Digital Image Processing

Chapter # 9 B
Morphological Image Processing

Contents

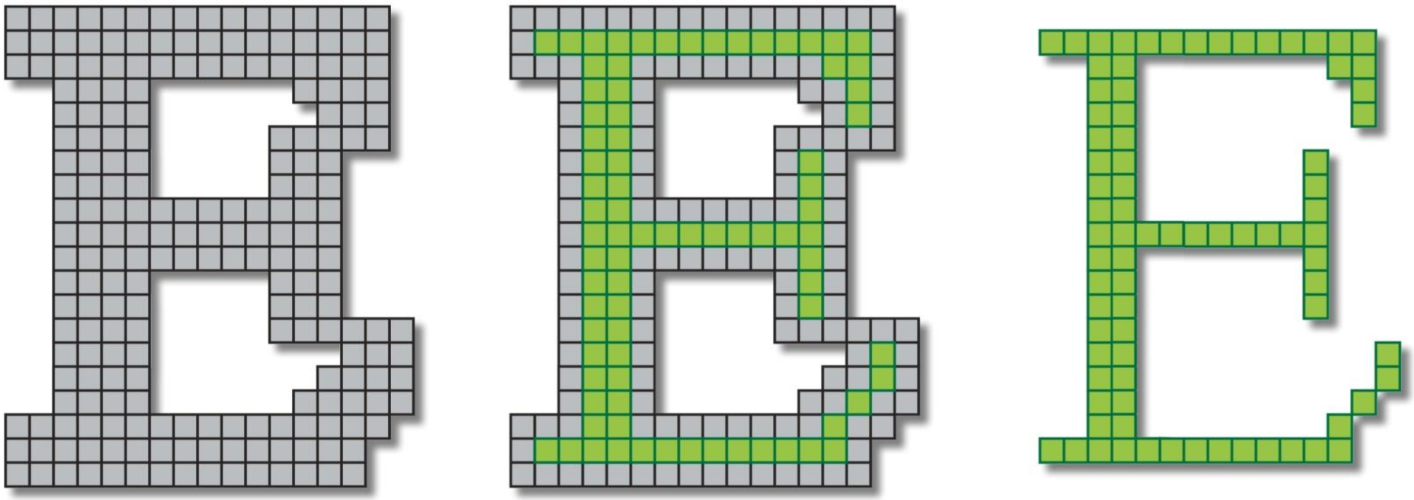
- Fundamental Operations
  - Erosion
  - Dilation
- Compound Operations
  - Opening
  - Closing

Fundamental Operations

- Fundamentally morphological image processing is very like spatial filtering
- The structuring element is moved across every pixel in the original image to give a pixel in a new processed image
- The value of this new pixel depends on the operation performed

There are two basic morphological operations: **erosion** and **dilation**

Erosion



Erosion

Definition 1:

The erosion of two sets A and B is defined as:

$$A! \quad B = \{z \mid (B)_z \subseteq A\}$$

i.e. The Erosion of *A* by *B* is the set of all points *z*, such that B, translated by *z*, is contained in *A*

Erosion

Definition 2:

Erosion of image *f* by structuring element *s* is given by $f \ominus s$

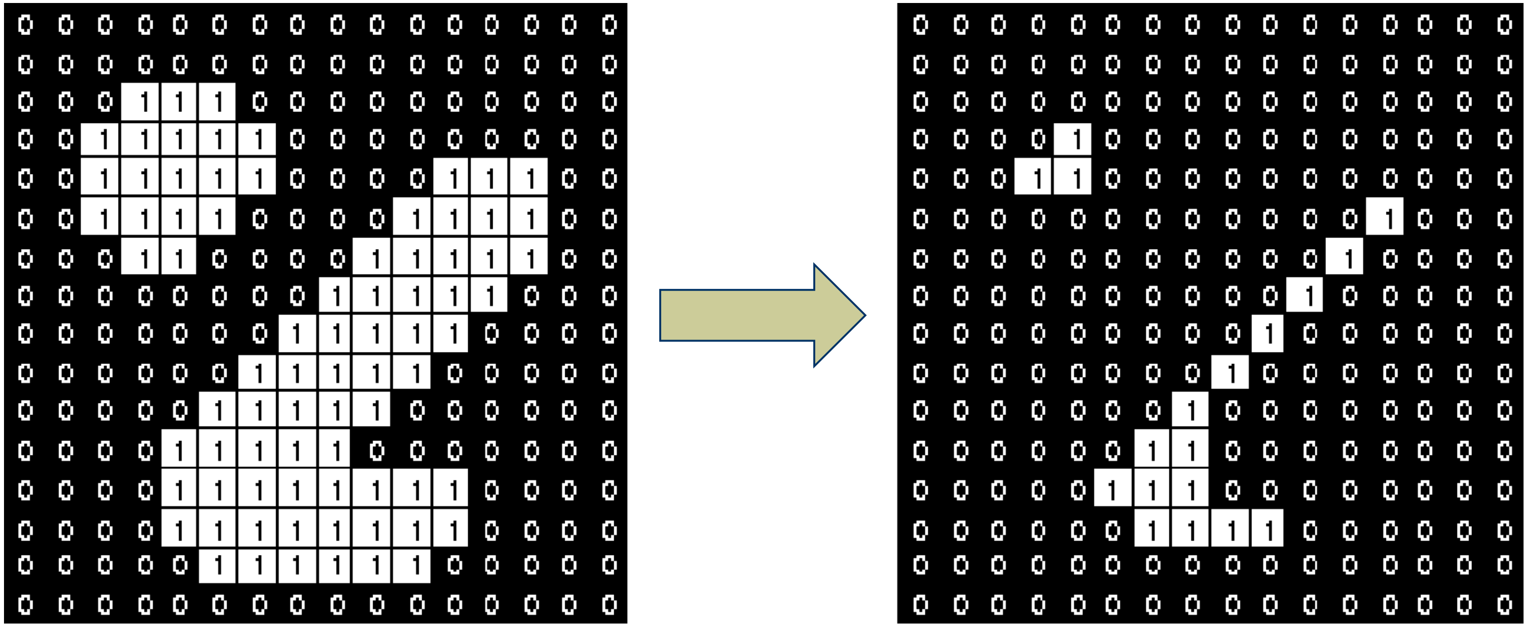
The structuring element s is positioned with its origin at *(x, y)* and the new pixel value is determined using the rule:

$$g(x, y) = \begin{cases} 1 \text{ if } s \text{ fits } f \\ 0 \text{ otherwise} \end{cases}$$

Erosion – How to compute

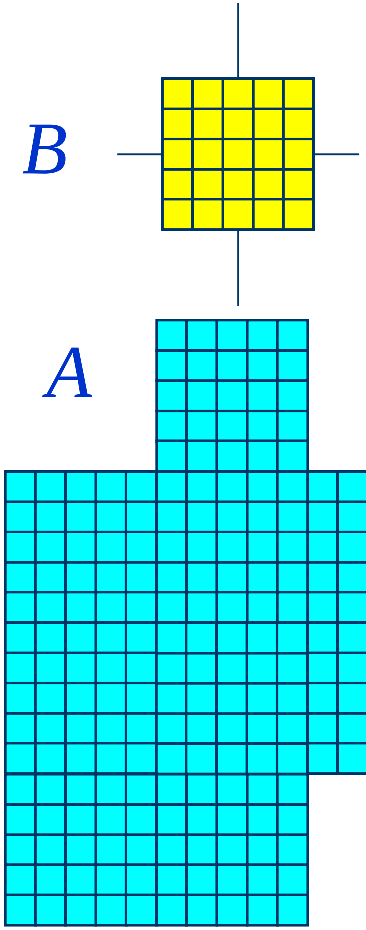
- For each foreground pixel (which we will call the *input pixel*)
  - Superimpose the structuring element on top of the input image so that the origin of the structuring element coincides with the input pixel position.
  - If *for every* pixel in the structuring element, the corresponding pixel in the image underneath is a foreground pixel, then the input pixel is left as it is.
  - If any of the corresponding pixels in the image are background, however, the input pixel is also set to background value.

