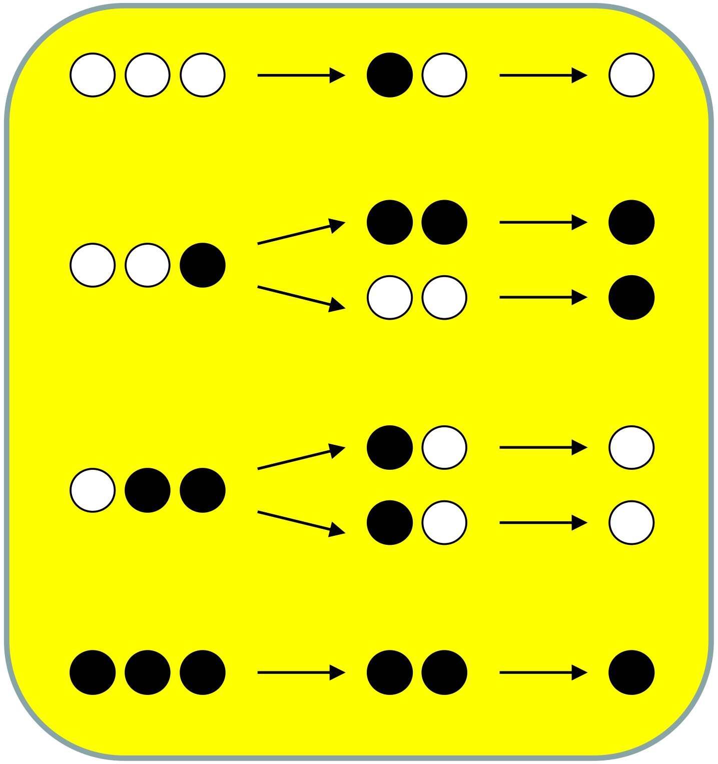

A THREE-BALL GAME

Implication

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Bi-conditional – if and only if

P	Q	$P \Leftrightarrow Q$
T	T	T
T	**F**	**F**
F	F	T
F	**T**	**F**

$P \Leftrightarrow Q$ means $P \Rightarrow Q \wedge Q \Rightarrow P$

- A compound proposition that is always true, irrespective of the truth values of the comprising propositions, is called a tautology.

  $$\mathbf{p \lor \neg p}$$

- The propositions **p** and **q** are called logically equivalent if $\mathbf{p} \Leftrightarrow \mathbf{q}$ is tautology.

- It is written as ,

$$\mathbf{p} \equiv \mathbf{q}$$

For example: $\neg\, (\mathbf{p} \vee \mathbf{q}) \equiv \neg\, \mathbf{p} \wedge \neg\, \mathbf{q}$

Some useful equivalences

p or true ≡ true	**p or false ≡ p**

Some useful equivalences

p or true ≡ true	p or false ≡ p
p and true ≡ p	p and false ≡ false

Some useful equivalences

p or true $\equiv$ true	p or false $\equiv$ p
p and true $\equiv$ p	p and false $\equiv$ false
true $\Rightarrow$ p $\equiv$ p	false $\Rightarrow$ p $\equiv$ true
p $\Rightarrow$ true $\equiv$ true	p $\Rightarrow$ false $\equiv$ not p

Some useful equivalences

p or true ≡ true	p or false ≡ p
p and true ≡ p	p and false ≡ false
true ⇒ p ≡ p	false ⇒ p ≡ true
p ⇒ true ≡ true	p ⇒ false ≡ not p
p or p ≡ p	p and p ≡ p

Some useful equivalences

p or true ≡ true	p or false ≡ p
p and true ≡ p	p and false ≡ false
true ⇒ p ≡ p	false ⇒ p ≡ true
p ⇒ true ≡ true	p ⇒ false ≡ not p
p or p ≡ p	p and p ≡ p

not not p ≡ p

Some useful equivalences

p or true ≡ true	p or false ≡ p
p and true ≡ p	p and false ≡ false
true ⇒ p ≡ p	false ⇒ p ≡ true
p ⇒ true ≡ true	p ⇒ false ≡ not p
p or p ≡ p	p and p ≡ p
not not p ≡ p	
p or not p ≡ true	p and not p ≡ false

Some useful equivalences

distributivity of

- and over or
- or over and
- or over $\Rightarrow$
- $\Rightarrow$ over and
- $\Rightarrow$ over or
- $\Rightarrow$ over $\Rightarrow$
- $\Rightarrow$ over $\Leftrightarrow$

associativity of

$\vee$, $\wedge$, and $\Leftrightarrow$

Commutativity of

$\vee$, $\wedge$, and $\Leftrightarrow$

Demorgan's law

- Implication
- if and only if

Logic problem for the day

Someone asks person A, “Are you a knight?” He replies, “If I am a knight then I’ll eat my hat”. Prove that A has to eat his hat.

- A is a knight: A
- A eats his hat: H

- A is a knight: A
- A eats his hat: H

- If I am a knight then I’ll eat my hat:

$$A \Rightarrow H$$

We have seen that $(X \Leftrightarrow S)$

Therefore

$(A \Leftrightarrow A \Rightarrow H)$

- Objective is to logically deduce H

Proof using equivalences

$A \Leftrightarrow (A \Rightarrow \mathbf{H})$

$\equiv \quad A \Leftrightarrow (\text{not } A \text{ or } \mathbf{H})$

$\equiv (A \text{ and } (\text{not } A \text{ or } H)) \text{ or}$

$(\text{not } A \text{ and not } (\text{not } A \text{ or } H))$

Proof using equivalences

A and (not A or H)

≡ (A and not A) or (A and H)

≡ false or (A and H)

≡ A and H

Proof using equivalences

not A and not (not A or H)

≡ not A and (A and not H)

≡ (not A and A) and not H

≡ false and not H

≡ false

Proof using equivalences

Hence

$A \Leftrightarrow$ (not A or **H)**

$\equiv$ **(**A and H) or false

$\equiv$ A and H