Digital Image Processing

Lecture # 3 B
Image Transformations

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Contents

- Histograms of:
  - Grayscale Images
  - Colored Images
- Contrast Stretching
- Histogram Equalization

Histogram of a Grayscale Image

- Let I be a 1-band (grayscale) image.
- $I(r,c)$ is an 8-bit integer between 0 and 255.
- Histogram, h_I, of I:
  - a 256-element array, h_I
  - $h_I(g)$ = number of pixels in I that have value g.

    for $g = 0, 1, 2, 3, \ldots, 255$

Histogram of a Grayscale Image

- Histogram of a digital image with gray levels in the range *[0,L-1]* is a discrete function

$$h(r_k) = n_k$$

Where

- $r_k = k^{th}$ gray level
- n_k = number of pixels in the image having gray level r_k
- $h(r_k)$ = histogram of an image having r_k gray levels

Normalized Histogram

- Dividing each of histogram at gray level r_k by the total number of pixels in the image, n

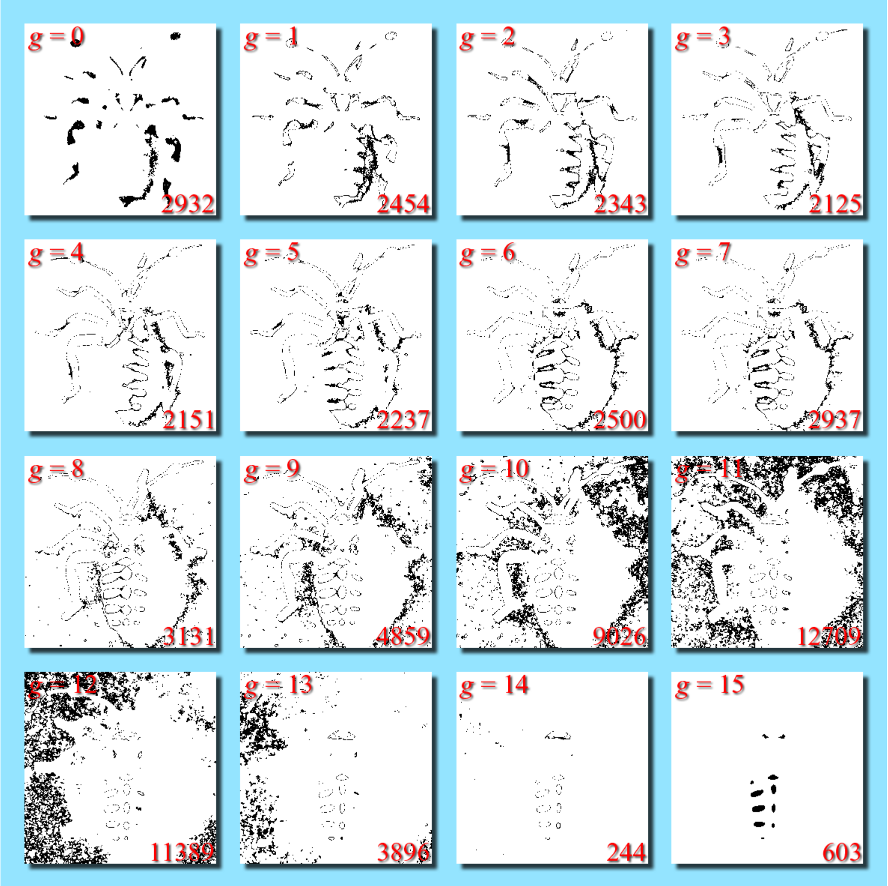
$$p(r_k) = n_k / n \quad \text{for } k = 0, 1, \ldots, L-1$$

- $p(r_k)$ gives an estimate of the probability of occurrence of gray level r_k
- The sum of all components of a normalized histogram is equal to 1