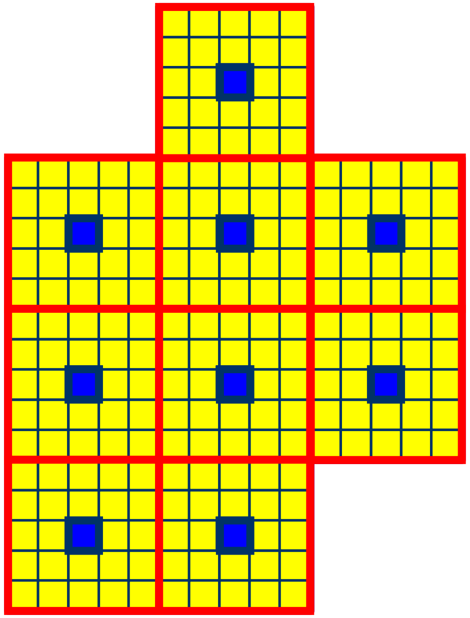
Erosion – How to compute



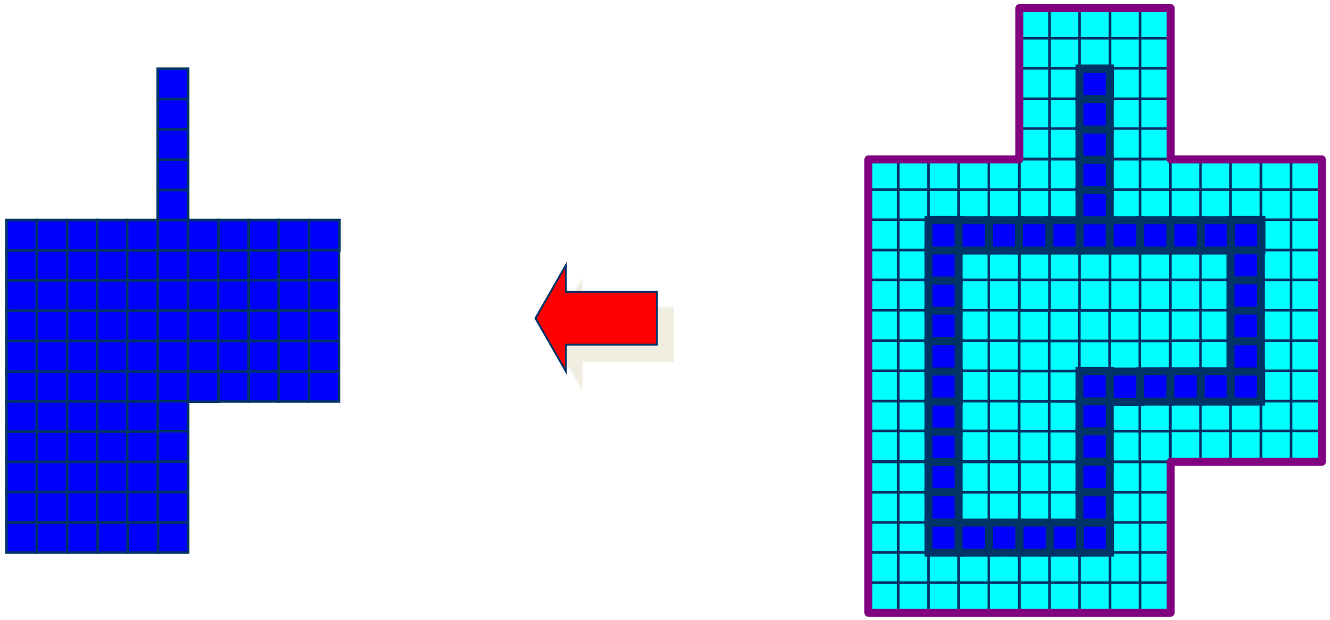
Erosion with a structuring element of size 3x3

