Digital Image Processing

Lecture # 3D
Spatial Filtering

Contents

- Sharpening Spatial Filters
- Image Enhancement using
  - 2^{nd} Derivative
  - 1^{st} Derivative
- Combining Spatial Enhancement Methods

Sharpening Spatial Filters

Previously we have looked at smoothing filters which remove fine detail

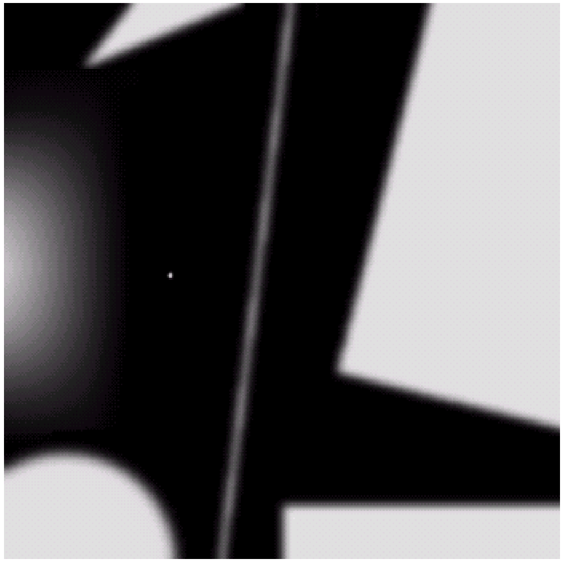
Sharpening spatial filters seek to highlight fine detail

- Remove blurring from images
- Highlight edges

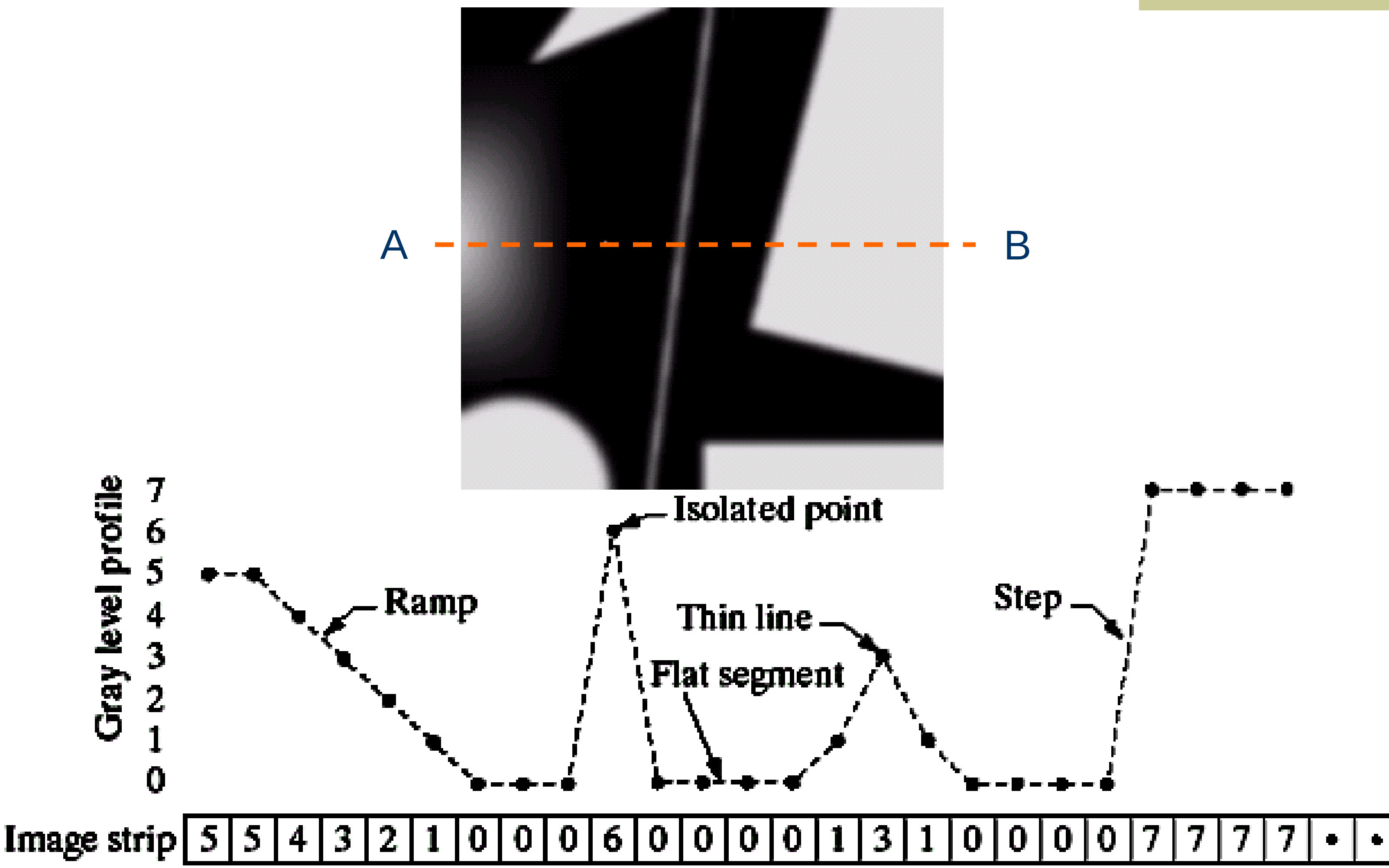
Sharpening filters are based on *spatial differentiation*

