TESTING TIME SERIES DATA FOR STATIONARITY

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Why Testing for Stationarity is Important

Testing data for stationarity is very important in research where the underlying variables based on time. Moreover time series data analysis has many applications in many areas including studying the relationship between wages and house prices, profits and dividends, and consumption and GDP. An important econometric task is determining the most appropriate form of the trend in the data. Many economic and financial time series exhibit trending behavior or non-stationarity in the mean. Leading examples are asset prices, exchange rates and the levels of macroeconomic aggregates like real GDP. In the beginning of the decade 1970s there was a great debate about this topic. Granger and Newbold (1974) were the researchers, who give the idea that the macroeconomic data as a rule contained stochastic trends, and this data is characterized by unit root, they also suggest that using these variables in econometric models may lead towards spurious regressions. So testing for stationarity is very important because the whole results of the regression might be fabricated. In simple words we can say that trended series is called non-stationary and with unit root and on the other hand non-trended series is a stationary series characterized by without unit root. In formal way the series is called stationary if it satisfies three conditions, otherwise it will be a non-stationary series.

i. Mean of Yt (E(Yt) remain same over time or time invariant. i.e.

   $E(Yt) = u, \forall t$

   Where the symbol $\forall$, is use for all and (u) is any scalar

ii Variance of Yt (V (Yt) is time invariant. i.e.

   $V(Yt) = \sigma^2, \forall t$

iii Cov of Yt and Yt-s (cov (Yt, Yt-s) is time invariant, but can be depend upon the lag length. i.e Cov(Yt, Yt-s) = ϒs

If the above conditions do not hold series is non-stationary. In stationary series there is no link between previous values. While the regression on trended (non-stationary) variables, is meaningless, and provides misleading and spurious results. Normally most of the time series data gives us meaningless results until appropriate econometrics and statistical tools were not applied. Now the question is that how to know the data is stationary or non-stationary. We use certain formal and informal tests as well to test the data for unit root. By plotting data into scatter plot graph stationary of the data can be checked. The graphical method called informal way to check the stationary while using E-views or other packages we can check the data for stationarity in a formal method.

There are different formal methods use to check the data for stationarity proposed by different researchers. But in this article our emphasis will endure on the test proposed by Dickey and Fuller (1979, 1981) they develop a formal test for stationarity. The significant thing in their test was that, testing for non-stationarity is equivalent to testing for the existence of unit root. Moreover critique is also there on Dickey and Fuller test that the power of the test is very low, about 30% it makes correct decisions. It is not considered an appropriate test and the entire tests used for unit root have low power.

In this article Household final consumption expenditure per capita[1] and GDP per capita[2] of India are used to test for stationarity. The data about these indicators covering the period of 1980-2009

