

# Digital Image Processing

## **Lecture # 9 C** **Morphological Image Processing**

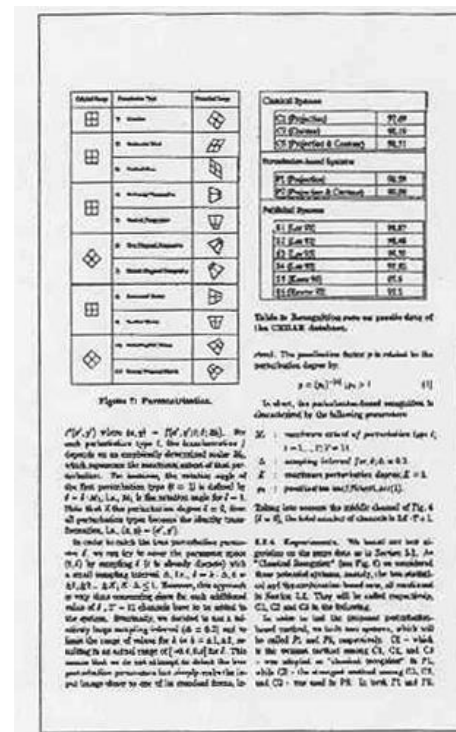
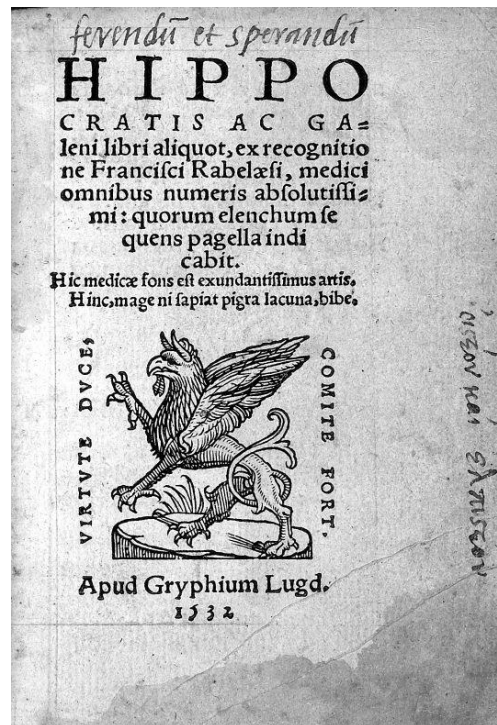


# Run Length Smoothing Algorithm



## Text/Graphics Segmentation

# Separate Text from Graphics



# Text/Graphics Segmentation

## Run Length Smoothing Algorithm – RLSA

- Change runs of white pixels of length below a threshold to black
- Black pixels remain unchanged
- Example:  $C=4$

$X = 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 1\ 0$

$Y = 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0$

# Horizontal RLSA

Algebraic	Boolean Taut	Boolean Prop
	1. Identity	
	2. Nullvalue Absor	
	3. Nullvalue Abs	
	4. Absorption/Propag	
	5. Nullval. Propagat	
	6. Null. Propag. Absorpt	
	7. Distrib. Propag. Abs	
	8. Associat. Absor	
	9. Associat. Abs	
	10. Associat. Propag	
	11. Assoc. Propag. Abs	

**Figure 7: Parental Utilization**

$(f', g')$  with  $(u, v) = (f', g')/(t, 2t)$ , and perturbation type 2, the branchwork depends on an empirically determined value which represents the maximum extent of the perturbation. For instance, the relation says that the perturbation type  $\theta = 1$  is defined  $\theta = 1/(M_1 \cdot L_1 \cdot N_1)$  is the relative angle for  $\theta$ . Thus, if  $\theta$  the perturbation degree  $\theta = 0$ , all perturbations type become the identity transformation, i.e.  $(x, y) = (f', g')$ .

In order to match the time your selected permutation  $\pi$  was used to solve the permutation problem  $(\pi, f)$  for sampling  $f$  is *already known*), you need a small sampling interval  $\Delta t$ ,  $f = \Delta t \cdot \Delta \omega$ ,  $\Delta \omega = \Delta f / \Delta t$ ,  $\Delta f / X \leq \Delta \omega \leq 1$ . However, this approach is very data consuming since the small additional value of  $\Delta t \cdot \pi = \pi$  obviously has to be added to the system. Alternatively, we decided to use a bidirectional large sampling interval ( $\Delta t = 0.2$ ) and to limit the range of values for  $f$  ( $\omega = \Delta \omega \cdot \Delta t$ ), resulting in an actual range of  $[-0.6, 0.6]$  for  $\Delta t$ . This means that we do not attempt to detect the true post-solution permutation but simply realize the input image above to one of its extended states.

Chemical Systems	
Q1 (Projected)	97.69
Q2 (Current)	95.19
Q3 (Projected & Current)	96.71

Thermalization Based Systems	
Q1 (Projected)	96.29
Q2 (Projected & Current)	95.29

Petroleum Systems	
Q1 (Est Q2)	96.87
Q2 (Est Q2)	95.48
Q3 (Est Q2)	96.75
Q4 (Est Q2)	97.85
Q5 (Known Q2)	97.8
Q6 (Known Q2)	97.5

Table 2. Renegotiating rate on possible date of the CHILAN database.

And, The partition factor  $p$  is related to the partition degree by

$$p = (2\pi)^{-d} \int_{\mathbb{R}^d} \exp(-i\mathbf{p} \cdot \mathbf{x}) \mu(\mathbf{x}) d\mathbf{x} \quad (6)$$

In short, the particularized complaint is characterized by the following premises:

$$Z_i = \text{number of nodes of perturbation type } i, \\ i = 1, \dots, 17 \neq 11$$

- $\Delta$  : sampling interval for  $\theta, \Delta = 0.2$
- $\bar{E}$  : maximum performance degree,  $\bar{E} = 1$
- $\alpha$  : modification factor (500000000)

Taking into account the middle channel of Fig. 6

• If  $\mathcal{H} = \mathbb{R}$ , the total number of characters is  $1 + T + 1$ .

3.1.4. *Experiments.* We tested our tool against guidelines on the paper Ada as is Section 5.1. An extended document for Fig. 6 is available.

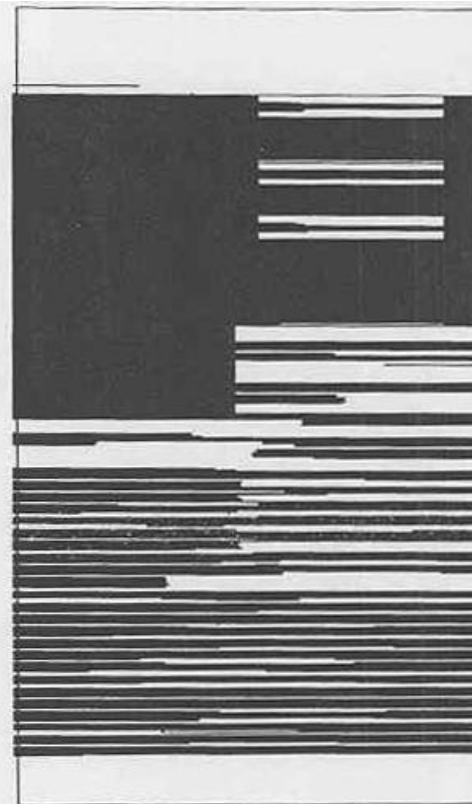
and the combination band near 600 cm<sup>-1</sup>.

CL, CF and CG is the following.

