

Loop Invariants

1

```
s = 0;  
for i := 1 to n do  
    s = s + a[i];
```

What is the 'loop invariant'?

s is the sum of elements from $a[1]$ to $a[i]$
immediately before i is incremented!

Weakest Precondition for While Statement

$\{P\} \text{ while } B \text{ do } S \{Q\}$

- ▶ Let W be **while** B **do** S
- ▶ condition for termination of the loop
 - $P_0 \equiv (\text{not } B)$
 - $P_1 \equiv B \text{ and } wp(S, P_0) \equiv wp(S, \text{not } B)$
 - $P_k \equiv B \text{ and } wp(S, P_{k-1})$
- ▶ $wp(W, \text{true}) \equiv wp(W, \text{not } B) \equiv \exists(k: k \geq 0: P_k)$

Weakest Precondition for While Statement

3

while B do S

The invariant condition

$\{I\} \text{ while } B \text{ do } S \{I \text{ and not } B\}$

```
a = 0;
```

```
i = 0;
```

```
while (i < N)
```

```
    a = a + i++;
```

Loop Invariant: $a = \sum_{i=0}^{N-1} i$

Quiz 2

- ▶ $\text{wp}(\text{if } x > 2 \text{ then } y := 1 \text{ else } y := -1, (y > 0))$
 - ▶ $\text{wp}(x := x + 2; y := y - 2, (x + y = 0))$
- ▶ Find the Precondition of following
 - ▶ $\text{WP}(\text{if } a < X \text{ and } a > Y \text{ then Swap}(), X = Y_0 \text{ and } Y = X_0)$

Quiz 2

- ▶ $WP(k:=5; x:=2*y; y:=x-4, w*y = z+x)$
- ▶ $WP(\text{if } x \text{ is even then } x:=x+1, x \text{ not } \% 2)$
- ▶ Find the pre condition for following.
 - ▶ $WP(\text{if } a>x \text{ Swap}(), (X=Y_0 \text{ and } Y=X_0))$