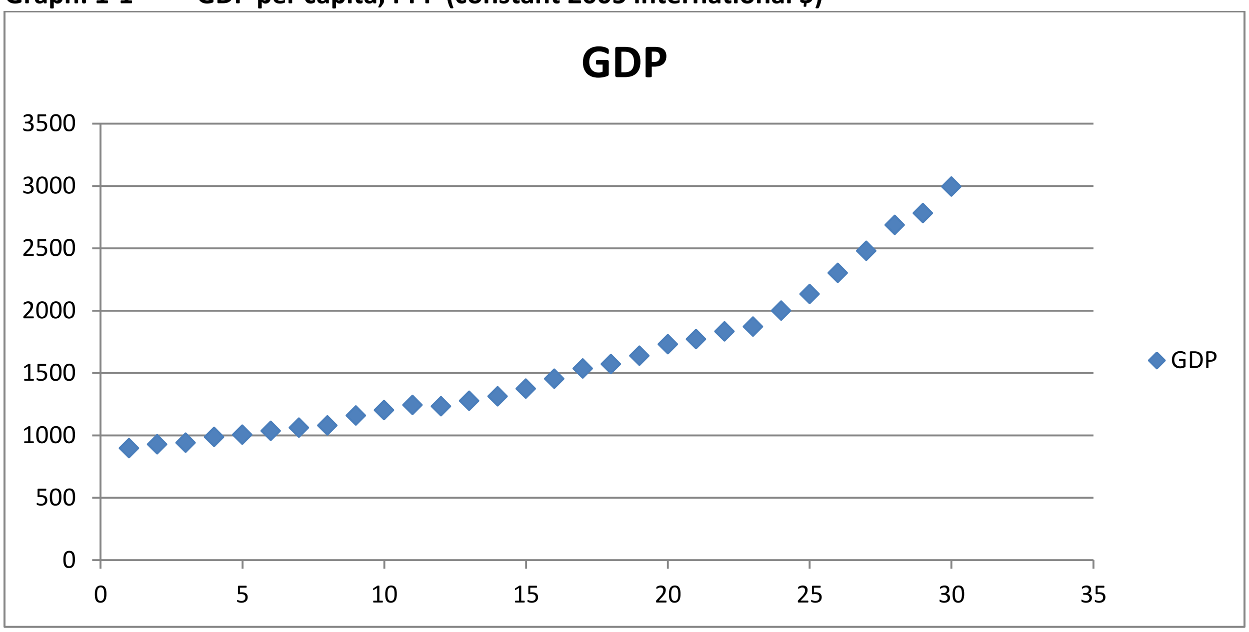
[1] Household final consumption expenditure per capita (constant 2000 US$)
[2] GDP per capita, PPP (constant 2005 international $)

was congregated from World Bank official web site[3]. Primarily two methods are used to test the data for stationarity the first one is informal method, based on simple graphical representation of the series and the latter is the formal method including Dickey and Fuller and Augmented Dickey and Fuller test using E-views.

1. Informal Method to Test the Data for Stationarity

With the help of Microsoft Excel the series of household final consumption is plotted. It is self-explanatory chart, it shows increasing trend in GDP per capita of India and this series might professed as non-stationary series and it has a unit root. Graph: 1-1 is presented below:

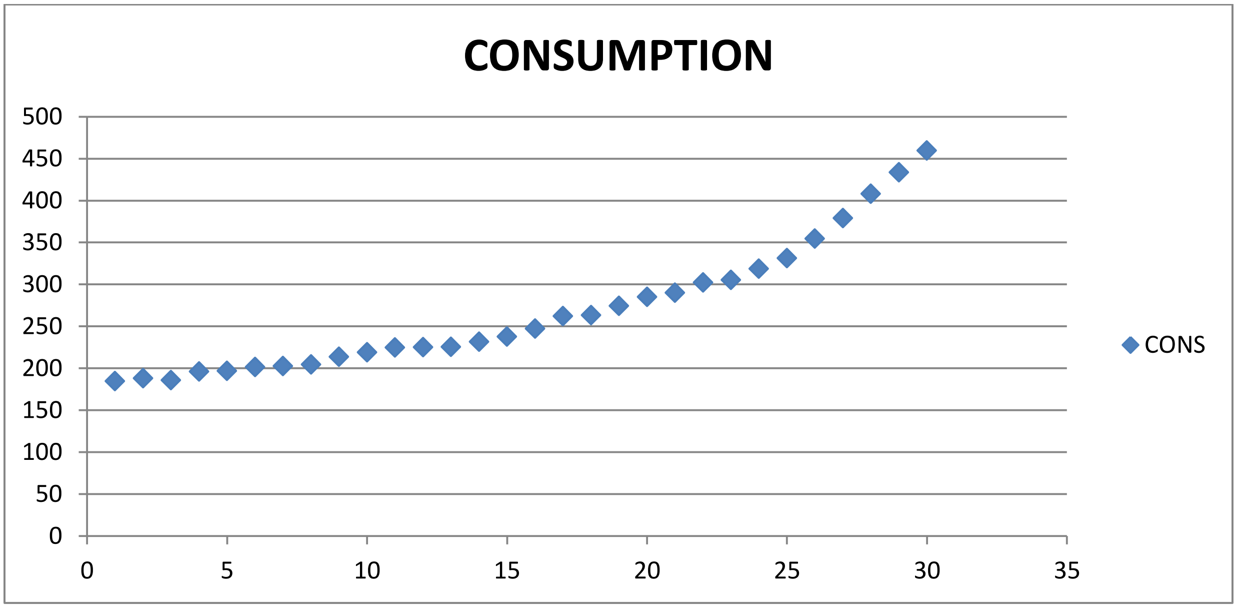
Graph: 1-1 GDP per capita, PPP (constant 2005 international $)