Histogram of a Grayscale Image

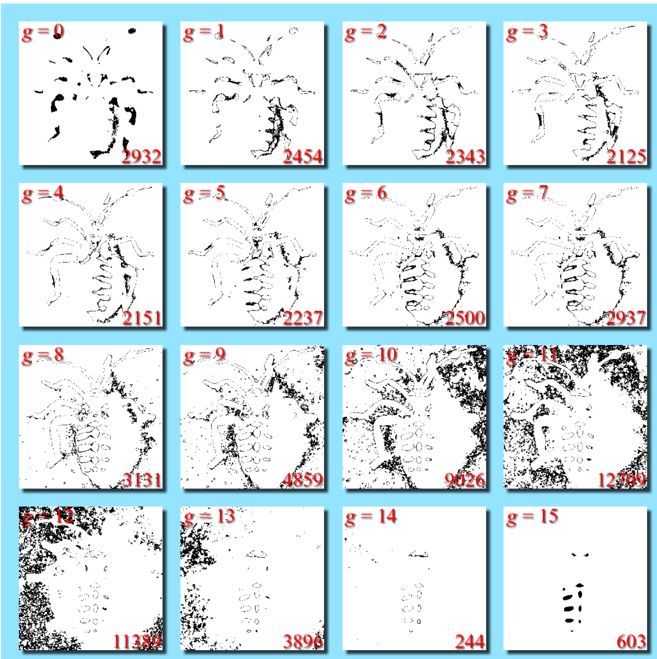
16-level (4-bit) image

lower RHC: number of pixels with intensity *g*

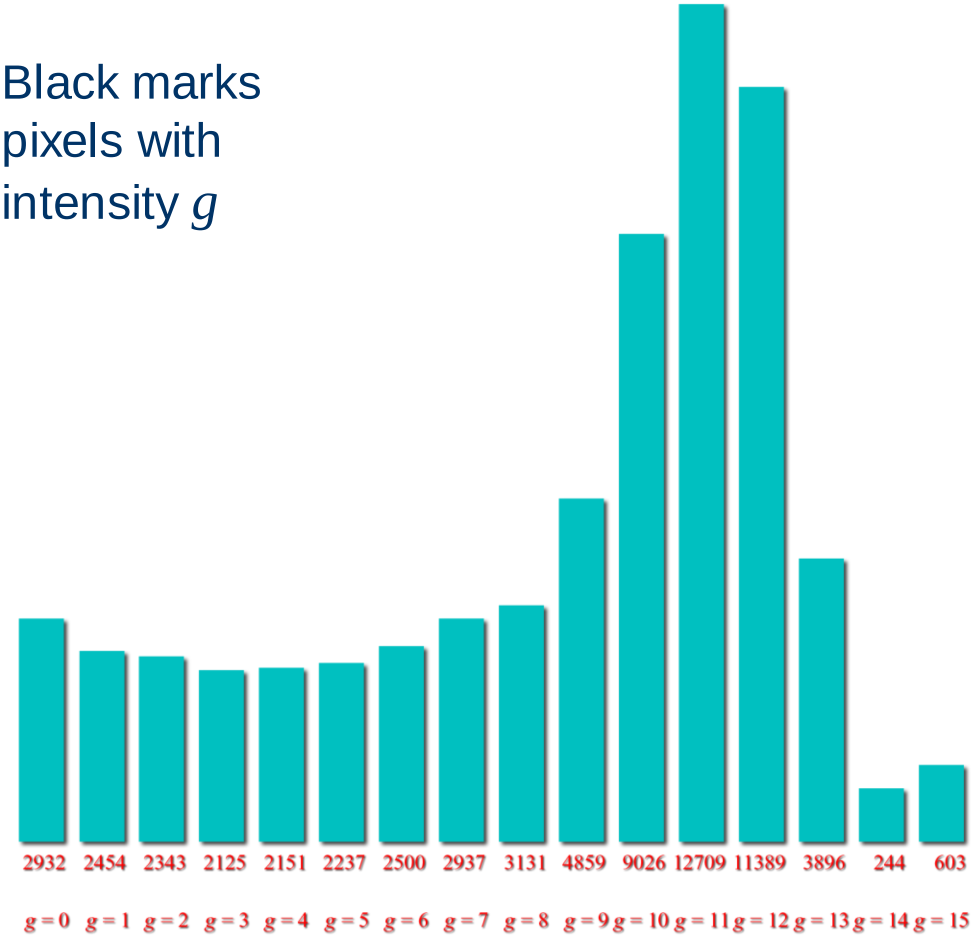


black marks pixels with intensity *g*

Histogram of a Grayscale Image

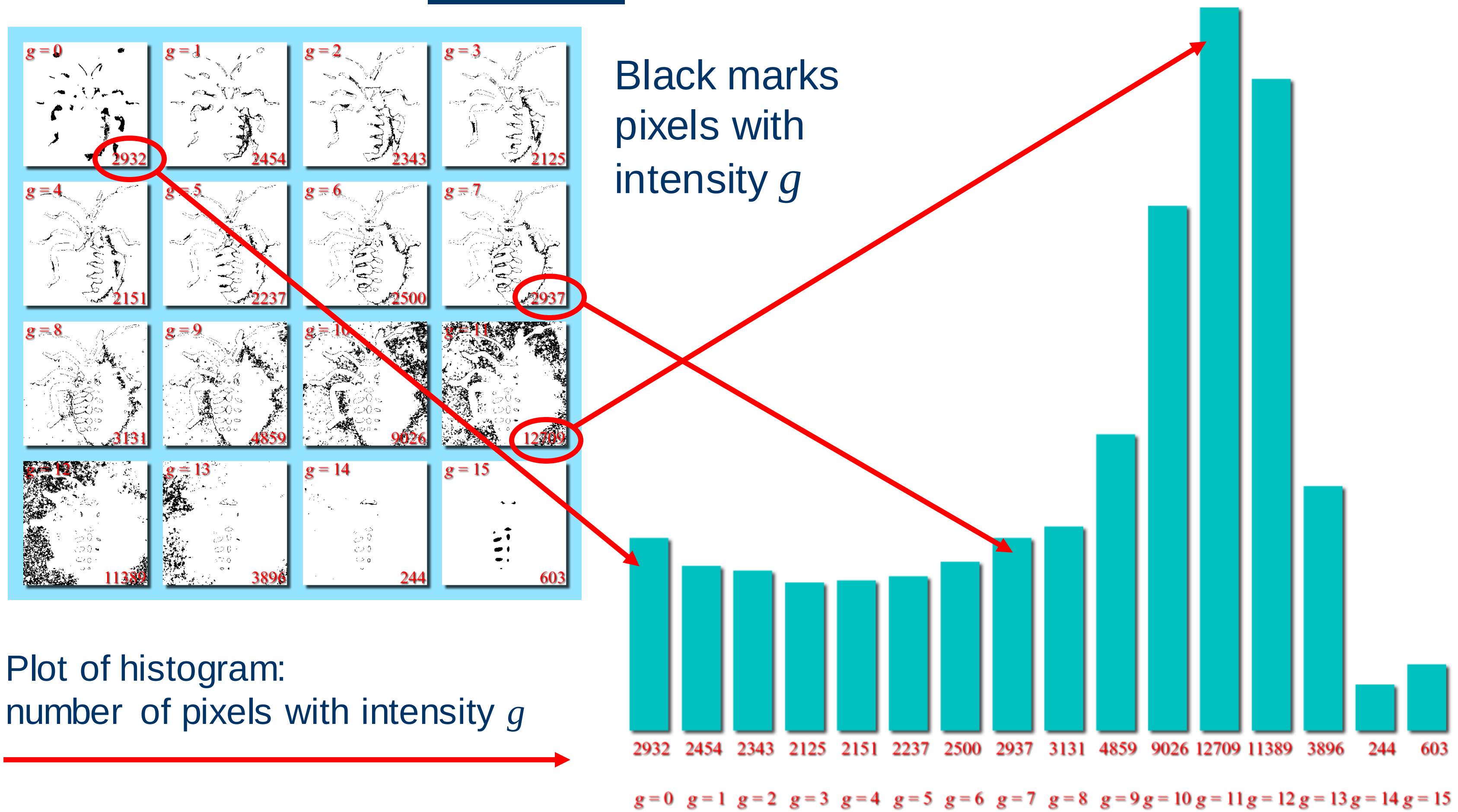


Black marks pixels with intensity *g*



Plot of histogram:
number of pixels with intensity *g*

Histogram of a Grayscale Image

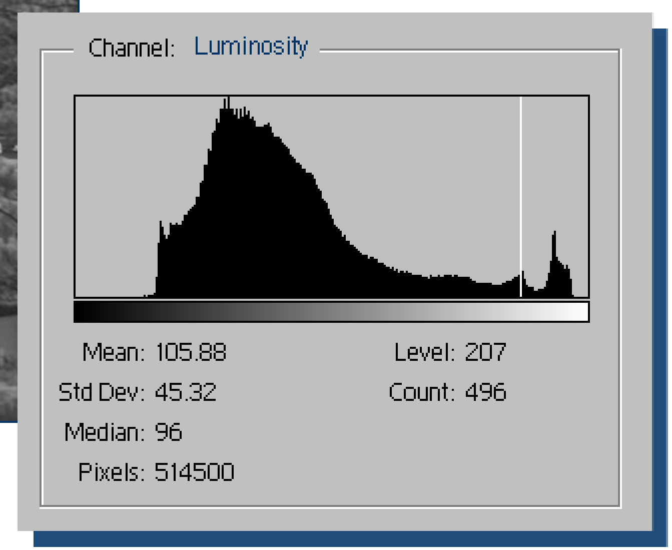


Black marks pixels with intensity g

Plot of histogram: number of pixels with intensity g

Histogram of a Grayscale Image

$h_I(g)$ = the number of pixels in I with gray level g

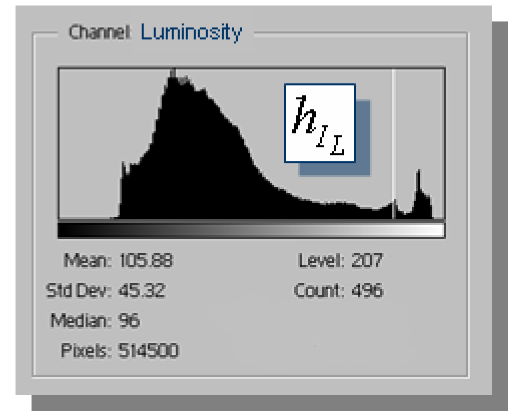


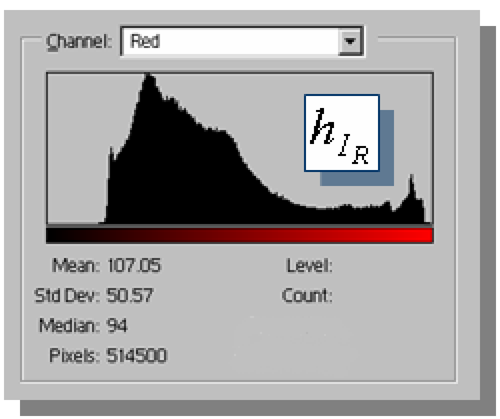
Histogram of a Color Image

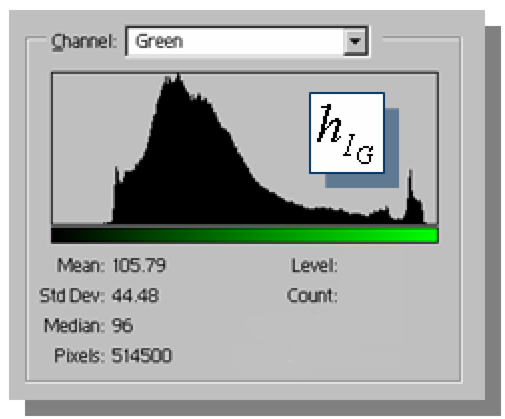
- If I is a 3-band image
- then I(r,c,b) is an integer between 0 and 255.
- I has 3 histograms:
  - $h_R(g)$ = # of pixels in I(:,:,1) with intensity value g
  - $h_G(g)$ = # of pixels in I(:,:,2) with intensity value g
  - $h_B(g)$ = # of pixels in I(:,:,3) with intensity value g

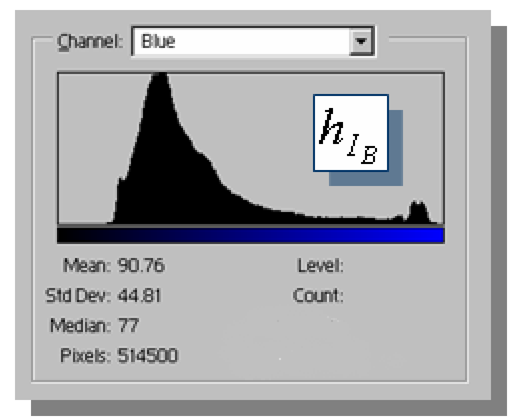
Histogram of a Color Image

There is one histogram per color band R, G, & B. Luminosity histogram is from 1 band = (R+G+B)/3