Spatial Differentiation

- Let's consider a simple 1 dimensional example



Spatial Differentiation



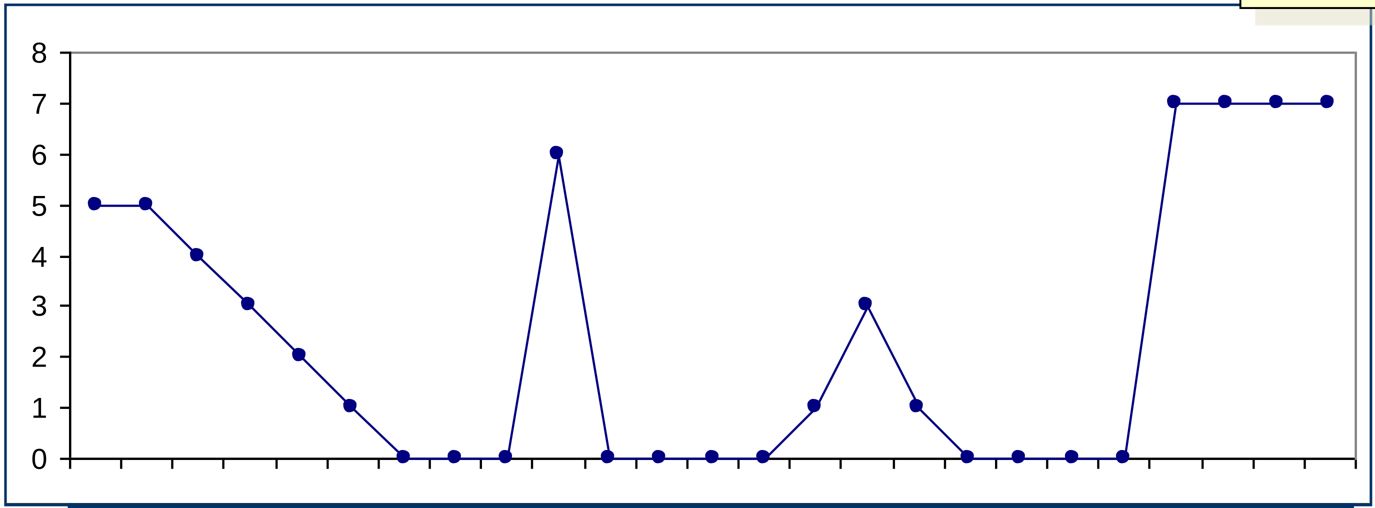
1st Derivative

The 1st derivative of a function is given by:

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

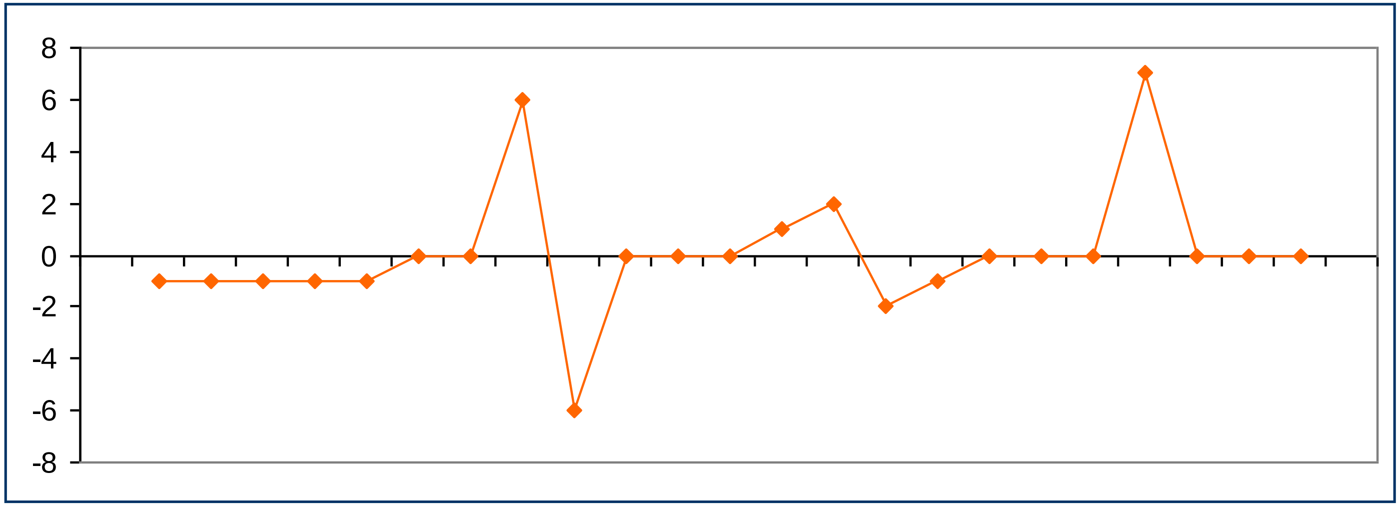
Its just the difference between subsequent values and measures the rate of change of the function

1^{st} Derivative



5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7

	-1	-1	-1	-1	-1	0	0	6	-6	0	0	0	1	2	-2	-1	0	0	0	7	0	0	0	



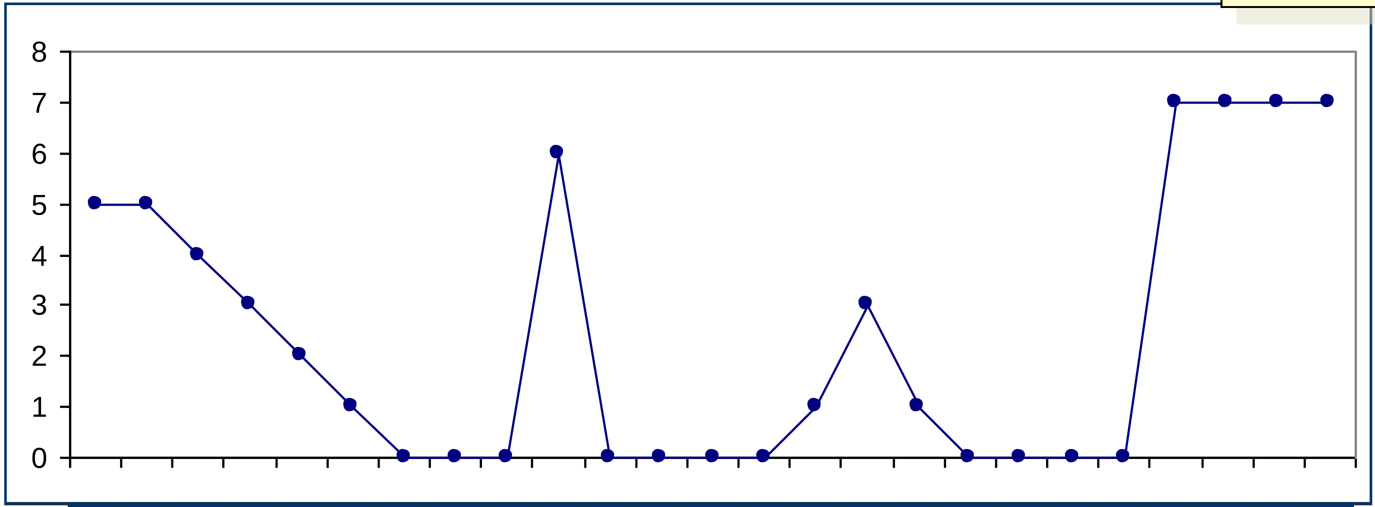
2^{nd} Derivative

The 2nd derivative of a function is given by:

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1) + f(x-1) - 2f(x)$$

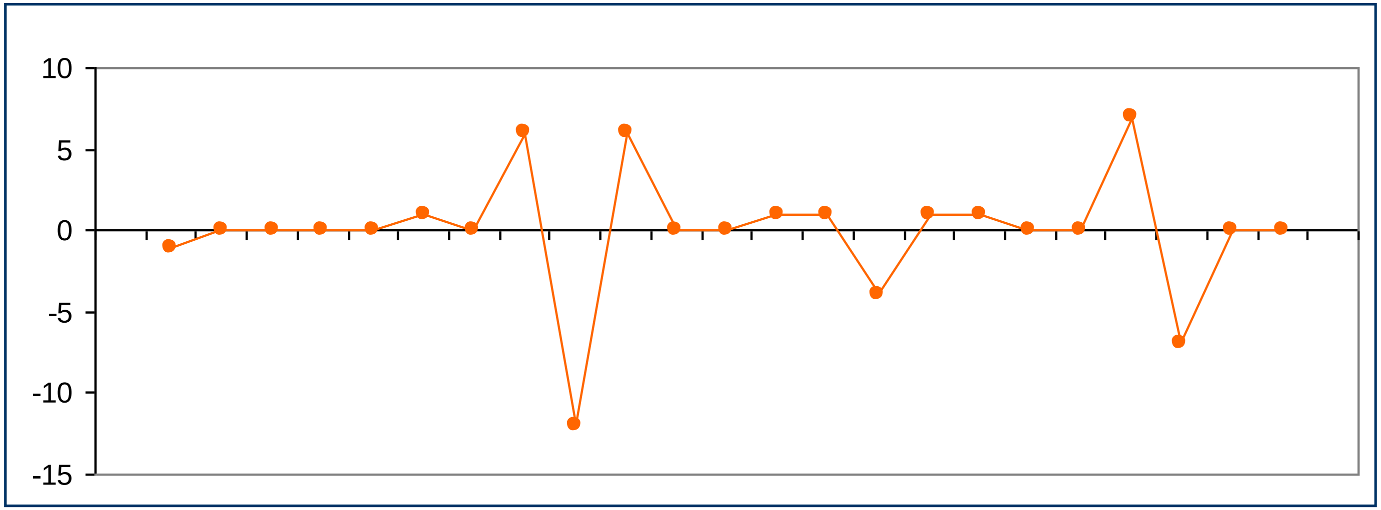
Simply takes into account the values both before and after the current value

2^{nd} Derivative



5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7

	-1	0	0	0	0	1	0	6	-12	6	0	0	1	1	-4	1	1	0	0	7	-7	0	0	



2^{nd} Derivative for Image Enhancement

The 2nd derivative is more useful for image enhancement than the 1st derivative - *Stronger response to fine detail*

We will come back to the 1st order derivative later on

The first sharpening filter we will look at is the *Laplacian*

Laplacian Filter

The Laplacian is defined as follows:

$$\boxed{\nabla^2 f = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y}}$$

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial^2 y} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

Laplacian Filter

So, the Laplacian can be given as follows:

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$$

Can we implement it using a filter/ mask?

0	1	0
1	-4	1
0	1	0

Laplacian Filter

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

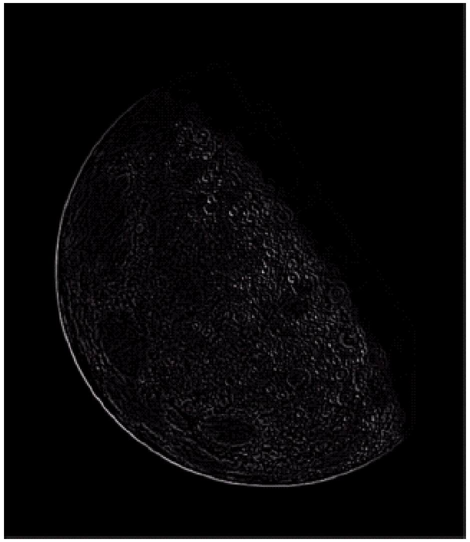
a	b
c	d

FIGURE 3.39
(a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4). (b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.

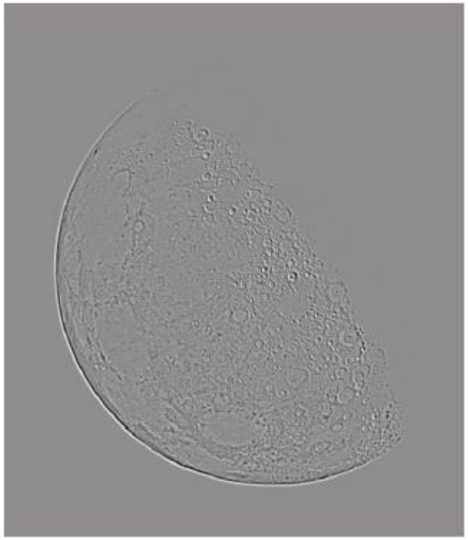
Laplacian Filter

Applying the Laplacian to an image we get a new image that highlights edges and other discontinuities

