using Kuhn – Tucker's method solve

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$$\text{Maximize } Z = 2x_1^2 + 12x_1x_2 - 7x_2^2$$

Subject to the constraints $2x_1 + 5x_2 \leq 98$ and $x_1, x_2 \geq 0$

(c) (i) Average mark scored by 32 boys is 72 with standard deviation of 8 while that for 36 girls is 70 with standard deviation of 6. Test at 1% LoS whether the boys perform better than the girls.

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(ii) If the first four moments of a distribution about the value 4 of the random variable are $-1.5, 17, -30$ and 108 then find first four raw moments.

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