Erosion

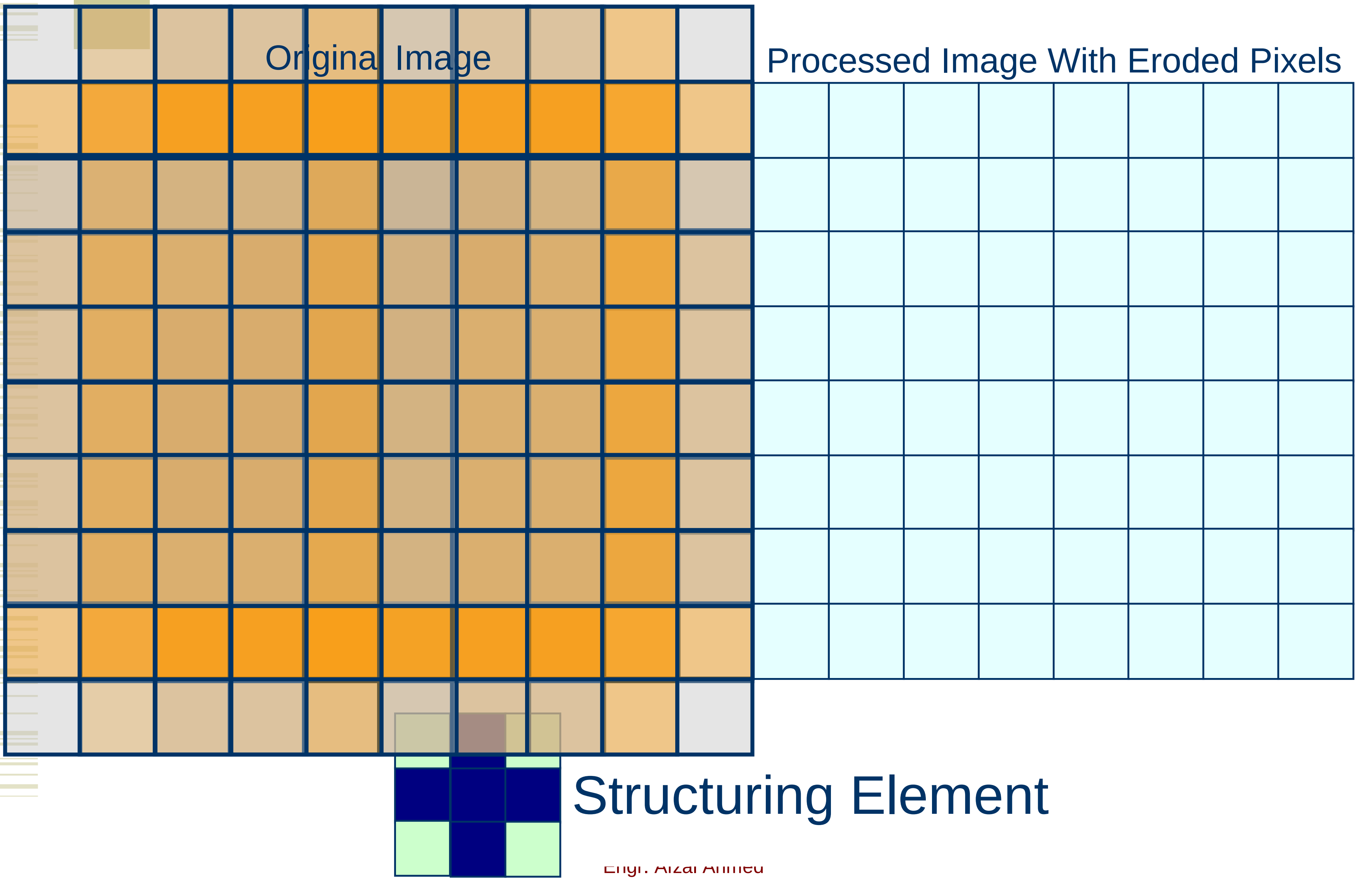




Erosion

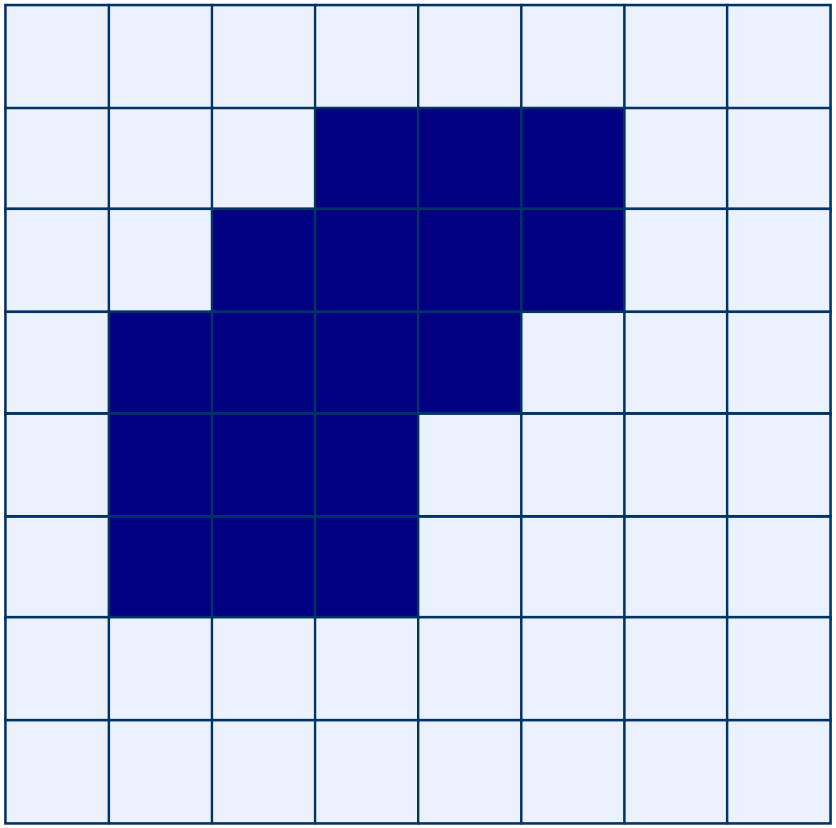


Erosion: Example

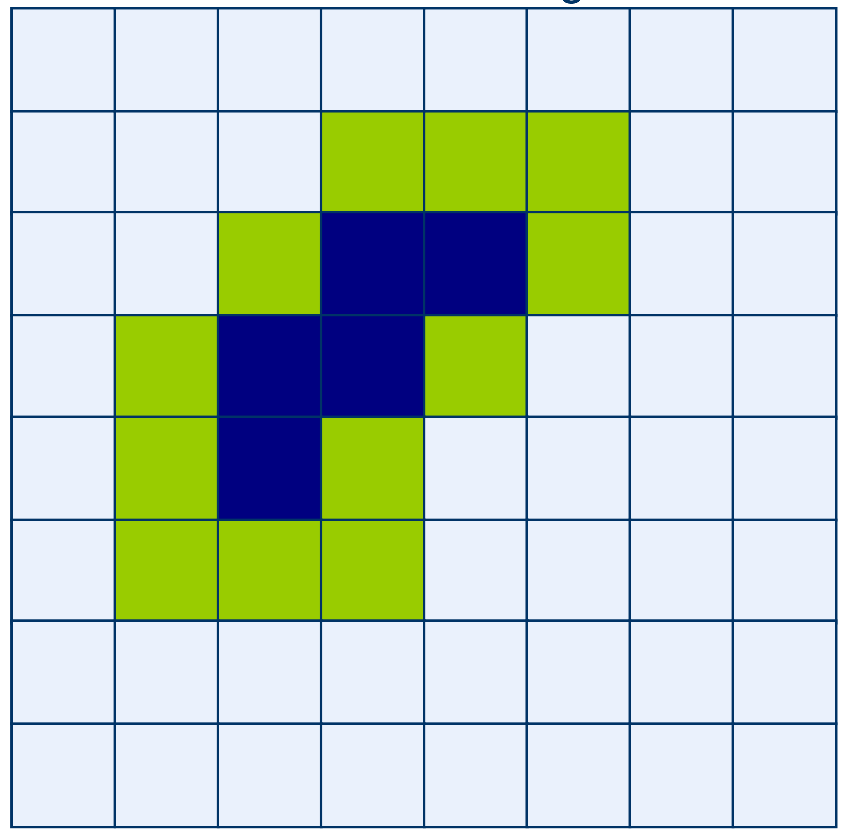


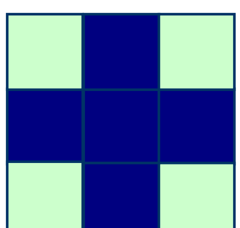
Erosion: Example

Original Image



Processed Image





Structuring Element

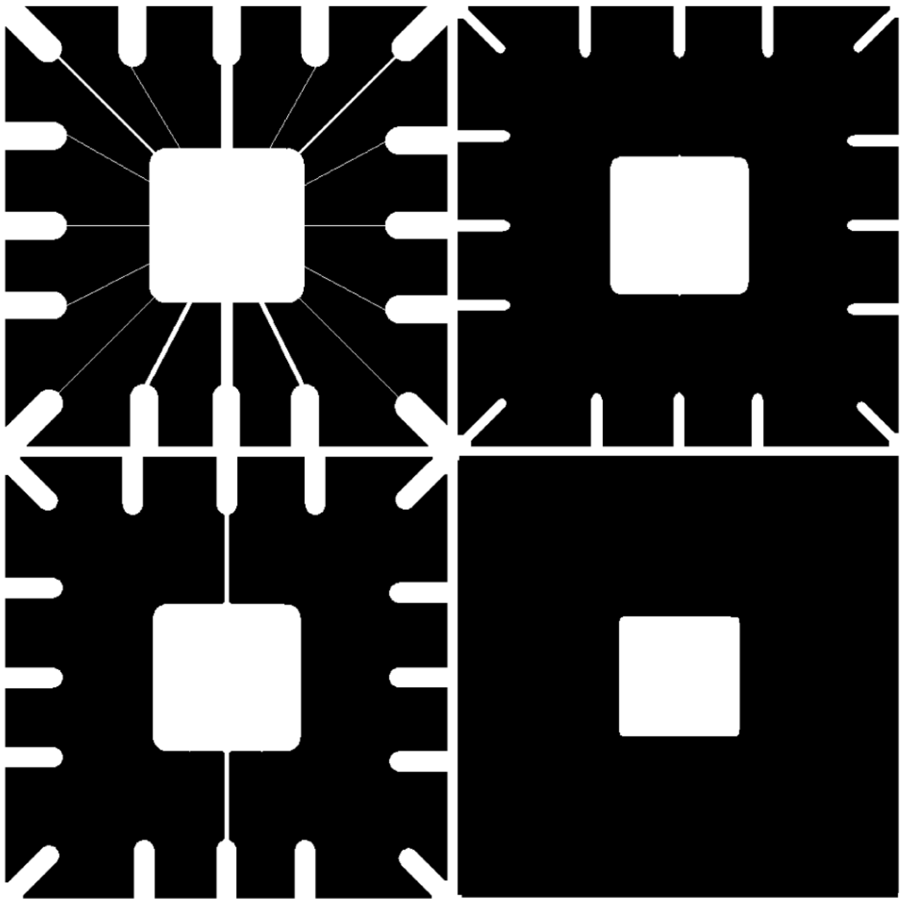
Erosion

- Effects
  - Shrinks the size of foreground (1-valued) objects
  - Smoothes object boundaries
  - Removes small objects

- Rule for Erosion
  In a binary image, if any of the pixel (in the neighborhood defined by structuring element) is 0, then output is 0

Erosion: Example 1

Original image



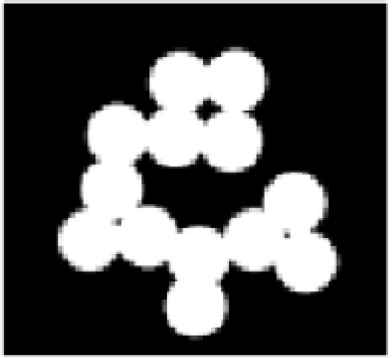
After erosion with a disc of radius 10

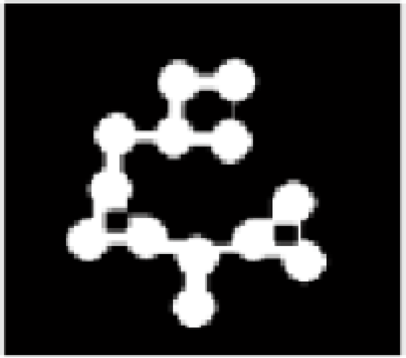
After erosion with a disc of radius 5

After erosion with a disc of radius 20

Erosion: Example 2

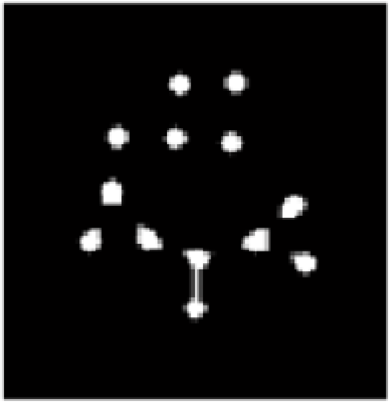
Original binary image *circles*

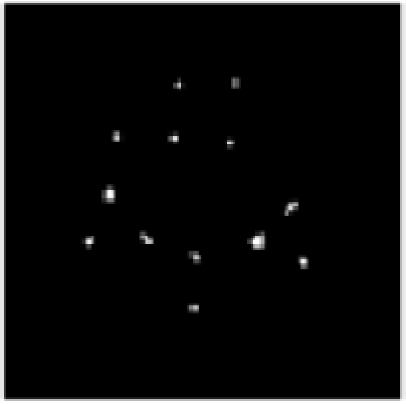




Erosion by 11x11 structuring element

Erosion by 21x21 structuring element

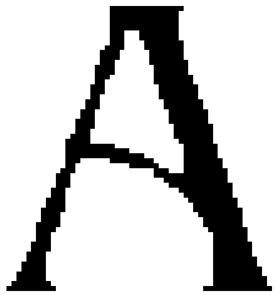




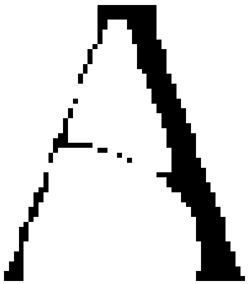
Erosion by 27x27 structuring element

Erosion: Example 3

Original image



Erosion by 3*3 square structuring element



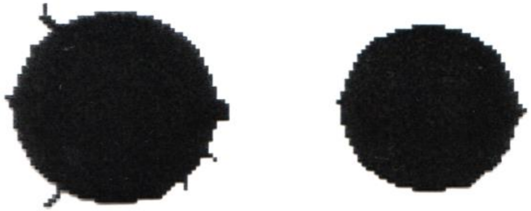
Erosion by 5*5 square structuring element

Note: In these examples a 1 refers to a black pixel!

