Image Compression

Chapter 8

Motivation

- Storage needed for a two-hour standard television movie (Color)
  - Image size = 720 x 480 pixels
  - Frame rate = 30 fps (frame per seconds)

$$30\,\frac{\text{frames}}{\text{sec}}\ \text{x}\ (720\ \text{x}\ 480)\ \frac{\text{pixels}}{\text{frame}}\ \text{x}\ 3\,\frac{\text{bytes}}{\text{pixel}} = 31{,}104{,}000\ \text{bytes/sec}$$

For 2 hour movie

$$31{,}104{,}000\,\frac{\text{bytes}}{\text{sec}}\ \text{x}\ (60^2)\ \frac{\text{sec}}{\text{hr}}\ \text{x}\ 2\ \text{hrs} = 2.24\ \text{x}\ 10^{11}\ \text{bytes} = 224\ \text{GB}$$

Image compression

- Principal objective
  To minimize the number of bits required to represent an image
- Applications
  - Transmission: Broadcast TV, remote sensing via satellite, military communications via aircraft, radar and sonar, teleconferencing, computer communications, …
  - Storage: Educational and business documents, medical images (CT, MRI and digital radiology), motion pictures, satellite images, weather maps, geological surveys, ...

Overview

- Image data compression methods fall into two common categories:
- Information preserving compression
  - Especially for image archiving (storage of legal or medical records)
  - Compress and decompress images without losing information
- Lossy image compression
  - Provide higher levels of data reduction
  - Result in a less than perfect reproduction of the original image

Data vs. Information

- Data are the means to convey information; various amounts of data may be used to represent the same amount of information

- Part of data may provide no relevant information: data redundancy

Relative data redundancy

- Let b and b' refer to amounts of data in two data sets that carry the same information

$$\text{Compression Ratio } (C) = \frac{b}{b'}$$

$$\text{Releative data redundancy } (R) = 1 - \frac{1}{C}$$

of the first dataset b

- if b = b', C = 1 and R = 0, relative to the second data set, the first set contains no redundant data
- if b >> b', C → ∞ and R → 1, relative to the second data set, the first set contains highly redundant data
- if b << b', C → 0 and R → -∞, relative to the second data set, the first set is highly compressed

C = 10 means 90% of the data in the first data set is redundant

Data redundancy

- Image compression techniques can be designed by reducing or eliminating the data redundancy

- Three basic data redundancies
  - Coding redundancy
  - Spatial and Temporal redundancy
  - Irrelevant information

Coding redundancy

- A natural m-bit coding method assigns m-bit to each gray level without considering the probability that gray level occurs

  → very likely to contain coding redundancy

- Basic concept
  - Utilize the probability of occurrence of each gray level (histogram) to determine length of code representing that particular gray level: **variable-length coding**
  - Assign shorter code words to the gray levels that occur most frequently or vice versa

Coding redundancy

Let $0 \le r_k \le 1$: Gray levels (discrete random variable)

$p_r(r_k)$: Propability of occurrence of r_k

n_k : Frequency of gray level r_k

n : Total number of pixels in the image

L : Total number of gray level

$l(r_k)$: Number of bits used to represent r_k

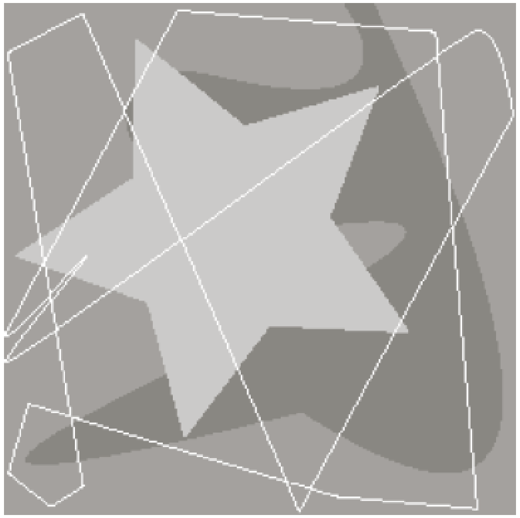
L_{avg} : Average length of code words assigned to gray levels

$$L_{avg} = \sum_{k=0}^{L-1} l(r_k) p_r(r_k) \quad where\ p_r(r_k) = \frac{n_k}{n}, k = 0, 1, 2, \ldots, L-1$$

Hence, the total number of bits required to code and *M*x*N* pixel image is MNL_{avg}

For a natural m-bit coding L_{avg}= m

Coding redundancy: Example

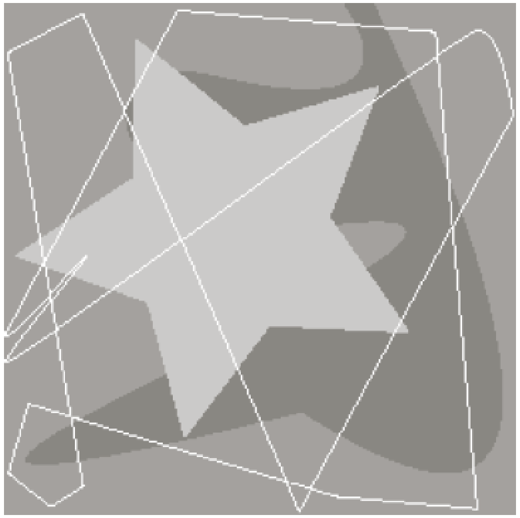


A computer generated (synthetic) 8-bit image
M = N = 256

- Code 1: Natural code (m = 8) is used, L_{avg} = 8 bits
- Code 2: Variable length code
  L_{avg} = (0.25)2 + 0.47(1) + 0.25(3) + 0.03(3)
  = 1.81 bits
- Compression Ratio = $\dfrac{256 \times 256 \times 8}{256 \times 256 \times 1.81} = 4.42$
- R = 1 – 1/4.42 = 0.774

r_k		$p_r(r_k)$	Code 1	$l_1(r_k)$	Code 2	$l_2(r_k)$
r_{87}	87	0.25	01010111	8	01	2
r_{128}	128	0.47	10000000	8	1	1
r_{186}	186	0.25	11000100	8	000	3
r_{255}	255	0.03	11111111	8	001	3
r_k for $k \neq 87, 128, 186, 255$		0	—	8	—	0

