Digital Image Processing

Chapter # 9 A

Morphological Image Processing

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Contents

- Introduction to Morphological Image Processing
- Set Theory – Recap
- Structuring Element
- Fit & Hit

Introduction

- Morphology

  A branch of biology which deals with the form and structure of animals and plants

- Mathematical Morphology
  - A tool for extracting image components that are useful in the representation and description of region shapes
  - The language of mathematical morphology is Set Theory

Morphology: Quick Example

Image after segmentation

Image after segmentation and morphological processing

Introduction

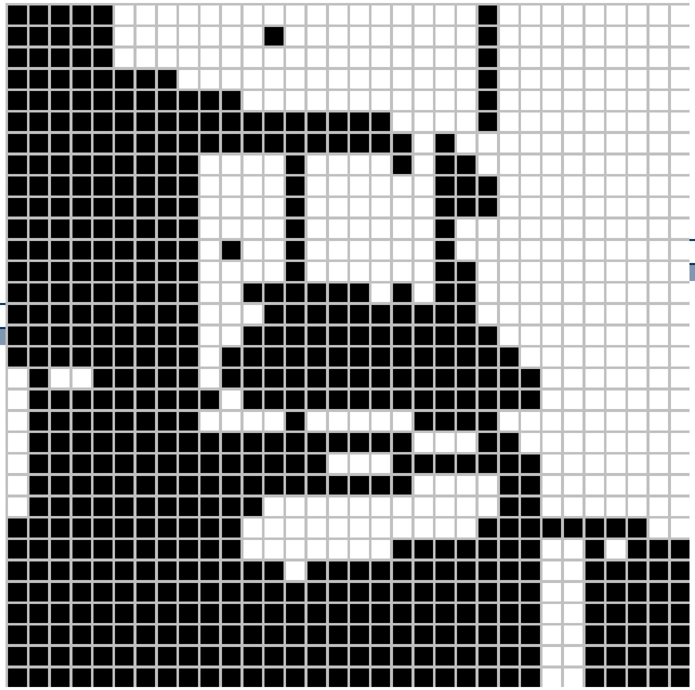
Morphological image processing describes a range of image processing techniques that deal with the shape (or morphology) of objects in an image

Sets in mathematical morphology represents objects in an image. E.g. Set of all white pixels in a binary image.

Introduction

background

$I(\mathbf{p}) = 0$



foreground:

$I(\mathbf{p}) = c$

This represents a digital image. Each square is one pixel.

Set Theory

- The set space of binary image is Z^2
  - Each element of the set is a 2D vector whose coordinates are the *(x,y)* of a black (or white, depending on the convention) pixel in the image
- The set space of gray level image is Z^3
  - Each element of the set is a 3D vector: *(x,y)* and intensity level.

NOTE:

Set Theory and Logical operations are covered in:
Section 9.1, Chapter # 9, 2nd Edition DIP by Gonzalez
Section 2.6.4, Chapter # 2, 3rd Edition DIP by Gonzalez

Set Theory

- Let A be a set in Z^2. if $a = (a_1, a_2)$ is an element of A, then we write

$$a \in A$$

- If a is not an element of A, we write

$$a \notin A$$

- Set representation

$$A = \{(a_1, a_2), (a_3, a_4)\}$$

- Empty or Null set

$$A = \varnothing$$

Set Theory

- **Subset:** if every element of A is also an element of another set B, the A is said to be a subset of B

$$A \subseteq B$$

- **Union:** The set of all elements belonging either to A, B or both

$$C = A \bigcup B$$

- **Intersection:** The set of all elements belonging to both A and B

$$D = A \bigcap B$$

Set Theory

- Two sets A and B are said to be disjoint or mutually exclusive if they have no common element