Original Image



Laplacian Filtered Image

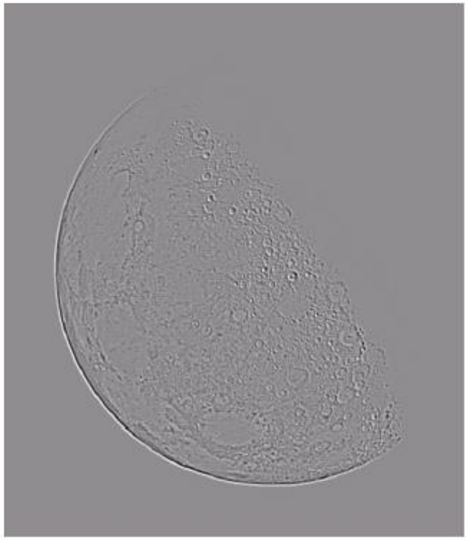


Laplacian Filtered Image Scaled for Display

Laplacian Image Enhancement

The result of a Laplacian filtering is not an enhanced image

To generate the final enhanced image



Laplacian
Filtered Image
Scaled for Display

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f, & w_5 < 0 \\ f(x,y) + \nabla^2 f, & w_5 > 0 \end{cases}$$

Laplacian Image Enhancement

Original Image

-

Laplacian Filtered Image

=

Sharpened Image

In the final sharpened image edges and fine detail are much more obvious

Laplacian Image Enhancement

Simplified Image Enhancement

- The entire enhancement can be combined into a single filtering operation

$$
\begin{aligned}
g(x, y) &= f(x, y) - \nabla^2 f \\
&= f(x, y) - [f(x+1, y) + f(x-1, y) \\
&\qquad + f(x, y+1) + f(x, y-1) \\
&\qquad - 4f(x, y)]
\end{aligned}
$$

Simplified Image Enhancement

- The entire enhancement can be combined into a single filtering operation

$$g(x,y) = f(x,y) - \nabla^2 f$$

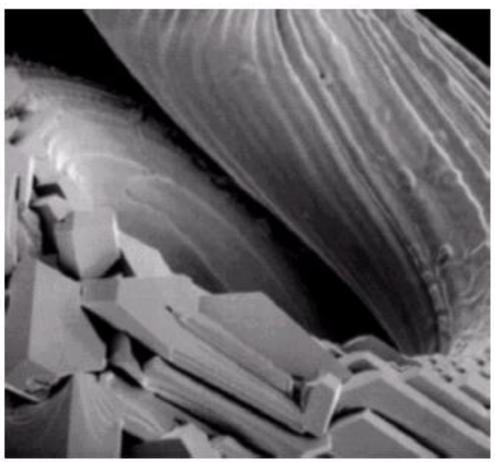
$$= 5f(x,y) - f(x+1,y) - f(x-1,y)$$

$$- f(x,y+1) - f(x,y-1)$$

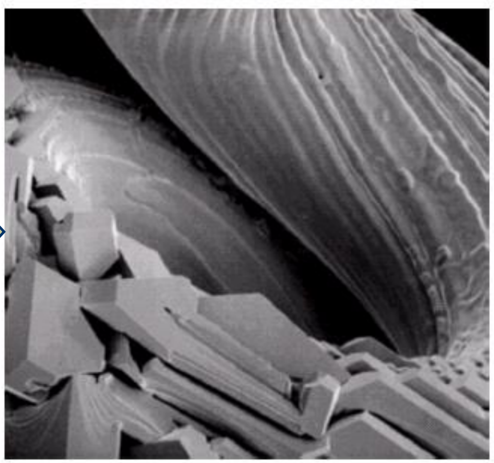
0	-1	0
-1	5	-1
0	-1	0

Simplified Image Enhancement

- This gives us a new filter which does the whole job for us in one step



0	-1	0
-1	5	-1
0	-1	0



Use of first derivatives for image enhancement: The Gradient

- The **gradient** of a function f(x,y) is defined as

$$\nabla \mathrm{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

Use of first derivatives for image enhancement: The Gradient

The magnitude of this vector is given by:

$$\begin{aligned}\nabla f &= mag(\nabla \mathrm{f}) \\ &= \left[G_x^2 + G_y^2\right]^{1/2} \\ &= \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2}\end{aligned}$$

For practical reasons this can be simplified as:

$$\nabla f \approx |G_x| + |G_y|$$

Gradient Operators

There is some debate as to how best to calculate these gradients

Simplest Operator

$$\frac{\partial f}{\partial y} = (z_8 - z_5), \frac{\partial f}{\partial x} = (z_6 - z_5)$$

$$\nabla f = \sqrt{(z_8 - z_5)^2 + (z_6 - z_5)^2}$$

$$\nabla f \approx |(z_8 - z_5)| + |(z_6 - z_5)|$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Gradient Operators