Erosion

Erosion can split apart joined objects

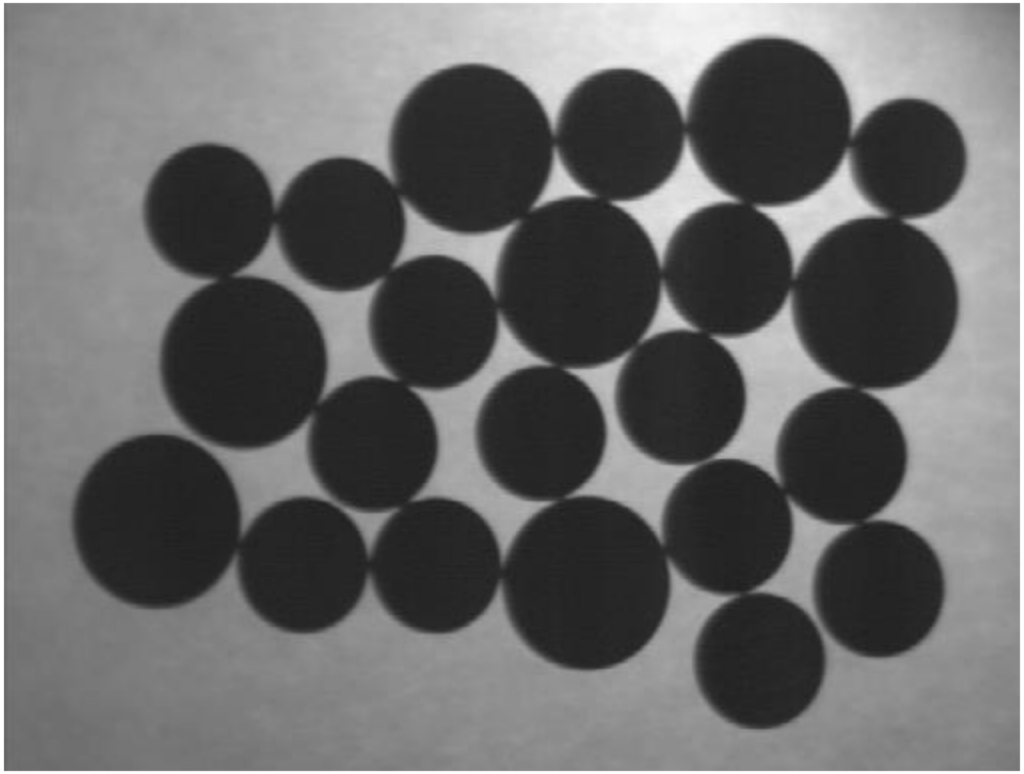
Erosion can strip away extrusions



Watch out: Erosion shrinks objects

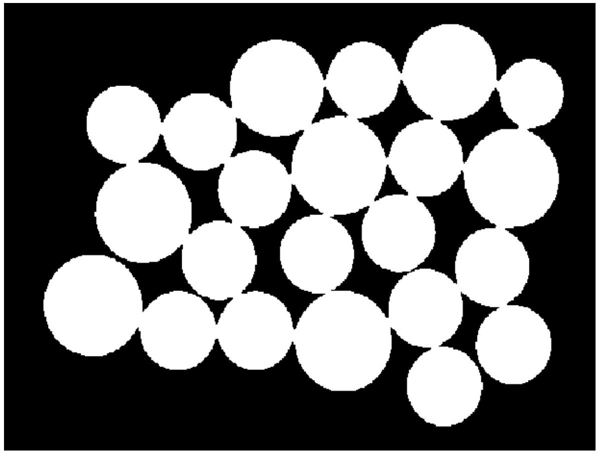
Exercise

Count the number of coins in the given image

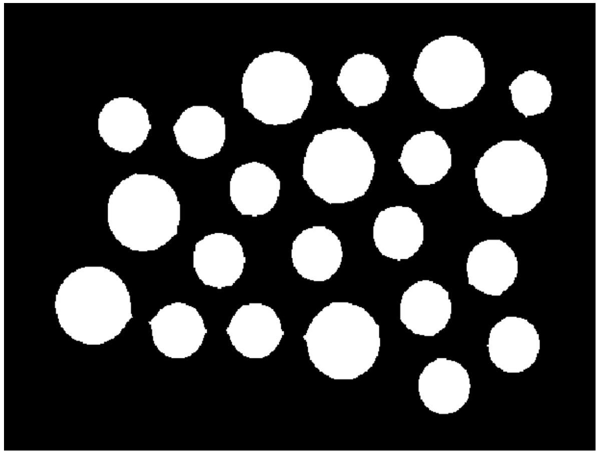


Exercise: Solution

Binarize the image

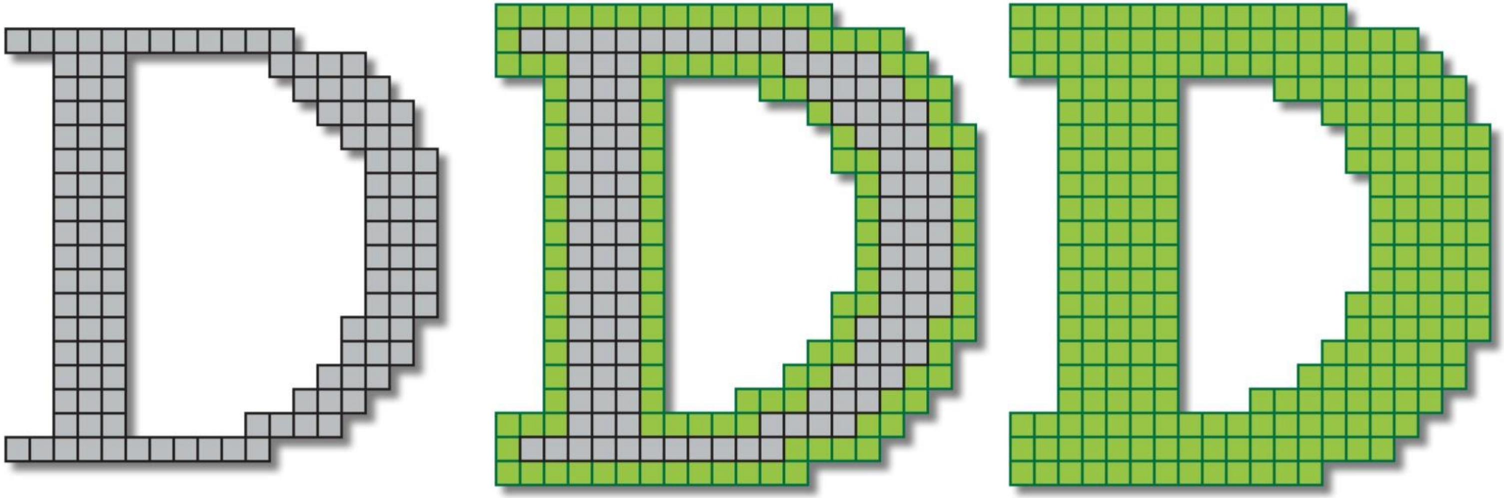


Perform Erosion



Use connected component labeling to count the number of coins

Dilation



Dilation

Definition 1:

The dilation of two sets A and B is defined as:

$$A \oplus B = \{z \mid (B)_z \cap A \neq \varnothing\}$$

i.e. when the reflection of set *B* about its origin is shifted by *z*, the dilation of *A* by *B* is the set of all displacements such that overlap *A* by at least one element

We will only consider symmetric SEs so reflection will have no effect

Dilation

Definition 2:

Dilation of image *f* by structuring element *s* is given by

$f \oplus s$

The structuring element s is positioned with its origin at *(x, y)* and the new pixel value is determined using the rule:

$$g(x,y) = \begin{cases} 1 \text{ if } s \text{ hits } f \\ 0 \text{ otherwise} \end{cases}$$

Dilation – How to compute

- For each background pixel (which we will call the *input pixel*)
  - Superimpose the structuring element on top of the input image so that the origin of the structuring element coincides with the input pixel position

  - If *at least one* pixel in the structuring element coincides with a foreground pixel in the image underneath, then the input pixel is set to the foreground value

  - If all the corresponding pixels in the image are background, however, the input pixel is left at the background value

Dilation: Example

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	1	1	1	0	0
0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0
0	0	0	1	1	0	0	0	0	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0	0	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0
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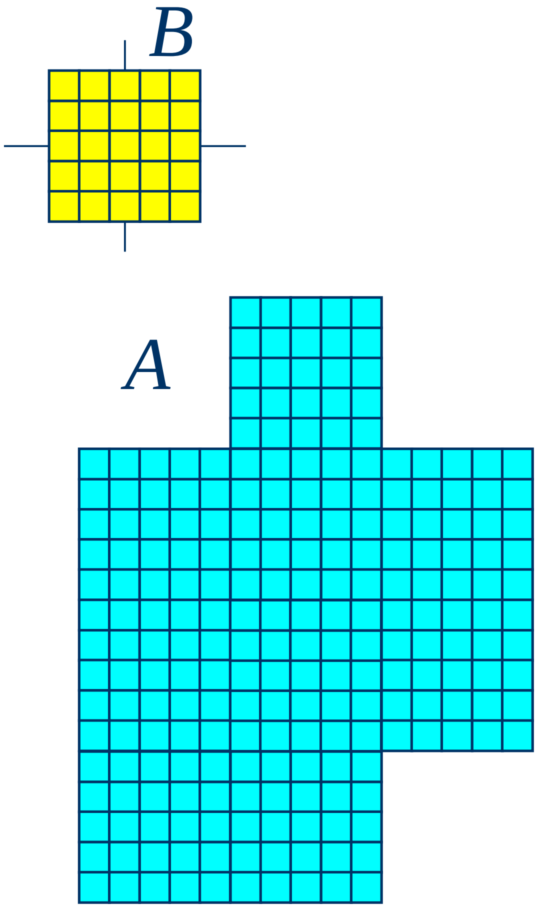
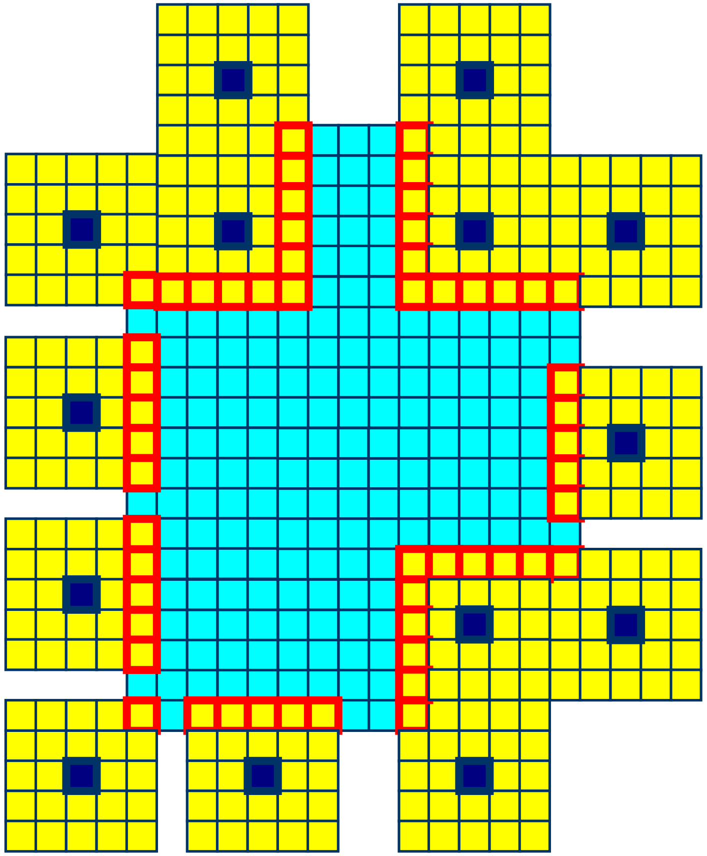
0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	1	1	1	1		0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0

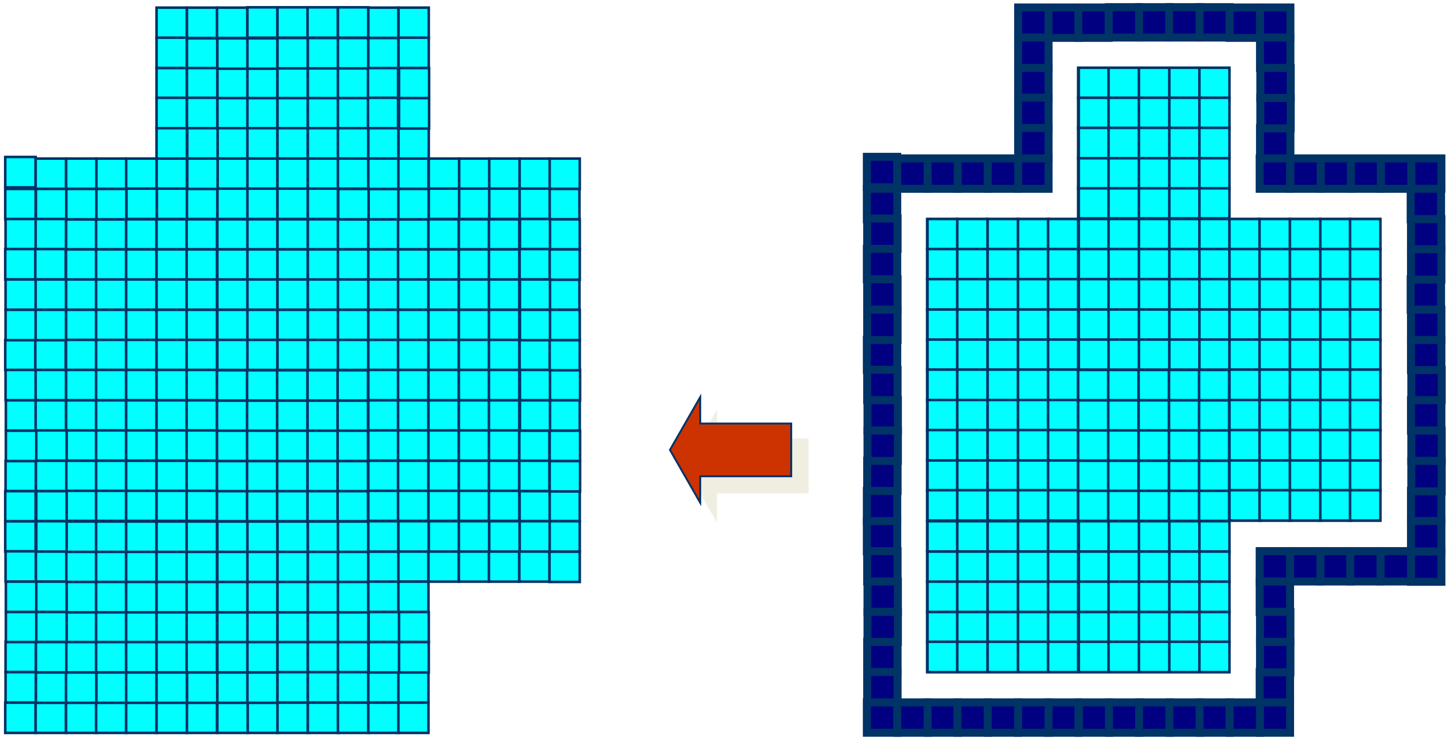
Effect of dilation using a 3×3 square structuring element