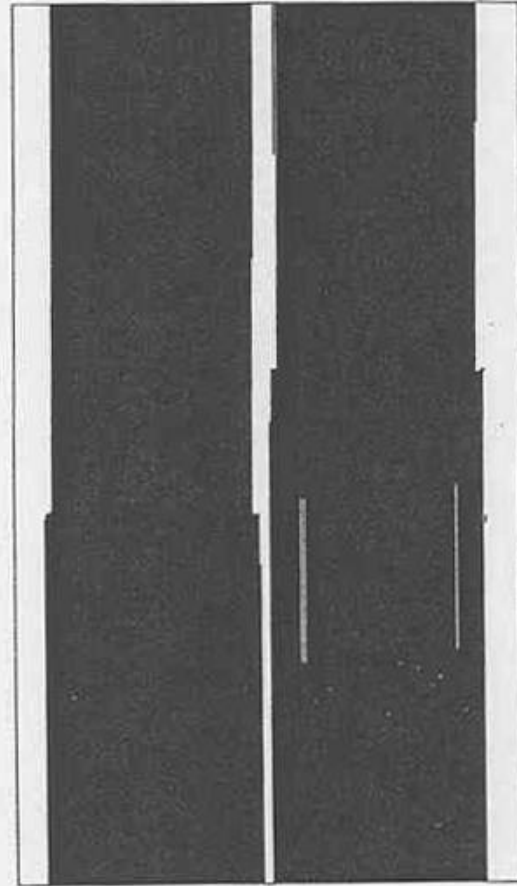
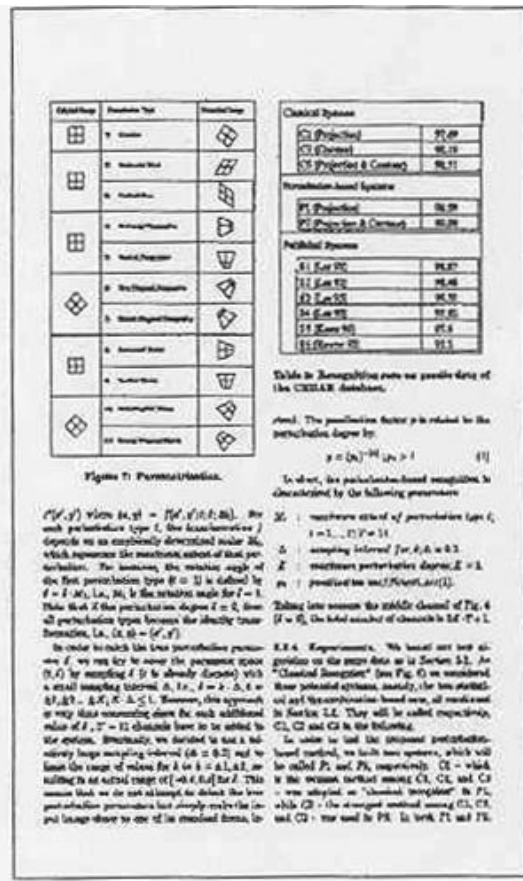
In order to test the proposed polygraphical-based method, we built two systems, which will be called *Ph* and *Ph-mimicking*. *Ph* is built

be called  $P_1$  and  $P_2$ , respectively. CE – which is the process carried among CE, CE, and CE – was referred as “chemical reaction” by  $P_1$ .

and C2 - the used by PK. In work 71 and 72.

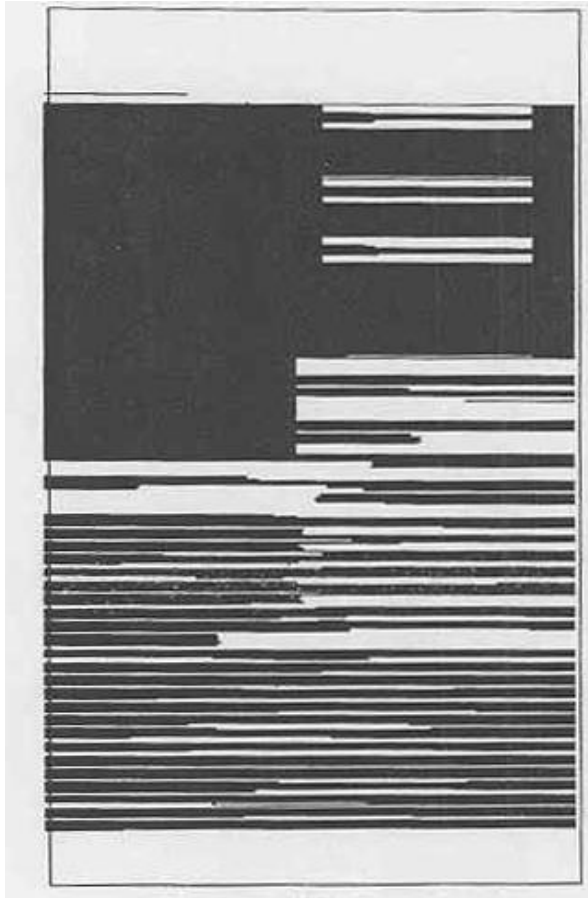


# Text/Graphics Segmentation

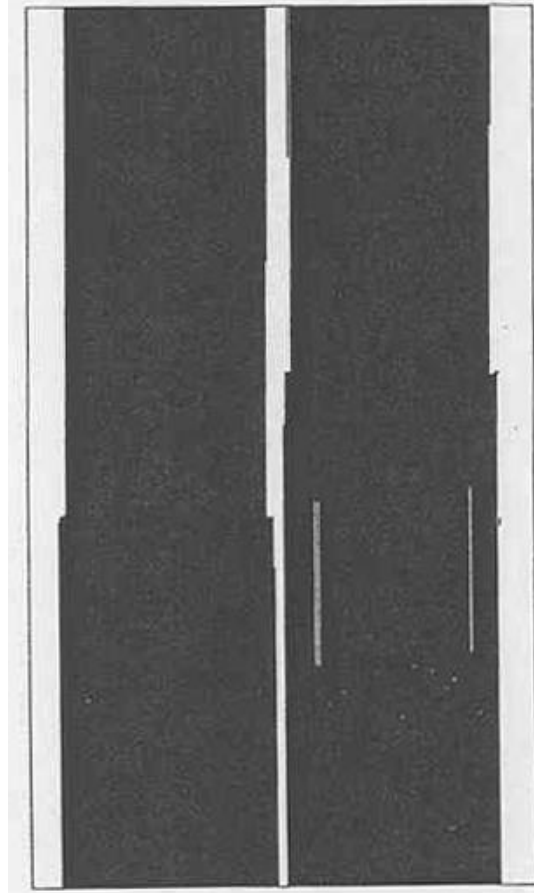


## Vertical RLSA

# Text/Graphics Segmentation



Horizontal RLSA

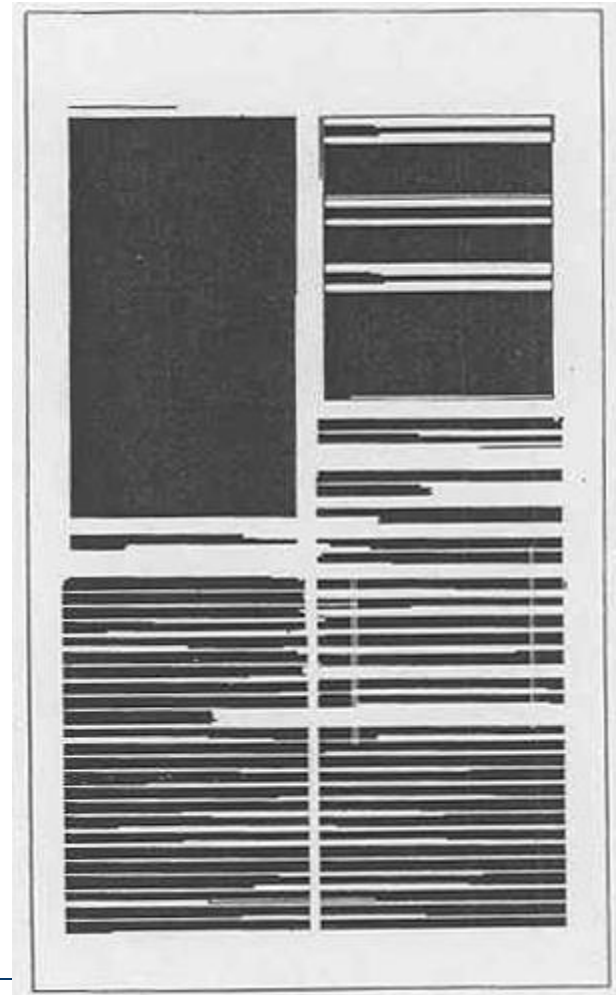


Vertical RLSA

# Text/Graphics Segmentation

## RLSA - Algorithm

- Result of ANDING horizontal and vertical RLS-ed images







# Text/Graphics Segmentation

26 NEZ, FOSSES NASALES. — RHINOSCOPIE.

culum de Thudichum a l'inconvénient de ne pouvoir s'appliquer des deux côtés; il faut un instrument spécial pour chaque narine. Je ne parlerai que pour mémoire du spéculum de Merz (d'Aix-la-Chapelle), à branches séparées: s'il permet au chirurgien de régler à sa volonté la dilatation de la narine, il présente l'extrême désavantage de nécessiter l'emploi des deux mains. C'est au spéculum de Duplay que la préférence doit être accordée comme au plus simple et au plus commode. Il se compose de deux valves, dont l'une, qui doit répondre à la cloison, est légèrement aplatie et fixe, tandis que l'autre valve, destinée à dilater la narine, est mobile et s'écarte à l'aide d'une pression exercée sur une petite pédale. L'écartement, produit au degré convenable, est maintenu à l'aide d'une vis (fig. 2). L'instrument est introduit fermé et poussé jusqu'à la limite de la portion cartilagineuse et de la portion osseuse: la valve externe (mobile) est écartée et la dilatation est portée au point voulu (Duplay, t. III, p. 749).