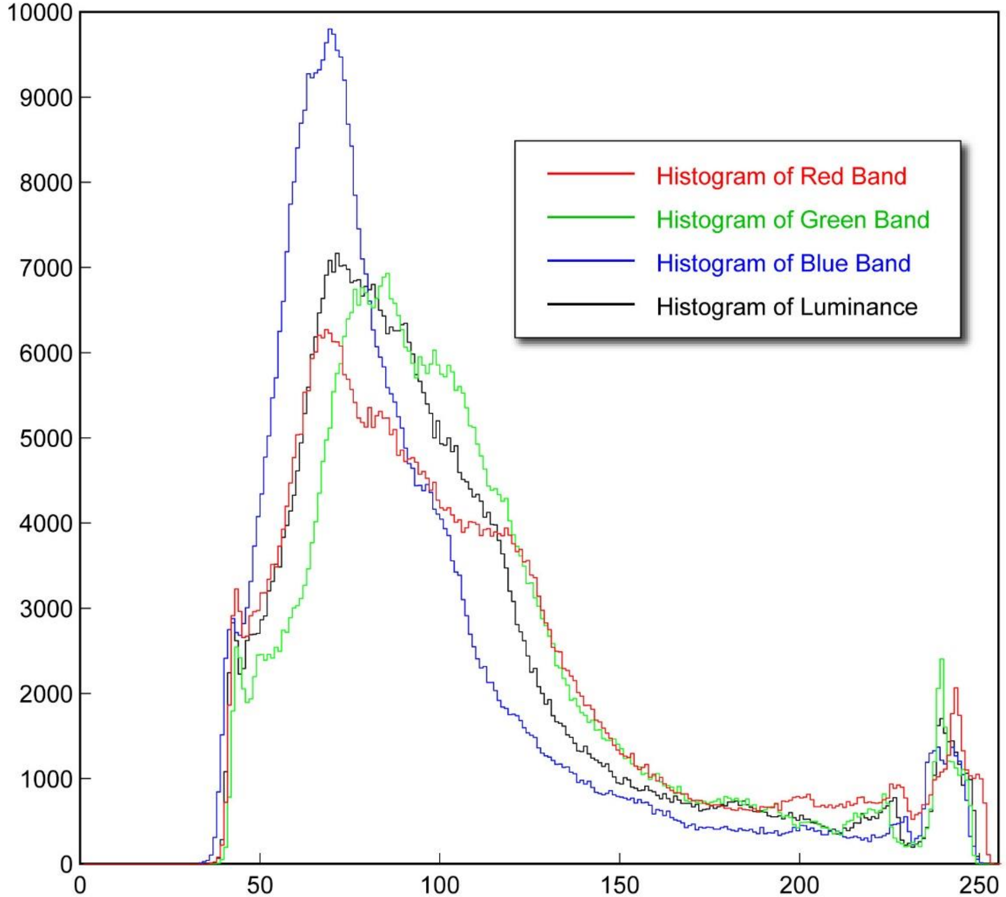




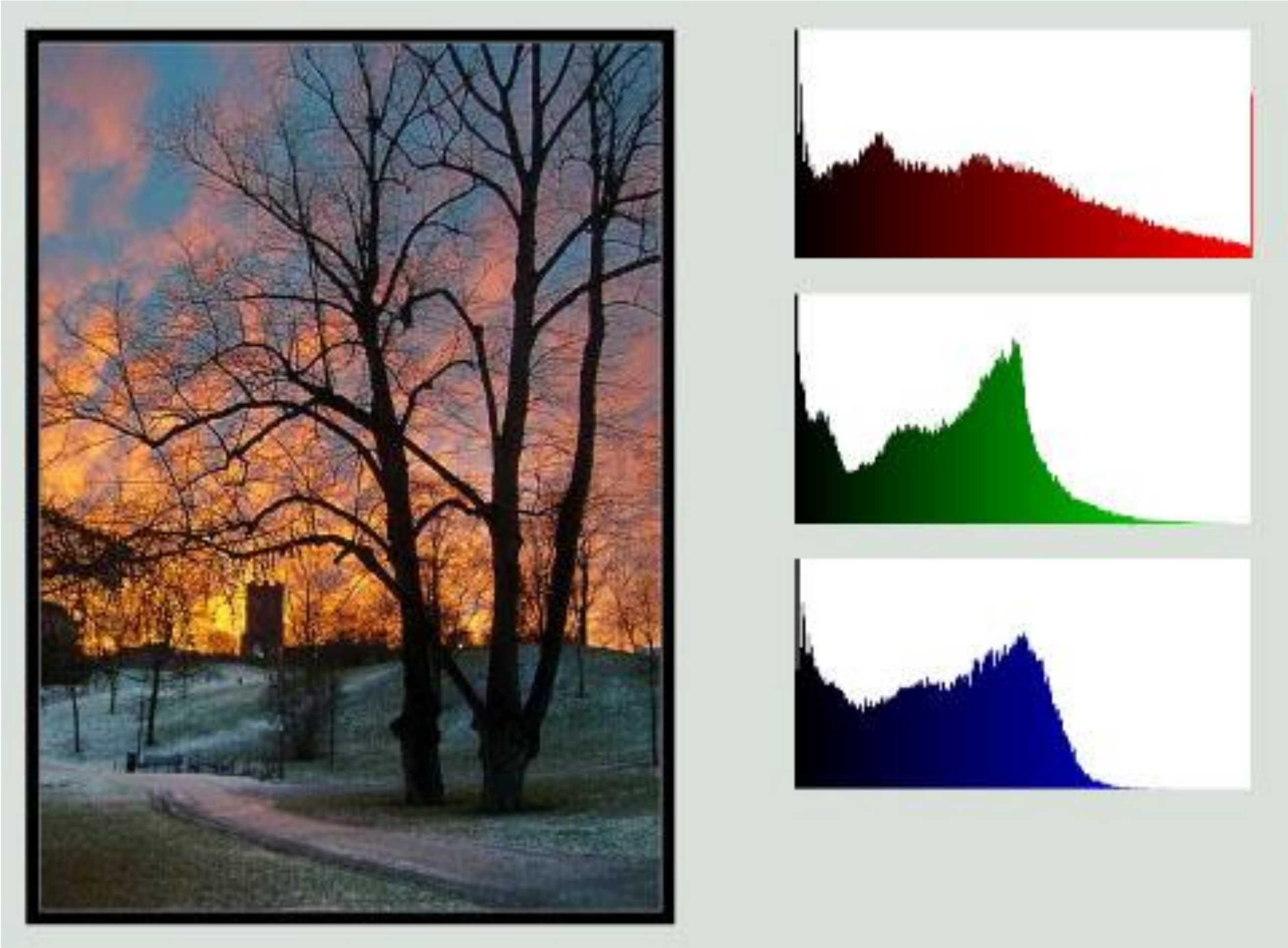




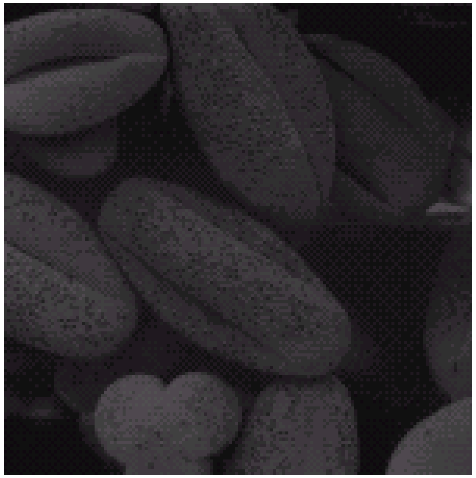
Histogram of a Color Image



Histogram of a Color Image



Histogram: Example

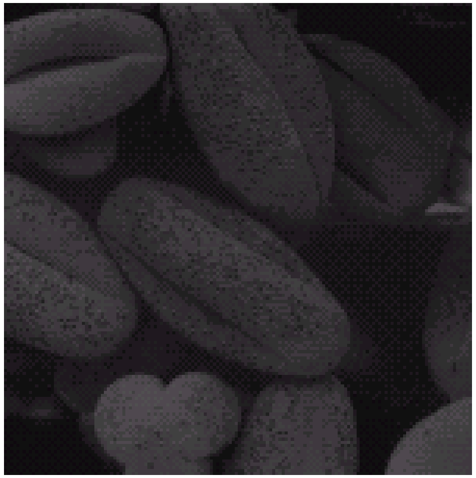


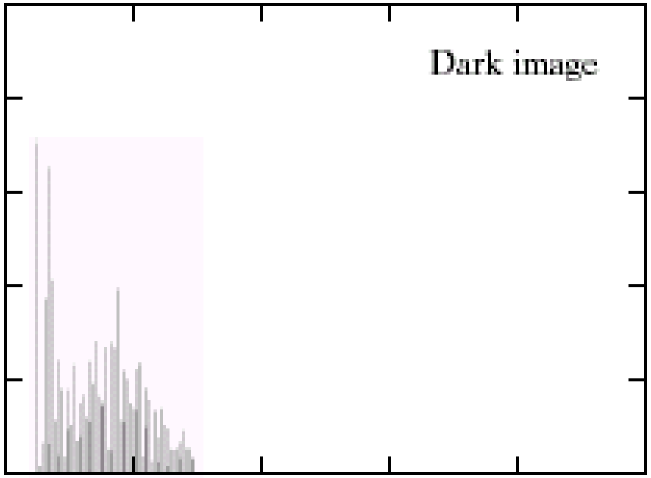
Dark Image

How would the histograms of these images look like?

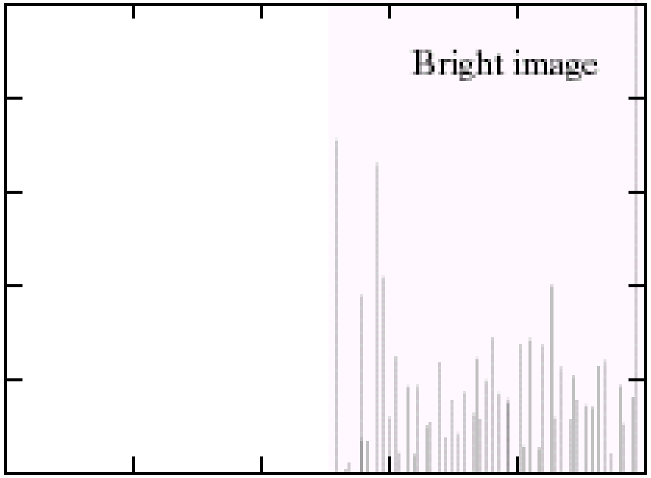
Bright Image

Histogram: Example



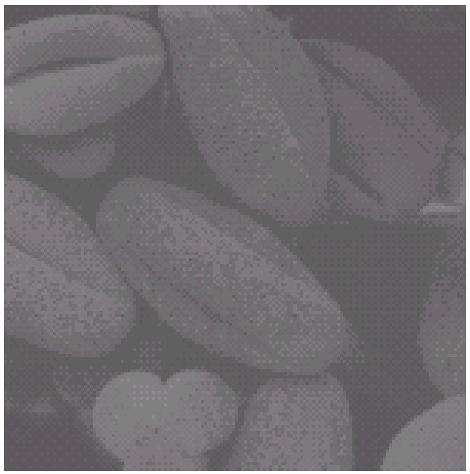


Dark image
Components of histogram are concentrated on the low side of the gray scale



Bright image
Components of histogram are concentrated on the high side of the gray scale

Histogram: Example

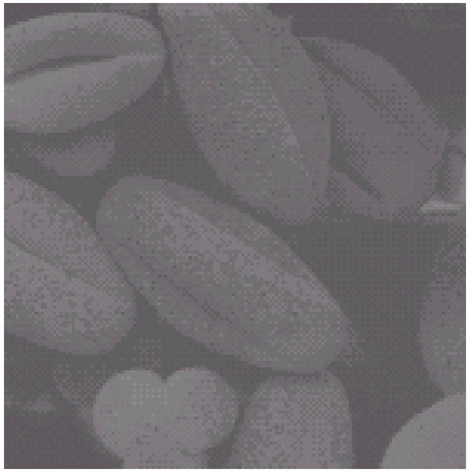


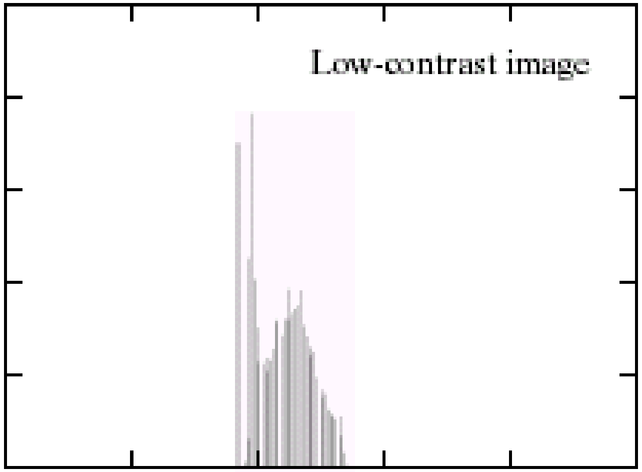
Low Contrast Image

How would the histograms of these images look like?

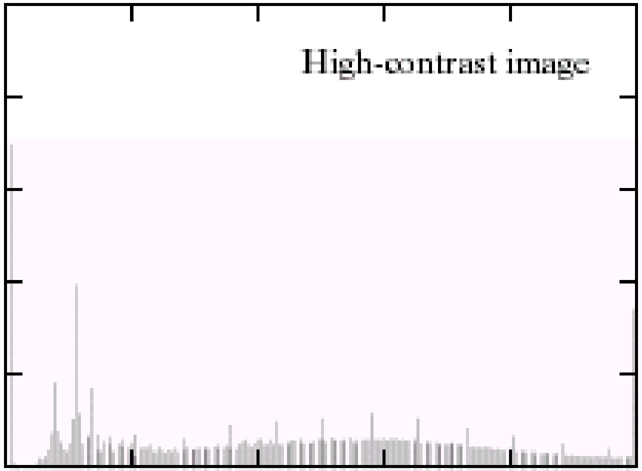
High Contrast Image

Histogram: Example





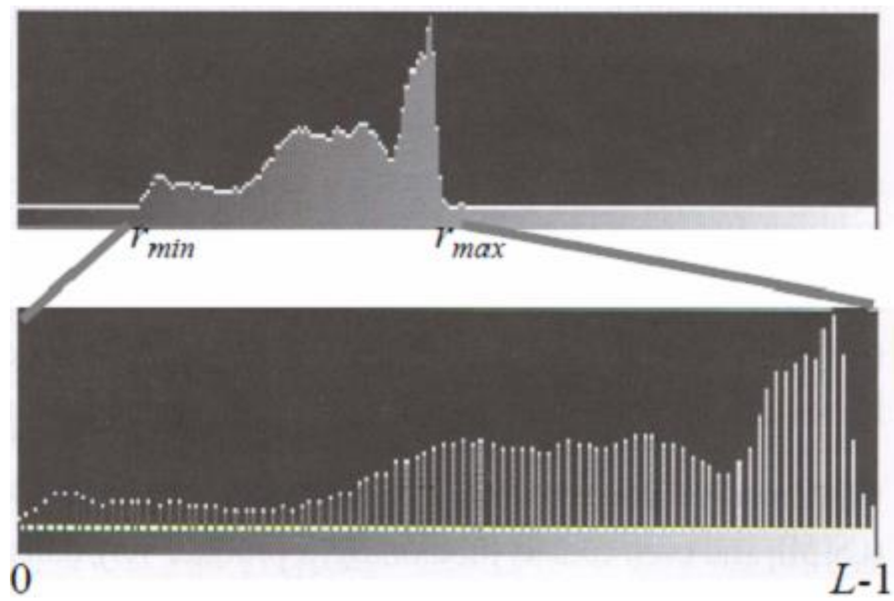
Low contrast image
Histogram is narrow and centered toward the middle of the gray scale