$$A \bigcap B = \varnothing$$

- Complement: The set of elements not contained in A

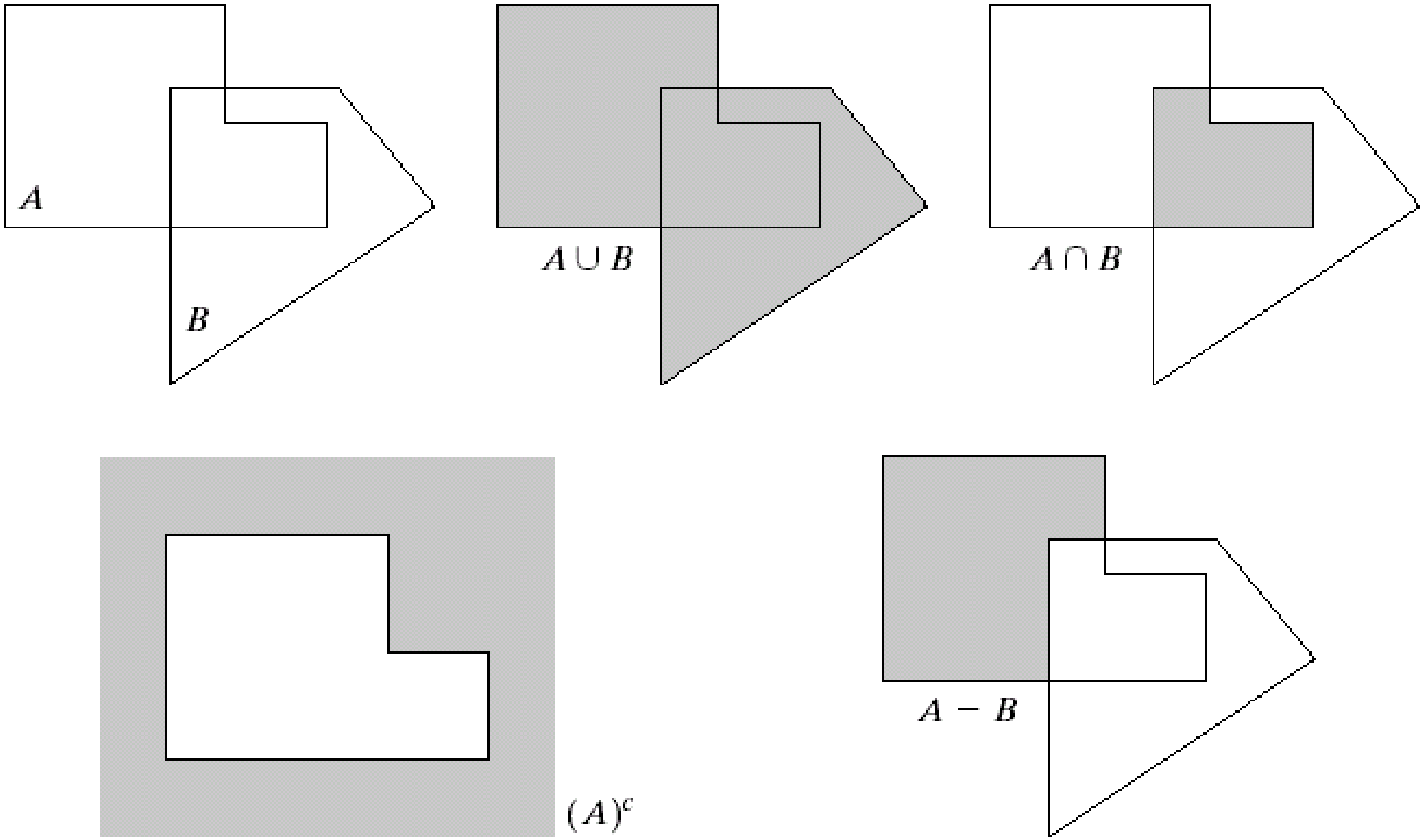
$$A^c = \{w \mid w \notin A\}$$

- Difference of two sets A and B, denoted by A – B, is defined as

$$A - B = \{w \mid w \in A, w \notin B\} = A \bigcap B^c$$

i.e. the set of elements that belong to A, but not to B

Set Theory



a b c
d e

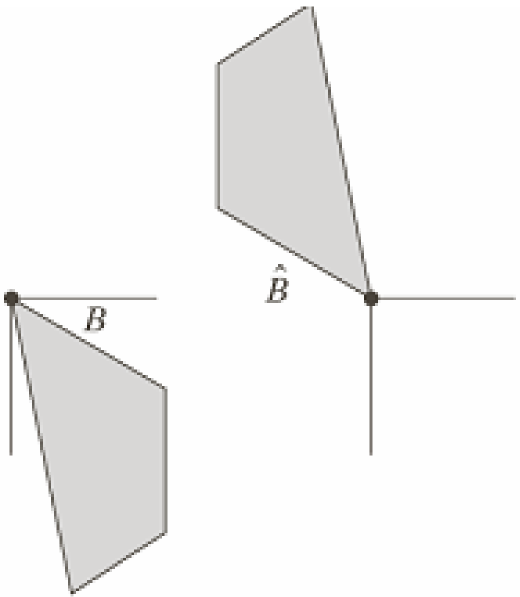
FIGURE 9.1
(a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

Set Theory

- Reflection of set B

$$B = \{w \mid w = -b, \quad \text{for } b \in B\}$$

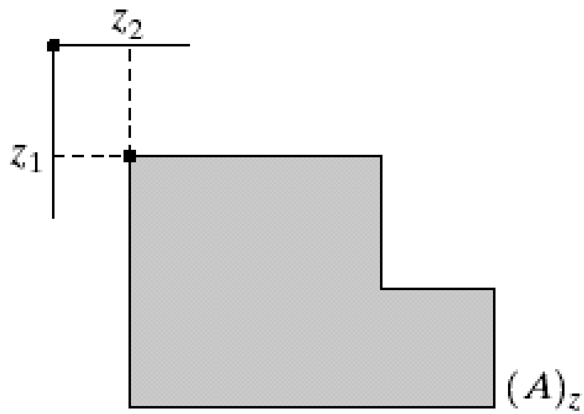
i.e. the set of element *w*, such that *w* is formed by multiplying each of two coordinates of all the elements of set *B* by -1



Set Theory