Coding redundancy: Example



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  $L_{avg} = (0.25)2 + 0.47(1) + 0.25(3) + 0.03(3)$
  $= 1.81$ bits
- Compression Ratio = $\dfrac{256 \times 256 \times 8}{256 \times 256 \times 1.81} = 4.42$

- $R = 1 - 1/4.42 = 0.774$

77.4% data in the image is redundant

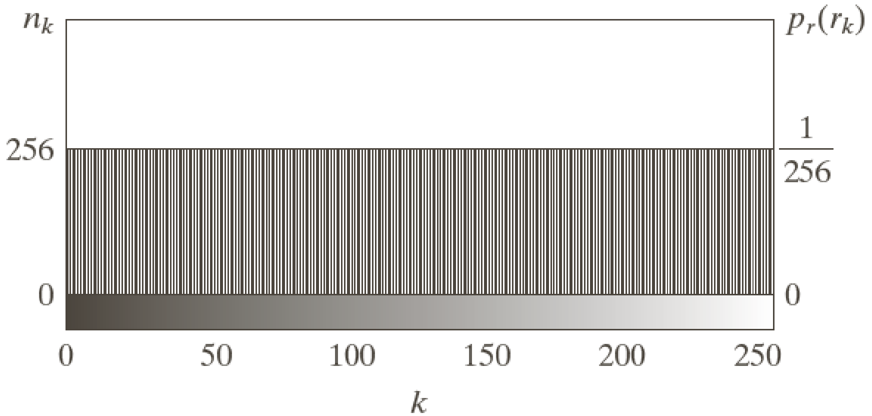
Spatial and Temporal redundancy

- Image features
  - All 256 gray levels are equally probable → uniform histogram

    (variable length coding can not be applied)
  - The gray levels of each line are selected randomly so pixels are independent of one another in vertical direction
  - Pixels along each line are identical, they are completely dependent on one another in horizontal direction

Spatial redundancy

A computer generated (synthetic) 8-bit image M = N = 256



Spatial and Temporal redundancy

- The spatial redundancy can be eliminated by using *run-length pairs (a mapping scheme)*
- **Run length pairs** has two parts
  - Start of new intensity
  - Number of consecutive pixels having that intensity
- Example (consider the image shown in previous slide)
  - Each 256 pixel line of the original image is replaced by a single 8-bit intensity value
  - Length of consecutive pixels having the same intensity = 256
  - Compression Ratio = $\boxed{\dfrac{256 \text{ x } 256 \text{ x } 8}{[256+256] \text{ x } 8} = 128}$

Spatial and Temporal redundancy

- In general, gray level of any given pixel can be reasonably predicted from the value of its neighbors (information carried by individual pixels is relatively small)
- To reduce the spatial (or temporal) redundancy, mapping is used.
  - Example: Map pixels of an image: f(x,y) to a sequence of pairs

    $(g_1,r_1), (g_2,r_2), \ldots, (g_i,r_i), \ldots$

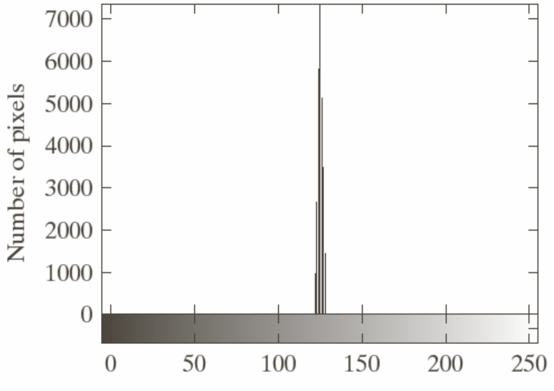
    g_i: i^{th} gray level r_i: run length of the i^{th} run

Irrelevant information

- The eye does not respond with equal sensitivity to all visual information
- Certain information has less relative importance than other information in normal visual processing
- The elimination of visually redundant data results in a loss of quantitative information → lossy data compression method

A computer generated (synthetic) 8-bit image M = N = 256

This image appears homogeneous so we can use its mean value to encode this image



Histogram of the image

Fidelity Criteria

- Quantify the nature and extent of information loss
- Level of information loss can be expressed as a function of the original (input) and compressed-decompressed (output) image

  Given an MxN image $f(x, y)$, its compressed-then-decompressed image $\hat{f}(x, y)$, then the error between corresponding values is given by

$$\boxed{e(x, y) = \hat{f}(x, y) - f(x, y)}$$

Total Error:

$$\boxed{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [\hat{f}(x, y) - f(x, y)]}$$

Fidelity Criteria

- Normally the objective fidelity criterion parameters are as follows:

Root mean square error:

$$e_{rms} = \left[\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [\hat{f}(x,y) - f(x,y)]^2 \right]^{\frac{1}{2}}$$

Mean-square signal-to-noise ratio

$$SNR_{ms} = \frac{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \hat{f}(x,y)^2}{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [\hat{f}(x,y) - f(x,y)]^2}$$

References

- Chapter #8: Digital Image Processing by Rafael C. Gonzales & Richard E. Woods