It might be possible to say that India is a developing country, and also a huge market for multinationals. It is the second largest country of the world with reference to population, with high birth rate. So it is expected increasing trends in GDP and consumption patterns.

[3] http://data.worldbank.org/indicator

Graph: 1-2 Household final consumption expenditure per capita (constant 2000 US$)



As we have noted earlier the series of GDP per capita showed increasing trends here graph 1-2 the series of household final consumption per capita also represents the non-stationary series, and exposes the increasing trend as discussed earlier.

2. **Dickey and Fuller Test** (Formal Test for Stationarity)

This section encompassed a formal test of stationarity i.e. Dickey and Fuller test. This test examine the null hypothesis of an autoregressive integrated moving average (ARIMA) against stationary and alternatively. Some basic information about this test is provided in first section; here we will discuss the methodology they use, and then carry on towards our real data analysis. The formal version of Dickey Fuller test is explained here:

Consider an AR (1) model: $Yt = \rho\, Y_{t-1} + \varepsilon$………………………………………….. (1)

Dickey Fuller suggest an alternative equation by subtracting Y_{t-1} from both sides of equation (1)

$Yt - Y_{t-1} = \rho\, Y_{t-1} - Y_{t-1} + \varepsilon$

$\Delta\, Yt = (\rho\text{-}1)\, Y_{t-1} + \varepsilon$

$\Delta\, Yt = \Upsilon\, Y_{t-1} + \varepsilon$…………………………………...… (2)

Equation two is without constant where, $\Upsilon = \rho\text{-}1$. Dickey and Fuller also suggest two alternate forms:

Constant only: $\Delta\, Yt = \alpha + \Upsilon\, Y_{t-1} + \varepsilon$………………………………….. (3)

Constant and Time Trend: $\Delta\, Yt = \alpha + \beta t + \Upsilon\, Y_{t-1} + \varepsilon$…………………………….. (4)

Testing the hypothesis

Ho: $\Upsilon = 0$

H1: $\Upsilon < 0$

Dickey-Fuller test with intercept was applied on both series to test the data for stationarity. The null hypothesis is tested via t-statistics which is given by this formula:

$$t = \frac{\gamma\hat{}-\gamma\text{Ho}}{SE(\gamma\hat{})}\text{……………………………………………………..} \quad (5)$$

If t calculated is greater than the critical value we do not reject our null hypothesis. In this situation the variable under consideration will be non-stationary and has a unit root. On the other hand if t calculated is less than the critical value we reject our null hypothesis. In this case the underlying series would be a stationary series and it does not have the unit root. Firs the series is tested on level if it does not become stationary than we travel further and test the series at first and second difference sequentially. There is another method to reject or not to reject the null hypothesis if the calculated value is on the right side of the critical value, on one sided tail (see figure2-1 in appendix) we do not reject null hypothesis and if the calculated value is on the left side of the critical value we reject the null hypothesis and conclude that the series does not has unit root. P-value is also used to reject or accept the null hypothesis if the p-value < .05 reject null hypothesis and vice versa.