- Translation of set A by point $z = (z_1, z_2)$, denoted $(A)_z$, is defined as

$$(A)_z = \{w \mid w = a + z, \quad \text{for } a \in A\}$$



Support of an Image

The support of a binary image, I, is

$$\operatorname{supp}\left(\mathrm{I}\right)=\left\{\,\mathbf{p}=(r,c)\,\middle|\,\mathrm{I}(\mathbf{p})=v_{\mathrm{f}g}\right\}.$$

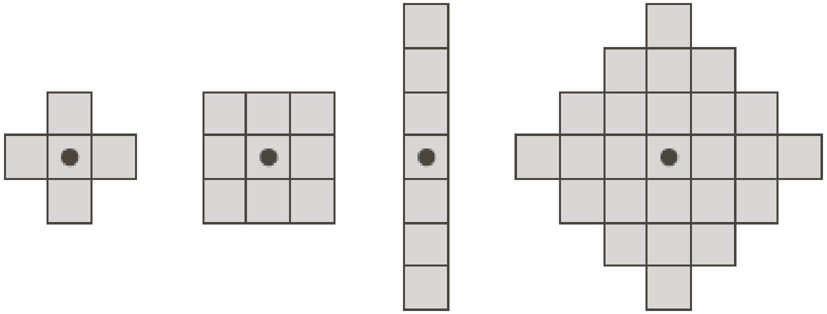
That is, the support of a binary image is the set of foreground pixel locations within the image plane.

The complement of the support is, therefore, the set of background pixel locations within the image plane.

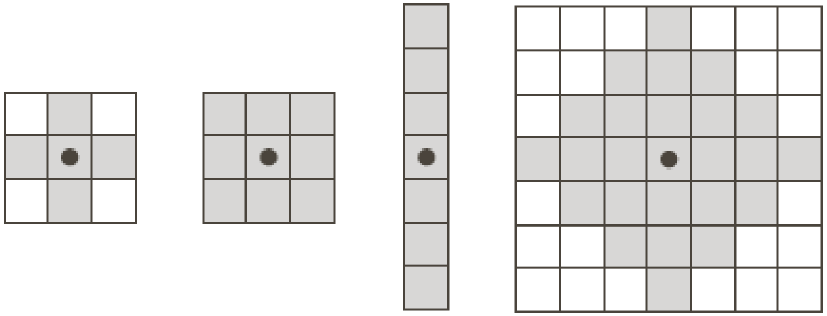
$$\left\{\operatorname{supp}\left(\mathrm{I}\right)\right\}^{\mathrm{C}}=\left\{\,\mathbf{p}=(r,c)\,\middle|\,\mathrm{I}(\mathbf{p})=v_{\mathrm{b}g}\right\}.$$

