Recap

- Reasoning
- Types of reasoning:
  - Deductive
  - Inductive
  - Adbuctive
  - Analogical
  - common-sense
  - non-monotonic reasoning
- Logic: syntax, semantics, Proof systems
- Rules of inference: Modus ponens, Modus tolens, And-introduction, And-elimination
- Inference example

Resolution

- **The deduction mechanism we discussed may be used in practical systems, but is not feasible. It uses a lot of inference rules, which introduces a large branch factor in the search for a proof.**
- **An alternative is Resolution, a strategy used to assert the determine the truth of an assertion.**
- **Only one Resolution rule:**

$$\frac{\begin{array}{l}\alpha \vee \beta \\ \neg\beta \vee \gamma\end{array}}{\alpha \vee \gamma}$$

Resolution Rule

α	β	γ	$\neg\beta$	$\alpha \vee \beta$	$\neg\beta \vee \gamma$	$\alpha \vee \gamma$
F	F	F	T	F	T	F
F	F	T	T	F	T	T
F	T	F	F	T	F	F
F	T	T	F	T	T	T
T	F	F	T	T	T	T
T	F	T	T	T	T	T
T	T	F	F	T	F	T
T	T	T	F	T	T	T

Conjunctive Normal Form

- ANDs of ORs
- Resolution requires all sentences to be converted into a special form called Conjunctive Normal Form (CNF)
- A sentence written in CNF looks like

$$(A \vee B) \wedge (B \vee \neg C) \wedge (D)$$

$$note : D = (D \vee \neg D)$$

- Outermost structure is made up of conjunctions. Inner units called clauses are made up of disjunctions

Conjunctive Normal Form

- **Clause**
  - A clause is the disjunction of many things. $(B \vee \neg C)$
- **Literals**
  - The units that make up a clause are called literals. And a literal is either a variable or the negation of a variable. So you get an expression where the negations are pushed in as tightly as possible, then you have ors, then you have ands.
- You can think of each clause as a requirement. Each clause has to be satisfied to satisfy the entire statement

Convert to CNF

- Eliminate arrows (implications)

$$A \rightarrow B = \neg A \vee B$$

Convert to CNF

- Drive in negations using De Morgan's Laws

$$\neg(A \vee B) = (\neg A \wedge \neg B)$$

$$\neg(A \wedge B) = (\neg A \vee \neg B)$$

Convert to CNF

- Distribute OR over AND

$$A \vee (B \wedge C)$$

$$= (A \vee B) \wedge (A \vee C)$$

Convert to CNF Example

$$(A \vee B) \rightarrow (C \rightarrow D)$$

$$1. \neg(A \vee B) \vee (\neg C \vee D)$$

$$2. (\neg A \wedge \neg B) \vee (\neg C \vee D)$$

$$3. (\neg A \vee \neg C \vee D) \wedge (\neg B \vee \neg C \vee D)$$

Resolution by Refutation

- Proof strategy called **Resolution Refutation**
  - Write all sentences in CNF
  - Negate the desired conclusion
  - Apply the resolution rule until you derive a contradiction or cannot apply the rule anymore.
- If we derive a contradiction, then the conclusion follows from the given axioms
- If we cannot apply anymore, then the conclusion cannot be proved from the given axioms

Resolution by Refutation

Step	Formula	Derivation
1	$P \vee Q$	Given
2	$\neg P \vee R$	Given
3	$\neg Q \vee R$	Given

Prove R

1	$P \vee Q$
2	$P \rightarrow R$
3	$Q \rightarrow R$

Resolution Example

Step	Formula	Derivation
1	$P \vee Q$	Given
2	$\neg P \vee R$	Given
3	$\neg Q \vee R$	Given
4	$\neg R$	Negated Conclusion

Prove R

1	$P \vee Q$
2	$P \rightarrow R$
3	$Q \rightarrow R$

Resolution Example

Step	Formula	Derivation
1	$P \vee Q$	Given
2	$\neg P \vee R$	Given
3	$\neg Q \vee R$	Given
4	$\neg R$	Negated Conclusion
5	$Q \vee R$	1,2 Resolution Rule

Prove R

1	$P \vee Q$
2	$P \rightarrow R$
3	$Q \rightarrow R$

Resolution Example

Step	Formula	Derivation
1	$P \vee Q$	Given
2	$\neg P \vee R$	Given
3	$\neg Q \vee R$	Given
4	$\neg R$	Negated Conclusion
5	$Q \vee R$	1,2
6	$\neg P$	2,4

Prove R

1	$P \vee Q$
2	$P \rightarrow R$
3	$Q \rightarrow R$

Resolution Example

Step	Formula	Derivation
1	$P \vee Q$	Given
2	$\neg P \vee R$	Given
3	$\neg Q \vee R$	Given
4	$\neg R$	Negated Conclusion
5	$Q \vee R$	1,2
6	$\neg P$	2,4
7	$\neg Q$	3,4

Prove R

1	$P \vee Q$
2	$P \rightarrow R$
3	$Q \rightarrow R$

Resolution Example

Step	Formula	Derivation
1	$P \vee Q$	Given
2	$\neg P \vee R$	Given
3	$\neg Q \vee R$	Given
4	$\neg R$	Negated Conclusion
5	$Q \vee R$	1,2
6	$\neg P$	2,4
7	$\neg Q$	3,4
8	R	5,7 Contradiction!

Prove R

1	$P \vee Q$
2	$P \rightarrow R$
3	$Q \rightarrow R$

Resolution Example

- Note that you could have come up with multiple ways of proving R

Step	Formula	
1	$P \vee Q$	Given
2	$\neg P \vee R$	Given
3	$\neg Q \vee R$	Given
4	$\neg R$	
5	$\neg Q$	3,4
6	P	1,5
7	**R**	2,6

Step	Formula	
1	$P \vee Q$	Given
2	$\neg P \vee R$	Given
3	$\neg Q \vee R$	Given
4	$\neg R$	
5	$Q \vee R$	1,2
6	$\neg P$	2,4
7	$\neg Q$	3,4
8	**R**	5,7

Resolution Example 2

1. (P→Q) →Q
2. P→R
3. ¬R → ¬Q

Convert to CNF:

$$1.(P \to Q) \to Q$$

$$= (\neg P \vee Q) \to Q$$

$$= \neg(\neg P \vee Q) \vee Q$$

$$= (P \wedge \neg Q) \vee Q$$

$$= (P \vee Q) \wedge (\neg Q \vee Q)$$

$$= (P \vee Q)$$

$$2.P \to R = \neg P \vee R$$

$$3.\neg R \to \neg Q = R \vee \neg Q$$

Resolution Example 2

Step	Formula	Derivation
1	$Q \vee P$	Given
2	$\neg P \vee R$	Given
3	$R \vee \neg Q$	Given
4	$\neg R$	
5	P	2,4
6	R	2,5

Step	Formula	Derivation
1	$Q \vee P$	Given
2	$\neg P \vee R$	Given
3	$R \vee \neg Q$	Given
4	$\neg R$	
5	$\neg Q$	3,4
6	P	1,5
7	R	2,6

Proof Strategies

- We may apply rules in an arbitrary order, but there are some rules of thumb
  - **Unit preference**: prefer using a clause with one literal. Produces shorter clauses
  - **Set of support**: try to involve the thing you are trying to prove. Chose a resolution involving the negated goal. These are relevant clauses. We move ‘towards solution’