Dilation

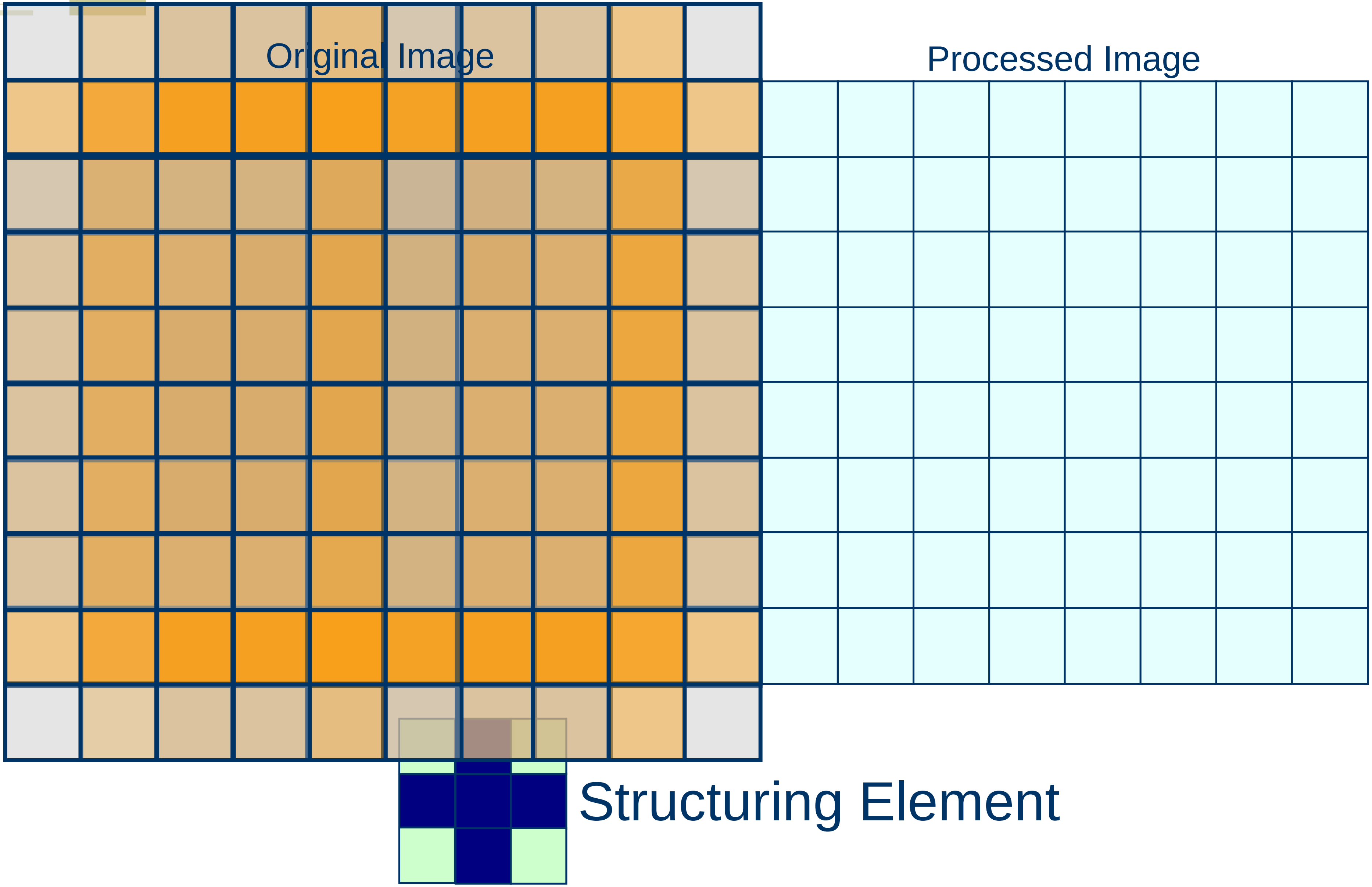




Dilation

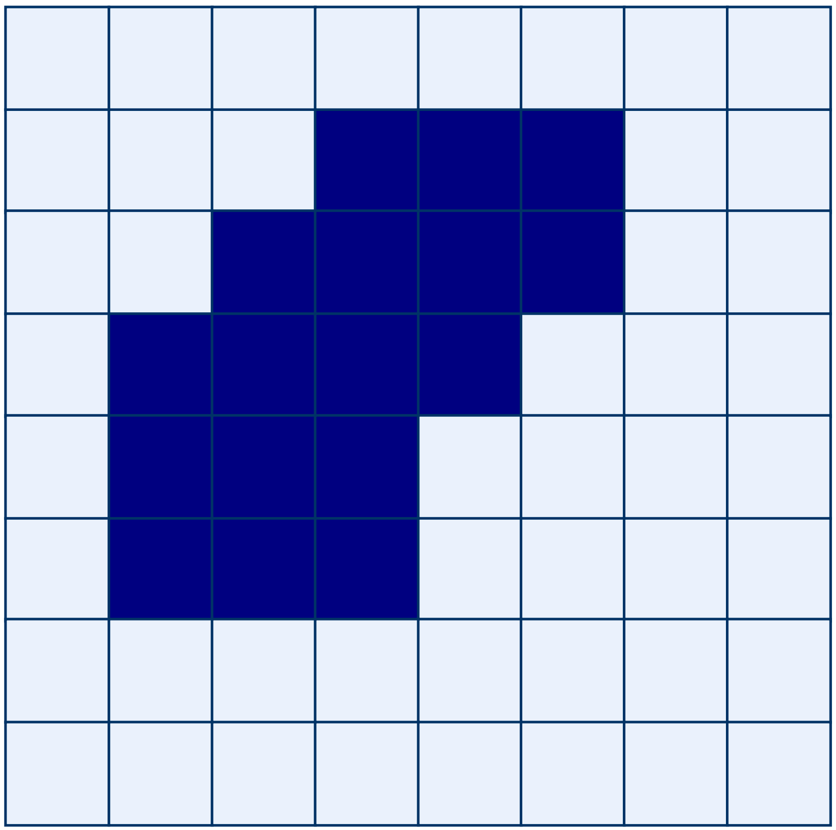


Dilation: Example

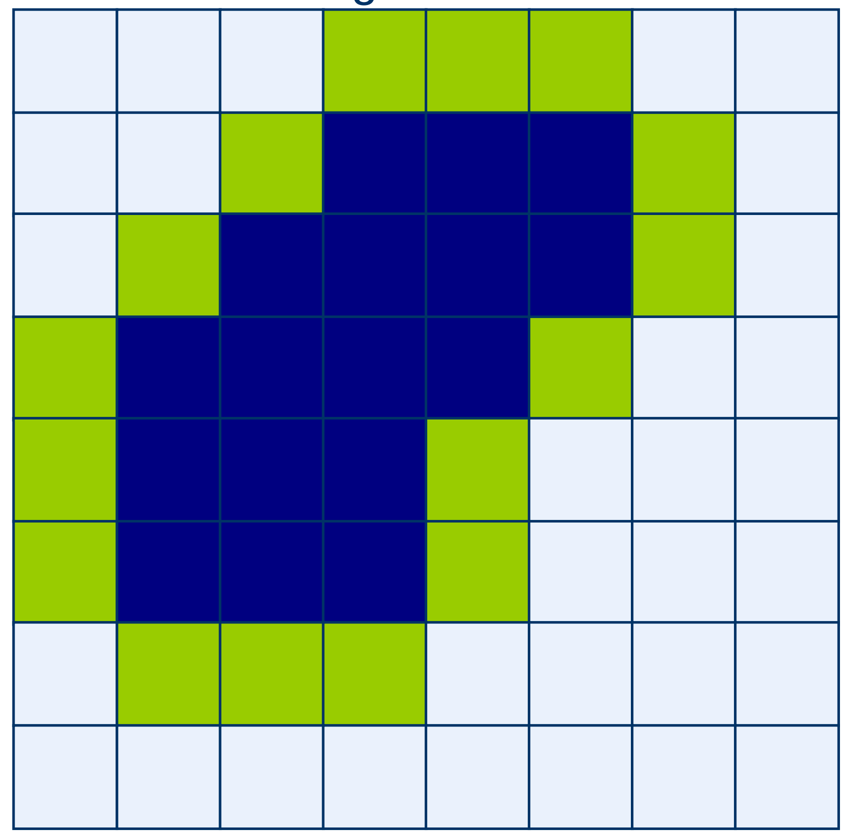


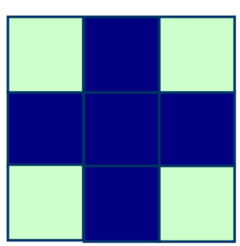
Dilation: Example

Original Image



Processed Image With Dilated Pixels





Structuring Element

Dilation

- Effects
  - Expands the size of foreground(1-valued) objects
  - Smoothes object boundaries
  - Closes holes and gaps

- Rule for Dilation

  In a binary image, if any of the pixel (in the neighborhood defined by structuring element) is 1, then output is 1

Dilation: Example 1

Original image

Dilation by 3*3 square structuring element

Dilation by 5*5 square structuring element

Note: In these examples a 1 refers to a black pixel!