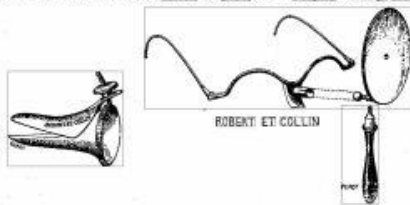
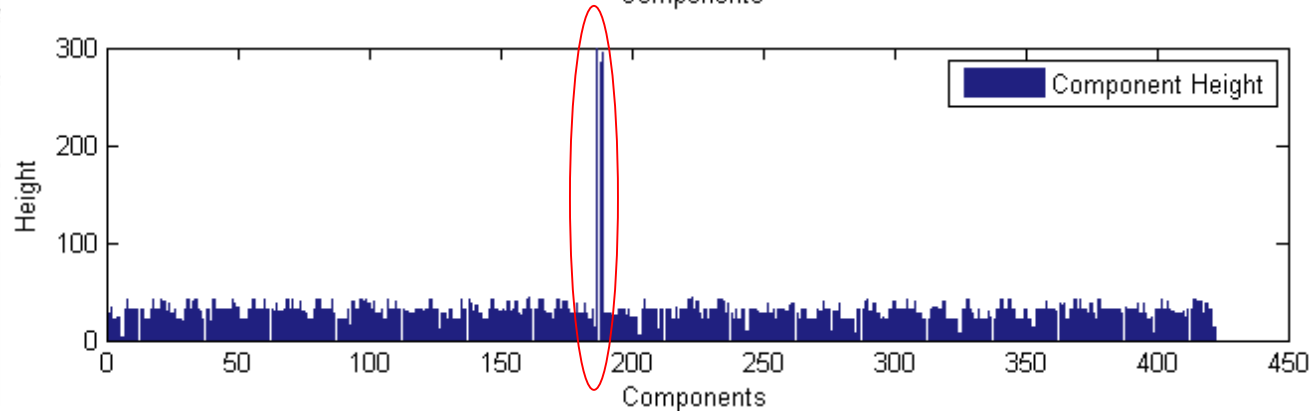
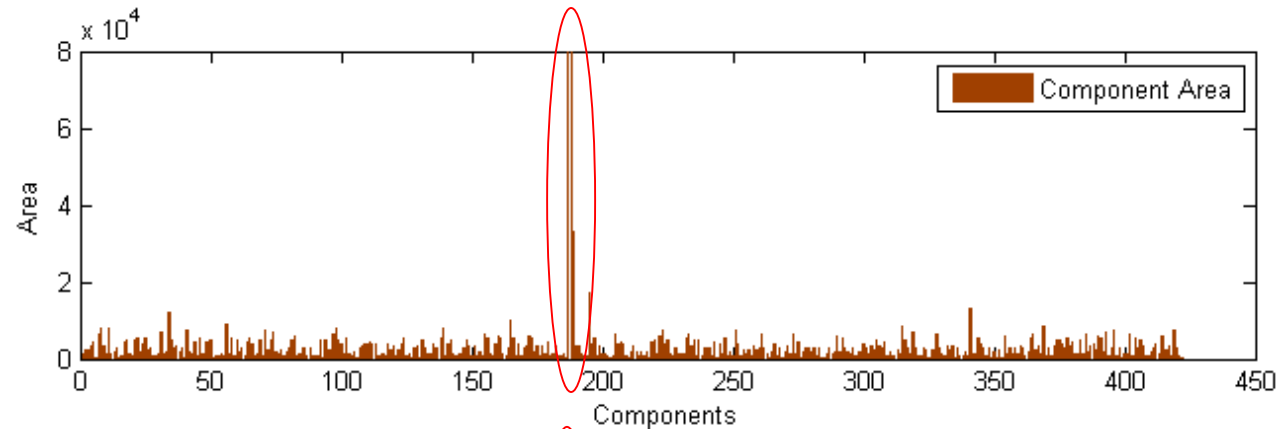


Fig. 2. — Spéculum de S. Duplay. Fig. 3. — Réflecteur de Semelroder (modifié par S. Duplay).

L'éclairage des fosses nasales a lieu par la lumière directe ou par la lumière réfléchie. Ce dernier procédé est en général préférable pour un examen minutieux et complet, parce qu'il permet de diriger et de concentrer la lumière à volonté et successivement sur les différents points et que l'opérateur n'est pas exposé à faire ombre avec sa tête, comme cela peut arriver en ayant recours à l'éclairage direct. La lumière peut être empruntée aux rayons du soleil ou (ce qui peut être pratiqué par tous les temps et dans tous les lieux) au foyer d'une forte lampe, munie au besoin de verres condensateurs. Pour réfléchir les rayons lumineux, on se servira ou bien d'un miroir concave à main, analogue à celui qui est en usage pour l'ophtalmoscopie, ou mieux du *miroir à fossette* de Duplay, qui a l'immense avantage de laisser libres les deux mains de l'opérateur (fig. 3).

La rhinoscopie antérieure permet le plus souvent l'examen direct d'une portion très-notable des fosses nasales. En variant la direction du spéculum et l'incidence des rayons lumineux, on peut successivement découvrir la cloison, les cornets inférieur et moyen et même, dans quelques cas, la paroi postérieure du pharynx (Duplay, t. III, p. 750).





# Line/Word Extraction

# Line and Word Extraction

## Line Extraction?

This is a text example,  
used to illustrate the principle  
of the X-Y-tree decomposition  
algorithm.

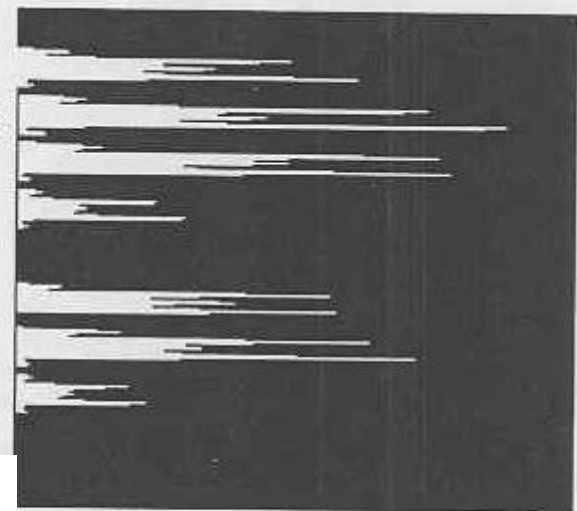
The text was scanned at  
the resolution of 300 dpi (dot  
per inch).

# Line and Word Extraction

## Line Extraction?

This is a text example,  
used to illustrate the principle  
of the X-Y-tree decomposition  
algorithm.

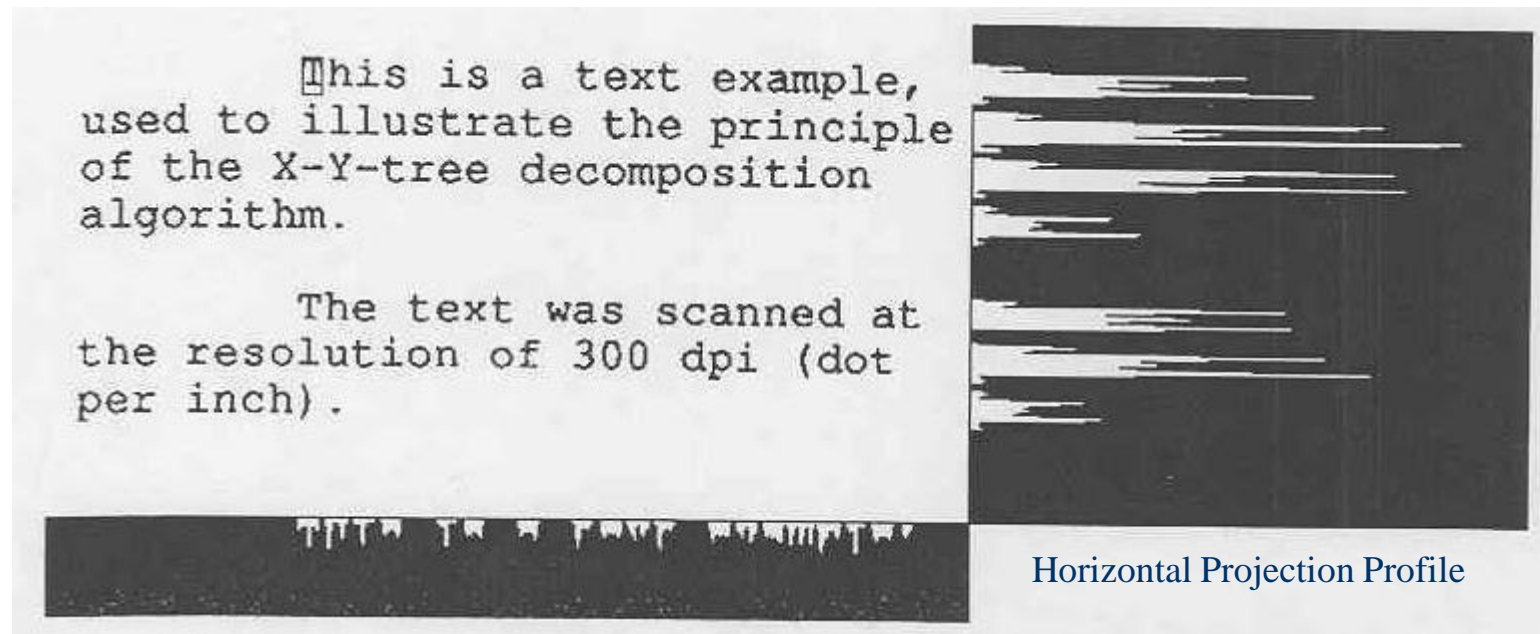
The text was scanned at  
the resolution of 300 dpi (dot  
per inch).



