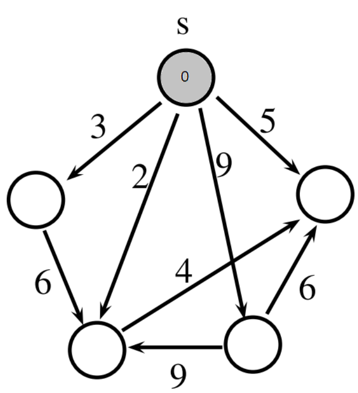
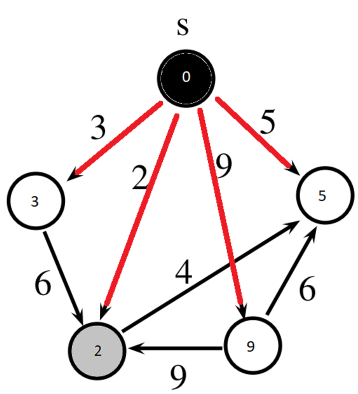
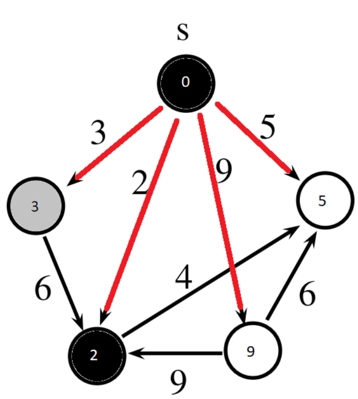
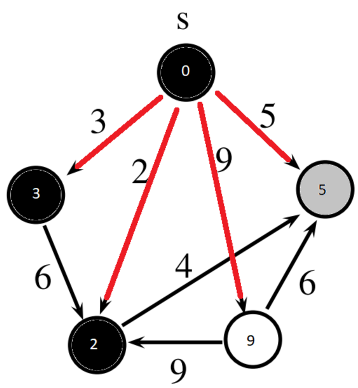
1 Problem 1 (60 points).

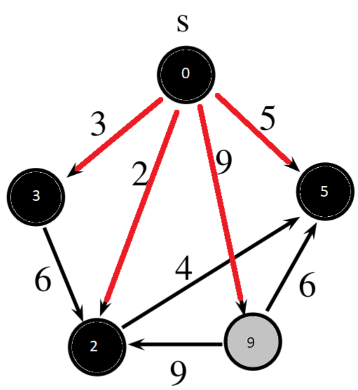
Trace the Dijkstra's algorithm to show the shortest paths from the vertex s to all the other vertices in the following connected directed graph. (See Figure 24.6 of CLRS, 3rd Ed. as an example.)