Structuring Element

A structuring element is a small image – used as a moving window



Example Structuring Elements



Structuring Elements converted to rectangular arrays

Structuring Element

For simplicity we will use rectangular structuring elements with their origin at the middle pixel

1	1	1
1	**1**	1
1	1	1

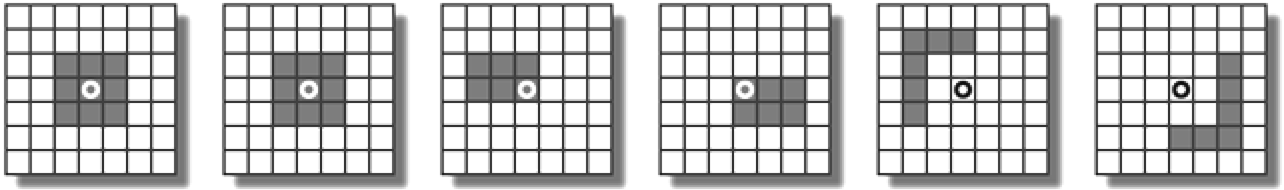
0	1	0
1	**1**	1
0	1	0

0	0	1	0	0
0	1	1	1	0
1	1	**1**	1	1
0	1	1	1	0
0	0	1	0	0

Structuring Element: Reflection

$$\widehat{B}(x,y) = B\left(-x,-y\right)$$

$\widehat{B}$ is B rotated by 180^{o} around its origin.



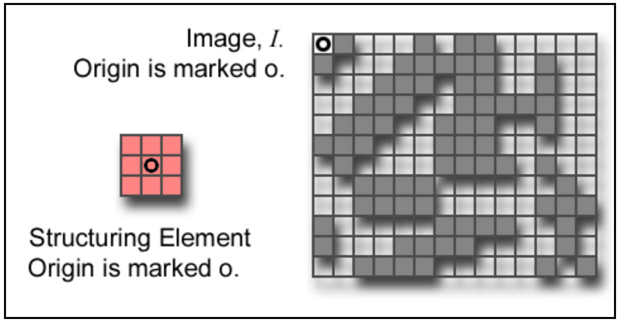
B_1 $\widehat{B}_1$ B_2 $\widehat{B}_2$ B_3 $\widehat{B}_3$

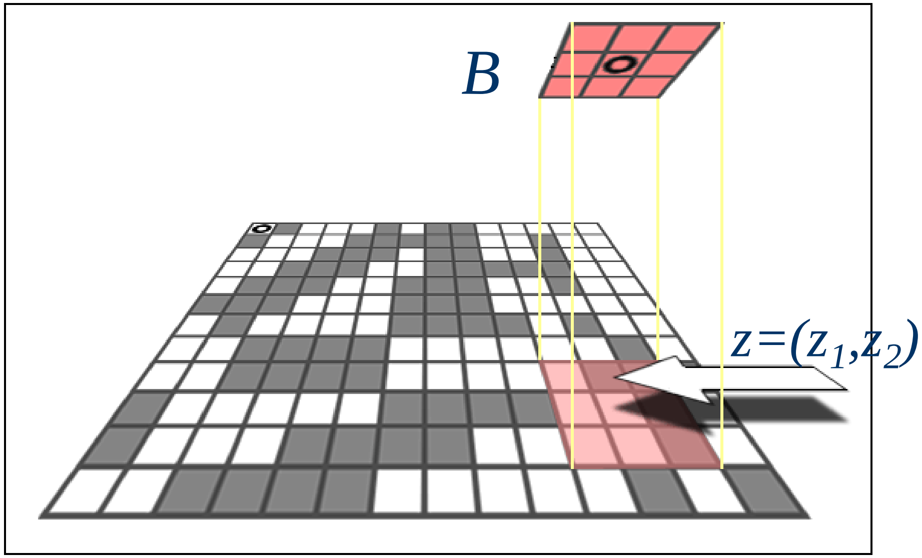
Structuring Element: Translation

Let I be an image and *B* a SE.