High contrast image
Histogram covers broad range of the gray scale and the distribution of pixels is not too far from uniform with very few vertical lines being much higher than the others

Contrast Stretching

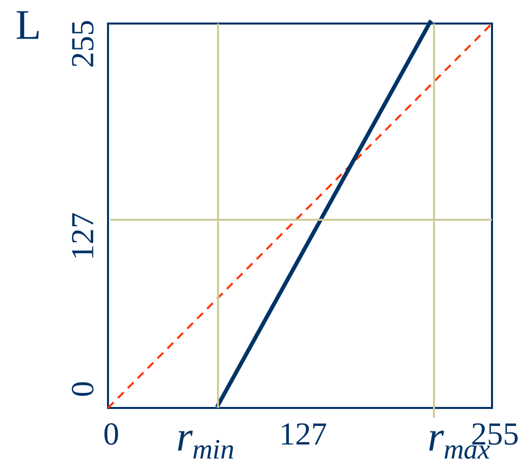
Improve the contrast in an image by `stretching' the range of intensity values it contains to span a desired range of values, *e.g.* the the full range of pixel values



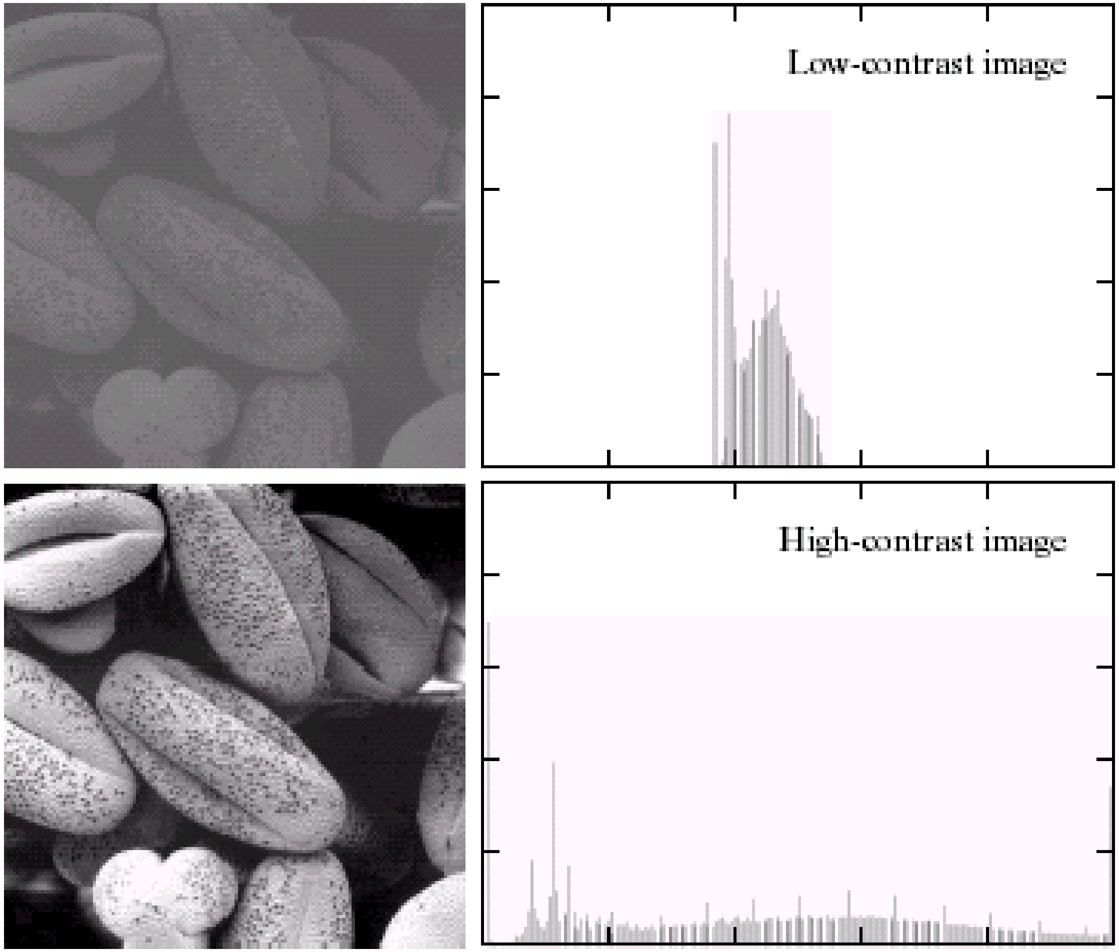
Contrast Stretching

If r_{max} and r_{min} are the maximum and minimum gray level of the input image and L is the total gray levels of output image, the transformation function for contrast stretch will be

$$s = T(r) = (r - r_{min})\left[\frac{L-1}{r_{max} - r_{min}}\right]$$

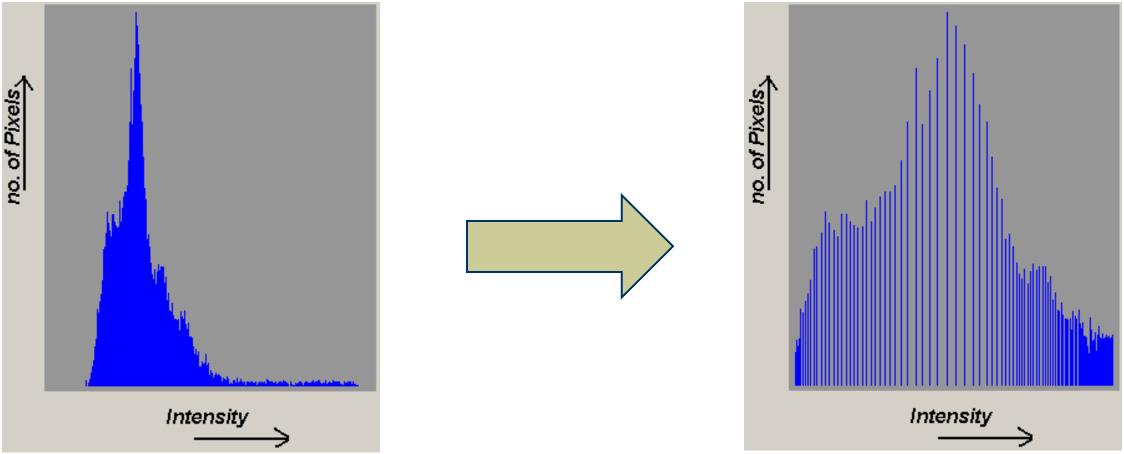


Contrast Stretching



Histogram Equalization

Histogram equalization re-assigns the intensity values of pixels in the input image such that the output image contains a uniform distribution of intensities



The Probability Distribution Function of an Image

$$\text{Let } A = \sum_{g=0}^{255} h_I(g)$$

Note that since $h_I(g)$ is the number of pixels in I with value g, A is the number of pixels in I. That is if I is R rows by C columns then $A = RC$.

Then,

$$p_I(g) = \frac{1}{A} h_I(g)$$

This is the probability that an arbitrary pixel from I has value g.

The Probability Distribution Function of an Image

- $p(g)$ is the fraction of pixels in an image that have intensity value g.
- $p(g)$ is the probability that a pixel randomly selected from the given image has intensity value g.
- Whereas the sum of the histogram $h(g)$ over all g from 0 to 255 is equal to the number of pixels in the image, the sum of $p(g)$ over all g is 1.
- p is the normalized histogram of the image

The Cumulative Distribution Function of an Image

Let **q** = $I(r,c)$ be the value of a randomly selected pixel from I. Let g be a specific gray level. The probability that $q \le g$ is given by

$$P_I(g) = \sum_{\gamma=0}^{g} p_I(\gamma) = \frac{1}{A}\sum_{\gamma=0}^{g} h_I(\gamma) = \frac{\sum_{\gamma=0}^{g} h_I(\gamma)}{\sum_{\gamma=0}^{255} h_I(\gamma)},$$

where $h_I(\gamma)$ is the histogram of image I.

This is the probability that any given pixel from I has value less than or equal to g.

The Cumulative Distribution Function of an Image

Let **q** = *I*(*r*,*c*) be the value of a randomly selected pixel from *I*. Let *g* be a specific gray level. The probability that q ≤ g is given by

$$P_I(g) = \sum_{\gamma=0}^{g} p_I(\gamma) = \frac{1}{RC}\sum_{\gamma=0}^{g} h_I(\gamma) = \frac{\sum_{\gamma=0}^{g} h_I(\gamma)}{\sum_{\gamma=0}^{255} h_I(\gamma)},$$

where $h_I(\gamma)$ is the histogram of image *I*.

Also called CDF for "Cumulative Distribution Function".

This is the probability that any given pixel from *I* has value less than or equal to g.

The Cumulative Distribution Function of an Image

- $P(g)$ is the fraction of pixels in an image that have intensity values less than or equal to g.
- $P(g)$ is the probability that a pixel randomly selected from the given band has an intensity value less than or equal to g.
- $P(g)$ is the cumulative (or running) sum of $p(g)$ from 0 through g inclusive.
- $P(0) = p(0)$ and $P(255) = 1$;

Histogram Equalization

Task: remap image I so that its histogram is as close to constant as possible

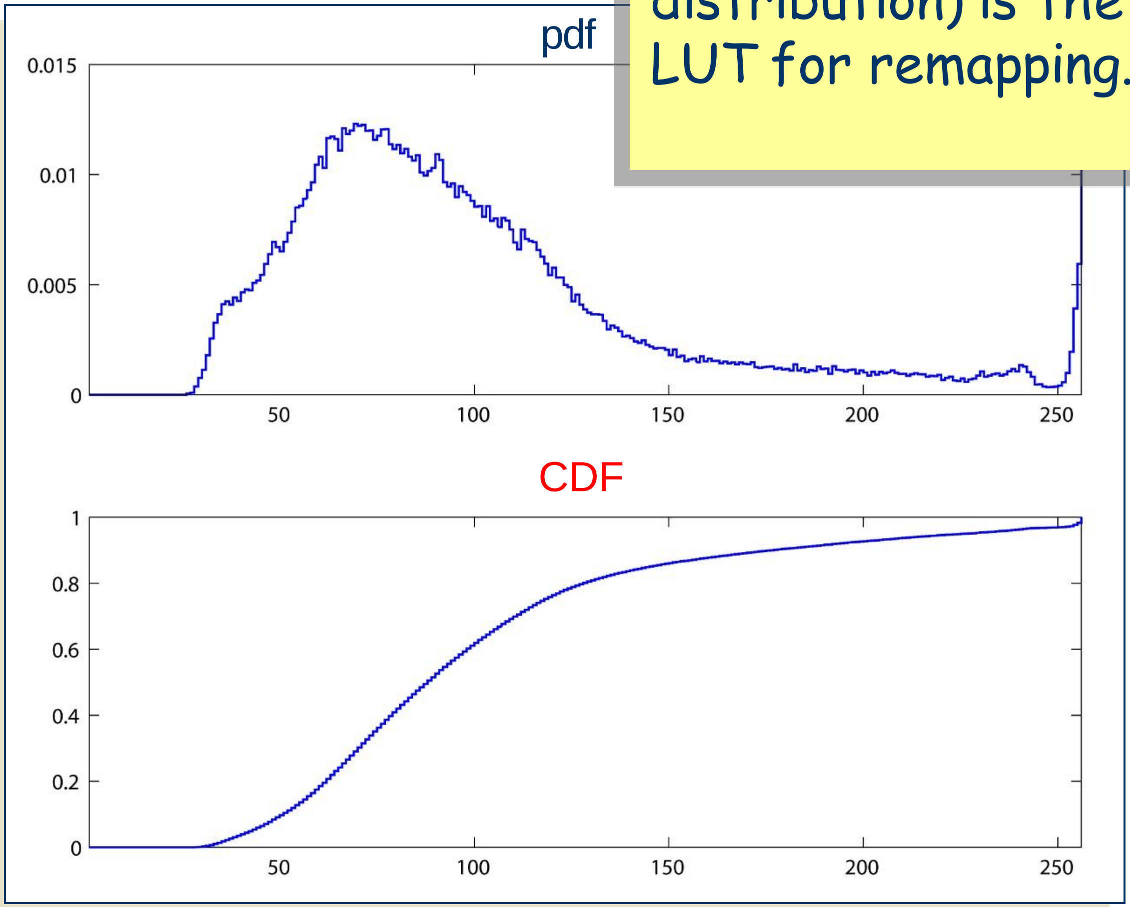
Let $P_I(\gamma)$

be the cumulative (probability) distribution function of I.