Table: 2-1, Dickey Fuller Test at Level depicts that the calculated t-value is greater than critical values at 1%, 5% and 10% significant levels. At levels both the underlying series are non-stationary. Hence we do not reject the null hypothesis and accept alternate hypothesis that the series has a unit root. Table: 2-2, named as Dickey Fuller Test at first difference provide similar type of results as in table: 2-1, DF test statistics-GDP and Consumption series are also non-stationary at first difference. Both the series are trended at first difference too, increasing trend was also found via scatter plot graph, as this method refer as the informal way to test the stationarity of the data.

Table: 2-1 Dickey-Fuller Test at Level

Variables	t-statistics	Level of significance	Critical value
DF test statistic-GDP	0.184098	1% Critical Value	-2.653401
		5% Critical Value	-1.953858
		10% Critical Value	-1.609571
DF test statistic-Cons	0.062812	1% Critical Value	-2.653401
		5% Critical Value	-1.953858

		10% Critical Value	-1.609571

Table: 2-2 Dickey-Fuller Test at First Difference

Variables	t-statistics	Level of significance	Critical value
DF test statistic-GDP	-0.673457	1% Critical Value	-2.653401
		5% Critical Value	-1.953858
		10% Critical Value	-1.609571
DF test statistic-Cons	-0.409076	1% Critical Value	-2.653401
		5% Critical Value	-1.953858
		10% Critical Value	-1.609571

Table: 2-3, Dickey Fuller Test at Second Difference reveals that both the series are stationary at second difference. Calculated t-value of GDP is (-9.128370) it less than the critical values at all significant levels. Similarly t-statistics of consumption series is (-8.512923) it is also less than the critical values at all significant levels. Therefore we reject our null hypothesis and conclude that both the series are stationary at 1% significant level at second difference, and both the series does not have the unit root. Our informal method to test stationarity also confirms the results of formal test i.e. Dickey Fuller test. Graphs of both the series at second difference do not demonstrate any kind of trend; there are fluctuations in the graphs. These fluctuations epitomize the stationary of underlying economic series.

Table: 2-3 Dickey-Fuller Test at Second Difference

Variables	t-statistics	Level of significance	Critical value
DF test statistic-GDP	-9.128370	1% Critical Value	-2.653401
		5% Critical Value	-1.953858

		10% Critical Value	-1.609571
		1% Critical Value	-2.653401
DF test statistic-Cons	-8.512923	5% Critical Value	-1.953858
		10% Critical Value	-1.609571

3. Augmented Dickey-Fuller Test

Augmented Dickey Fuller Test is the extended version of simple Dickey Fuller test. Because of the error term unlikely to be white noise[4]. They extended their test by including extra lagged in terms of the dependent variables in order to eliminate the problem of autocorrelation. Normally we use Augmented Dickey-Fuller test instead of simple Dickey-Fuller test. In simple words by including the lagged values of dependent variable to the existing model, and continue this procedure up till where the autocorrelation eliminated. It can be illustrated as:

$$Yt = \beta_1 + \beta_2\, Yt + \varepsilon_t \ldots\ldots (6)$$

$$Yt = \beta_1 + \beta_2\, Yt + \beta_3\, Y_{t-1} + \varepsilon_t \ldots\ldots (7)$$

$$Yt = \beta_1 + \beta_2\, Yt + \beta_3\, Y_{t-1} + \beta_4\, Y_{t-2} + \varepsilon_t \ldots\ldots (8)$$

Now,

$$\Delta Yt = \Upsilon\, Y_{t-1} + \beta_1\, \Delta Y_{t-1} + \varepsilon_t \ldots\ldots (9)$$

$$\Delta Y_t = \Upsilon\, Y_{t-1} + \beta_1\, \Delta Y_{t-1} + \beta_2\, \Delta Y_{t-2} \ldots\ldots + \beta_p\, \Delta Y_{t-p} + \varepsilon_t \ldots\ldots (10)$$