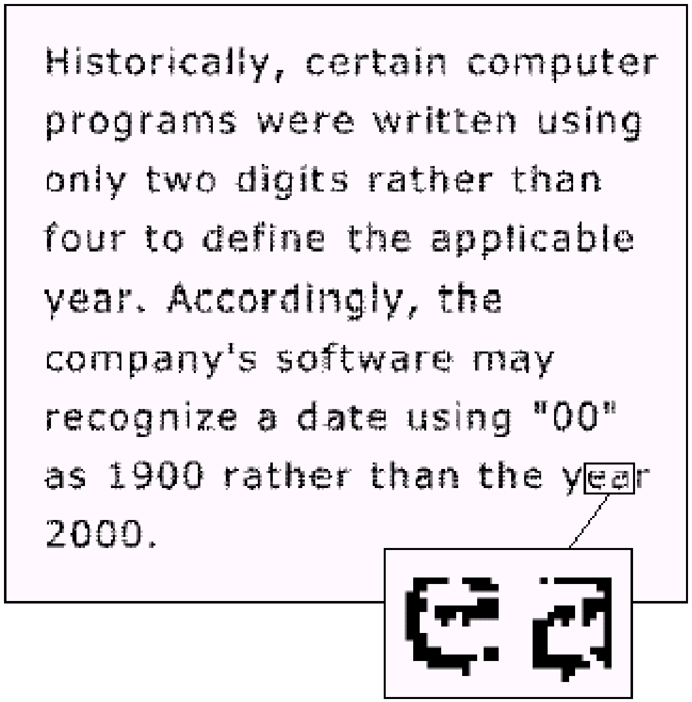
Dilation: Example 2

Original (178x178)

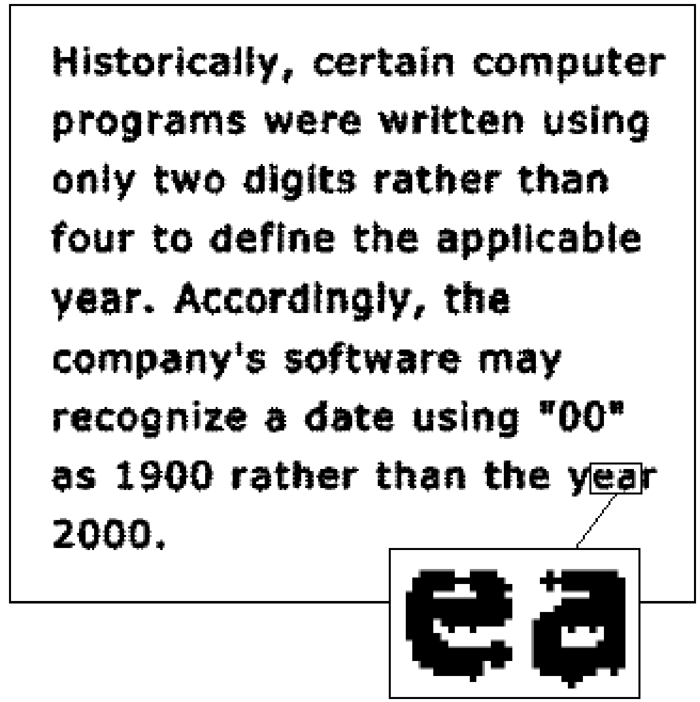
dilation with
3x3 structuring element

dilation with
7x7 structuring element

Dilation: Example 3



Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

Dilation

Dilation can repair breaks

Dilation can repair intrusions

Watch out: Dilation enlarges objects

Duality relationship between Dilation and Erosion

- Dilation and erosion are duals of each other:

$$(A!\ \ B)^c = A^c \oplus B$$

- For a symmetric structuring element:

$$(A!\ \ B)^c = A^c \oplus B$$

It means that we can obtain erosion of an image A by B simply by dilating its background (i.e. A^c) with the same structuring element and complementing the result.

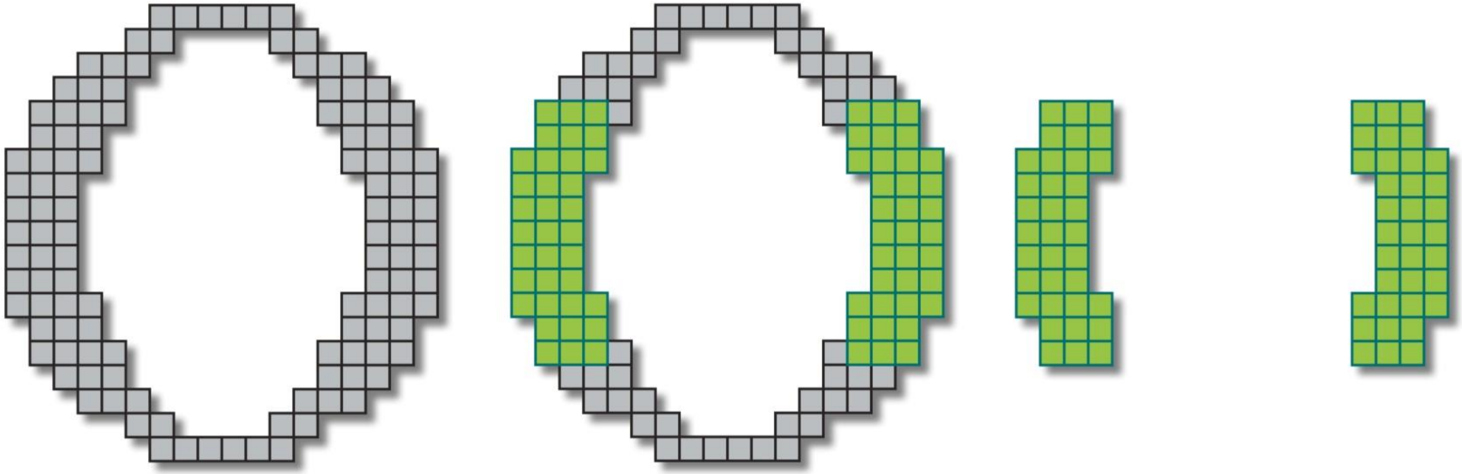
Compound Operations

- More interesting morphological operations can be performed by performing combinations of erosions and dilations

The most widely used of these *compound operations* are:

  - Opening
  - Closing

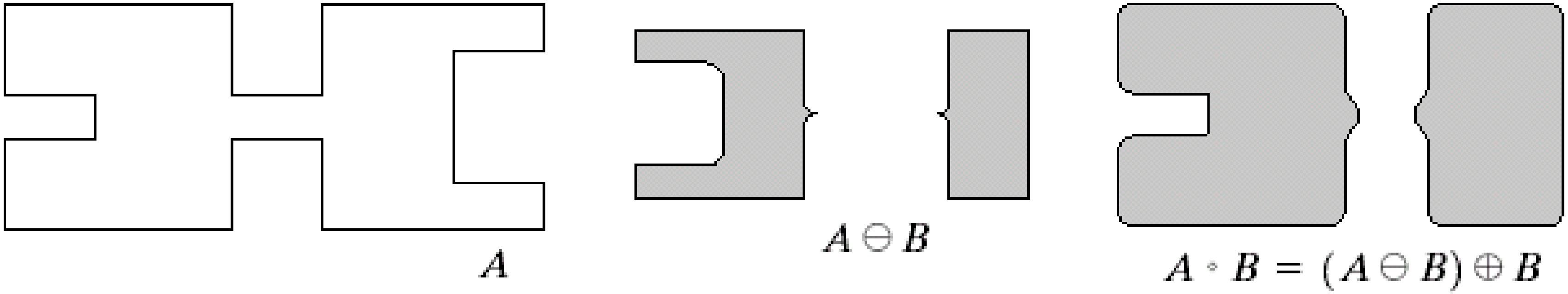
Opening



Opening

The opening of image *f* by structuring element *s*, denoted by $f \circ s$ is simply an erosion followed by a dilation

$$f \circ s = (f \ominus s) \oplus s$$



Original shape | After erosion | After dilation (opening)