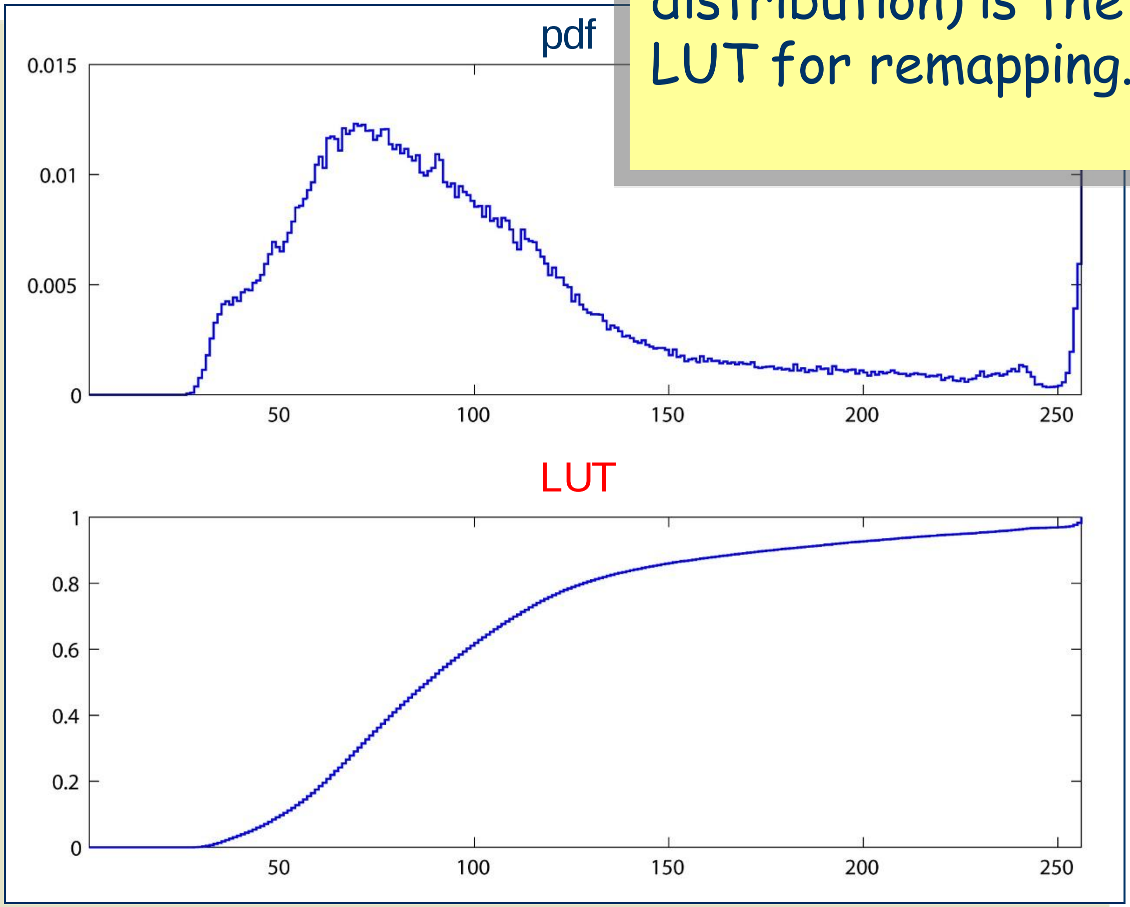
The CDF itself is used as the LUT.

Histogram Equalization

The CDF (cumulative distribution) is the LUT for remapping.

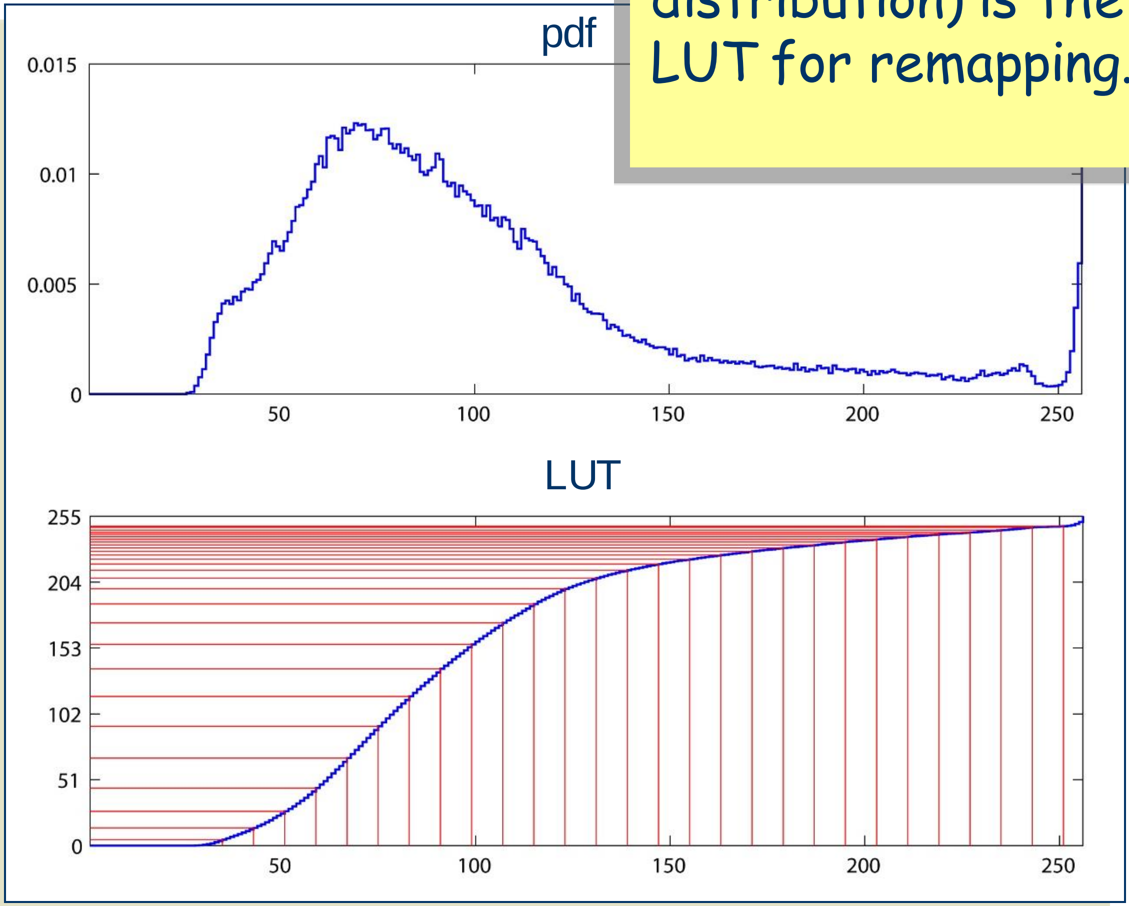


Histogram Equalization



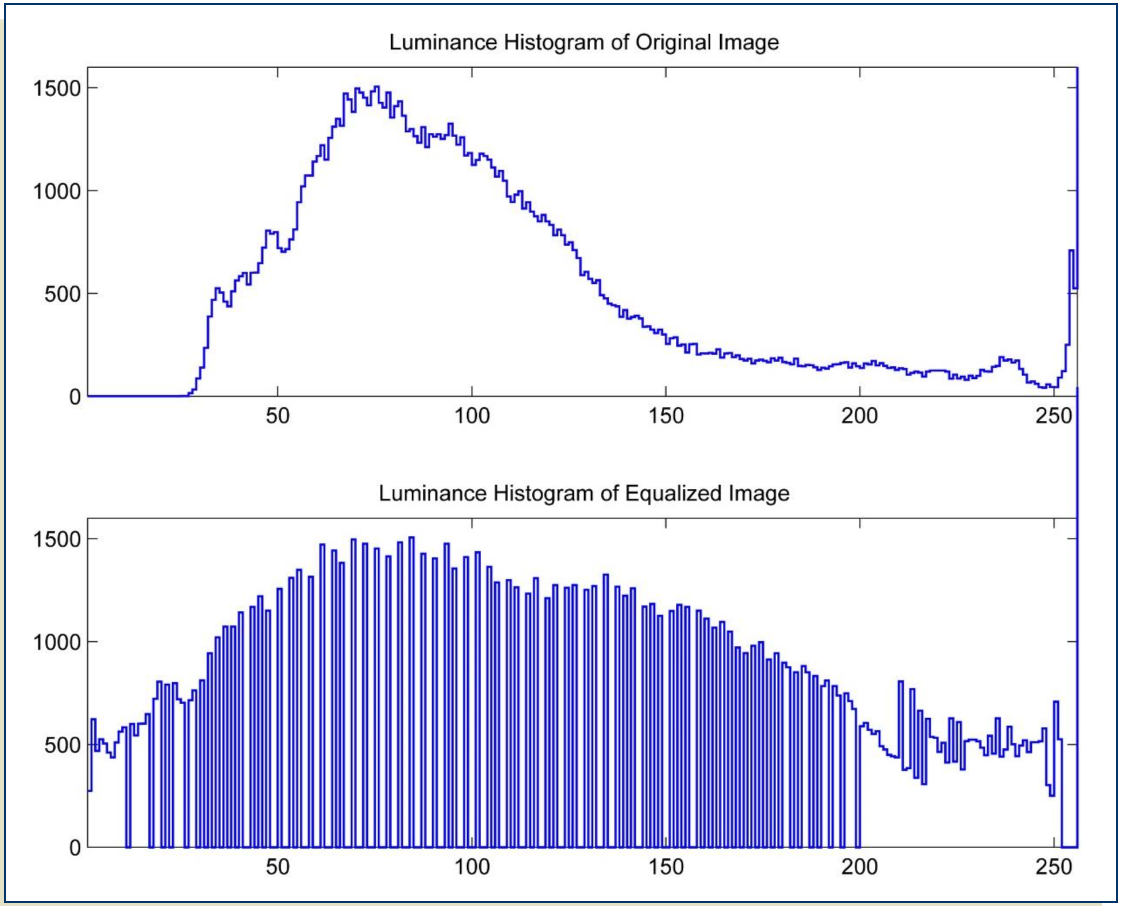
The CDF (cumulative distribution) is the LUT for remapping.