Prewitt Operator

$$\nabla f \approx \left|(z_7 + z_8 + z_9) - (z_1 + z_2 + z_3)\right| + \left|(z_3 + z_6 + z_9) - (z_1 + z_4 + z_7)\right|$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

$$\frac{\partial f}{\partial y} =$$

-1	**-1**	**-1**
0	**0**	**0**
1	**1**	**1**

Extract horizontal edges

$$\frac{\partial f}{\partial x} =$$

-1	**0**	**1**
-1	**0**	**1**
-1	**0**	**1**

Extract vertical edges

Gradient Operators

Sobel Operator

$$\frac{\partial f}{\partial y} = \begin{array}{|c|c|c|}\hline -1 & -2 & -1 \\ \hline 0 & 0 & 0 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

Extract horizontal edges

$$\frac{\partial f}{\partial x} = \begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array}$$

Extract vertical edges

Emphasize more the current point (x direction)

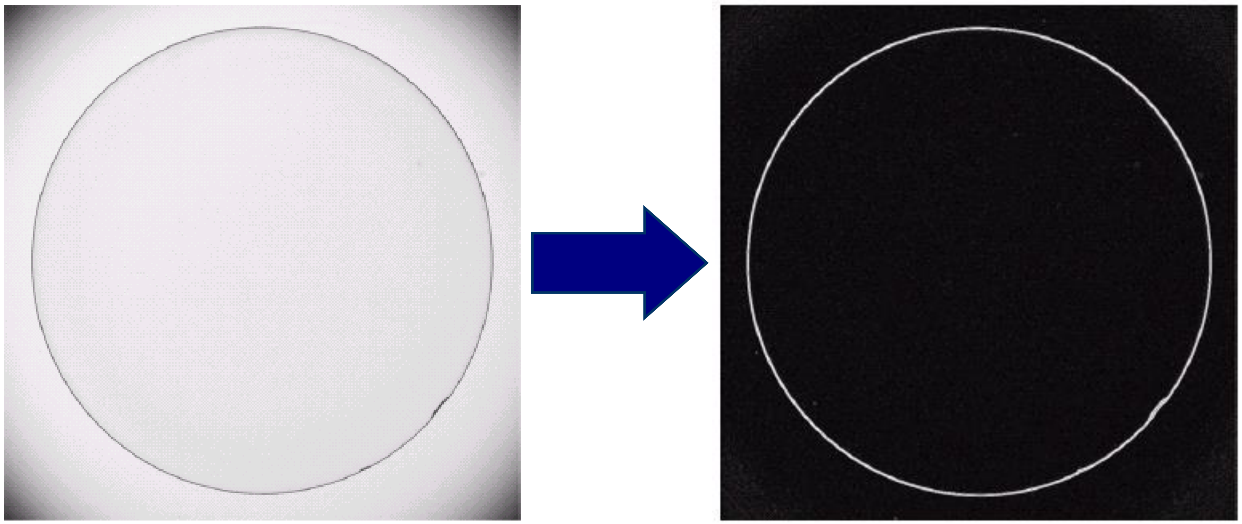
$$\nabla f \approx \left|(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)\right| + \left|(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)\right|$$

Emphasize more the current point (y direction)

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Pixel Arrangement

Sobel Operator: Example



An image of a contact lens which is enhanced in order to make defects more obvious

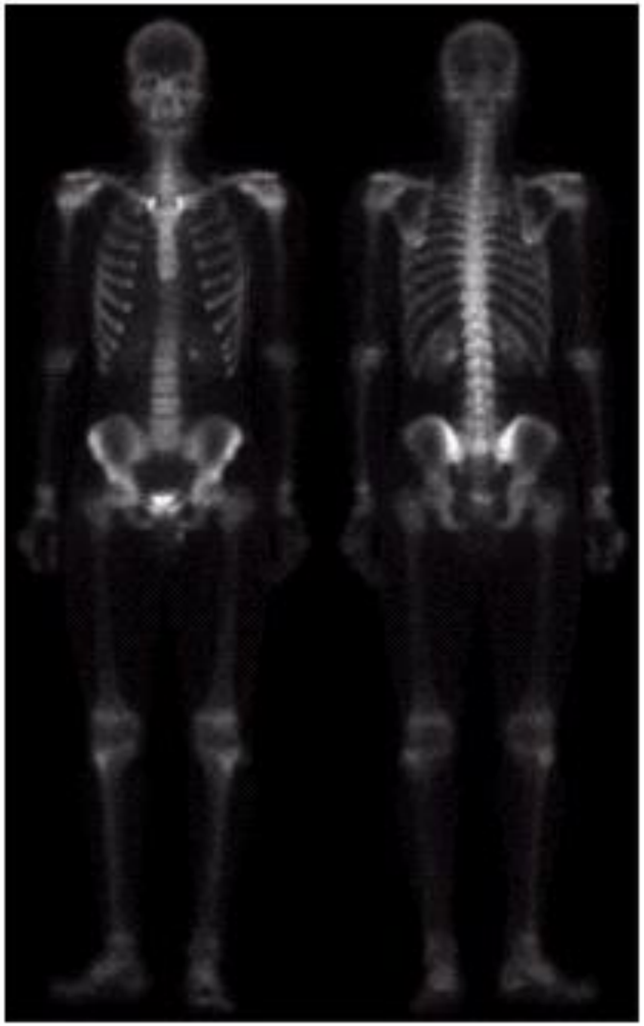
Sobel filters are typically used for edge detection