Continue this process up till, where autocorrelation eliminated. This expression could be written as:

[4] It is actually related with stationarity, except one thing. "The third condition of stationary series, $cov(Y_t, Y_{t-1} = 0)$" IID, Identical Independence Distribution. There is a slight difference between stationary and IID.

$$\Delta Yt = \Upsilon Yt - 1 + \sum_{i=1}^{p} \beta 1 \; \Delta Yt - 1 + \varepsilon t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(A)}$$

$$\Delta Yt = \alpha + \; \Upsilon Yt - 1 + \sum_{i=1}^{p} \beta 1 \; \Delta Yt - 1 + \varepsilon t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(B)}$$

$$\Delta Yt = \alpha + \beta t + \Upsilon Yt - 1 + \sum_{i=1}^{p} \beta 1 \; \Delta Yt - 1 + \varepsilon t \ldots\ldots\ldots\ldots\ldots\ldots\ldots.\ldots\ldots\ldots\ldots\ldots \text{(C)}$$

Some common assumptions of ordinary least square (OLS) are discussed here:

1. ε must be independent
2. There should be no heteroskedasticity[5], should homogeneity.
3. There should no structural break, co-efficient should stable.
4. Error term should be normally distributed.

Testing for stationarity in Augmented Dickey-Fuller test follows the same procedure as in simple Dickey-Fuller test. First, stationary is checked at level than at first difference and finally on second difference. At first difference the equation will be as follows:

$$\Delta 2Yt = \Upsilon Yt - 1 + \sum_{i=1}^{p} \beta 1 \; \Delta 2Yt - 1 + \varepsilon t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(a)}$$

$$\Delta 2Yt = \alpha + \; \Upsilon Yt - 1 + \sum_{i=1}^{p} \beta 1 \; \Delta 2Yt - 1 + \varepsilon t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(b)}$$

$$\Delta 2Yt = \alpha + \; \Upsilon Yt - 1 + \; \beta t + \sum_{i=1}^{p} \beta 1 \; \Delta 2Yt - 1 + \varepsilon t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(c)}$$

[5] Normally heteroskedasticity is defined as unequal spread. In econometrics the measure we usually use for spread is the variance, and therefore heteroskedasticity deals with unequal variance.

If series is still not-stationary we can use same equations by replacing[6] Δ^2 with Δ^3 rest process will be same.

Testing for hypothesis using ADF

$\mathbf{H_0}$ **:** $\Upsilon = 0$
$\mathbf{H_1}$ **:** $\Upsilon < 0$

Table: 3-1 demonstrates the output of Augmented Dickey Fuller test at Levels. Variables of concern are tested for stationary via E-Views. At levels both the variables are non-stationary as the t-statistics of GDP is (7.645804) and the t-value of consumption is (8.198867), these values are much higher than the critical values at all significant levels. So we do not reject our null hypothesis and conclude that both the series has a unit root. Simple Dickey-Fuller test also provide same kind of results at levels.

Table: 3-1 Augmented Dickey Fuller Test at Level

Variables	t-statistics	Level of significance	Critical value
ADF test statistic-GDP	7.645804	1% Critical Value	-3.679322
		5% Critical Value	-2.967767
		10% Critical Value	-2.622989
ADF test statistic-Cons	8.198867	1% Critical Value	-3.679322
		5% Critical Value	-2.967767
		10% Critical Value	-2.622989

Now the series are not stationary at level, we continue our process to ADF at first difference. Table: 3-2 denotes the results as of ADF at level. Because the calculated t values of GDP and

[6] $\Delta^3 Y_t$ use for second difference.

consumption (-0.600268) (-0.361552) respectively are higher than the critical values at all significant levels. Therefore we do not reject the null hypothesis. Hence it is possible to say that the series of GDP and consumption are not stationary at first difference and has a unit root.