Horizontal Projection Profile

# Line and Word Extraction

## Projection Profiles



# Word Extraction

## Word Extraction using RLSA

l'expérimentation physiologique

~~l'expérimentation~~ ~~physiologique~~

# Hit-or-Miss Transform

- ◆ A tool for shape detection or for the detection of a *disjoint region* in an image
- ◆ Idea
  - Suppose we have a binary image that contains certain shapes (circles, squares, lines, etc,....) called image A
  - We use another image or matrix to search image A for a particular pattern of bits. We will call this pattern “*shape B*”
  - We then search image A for shape B
  - Whenever there is a ‘hit’, we indicate where the center of shape B was on image A.



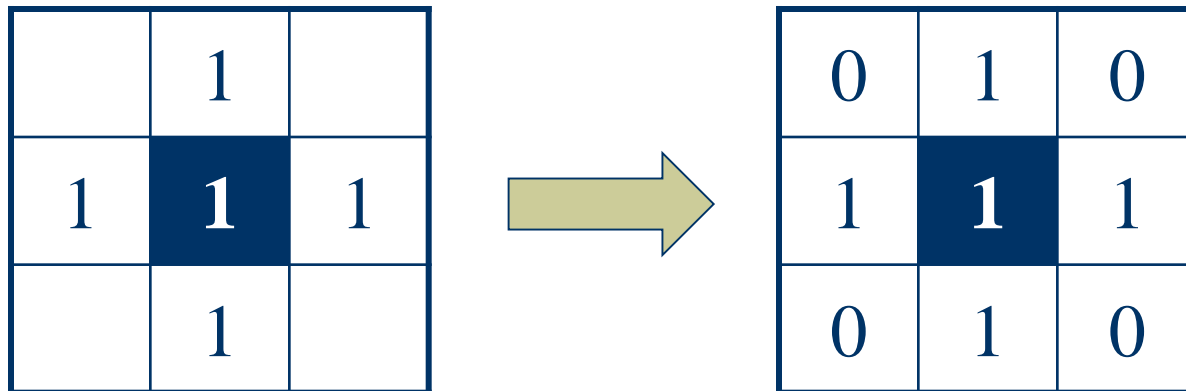
# Hit-or-Miss Transform

- ◆ A tool for shape detection or for the detection of a *disjoint region* in an image
- ◆ Idea
  - **Watch out:** We actually look for '*fits*' but we will be calling them '*hits*' when talking about hit-or-miss transform
  - We have a particular pattern of bits. We will call this pattern "shape B"
  - We then search image A for shape B
  - Whenever there is a 'hit', we indicate where the center of shape B was on image A.

# Hit-or-Miss Transform

- ♦ Structuring Element

So far we have considered the SEs where 0s are treated as Don't Cares i.e. we focus on the 1s only



# Hit-or-Miss Transform

- ◆ Extended Structuring Element

Now we will distinguish between the 0s and the Don't cares

1	1	1
×	0	×
×	0	×

E.g. For a 'fit' the 0s of SE should match with 0s of the underlying image

# Hit-or-Miss Transform

- Extended Structuring Element: Example

0	0	0	0	0	0	0	0
0	0	1	1	1	0	0	0
0	0	0	0	1	1	1	0
0	1	0	0	1	1	1	1
0	1	1	1	1	1	0	1
0	0	1	1	1	1	0	1

!

1	1	1
×	0	×
×	0	×

Erosion Recap: Slide the SE on the image and look for the 'fits'

# Hit-or-Miss Transform

- Extended Structuring Element: Example

0	0	0	0	0	0	0	0
0	0	1	1	1	0	0	0
0	0	0	0	1	1	1	0
0	1	0	0	1	1	1	1
0	1	1	1	1	1	0	1
0	0	1	1	1	1	0	1

'Fit' encountered

!

1	1	1
×	0	×
×	0	×

# Hit-or-Miss Transform

- Extended Structuring Element: Example

0	0	0	0	0	0	0	0
0	0	1	1	1	0	0	0
0	0	0	0	1	1	1	0
0	1	0	0	1	<b>1</b>	<b>1</b>	<b>1</b>
0	1	1	1	1	1	<b>0</b>	1
0	0	1	1	1	1	<b>0</b>	1

!

1	1	1
×	0	×
×	0	×

'Fit' encountered

# Hit-or-Miss Transform

- Extended Structuring Element: Example

0	0	0	0	0	0	0	0
0	0	1	1	1	0	0	0
0	0	0	0	1	1	1	0
0	1	0	0	1	1	1	1
0	1	1	1	1	1	0	1
0	0	1	1	1	1	0	1

!

1	1	1
×	0	×
×	0	×

=

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	<b>1</b>	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	<b>1</b>	0
0	0	0	0	0	0	0	0

What we actually did???

# Hit-or-Miss Transform

We have searched the pattern in the structuring element in the image

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0

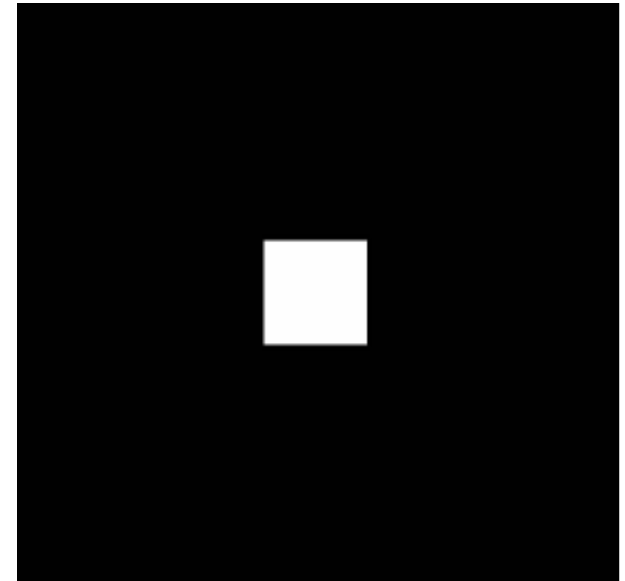
**Output:** The center of the pattern is 1 and rest everything is 0



# Hit-or-Miss Transform

Example: Search a 100x100 pixel square in an image

- ♦ How do we search?
  - Take an image of size 100x100 (B) representing a white square
  - We search the pattern in the input image (A)
  - If found, we have a “fit”. We mark the center of the “fit” with a white pixel
  - In the above example, there would be only 1 fit

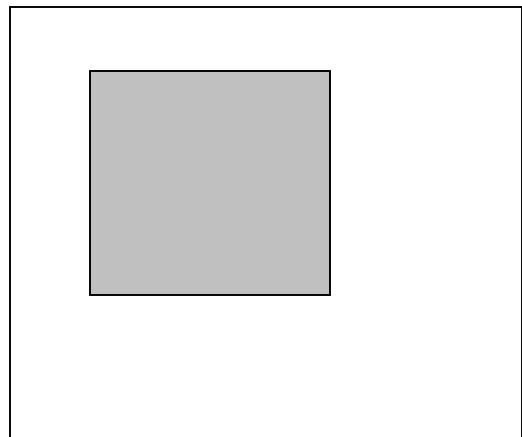


$(A \text{ ! } B)$

# Hit-or-Miss Transform

Do you find any problem with this?