Combining Spatial Enhancement Methods

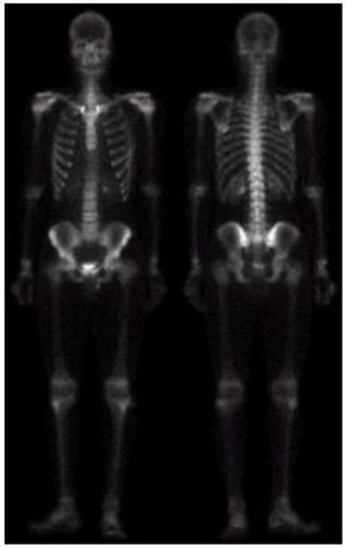
Successful image enhancement is typically not achieved using a single operation

Rather we combine a range of techniques in order to achieve a final result

This example will focus on enhancing the bone scan

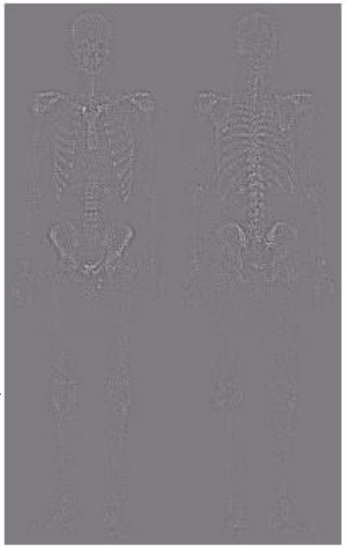


Combining Spatial Enhancement Methods



(a)

Laplacian filter of bone scan (a)



(b)

Sharpened version of bone scan achieved by subtracting (a) and (b)

(c)

Sobel filter of bone scan (a)

(d)

Combining Spatial Enhancement Methods

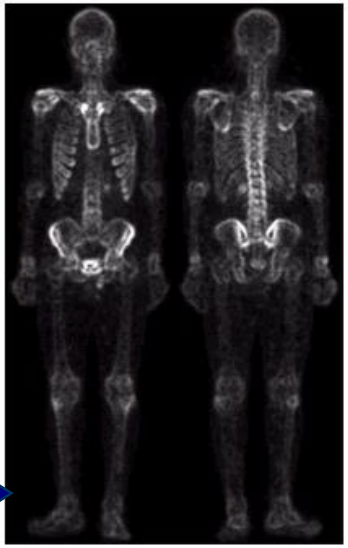


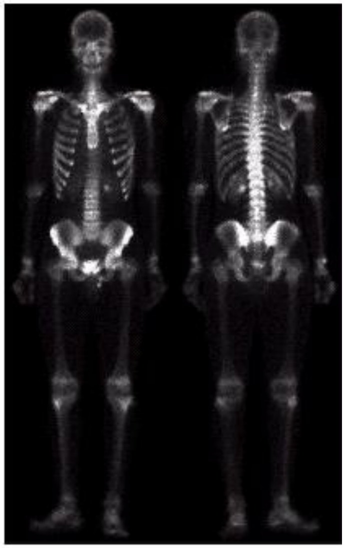
Image (d) smoothed with a 5*5 averaging filter

The product of (c) and (e) which will be used as a mask

(e)

Sharpened image which is sum of (a) and (f)

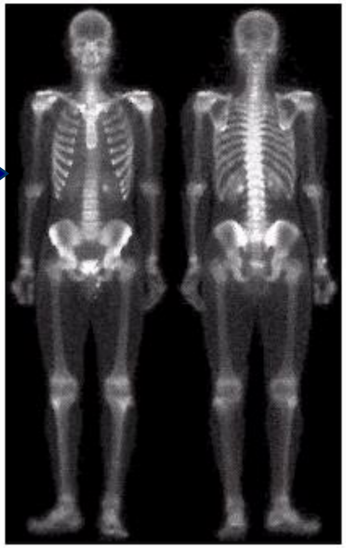
(f)