Histogram Equalization

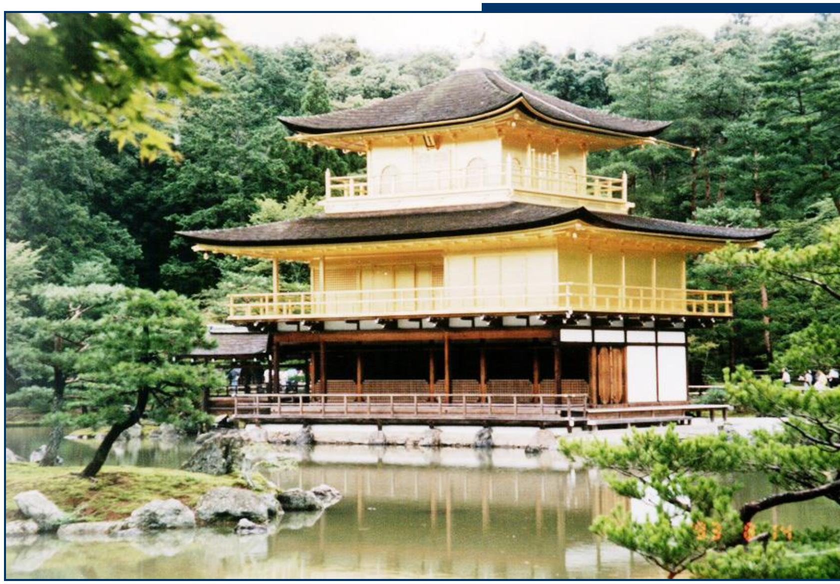


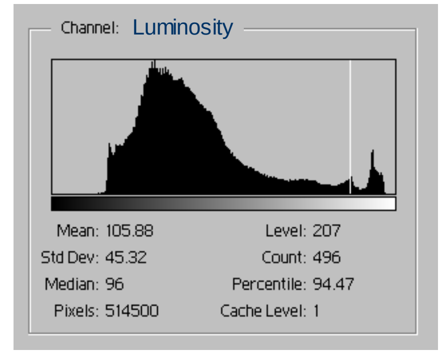
The CDF (cumulative distribution) is the LUT for remapping.

Histogram Equalization

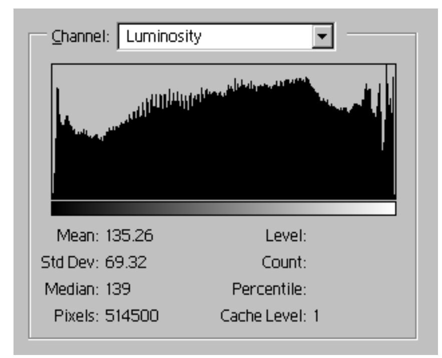


Histogram Equalization





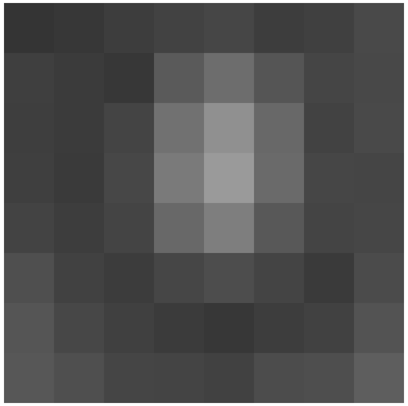
before



after

$$J(r,c) = 255 \cdot P_I[I(r,c)]$$

Histogram Equalization: Example



$$\begin{bmatrix} 52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\ 63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\ 62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\ 63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\ 67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\ 79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\ 85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\ 87 & 79 & 69 & 68 & 65 & 76 & 78 & 94 \end{bmatrix}$$

An 8x8 image

Histogram Equalization: Example

Take out a paper and fill in the following table/histogram

Value	Count	Value	Count	Value	Count	Value	Count	Value	Count
52		64		72		85		113	
55		65		73		87		122	
58		66		75		88		126	
59		67		76		90		144	
60		68		77		94		154	
61		69		78		104			
62		70		79		106			
63		71		83		109			

Image Histogram (Non-zero values)

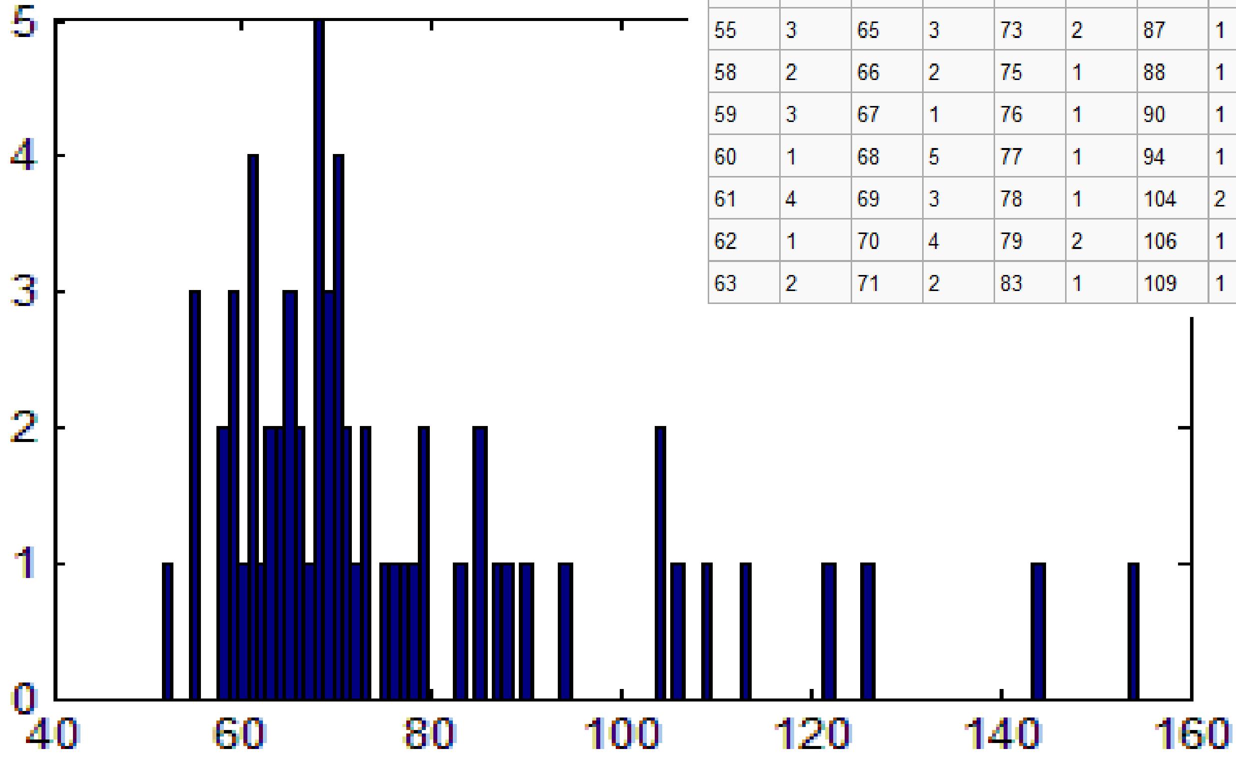
Histogram Equalization: Example

Image Histogram (Non-zero values shown)

Value	Count	Value	Count	Value	Count	Value	Count	Value	Count
52	1	64	2	72	1	85	2	113	1
55	3	65	3	73	2	87	1	122	1
58	2	66	2	75	1	88	1	126	1
59	3	67	1	76	1	90	1	144	1
60	1	68	5	77	1	94	1	154	1
61	4	69	3	78	1	104	2		
62	1	70	4	79	2	106	1		
63	2	71	2	83	1	109	1		

Histogram Equalization: Example

Value	Count	Value	Count	Value	Count	Value	Count	Value	Count
52	1	64	2	72	1	85	2	113	1
55	3	65	3	73	2	87	1	122	1
58	2	66	2	75	1	88	1	126	1
59	3	67	1	76	1	90	1	144	1
60	1	68	5	77	1	94	1	154	1
61	4	69	3	78	1	104	2		
62	1	70	4	79	2	106	1		
63	2	71	2	83	1	109	1		



Histogram Equalization: Example

Image Histogram/Prob Mass Function

Value	Count	Value	Count	Value	Count	Value	Count	Value	Count
52	1	64	2	72	1	85	2	113	1
55	3	65	3	73	2	87	1	122	1
58	2	66	2	75	1	88	1	126	1
59	3	67	1	76	1	90	1	144	1
60	1	68	5	77	1	94	1	154	1
61	4	69	3	78	1	104	2		
62	1	70	4	79	2	106	1		
63	2	71	2	83	1	109	1		

Cumulative Distribution Function (cdf)

Value	cdf	Value	cdf	Value	cdf	Value	cdf	Value	cdf
52		64		72		85		113	
55		65		73		87		122	
58		66		75		88		126	
59		67		76		90		144	
60		68		77		94		154	
61		69		78		104			
62		70		79		106			
63		71		83		109			