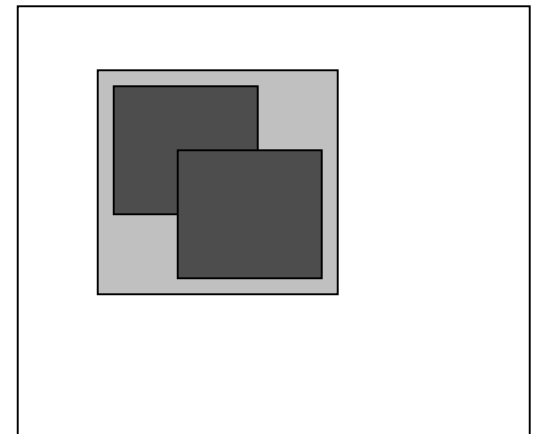
If we search for a 100x100 pixel square in an image we will have a positive response for all squares greater than 100x100 as well



Image



Shape to pattern



The pattern will fit at different places

$(A \neq B) \cap$  Need some thing here

# Hit-or-Miss Transform

We will limit our discussion to this simple version only

$$A \# B = (A ! B)$$

# Morphological Algorithms

Using the simple technique we have looked at so far we can begin to consider some more interesting morphological algorithms

We will look at:

- Boundary extraction
- Region filling
- Extraction of connected components

There are lots of others as well though:

- Thinning/thickening
- Skeletonization

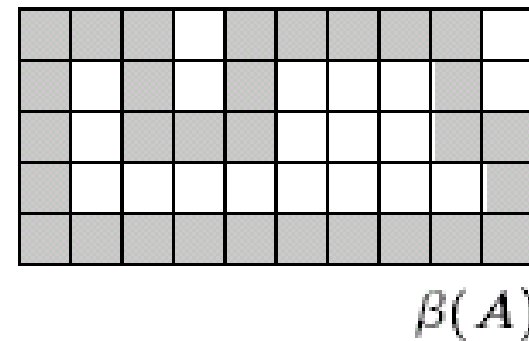
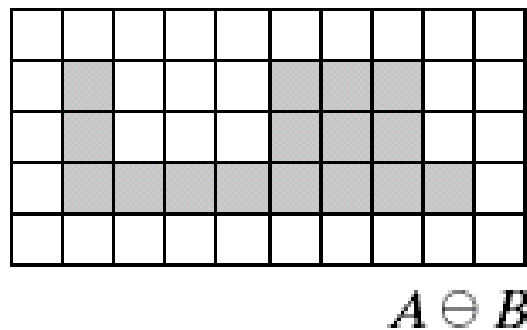
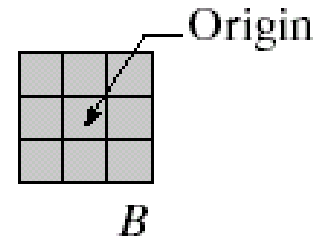
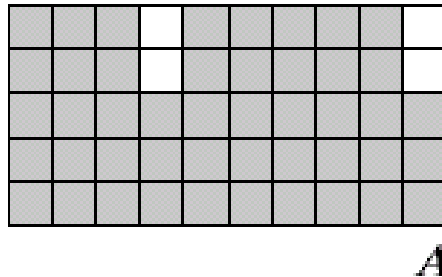
# Boundary Extraction

The boundary of set  $A$  denoted by  $\beta(A)$  is obtained by first eroding  $A$  by a suitable structuring element  $B$  and then taking the difference between  $A$  and its erosion.

$$\beta(A) = A - (A \ominus B)$$

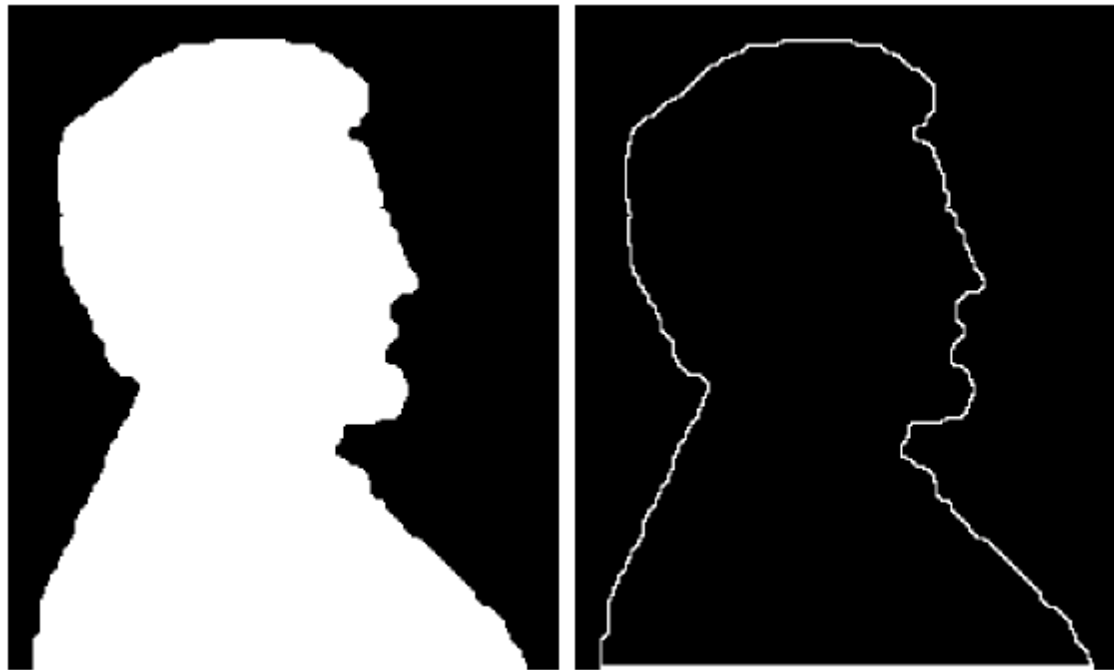
# Boundary Extraction

$$\beta(A) = A - (A \circledast B)$$



# Boundary Extraction

A simple image and the result of performing boundary extraction using a square  $3 \times 3$  structuring element

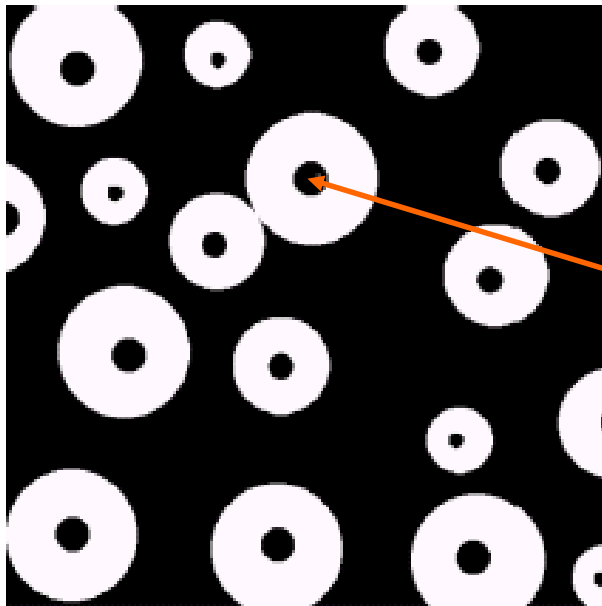


Original Image

Extracted Boundary

# Region (hole) Filling

Given a pixel inside a boundary, *region filling* attempts to fill that boundary with object pixels (1s)



Given a point inside here, can we fill the whole circle?



# Region Filling

Let  $A$  is a set containing a subset whose elements are 8-connected boundary points of a region, enclosing a background region i.e. hole

If all boundary points are labeled 1 and non boundary points are labeled 0, the following procedure fills the region:

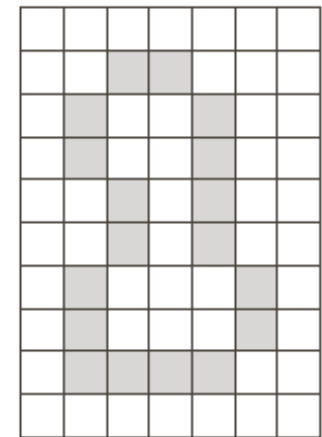
Inside the boundary

- ♦ Start from a known point  $p$  and taking  $X_0 = p$ ,
- ♦ Then taking the next values of  $X_k$  as:

$$X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1, 2, 3, \dots$$

$B$  is suitable structuring element

- ♦ Terminate iterations if  $X_{k+1} = X_k$
- ♦ The set union of  $X_k$  and  $A$  contains the filled set and its boundaries.