Result of applying a power-law trans. to (g)

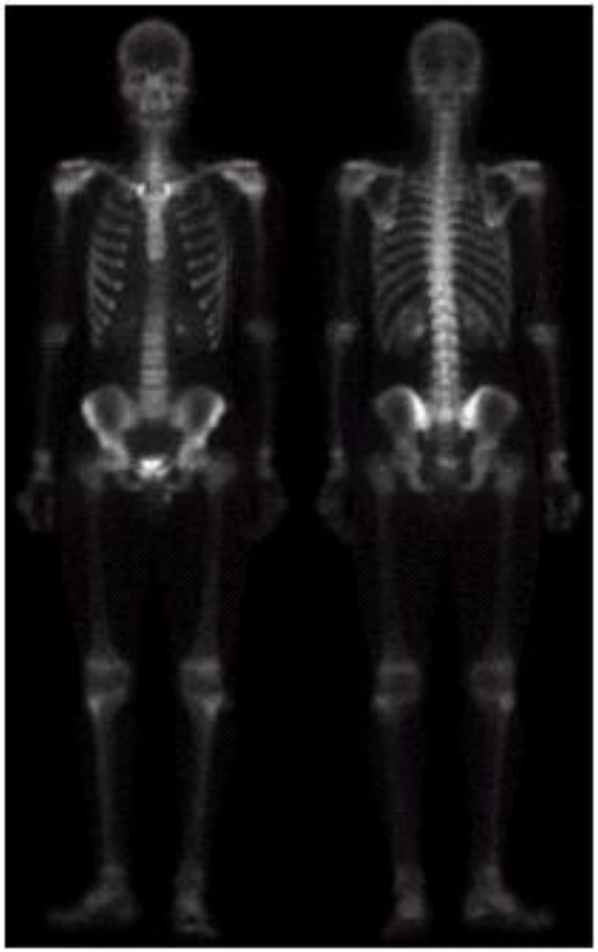
(g)

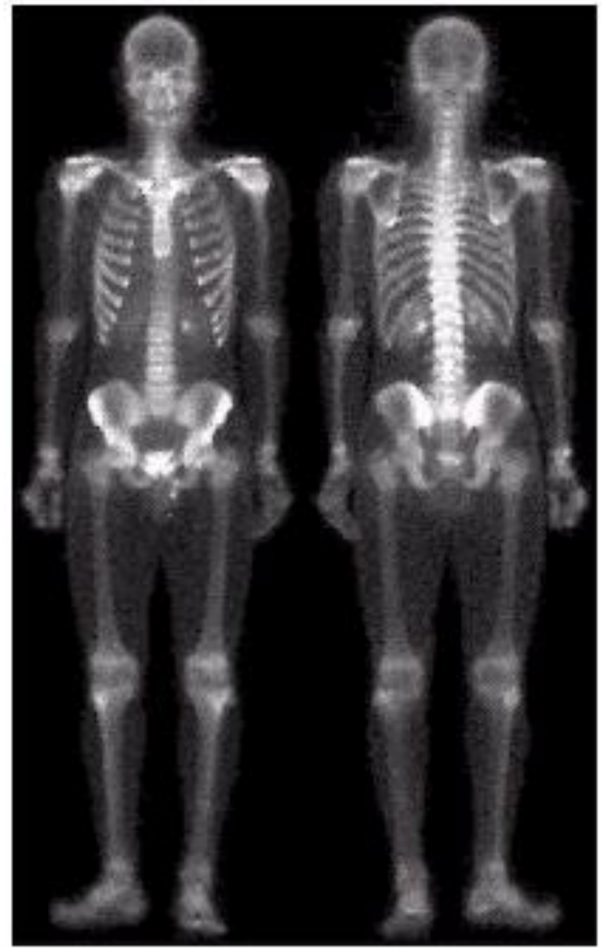
(h)



Combining Spatial Enhancement Methods

Compare the original and final images





Acknowledgements

- Digital Image Processing", Rafael C. Gonzalez & Richard E. Woods, Addison-Wesley, 2002
- Peters, Richard Alan, II, Lectures on Image Processing, Vanderbilt University, Nashville, TN, April 2008
- Brian Mac Namee, Digitial Image Processing, School of Computing, Dublin Institute of Technology
- Computer Vision for Computer Graphics, Mark Borg

Material in these slides has been taken from, the following resources