Rules of Inference and Logical Deduction

Introduction
Elimination

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Rules of Inference and Logical Deduction

Introduction
$\frac{p \quad q}{p \text{ and } q}$

Rules of Inference and Logical Deduction

Introduction	
$\frac{p}{p \text{ or } q}$	$\frac{q}{p \text{ or } q}$

Rules of Inference and Logical Deduction

Elimination	
$\frac{p \text{ and } q}{p}$	$\frac{p \text{ and } q}{q}$

Rules of Inference and Logical Deduction

Introduction
$\dfrac{[p]\ q}{p \Rightarrow q}$

Rules of Inference and Logical Deduction

Elimination
$\dfrac{p \quad p \Rightarrow q}{q}$

Rules of Inference and Logical Deduction

Elimination	
$\dfrac{p \quad \text{not } p}{\text{false}}$	$\dfrac{\text{false}}{p}$

Does the Superman Exist?

If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil, he would be incapable; if he were unwilling to prevent evil, he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither incapable nor malevolent. Therefore Superman does not exist.

Inference and deduction

- Superman exists X
- Superman is willing to prevent evil W
- Superman is able to prevent evil A
- Superman is malevolent M
- Superman is incapable I
- Superman prevents evil E

Inference and Deduction

- Our objective is to prove the proposition:

((W and A) $\Rightarrow$ E)
and ((not A) $\Rightarrow$ I)
and ((not W) $\Rightarrow$ M)
and (not E)
and (X $\Rightarrow$ not (I or M))
$\Rightarrow$ not X

1. Assume

   ((W and A) ⇒ E)

   and ((not A) ⇒ I)

   and ((not W) ⇒ M)

   and (not E)

   and (X ⇒ not (I or M))

The objective is now to prove *not X*

2. **Assume X**

2. Assume X

   Use Elimination Rules to break No. 1 down into 5 premises
3. (W and A) $\Rightarrow$ E
4. (not A) $\Rightarrow$ I
5. (not W) $\Rightarrow$ M
6. not E
7. X $\Rightarrow$ not (I or M)

Now application of elimination on 2 and 7 derives another simple proposition

2. Assume X

7. X $\Rightarrow$ not (I or M)

8. not (I or M)

Now application of elimination on 2 and 7 derives another simple proposition

2. Assume X

7. X $\Rightarrow$ not (I or M)

8. not (I or M)

Now proving I or M will result in a contradiction

We will now analyze W

9. Assume not W
10. M (from 5 and 9)
11. I or M (from 10 - introduction)

12. Assume W
13. Assume A
14. W and A (12 and 13)
15. E (3 and 14)
16. false (6 and 15)
17. I or M (16)

18. Assume not A
19. I (4 and 18)
20. I or M (19)
21. I or M (17 and 20)
22. I or M (11 and 21)
23. false (contradiction 8 and 22)
24. not X (2 and 23)

Logic problem for the day

On the island of knights and knaves, it is rumored that there is gold buried on the island. You ask one of the natives, A, whether there is gold on the island. He makes the following response: “There is gold on this island if and only if I am a knight.”

Logic problem for the day

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The problem is as follows:

a) Can it be determined whether A is a knight or a knave?

b) Can it be determined whether there is gold on the island?