$A$

# Region Filling

0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	1	0	0	0	1	0
0	1	0	0	0	1	0
0	1	0	0	0	1	0
0	1	1	1	1	1	0
0	0	0	0	0	0	0

A

1	1	1	1	1	1	1
1	0	0	0	0	0	1
1	0	1	1	1	0	1
1	0	1	1	1	0	1
1	0	1	1	1	0	1
1	0	0	0	0	0	1
1	1	1	1	1	1	1

$A^c$

0	1	0
1	1	1
0	1	0

B

# Region Filling

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$X_0$

0	1	0
1	1	1
0	1	0

$B$

0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	1	1	0	0	0
0	0	1	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$(X_0 \oplus B)$

$$X_k = (X_{k-} \oplus B) \cap A^c$$

$$k = 1, 2, 3, \dots$$

# Region Filling

0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	1	1	0	0	0
0	0	1	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$$X_1 = (X_0 \oplus B)$$

1	1	1	1	1	1	1
1	0	0	0	0	0	1
1	0	1	1	1	0	1
1	0	1	1	1	0	1
1	0	1	1	1	0	1
1	0	0	0	0	0	1
1	1	1	1	1	1	1

$$A^c$$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	1	1	0	0	0
0	0	1	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$$X_1 = (X_0 \oplus B) \cap A^c$$

$$X_k = (X_{k-1} \oplus B) \cap A^c$$

$$k = 1, 2, 3, \dots$$

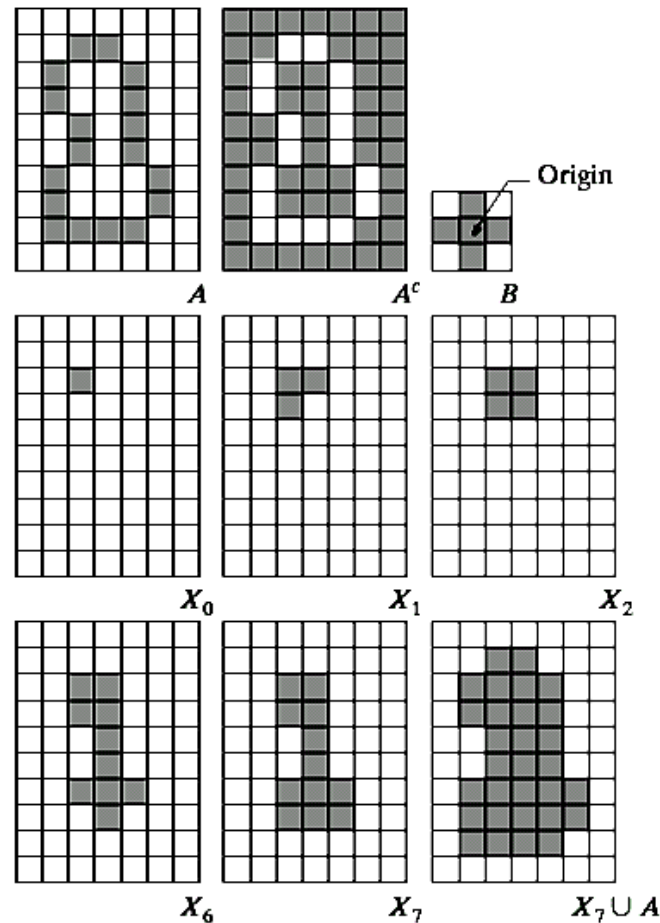
# Region Filling

$$X_k = (X_{k-1} \oplus B) \cap A^c$$

$$k = 1, 2, 3, \dots$$

## NOTE:

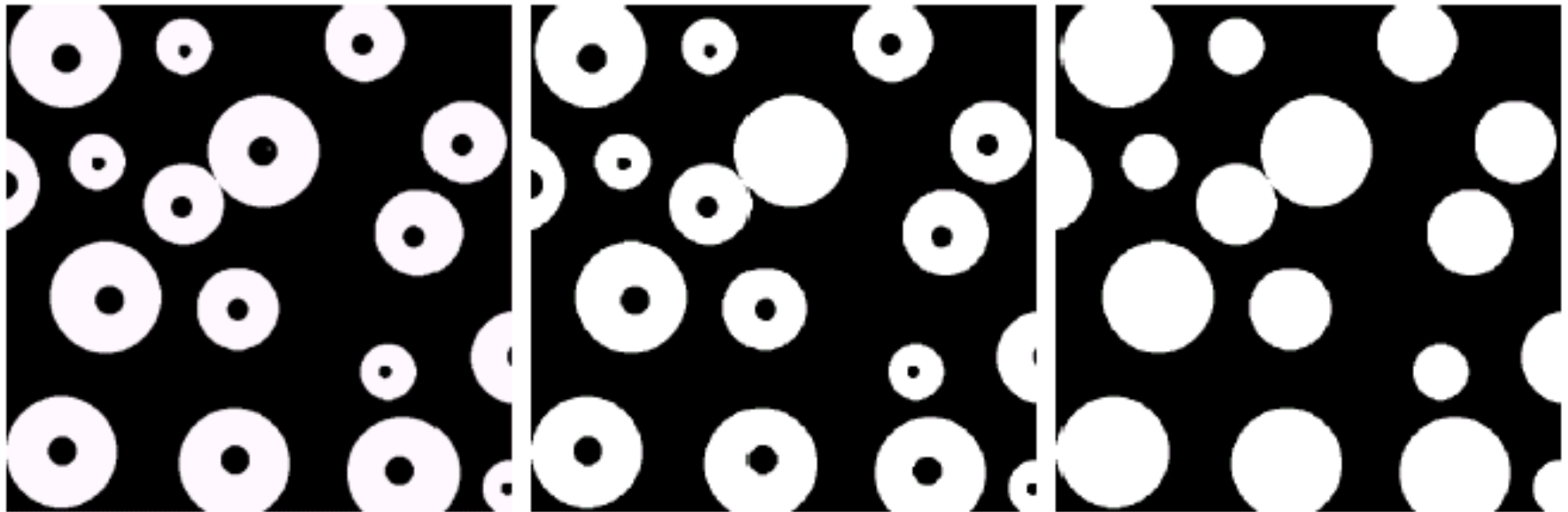
The intersection of dilation and the complement of A limits the result to inside the region of interest



a	b	c
d	e	f
g	h	i

**FIGURE 9.15** Hole filling. (a) Set A (shown shaded). (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

# Region Filling: Example



Original Image

One Region  
Filled

All Regions  
Filled

# Extraction of Connected Components (CCs)

Let  $Y$  represents a connected component contained in  $A$  and the point  $p$  of the  $Y$  is known.

The following procedure iteratively finds all the elements of  $Y$ :

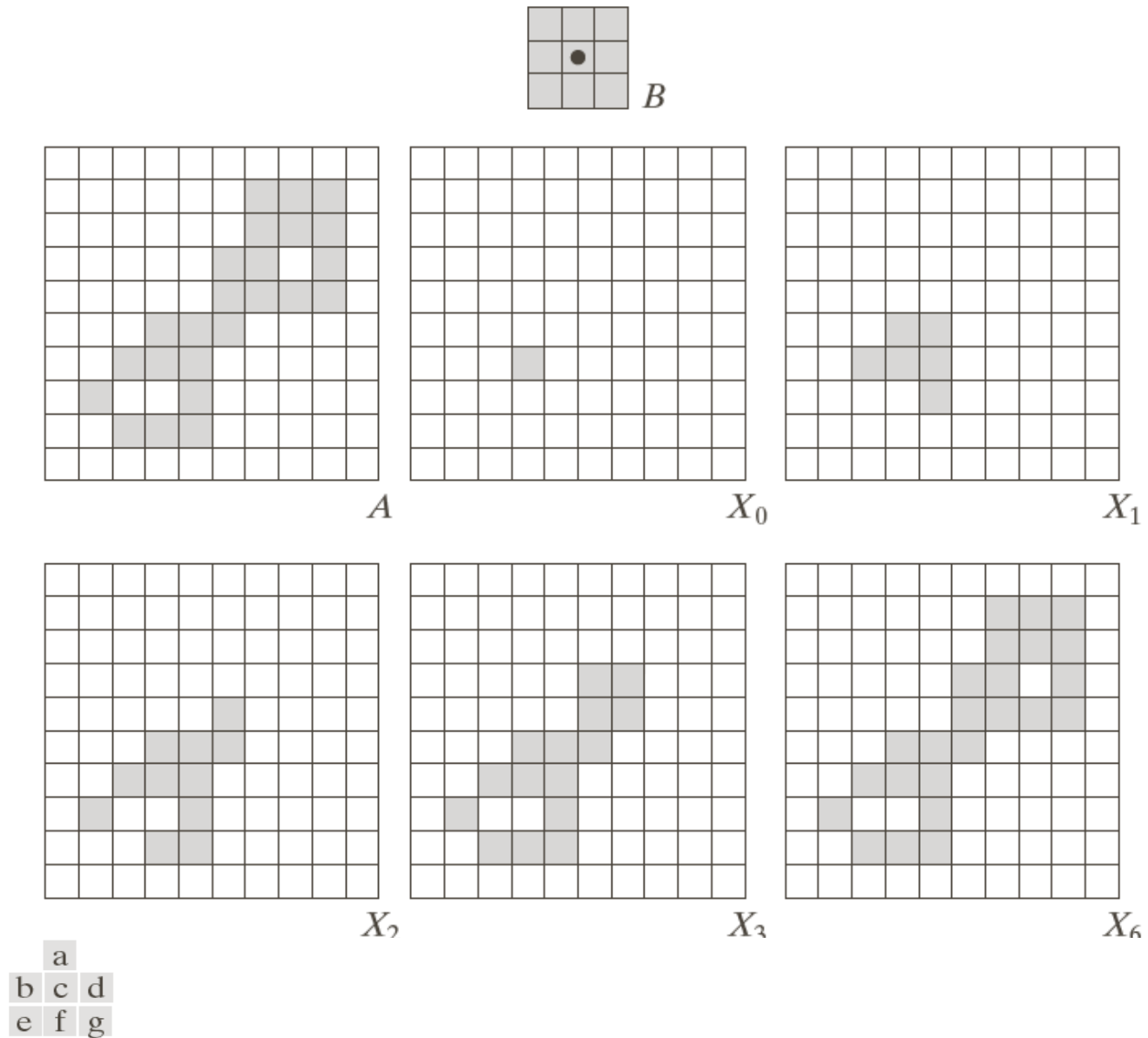
- ♦ Start from a known point  $p$  and taking  $X_0 = p$ ,
- ♦ Then taking the next values of  $X_k$  as:

$$X_k = (X_{k-1} \oplus B) \cap A \quad k = 1, 2, 3, \dots$$

$B$  is a suitable structuring element

- ♦ Algorithm converges if  $X_k = X_{k-1}$
- ♦ The component  $Y$  is given as  $Y = X_k$

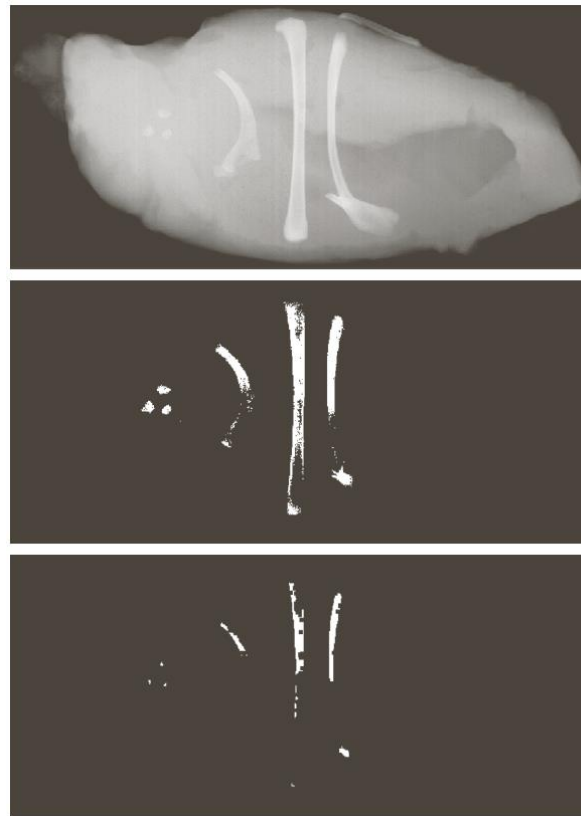
# Extraction of CCs: Example



**FIGURE 9.17** Extracting connected components. (a) Structuring element. (b) Array containing a set with one connected component. (c) Initial array containing a 1 in the region of the connected component. (d)–(g) Various steps in the iteration of Eq. (9.5-3).



# Extraction of CCs: Example



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

a  
b  
c d

**FIGURE 9.18**

(a) X-ray image of chicken file with bone fragments.

(b) Thresholded image. (c) Image eroded with a  $5 \times 5$  structuring element of 1s.

(d) Number of pixels in the connected components of (c).

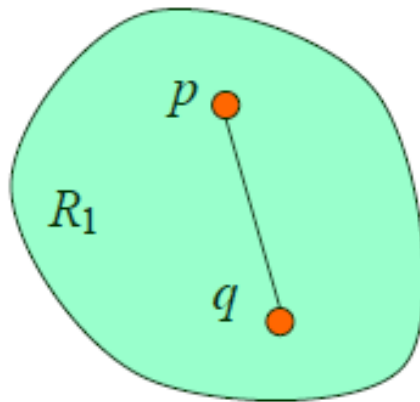
(Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, [www.ntbxray.com](http://www.ntbxray.com).)

# Convex Hull

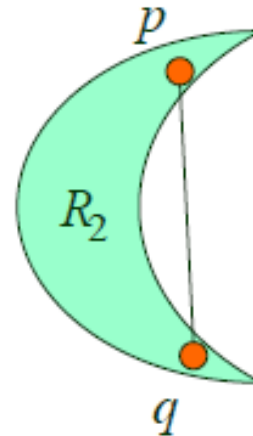
- ◆ **Convex set**

A set  $A$  is said to be convex if any straight line segment joining two points of  $A$  lies within  $A$

- ◆ Example:  $R_1$  is convex as line segment  $pq$  lies within set  $R_1$



Convex



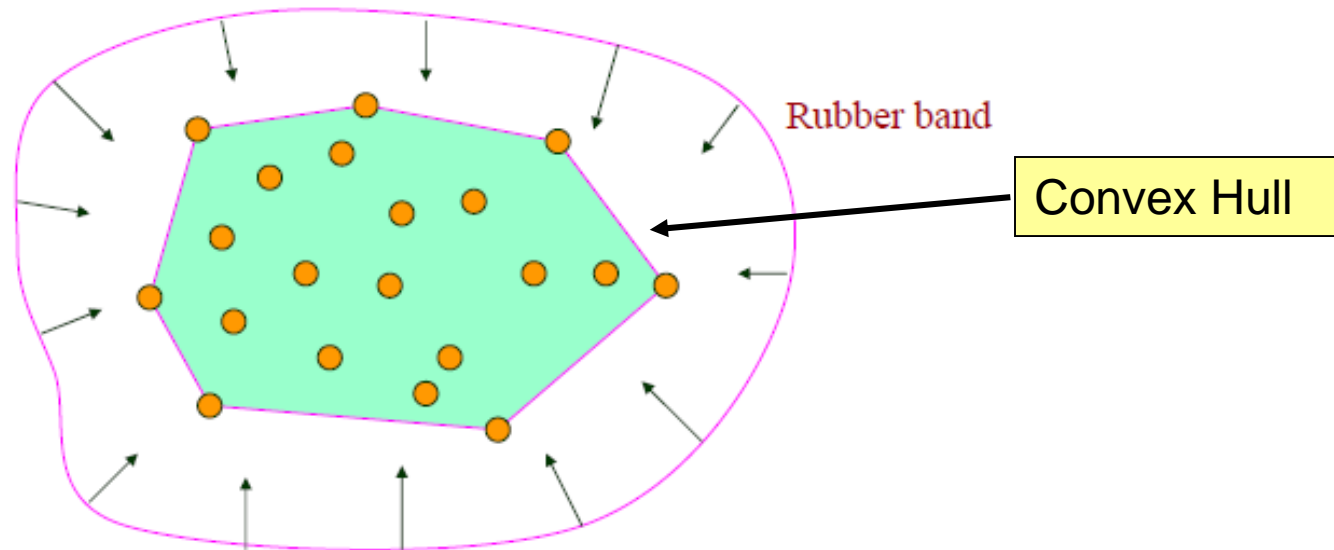
Concave

# Convex Hull

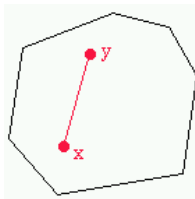
- ◆ **Convex Hull**

Convex hull  $H$  of a set  $S$  is the smallest convex set containing  $S$

- ◆ Rubber band example:



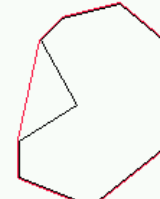
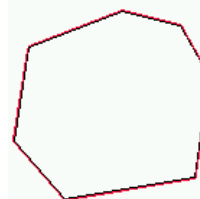
# Convex Hull



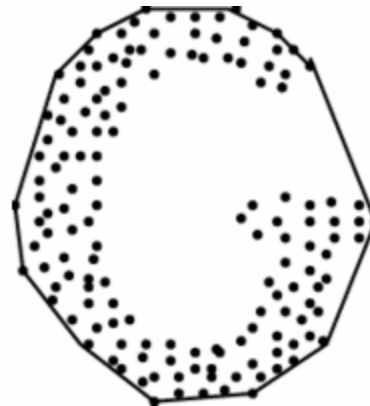
A convex polygon



A non-convex polygon



Convex hulls are in red.



