

Priority Queues: Introduction

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Data Structures Fundamentals
Algorithms and Data Structures

Outline

- 1 Overview
- 2 Naive Implementations

Learning objectives

You will be able to:

- Implement a priority queue
- Explain what is going on inside built-in implementations:
 - C++: `priority_queue`
 - Java: `PriorityQueue`
 - Python: `heapq`

Queue



A **queue** is an abstract data type supporting the following main operations:

- **PushBack(*e*)** adds an element to the back of the queue;
- **PopFront()** extracts an element from the front of the queue.

Priority Queue (Informally)

A **priority queue** is a generalization of a queue where each element is assigned a priority and elements come out in order by priority.

Priority Queues: Typical Use Case

Scheduling jobs

- Want to process jobs one by one in order of decreasing priority. While the current job is processed, new jobs may arrive.

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Scheduling jobs

- Want to process jobs one by one in order of decreasing priority. While the current job is processed, new jobs may arrive.
- To add a job to the set of scheduled jobs, call `Insert(job)`.
- To process a job with the highest priority, get it by calling `ExtractMax()`.

Priority Queue (Formally)

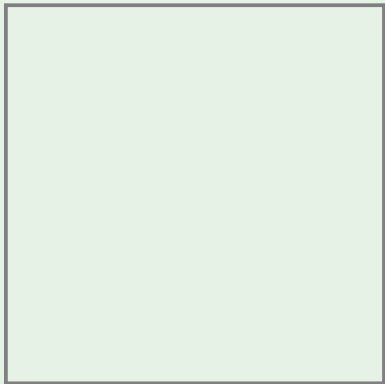
Definition

Priority queue is an abstract data type supporting the following main operations:

- $\text{Insert}(p)$ adds a new element with priority p
- $\text{ExtractMax}()$ extracts an element with maximum priority

Example

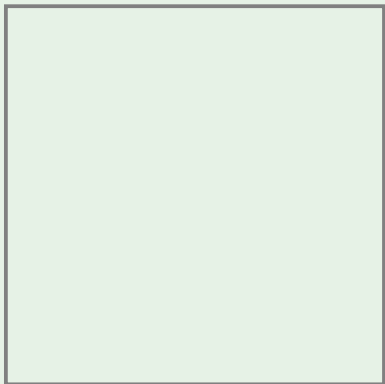
Contents:



Queries:

Example

Contents:



Queries:

Insert(5)

Example

Contents:

5

Queries:

Example

Contents:

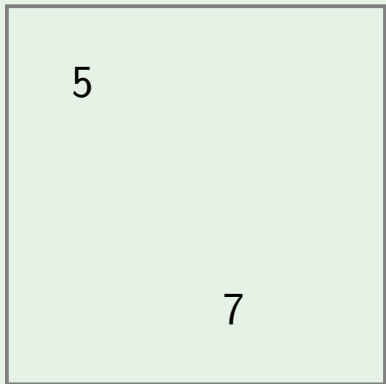
5

Queries:

Insert(7)

Example

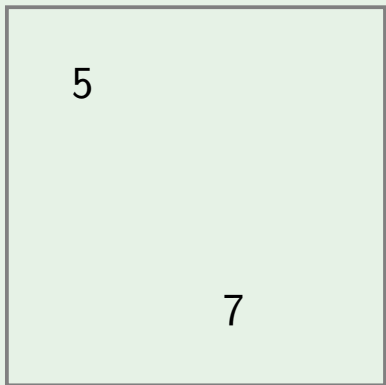
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Queries:

Example

Contents:

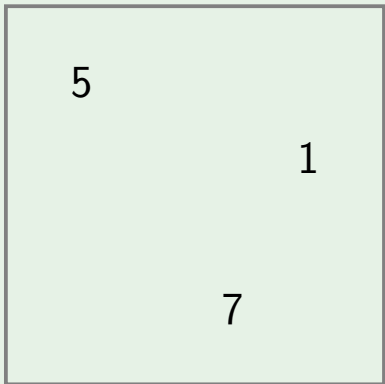


Queries:

Insert(1)