Table: 3-2 Augmented Dickey Fuller Test at First Difference

Variables	t-statistics	Level of significance	Critical value
ADF test statistic-GDP	-0.600268	1% Critical Value	-3.699871
		5% Critical Value	-2.976263
		10% Critical Value	-2.627420
ADF test statistic-Cons	-0.361552	1% Critical Value	-3.699871
		5% Critical Value	-2.976263
		10% Critical Value	-2.627420

Data about certain indicators of study does not become stationary at level and at first difference, so we carry on our analysis towards second difference. Table: 3-3 Augmented Dickey Fuller Test at Second Difference depicts the output of ADF at second difference with intercept. Here the series become stationary at 1% level of significance as the t-value of GDP is (-8.935281) and the critical value at 1% significance level is (-3.699871). The t-value is less than the critical value, it falls at left hand side of the critical value so we can reject null hypothesis and it might be possible to say that the series of GDP does not have a unit root at second difference.

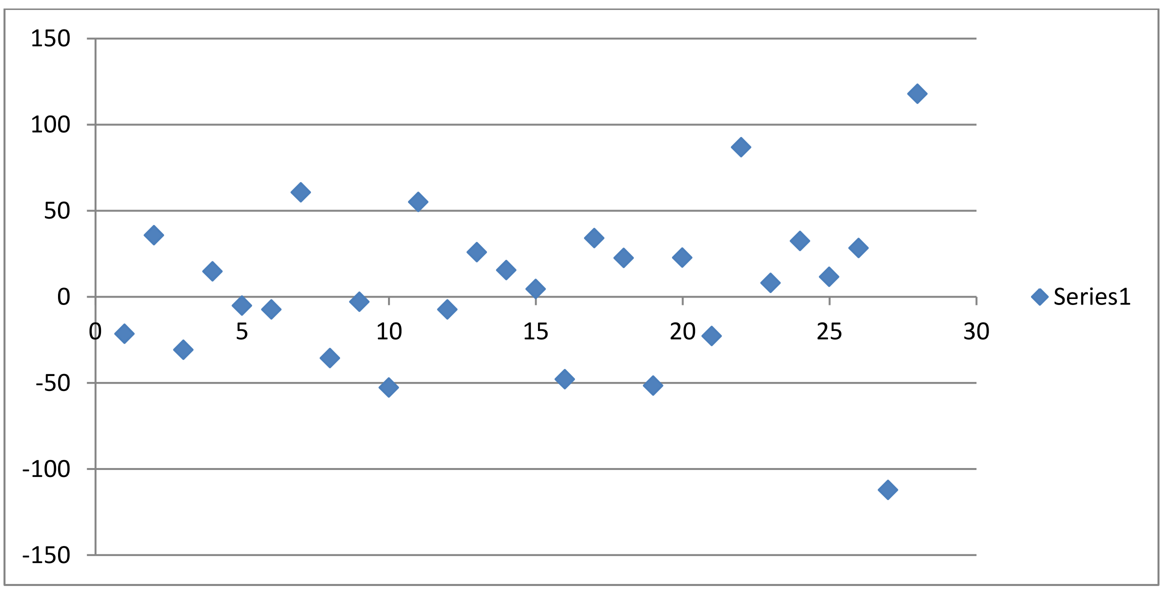
Table: 3-3 Augmented Dickey Fuller Test at Second Difference

Variables	t-statistics	Level of significance	Critical value
ADF test statistic-GDP	-8.935281	1% Critical Value	-3.699871
		5% Critical Value	-2.976263
		10% Critical Value	-2.627420