Opening: Example

Original Image

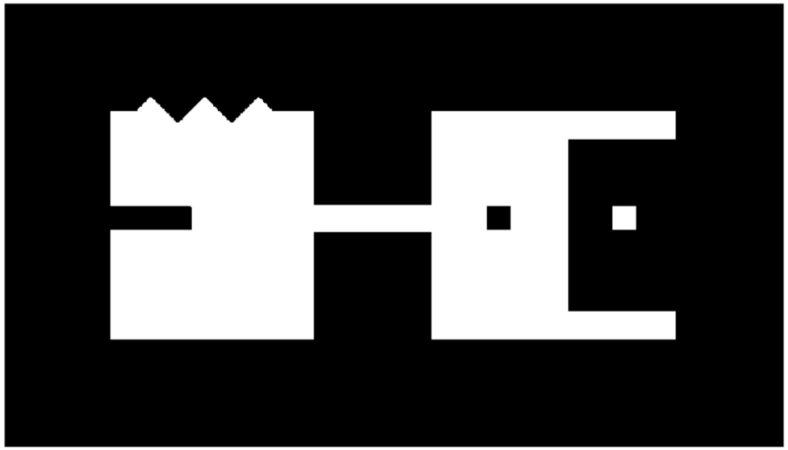


Image After Opening

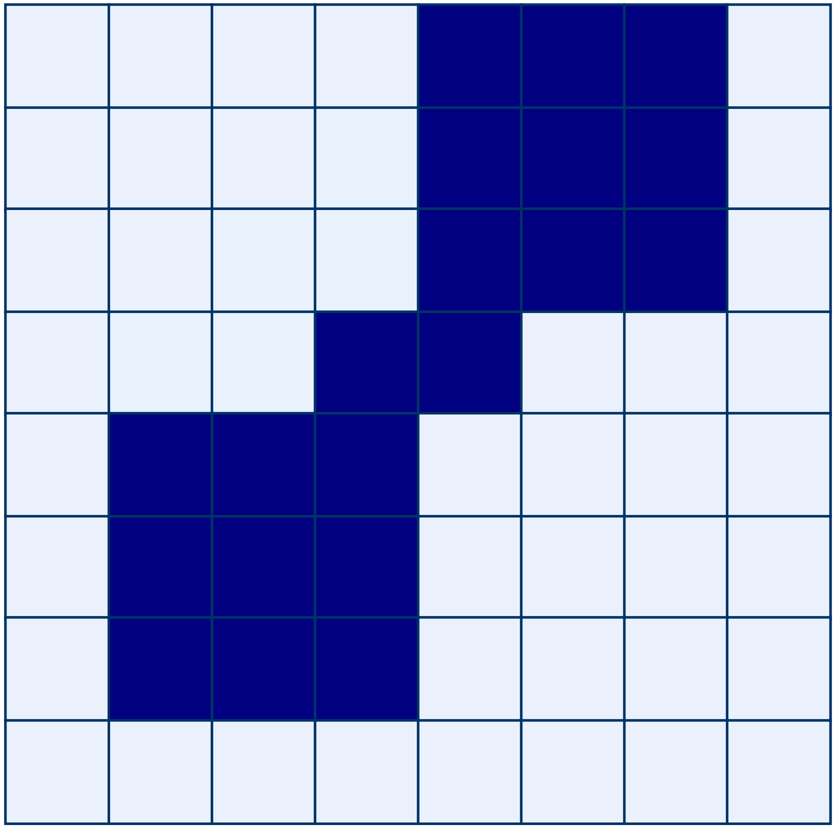
Opening

Breaks narrow joints

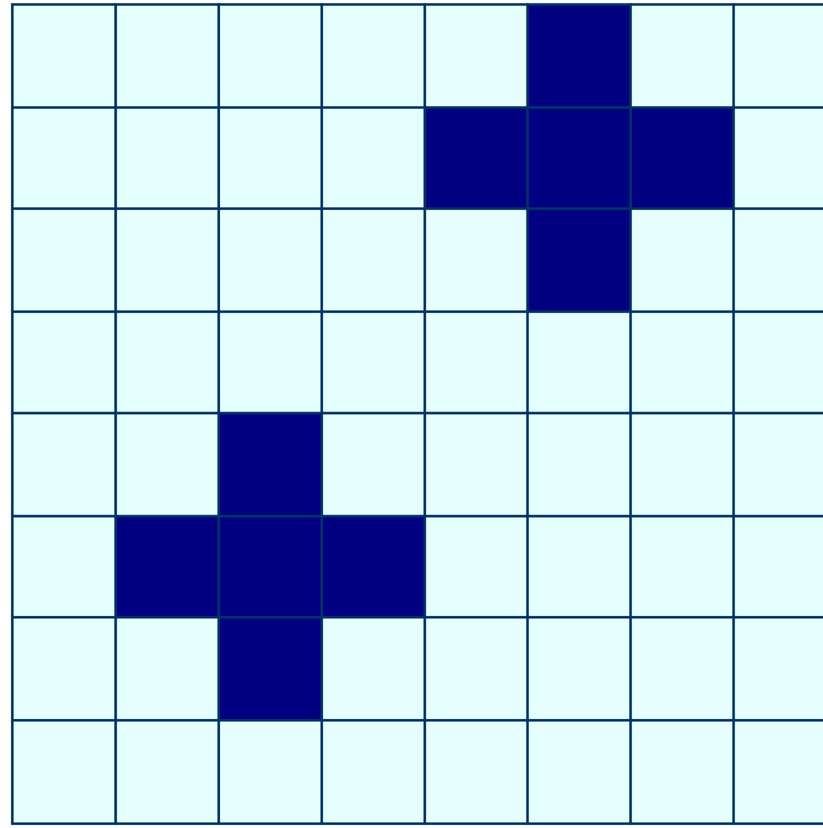
Removes ‘Salt’ noise

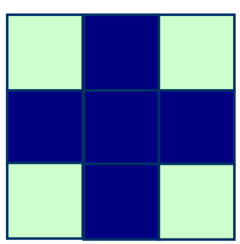
Opening: Example

Original Image



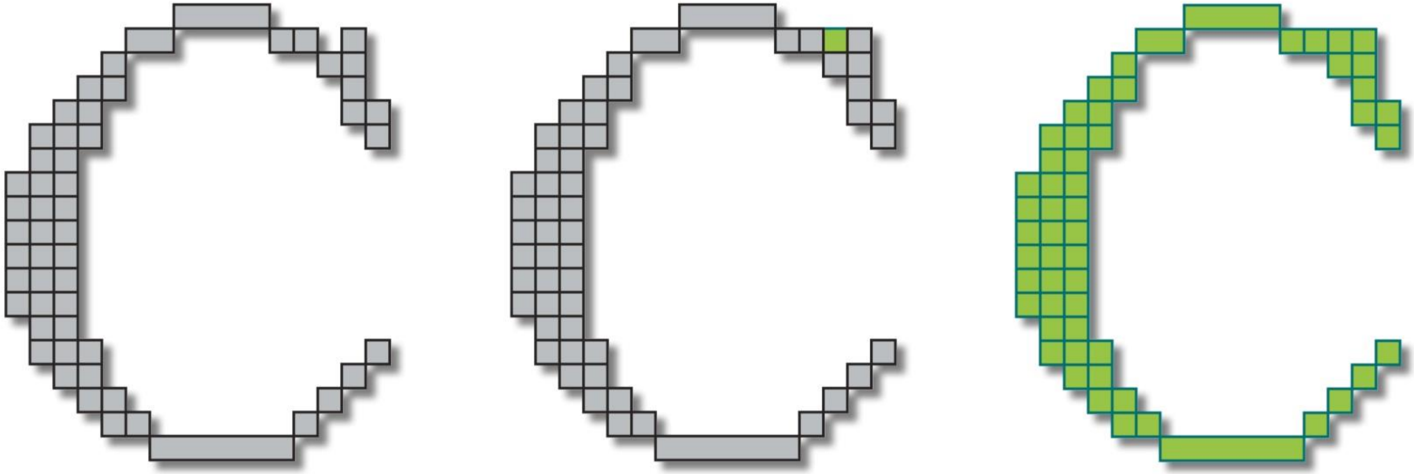
Processed Image





Structuring Element

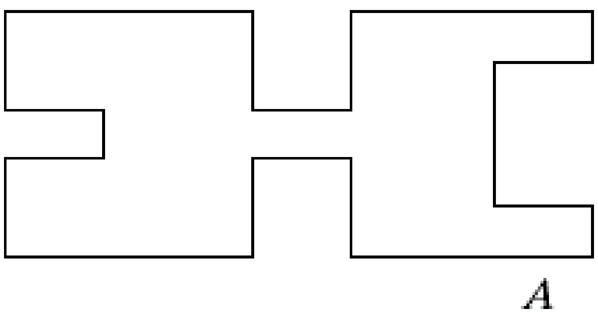
Closing



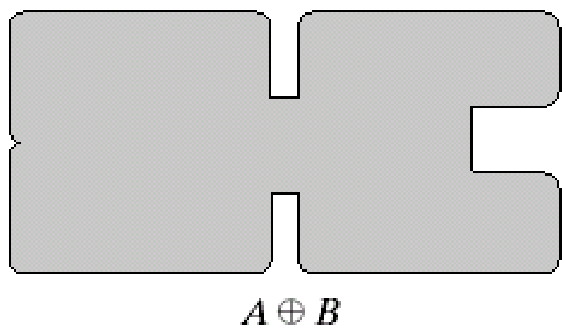
Closing

The closing of image *f* by structuring element *s*, denoted by $f \bullet s$ is simply a dilation followed by an erosion

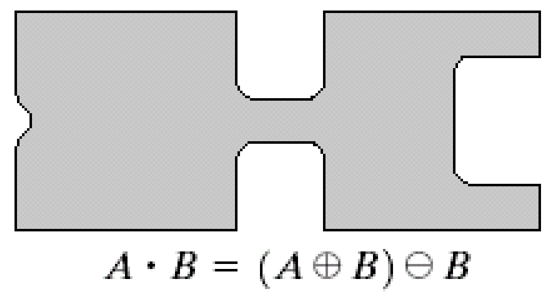
$$f \bullet s = (f \oplus s) \ominus s$$



Original shape



After dilation



After erosion (closing)