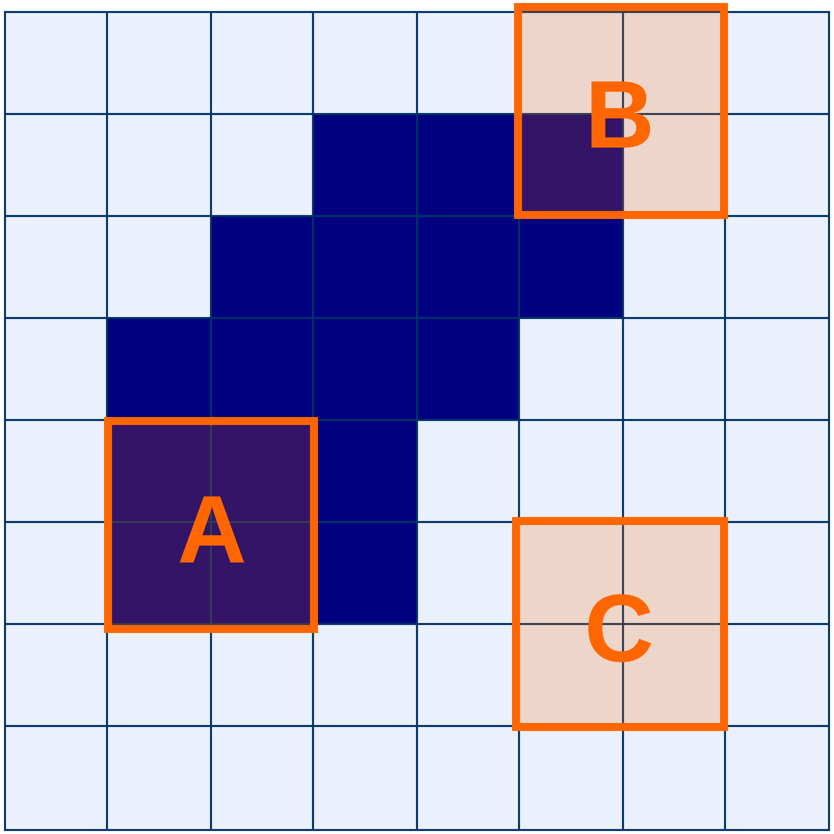
(B)_z means that *B* is moved so that its origin coincides with location z in S_P.

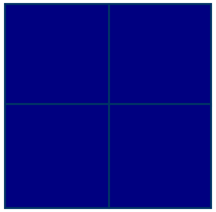
(B)_z is the *translate* of *B* to location *z* in S_P.





Structuring Elements: Hits & Fits





Structuring Element

Fit: All *on pixels* in the structuring element cover *on pixels* in the image

Hit: Any *on pixel* in the structuring element covers an *on pixel* in the image

All morphological processing operations are based on these simple ideas

Fitting & Hitting

0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	0	0	0	0	0
0	0	1	1 **B**	1	1	1	0	0 **C**	0	0	0
0	1	1	1	1	1	1	1	0	0	0	0
0	1	1	1	1	1	1	1	0	0	0	0
0	0	1	1	1	1	1	1	0	0	0	0
0	0	1	1	1	1	1	1	1	0	0	0
0	0	1	1	1	1	1	1 **A**	1	1	1	0
0	0	0	0	0	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0

1	1	1
1	1	1
1	1	1

Structuring Element 1

0	1	0
1	1	1
0	1	0

Structuring Element 2

Acknowledgements

- Digital Image Processing", Rafael C. Gonzalez & Richard E. Woods, Addison-Wesley, 2002
- Peters, Richard Alan, II, Lectures on Image Processing, Vanderbilt University, Nashville, TN, April 2008
- Brian Mac Namee, Digitial Image Processing, School of Computing, Dublin Institute of Technology
- Computer Vision for Computer Graphics, Mark Borg

Material in these slides has been taken from, the following resources