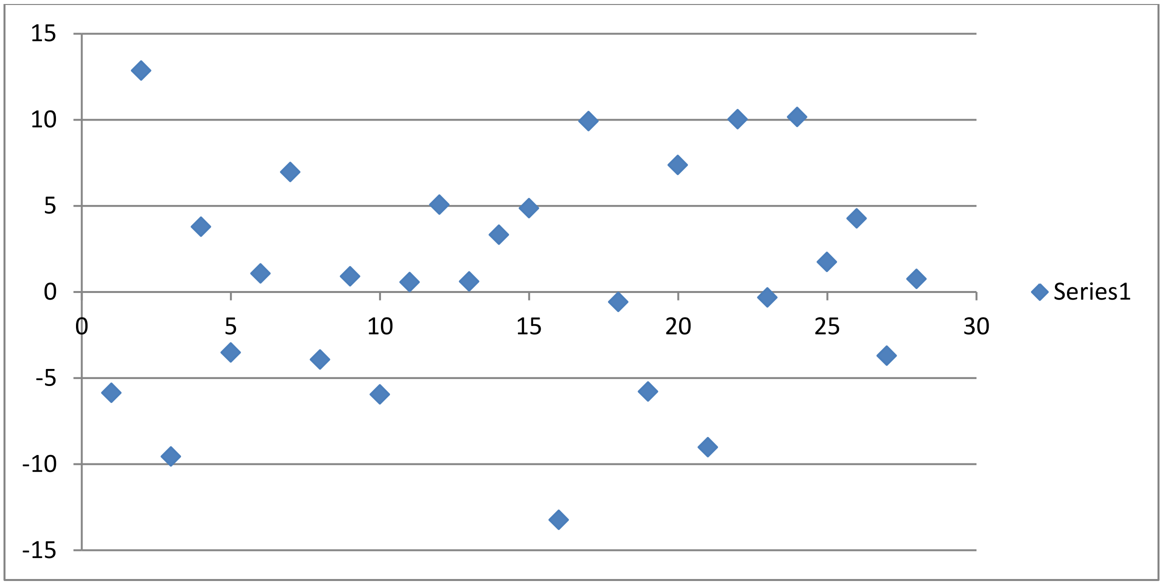
		1% Critical Value	-3.699871
ADF test statistic-Cons	-10.25461	5% Critical Value	-2.976263
		10% Critical Value	-2.627420

Calculated value of consumption is (-10.25461) and critical value at 1% significant level is (-3.699871), so we reject null hypothesis and conclude that the consumption series does not have a unit root and is stationary at second difference at 1% significant level. Series of data at second difference were also plotted on graphs, these also does not show any kind of severe trend.

Graph: 3-1 GDP Graph ADF at Second Difference



Graph: 3-2 Consumption ADF at Second Difference



Conclusion

This study was based on to test the data for stationarity. Testing the data for stationarity is the first step of data analysis in economics and finance research. Without doing so we cannot apply the appropriate statistical and econometrics tools to make decisions. We use informal and formal method to test the data for stationarity. Informal methods encompass of charts and diagrams while the formal way to test the stationary we use Dickey-Fuller and Augmented Dickey-Fuller test. We know that Augmented Dickey-Fuller test is commonly used to test the unit root.

Real data of Gross Domestic Product (GDP) per capita and consumption per capita of India for the period of 1980-2009 was selected for this purpose. Section one encompasses, scatter plots and Dickey-Fuller test. Scatter plots portray the increasing trend in both the series. While the first formal test i.e. Dickey-Fuller test reports that the series are non-stationary at level and at first difference. But at second difference GDP and consumption become stationary at 1% significant level. Section two comprises of Augmented Dickey-Fuller test, it does not contain the different findings as compared to Dickey-Fuller test. We also applied Augmented Dickey-

Fuller test with time and trend at back end, it also gives almost similar results. There are some critiques on Augmented Dickey-Fuller test as well, but besides these critiques this test is considered most important and widely used in research where the time series data is involved.

References

DICKEY D., FULLER W. (1979). – « Distribution of the Estimator for Autoregressive Time series with a Unit Root », *Journal of the American Statistical Association*, 74, pp. 427-431.

GRANGER, C.W.J. and NEWBOLD (1974). – « Economic Forecasting: The atheist's Viewpoint, in G.A Renton (ed.) », *Modeling the economy*. London: Heinemann.