Closing: Example

Original Image

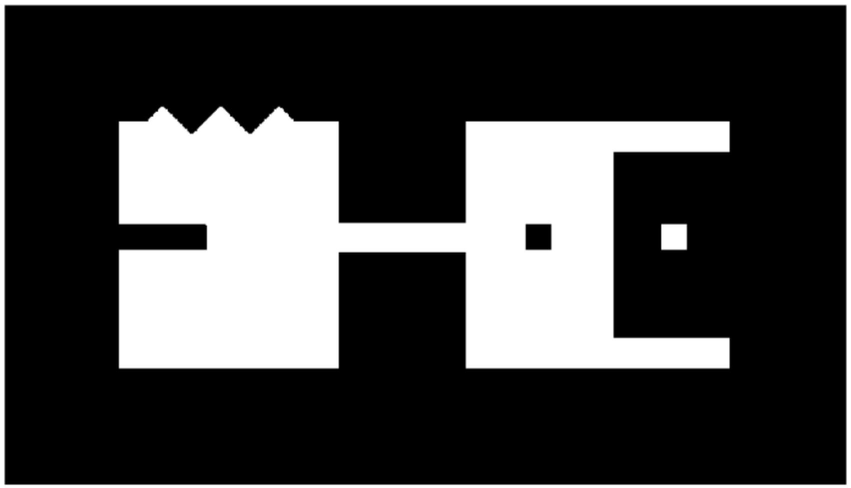


Image After Closing

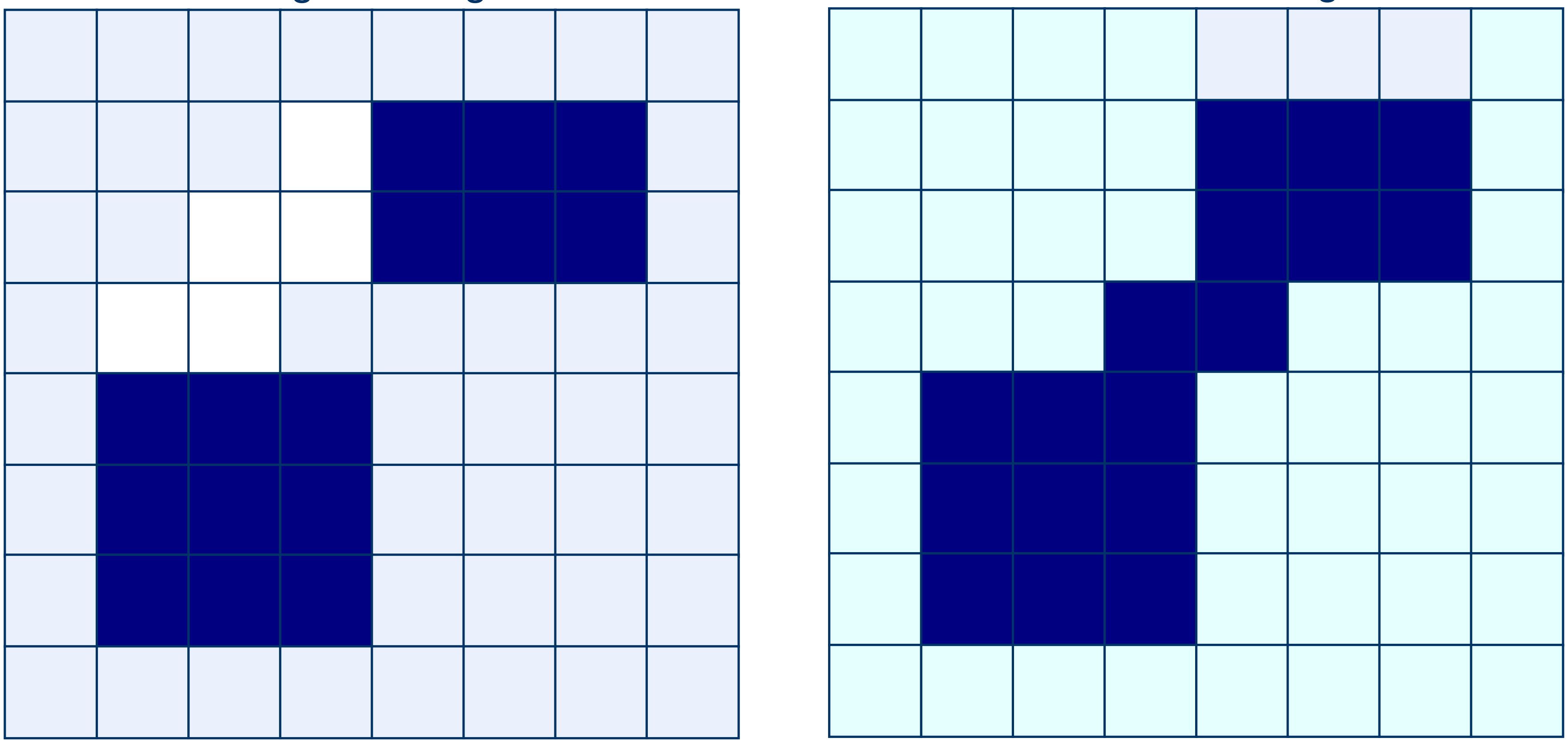
Closing

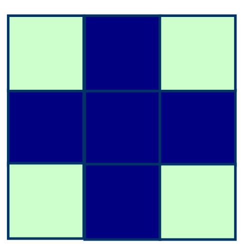
- Eliminates small holes
- Fills gaps
- Removes 'Pepper' noise

Closing: Example

Original Image

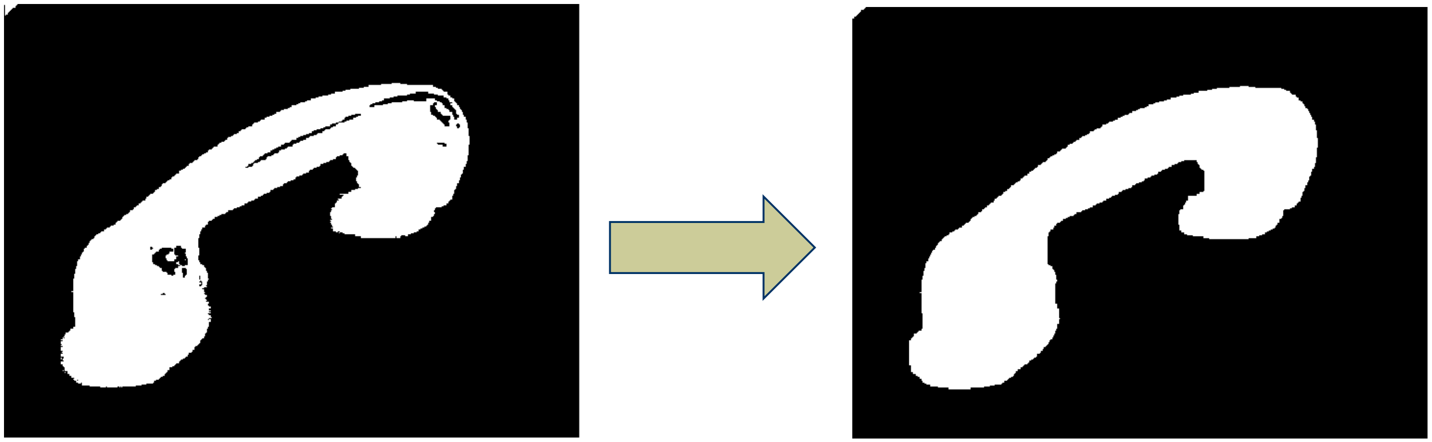
Processed Image





Structuring Element

Closing



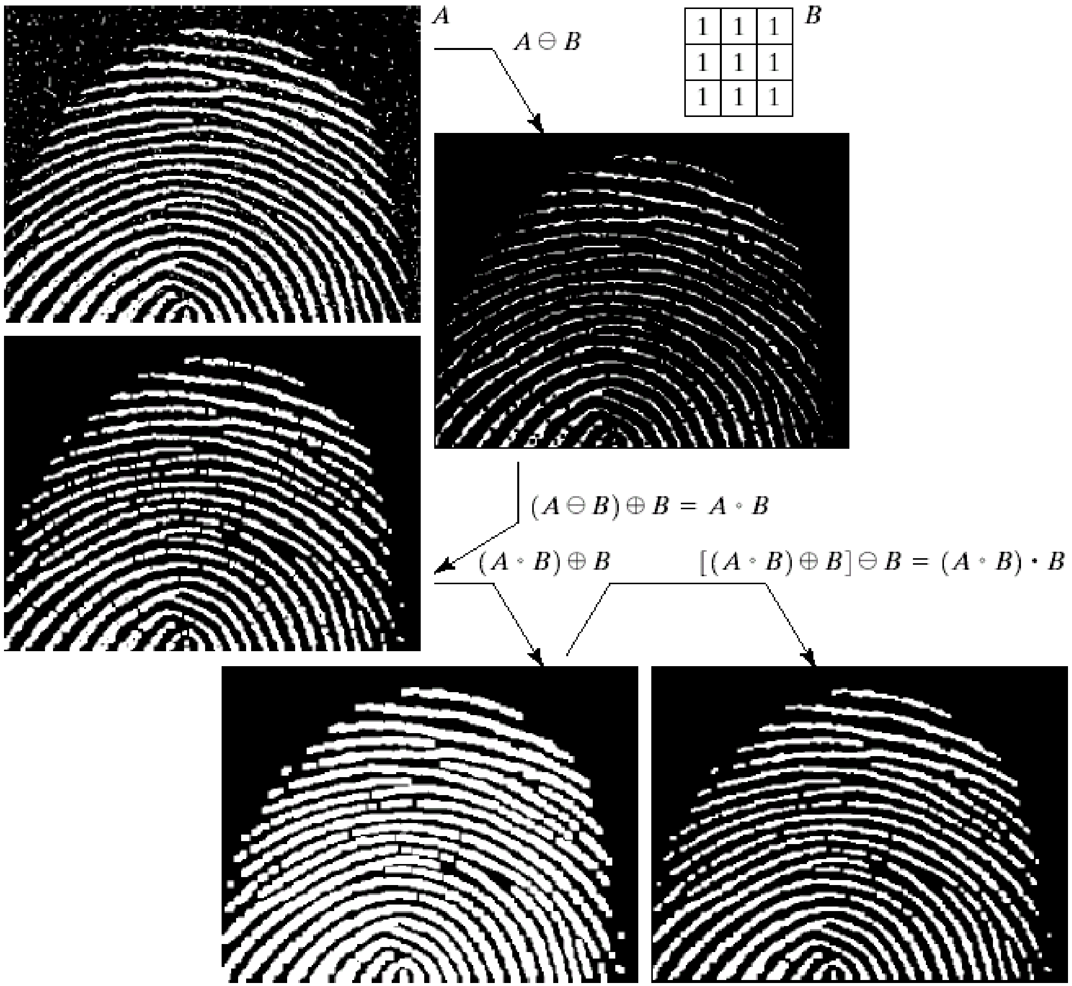
Opening & Closing

- Opening and closing are duals of each others with respect to set complementation and reflection

$$(A \bullet B)^c = (A^c \circ B)$$

$$(A \circ B)^c = (A^c \bullet B)$$

Morphological Processing Example



a b
d c
e f

FIGURE 9.11
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of A.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)

Acknowledgements

- Digital Image Processing", Rafael C. Gonzalez & Richard E. Woods, Addison-Wesley, 2002
- Peters, Richard Alan, II, Lectures on Image Processing, Vanderbilt University, Nashville, TN, April 2008
- Brian Mac Namee, Digitial Image Processing, School of Computing, Dublin Institute of Technology
- Computer Vision for Computer Graphics, Mark Borg

Material in these slides has been taken from, the following resources