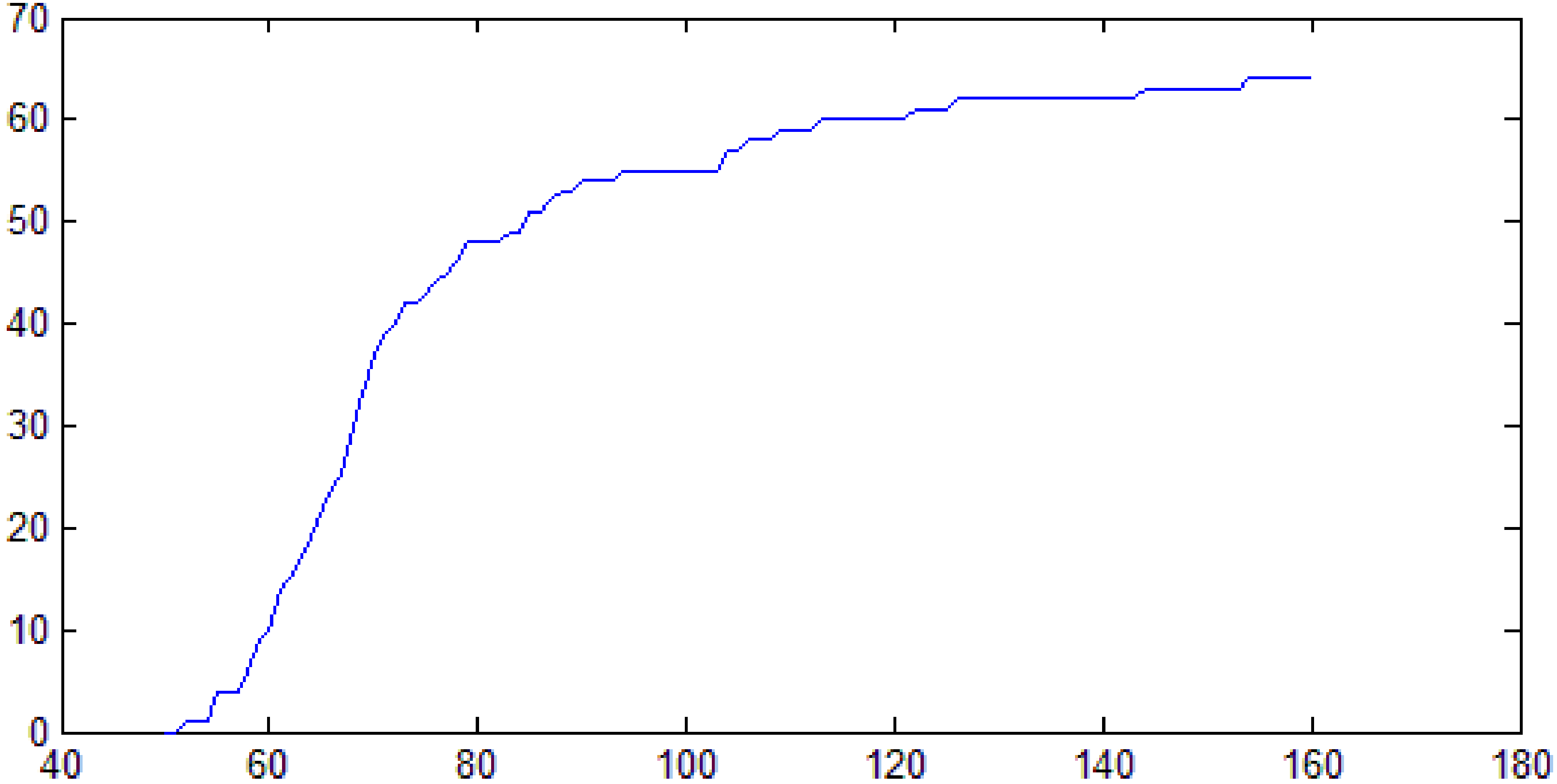
Histogram Equalization: Example

Cumulative Distribution Function (cdf)

Value	cdf	Value	cdf	Value	cdf	Value	cdf	Value	cdf
52	1	64	19	72	40	85	51	113	60
55	4	65	22	73	42	87	52	122	61
58	6	66	24	75	43	88	53	126	62
59	9	67	25	76	44	90	54	144	63
60	10	68	30	77	45	94	55	154	64
61	14	69	33	78	46	104	57		
62	15	70	37	79	48	106	58		
63	17	71	39	83	49	109	59		

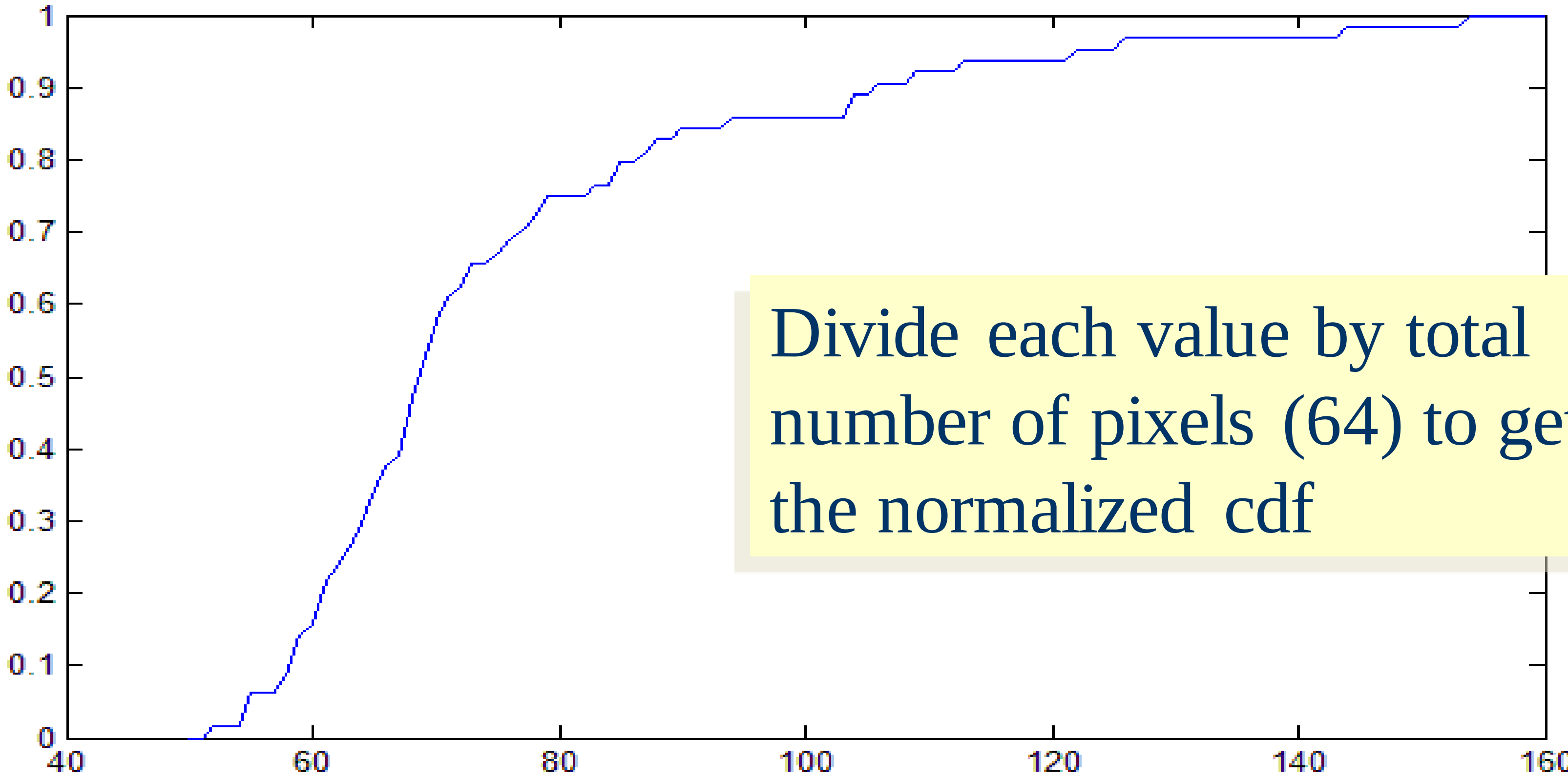
Histogram Equalization: Example

Cumulative Distribution Function (cdf)



Histogram Equalization: Example

Normalized Cumulative Distribution Function (cdf)



Divide each value by total number of pixels (64) to get the normalized cdf

Histogram Equalization: Example

Value	cdf	Value	cdf	Value	cdf	Value	cdf	Value	cdf
52	1	64	19	72	40	85	51	113	60
55	4	65	22	73	42	87	52	122	61
58	6	66	24	75	43	88	53	126	62
59	9	67	25	76	44	90	54	144	63
60	10	68	30	77	45	94	55	154	64
61	14	69	33	78	46	104	57		
62	15	70	37	79	48	106	58		
63	17	71	39	83	49	109	59		

$$J(r,c) = 255\,P_I[I(r,c)]$$

If cdf is normalized

$$s = round(255\,cdf(r))$$

If cdf is NOT normalized

$$s = round\left(255\,\frac{cdf(r)}{M \times N}\right)$$

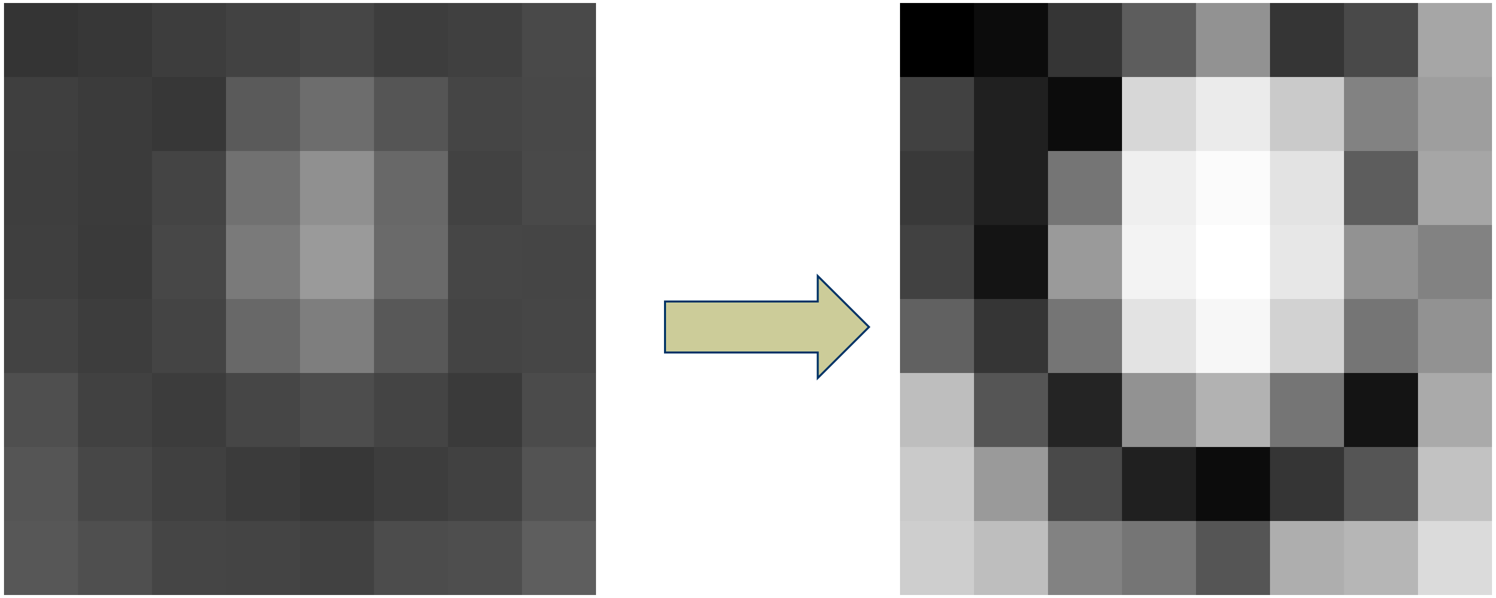
$$s = round(255(46/64))$$

$$s = 183$$

$$\begin{bmatrix} 52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\ 63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\ 62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\ 63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\ 67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\ 79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\ 85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\ 87 & 79 & 69 & 68 & 65 & 76 & 78 & 94 \end{bmatrix} \rightarrow 183$$