APPENDICES

Appendix Table-1
Least Square
Dickey-Fuller Test (GDP)

DF-GLS Test Equation on GLS Detrended Residuals
Dependent Variable: D(GLSRESID)
Method: Least Squares
Sample (adjusted): 1983 2009
Included observations: 27 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
GLSRESID(-1)	0.003715	0.020181	0.184098	0.8555
D(GLSRESID(-1))	0.382699	0.196755	1.945057	0.0636
D(GLSRESID(-2))	0.656065	0.210742	3.113121	0.0047

DF-GLS Test Equation on GLS Detrended Residuals
Dependent Variable: D(GLSRESID)
Method: Least Squares

Variable	Coefficient	Std. Error	t-Statistic	Prob.
GLSRESID(-1)	-0.105257	0.156294	-0.673457	0.5068
D(GLSRESID(-1))	-0.584431	0.202743	-2.882622	0.0080

DF-GLS Test Equation on GLS Detrended Residuals
Dependent Variable: D(GLSRESID)
Method: Least Squares

Variable	Coefficient	Std. Error	t-Statistic	Prob.
GLSRESID(-1)	-1.644567	0.180160	-9.128370	0.0000

Least Square Consumption (DF)

DF-GLS Test Equation on GLS Detrended Residuals
Dependent Variable: D(GLSRESID)
Method: Least Squares

Variable	Coefficient	Std. Error	t-Statistic	Prob.
GLSRESID(-1)	0.001392	0.022161	0.062812	0.9504
D(GLSRESID(-1))	0.449587	0.171200	2.626096	0.0148
D(GLSRESID(-2))	0.627739	0.182782	3.434365	0.0022

DF-GLS Test Equation on GLS Detrended Residuals
Dependent Variable: D(GLSRESID)
Method: Least Squares

Variable	Coefficient	Std. Error	t-Statistic	Prob.
GLSRESID(-1)	-0.059873	0.146361	-0.409076	0.6860
D(GLSRESID(-1))	-0.530184	0.183378	-2.891208	0.0078

DF-GLS Test Equation on GLS Detrended Residuals
Dependent Variable: D(GLSRESID)
Method: Least Squares

Variable	Coefficient	Std. Error	t-Statistic	Prob.
GLSRESID(-1)	-1.463864	0.171958	-8.512923	0.0000

Appendix Table-2
Least Square
Augmented Dickey-Fuller

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(GDP)
Method: Least Squares
Sample (adjusted): 1981 2009
Included observations: 29 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
GDP(-1)	0.089699	0.011732	7.645804	0.0000
C	-65.29079	19.05187	-3.427001	0.0020

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(GDP,2)
Method: Least Squares

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(GDP(-1))	-0.093481	0.155733	-0.600268	0.5540
D(GDP(-1),2)	-0.602272	0.202178	-2.978925	0.0065
C	15.26094	12.96260	1.177305	0.2506

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(GDP,3)
Method: Least Squares

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(GDP(-1),2)	-1.648544	0.184498	-8.935281	0.0000
C	8.954292	7.494656	1.194757	0.2434

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(CONS)
Method: Least Squares

Variable	Coefficient	Std. Error	t-Statistic	Prob.
CONS(-1)	0.107370	0.013096	8.198867	0.0000
C	-18.63444	3.540483	-5.263248	0.0000

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(CONS,2)
Method: Least Squares

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(CONS(-1))	-0.051773	0.143197	-0.361552	0.7209
D(CONS(-1),2)	-0.563873	0.180742	-3.119770	0.0047
C	1.985283	1.609884	1.233184	0.2294

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(CONS,3)
Method: Least Squares

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(CONS(-1),2)	-1.595374	0.155576	-10.25461	0.0000
C	1.540527	1.020295	1.509885	0.1436

Figure: 2-1