Example

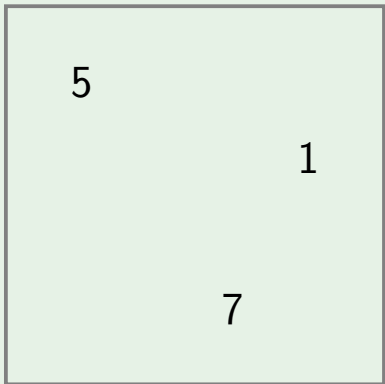
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Queries:

Example

Contents:

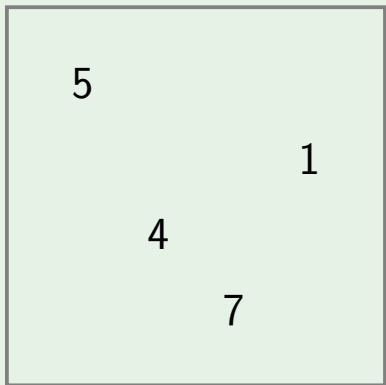


Queries:

Insert(4)

Example

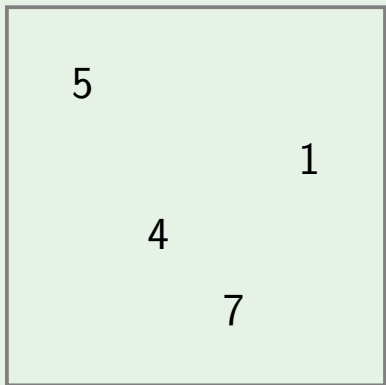
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Queries:

Example

Contents:

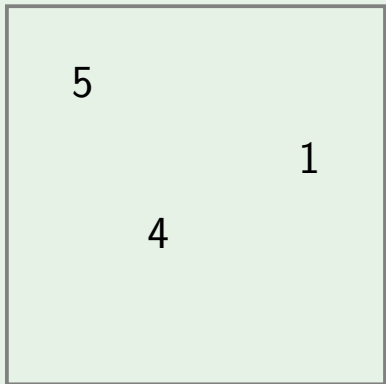


Queries:

`ExtractMax()` \rightarrow 7

Example

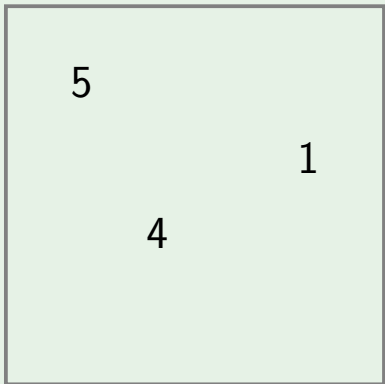
Contents:



Queries:

Example

Contents:

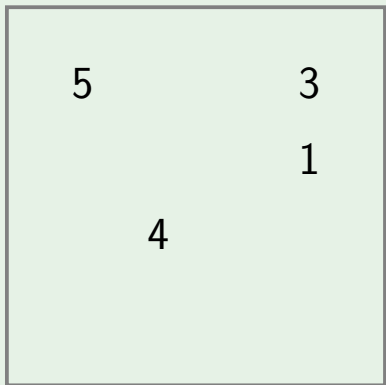


Queries:

Insert(3)

Example

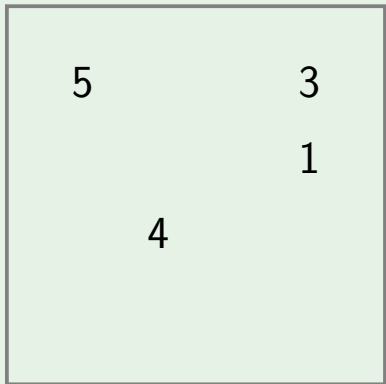
Contents:



Queries:

Example

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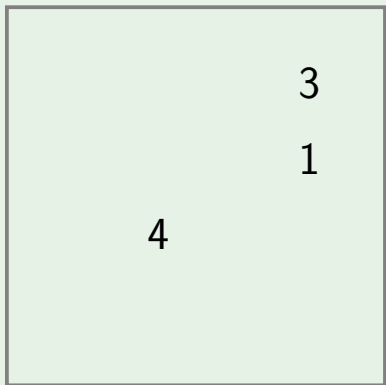


Queries:

`ExtractMax()` \rightarrow 5

Example

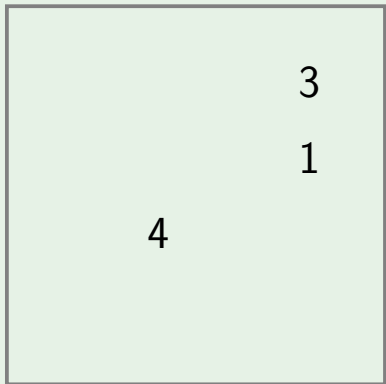
Contents:



Queries:

Example

Contents:

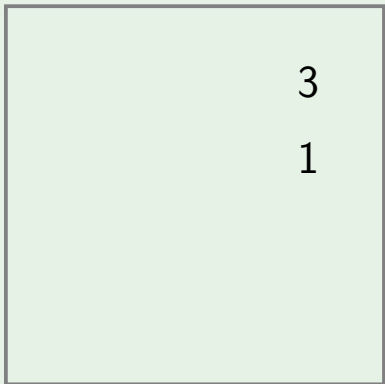


Queries:

`ExtractMax()` \rightarrow 4

Example

Contents:



Queries:

Additional Operations

- `Remove(it)` removes an element pointed by an iterator *it*
- `GetMax()` returns an element with maximum priority (without changing the set of elements)
- `ChangePriority(it, p)` changes the priority of an element pointed by *it* to *p*

Algorithms that Use Priority Queues

- Dijkstra's algorithm: finding a shortest path in a graph

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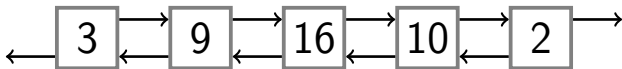
- Dijkstra's algorithm: finding a shortest path in a graph
- Prim's algorithm: constructing a minimum spanning tree of a graph
- Huffman's algorithm: constructing an optimum prefix-free encoding of a string
- Heap sort: sorting a given sequence

Outline

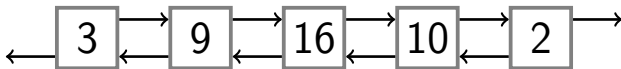
1 Overview

2 Naive Implementations

Unsorted Array/List

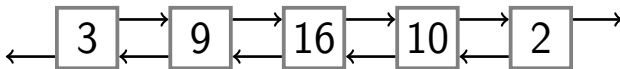


Unsorted Array/List



- `Insert(e)`
 - add `e` to the end
 - running time: $O(1)$

Unsorted Array/List



- `Insert(e)`
 - add e to the end
 - running time: $O(1)$
- `ExtractMax()`
 - scan the array/list
 - running time: $O(n)$

Sorted Array

2	3	9	10	16				
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Sorted Array

2	3	9	10	16				
---	---	---	----	----	--	--	--	--

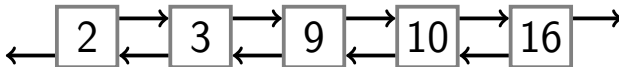
- ExtractMax()
 - extract the last element
 - running time: $O(1)$

Sorted Array

2	3	9	10	16				
---	---	---	----	----	--	--	--	--

- `ExtractMax()`
 - extract the last element
 - running time: $O(1)$
- `Insert(e)`
 - find a position for e ($O(\log n)$ by using binary search), shift all elements to the right of it by 1 ($O(n)$), insert e ($O(1)$)
 - running time: $O(n)$

Sorted List

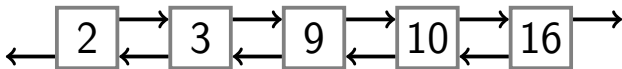


Sorted List



- `ExtractMax()`
 - extract the last element
 - running time: $O(1)$

Sorted List



- `ExtractMax()`
 - extract the last element
 - running time: $O(1)$
- `Insert(e)`
 - find a position for e ($O(n)$; note: cannot use binary search), insert e ($O(1)$)
 - running time: $O(n)$

Summary

	Insert	ExtractMax
Unsorted array/list	$O(1)$	$O(n)$
Sorted array/list	$O(n)$	$O(1)$

Summary

	Insert	ExtractMax
Unsorted array/list	$O(1)$	$O(n)$
Sorted array/list	$O(n)$	$O(1)$
Binary heap	$O(\log n)$	$O(\log n)$
