CSE 378 HW 1.3
Jia Chen 111118311

When $q = (w, [v_1, v_2, v_3])$, we assume our rotation matrix is

$$\begin{bmatrix} m_{00} & m_{01} & m_{02} \\ m_{10} & m_{11} & m_{12} \\ m_{20} & m_{21} & m_{22} \end{bmatrix}.$$

We know if we want transform point P_1 to point P_2 we use

$P2 = q * P1 * q'$

And now we want to find the ~~trans~~ rotation matrix that does the same thing, which is expressed as

$P2 = [M] P1$.

Now assume $P1 = (0, x, y, z)$ then

$$q * P1 * q' = (-v_1 * x - v_2 * y - v_3 * z + i(w * x + v_2 * z - v_3 * y) + j(w * y - v_1 * z + v_3 * x) + k(w * z + v_1 * y - v_2 * x)) * (w - v_1 i - v_2 j - v_3 k).$$

$$= -v_1 * v_2 * x - v_2 * v_3 * y - v_3 v_2 z + w v_1 x + v_2 v_1 z - v_3 v_1 y + w v_2 y - v_1 v_2 z + v_3 v_2 x + w v_3 z + v_1 v_3 y - v_2 v_3 x$$

$$+ i(w^2 x + v_2 w z - v_3 w y + v_1^2 x + v_2 v_1 y + v_3 v_1 z - w v_3 y + v_1 v_3 z - v_3^2 x + w v_2 z + v_1 v_2 y - v_2^2 x)$$

$$+ j(v_1 v_2 x + v_2^2 y + v_3 v_2 z + w v_3 x + v_2 v_3 z - v_3^2 y + w^2 y - v_1 w z + v_3 w x - w v_1 z - v_1^2 y + v_2 v_1 x)$$

$$+ k(v_1 v_3 x + v_2 v_3 y + v_3^2 z - w v_2 x - v_2^2 z + v_3 v_2 y + w v_1 y - v_1^2 z + v_3 v_1 x + w^2 z + v_1 w y - v_2 w x)$$

After grouping coefficients of x, y, and z terms as each columns for each row, where i, j, k are rows, we get:

$$\Rightarrow \begin{bmatrix} w^2 + v_1^2 - v_3^2 - v_2^2 & v_3 w + v_2 v_1 - w v_3 + v_1 v_2 & v_2 w + v_3 v_1 + v_1 v_3 + w v_2 \\ v_1 v_2 + w v_3 + v_3 w + v_2 v_1 & v_2^2 + \cancel{v_3} - v_3^2 + w^2 - v_1^2 & w_3 v_2 + v_2 v_3 - v_1 w - w v_1 \\ v_1 v_3 + w v_2 + v_3 v_1 - v_2 w & v_2 v_3 + v_3 v_2 + w v_1 + v_1 w & v_3^2 - v_2^2 - v_1^2 + w^2 \end{bmatrix}$$