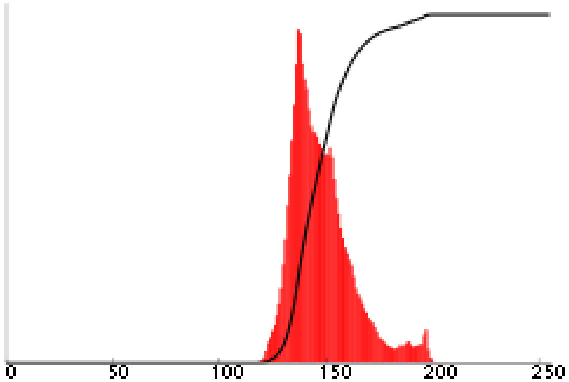
Original Image

Histogram Equalization: Example



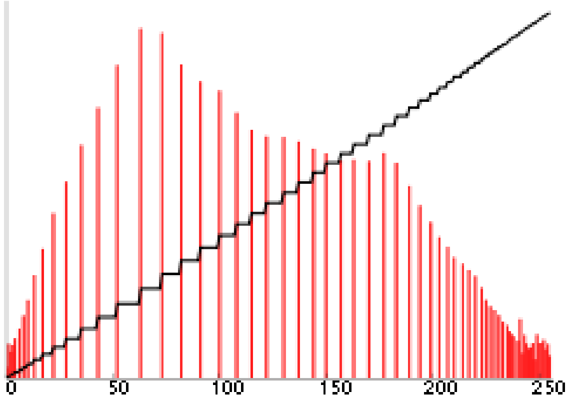
Histogram Equalization: Example

Original Image



Corresponding histogram (red) and cumulative histogram (black)

Image after histogram equalization

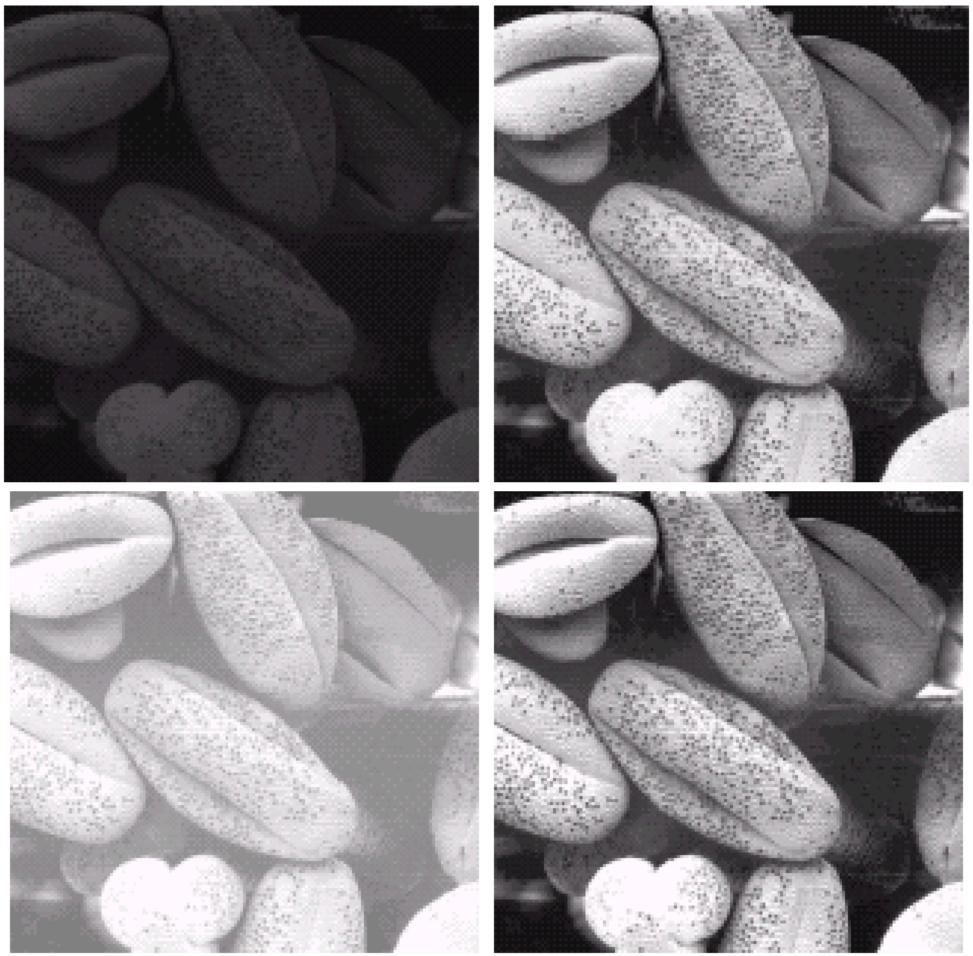


Corresponding histogram (red) and cumulative histogram (black)

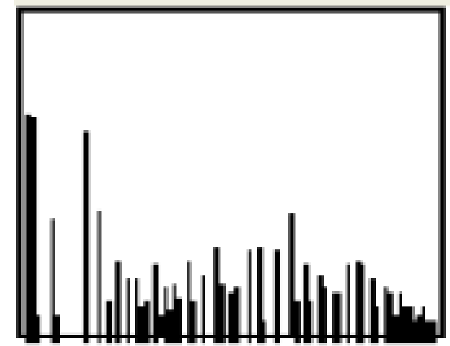
Histogram Equalization: Example

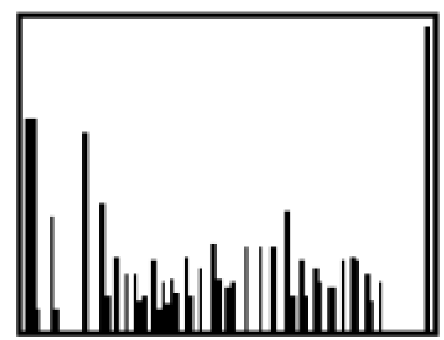
Dark image

Bright image



Equalized Histogram

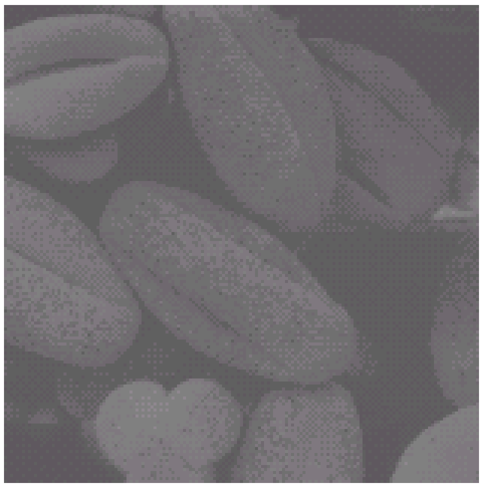




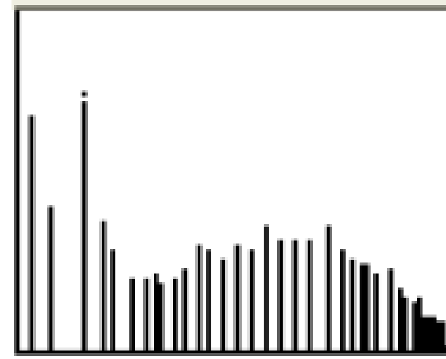
Equalized Histogram

Histogram Equalization: Example

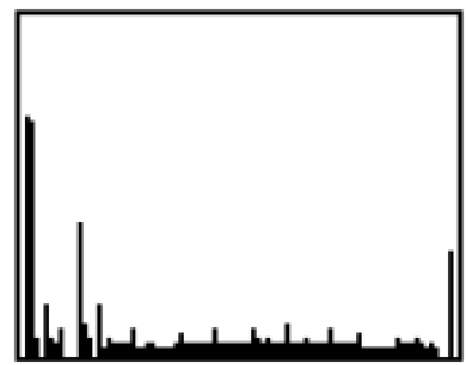
Low contrast



Equalized Histogram



High Contrast



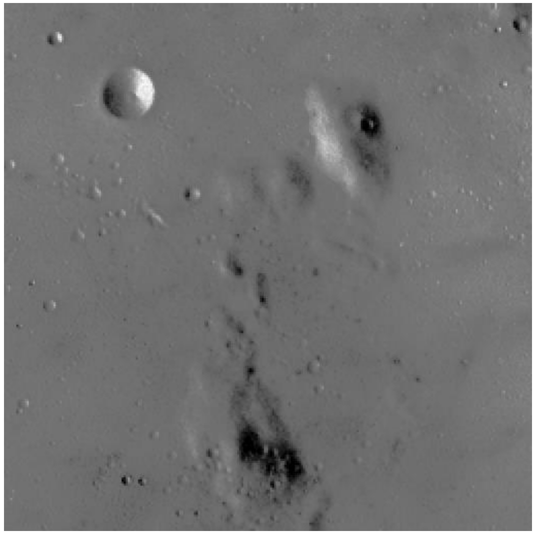
Equalized Histogram

Histogram Equalization vs. Contrast Stretching

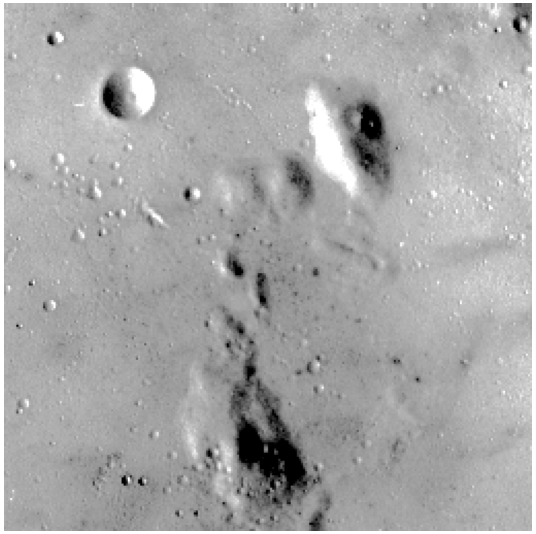
Histogram equalization is sophisticated version of contrast stretching

Contrast Stretching – Linear Transformation
Enhancement is less harsh

Histogram Equalization vs. Contrast Stretching



Original Image



Contrast Stretching

Histogram Equalization

Acknowledgements

- Digital Image Processing", Rafael C. Gonzalez & Richard E. Woods, Addison-Wesley, 2002
- Peters, Richard Alan, II, Lectures on Image Processing, Vanderbilt University, Nashville, TN, April 2008
- Brian Mac Namee, Digitial Image Processing, School of Computing, Dublin Institute of Technology
- Computer Vision for Computer Graphics, Mark Borg
- Web Resource: HIPR2

Material in these slides has been taken from, the following resources