# Associativity of Equivalence

$A \equiv B \equiv C$

# Associativity of Equivalence

$A \equiv B \equiv C$

can be evaluated as

$(A \equiv B) \equiv C$

or

$A \equiv (B \equiv C)$

# Even and odd numbers

m+n is even ≡ m is even ≡ n is even

m+n is even ≡ (m is even ≡ n is even)

# $A \equiv B \equiv C$

| A | B | C | $A \equiv B$ | $(A \equiv B) \equiv C$ |
|---|---|---|---|---|
| F | F | F | T | F |
| F | F | T | T | T |
| F | T | F | F | T |
| F | T | T | F | F |
| T | F | F | F | T |
| T | F | T | F | F |
| T | T | F | T | F |
| T | T | T | T | T |

There is gold on the island G

A is a knight A

There is gold on the island G

A is a knight A

$A \equiv (A \equiv G)$

$(A \equiv A) \equiv G$

$true \equiv G$

G

# Properties of equivalence

constant  true

$true \equiv p \equiv p$

$true = (p \equiv p)$

$(true \equiv p) = p$

# Properties of equivalence

Symmetry

$(p \equiv q) = (q \equiv p)$

# Properties of equivalence

$$p \equiv p \equiv q \equiv p \equiv r \equiv q$$

$$p \equiv p \equiv p \equiv q \equiv q \equiv r$$

$$\text{true} \equiv p \equiv \text{true} \equiv r$$

$$p \equiv r$$

# Properties of equivalence

$$p \equiv p \equiv q \equiv p \equiv r \equiv q$$

$$p \equiv p \equiv p \equiv q \equiv q \equiv r$$

$$\text{true} \equiv p \equiv \text{true} \equiv r$$

$$p \equiv r$$

Replace the term which is repeated odd number of times by a single occurrence of the term and any terms which is repeated an even number of times by removing all occurrences.

A is a knight A

A says “I am a knight.” A

A is a knight A

A says “I am a knight.” A

$A \equiv A$

Since this is always true, no meaningful conclusion can be made.

A is a knight  A

B is a knight  B

A says “I am the same type as B”  $A \equiv B$

A is a knight A

B is a knight B

A says “I am the same type as B” $A \equiv B$

$A \equiv A \equiv B$

B

# Island of knights and knaves

Suppose A is the proposition “person A is a knight” and suppose A makes a statement S. Then A is true is the same as S is true. That is:

$A \equiv S$

# Logic problem for the day

A tourist comes to a fork in the road, where one branch leads to a restaurant and one does not. A native of the island is standing at the fork. Formulate a single yes no question that the tourist can ask such that the answer will be yes if the left fork leads to the restaurant, and otherwise the answer will be no.