# Convex Hull

- ♦ To find the Convex Hull  $C(A)$  of a set  $A$  the following simple morphological algorithm can be used:
- ♦ Let  $B^i$ , where  $i = 1, 2, 3, 4$ , represent four structuring elements
- ♦ Implement:

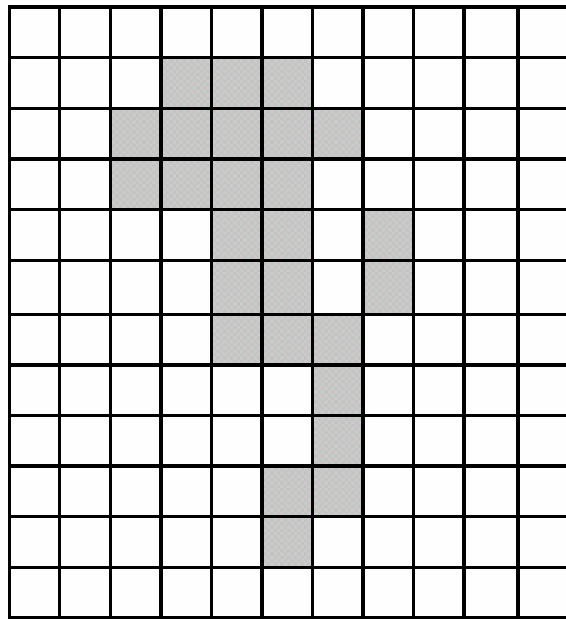
$$X_k^i = (X_{k-1}^i \# B^i) \cup A \quad i = 1, 2, 3, 4 \quad \text{and} \quad k = 1, 2, 3, \dots$$

- ♦ Starting with:  $X_0^i = A$
- ♦ Repeat 2<sup>nd</sup> step until convergence, i.e.  $D^i = X_{conv}^i \rightarrow X_k^i = X_{k+1}^i$
- ♦ Convex Hull  $C(A)$  is given by:

$$C(A) = \bigcup_{i=1}^4 D^i$$

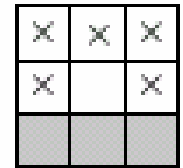
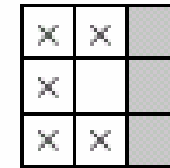
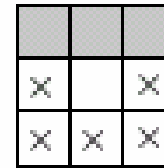
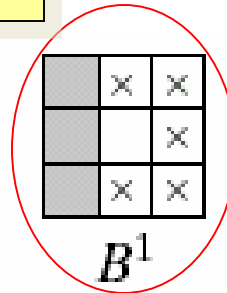
# Convex Hull

Pick the first SE



A

Start At:



$$X_0^1 = A$$

Find:

$$X_1^1 = (X_0^1 \# B^1) \cup A$$

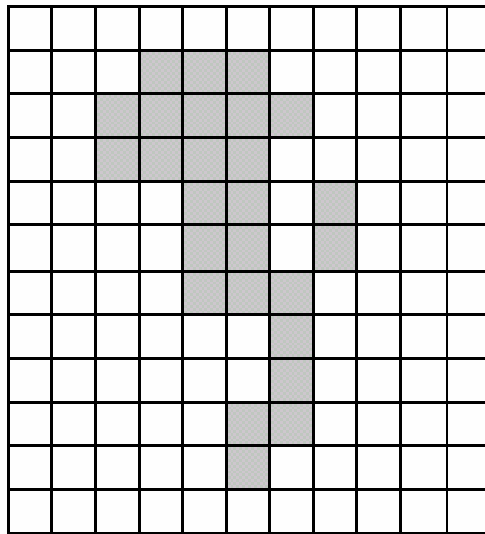
$$X_2^1 = (X_1^1 \# B^1) \cup A$$

⋮

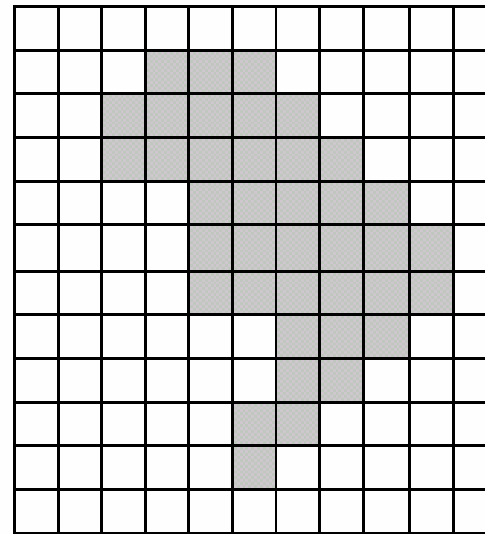
Until Convergence  $X_k^1 = (X_{k-1}^1 \# B^1) \cup A$

# Convex Hull

$$X_k^1 = (X_{k-1} \# B^1) \cup A$$



$X_0^1 = A$



$X_4^1$

Call it  $D^1$

Convergence after four iterations

# Convex Hull

Repeat the same process for  $B^2$ ,  $B^3$  and  $B^4$

x		x
x	x	x

$B^2$

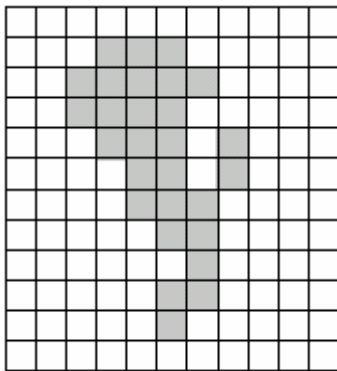
x	x	
x		
x	x	

$B^3$

x	x	x
x		x

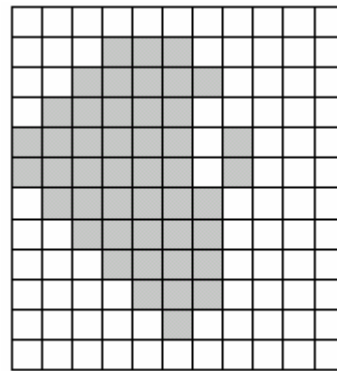
$B^4$

$$X_k^i = (X_{k-1} \# B^i) \cup A \quad i = 2, 3, 4$$



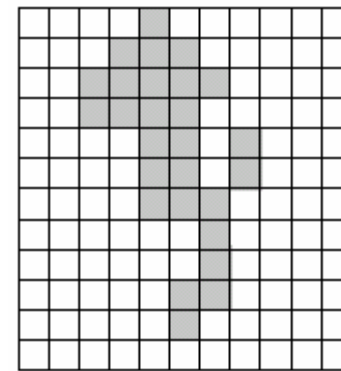
$X_2^2$

$D^2$



$X_8^3$

$D^3$



$X_2^4$

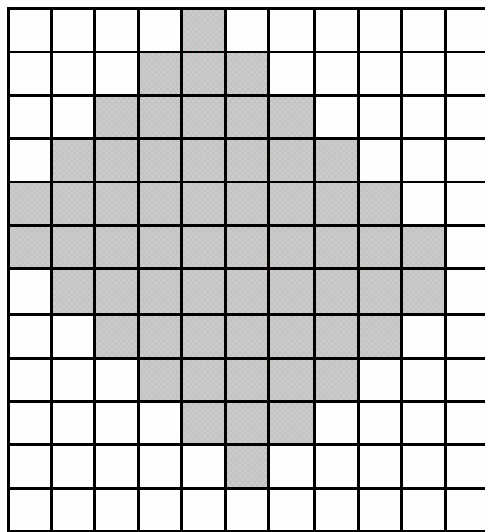
$D^4$



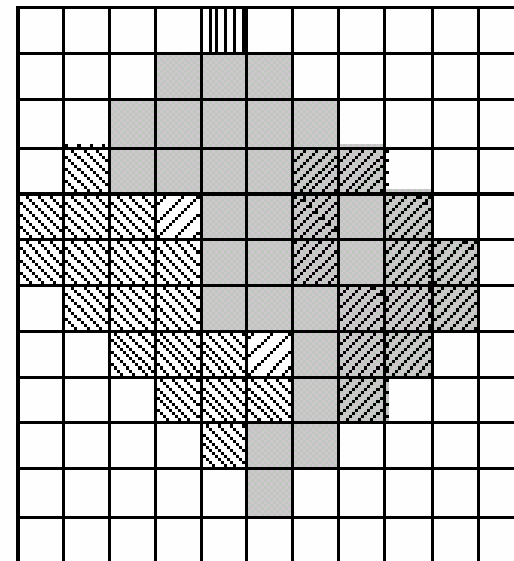
# Convex Hull





Take the union of all  $D^i$  to get the convex hull of  $A$

$$C(A) = \bigcup_{i=1}^4 D^i$$



$C(A)$



-   $B^1$
-   $B^2$
-   $B^3$
-   $B^4$

# Acknowledgements

- ♦ Digital Image Processing”, Rafael C. Gonzalez & Richard E. Woods, Addison-Wesley, 2002
- ♦ Peters, Richard Alan, II, Lectures on Image Processing, Vanderbilt University, Nashville, TN, April 2008
- ♦ Brian Mac Namee, Digital Image Processing, School of Computing, Dublin Institute of Technology
- ♦ Computer Vision for Computer Graphics, Mark Borg