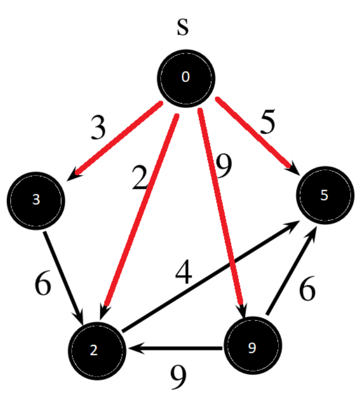






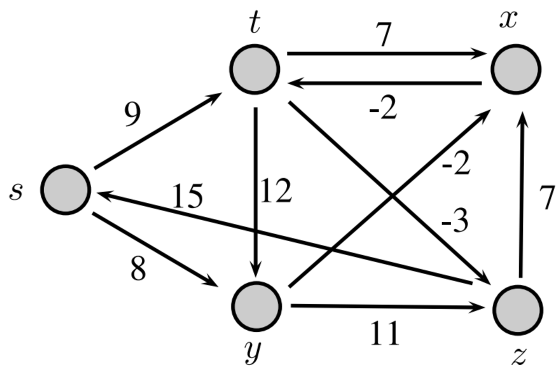


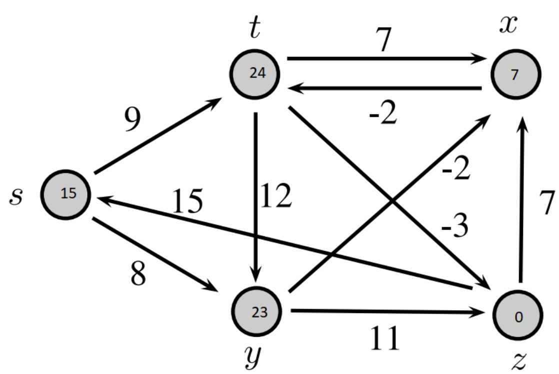


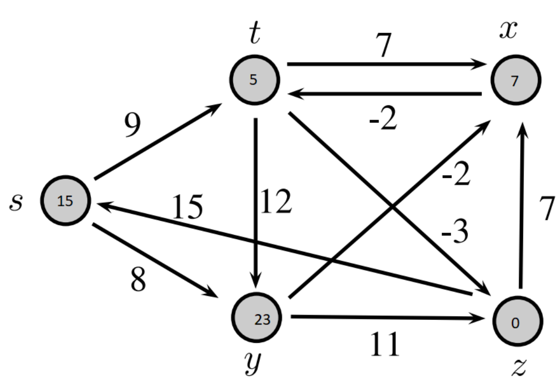


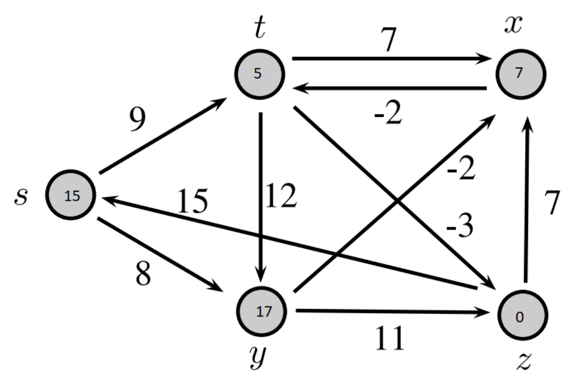
2 Problem 2 (40 points).

Show the trace of the Bellman-Ford algorithm on the following directed graph, using vertex z as the source. In each pass, relax edges in the order of (t, x),(t, y),(t, z),(x, t),(y, x),(y, z),(z, x),(z, s),(s, t),(s, y). Show the d values after each pass. (A similar trace which starts from vertex s can be found in Figure 24.4 of CLRS, 3rd Ed., but in this homework problem you start from vertex z and also do not need to show the π values.)









3 Problem 3 (20 points).

Give an $O(|V| * |E|)$-time algorithm for computing the transitive closure of a directed graph $G = (V, E)$. You can assume $|E| \geq |V|$.

BFS can be used to more efficiently search through the graph and return any reachable vertices from every source. Since BFS can keep track of completely visited nodes, we can avoid going over the same path twice in the case of circles, while making sure all reachable nodes from that source point are searched. The found nodes for each source can then be stored in a 2D array in the form of transitive closure values 0 or 1, only changing to a 1 any time a node is found in that BFS search from the starting node.

The reason this idea has a time complexity of $O(|V| * |E|)$ is because the whole algorithm is run for each vertex, and each vertex only goes over each edge up to exactly once, since visited nodes are kept track of so extra runs of the same edges are avoided.

4 Problem 4 (20 points).

Given the following example $D^{(0)}$, calculate $D^{(1)}$, $D^{(1)}$, $D^{(1)}$, $D^{(1)}$

$$D^{(0)} = \begin{pmatrix} 0 & 5 & \infty & 3 \\ \infty & 0 & -1 & \infty \\ 6 & \infty & 0 & \infty \\ \infty & 2 & 7 & 0 \end{pmatrix}$$

$$D^{(1)} = \begin{pmatrix} 0 & 5 & \infty & 3 \\ \infty & 0 & -1 & \infty \\ 6 & 30 & 0 & 9 \\ \infty & 2 & 7 & 0 \end{pmatrix}$$

$$D^{(2)} = \begin{pmatrix} 0 & 5 & 4 & 3 \\ \infty & 0 & -1 & \infty \\ 6 & 30 & 0 & 9 \\ \infty & 2 & 1 & 0 \end{pmatrix}$$

$$D^{(3)} = \begin{pmatrix} 0 & 5 & 4 & 3 \\ 5 & 0 & -1 & 8 \\ 6 & 30 & 0 & 9 \\ 7 & 2 & 1 & 0 \end{pmatrix}$$

$$D^{(4)} = \begin{pmatrix} 0 & 5 & 4 & 3 \\ 5 & 0 & -1 & 8 \\ 6 & 11 & 0 & 9 \\ 7 & 2 & 1 & 0 \end{pmatrix}$$