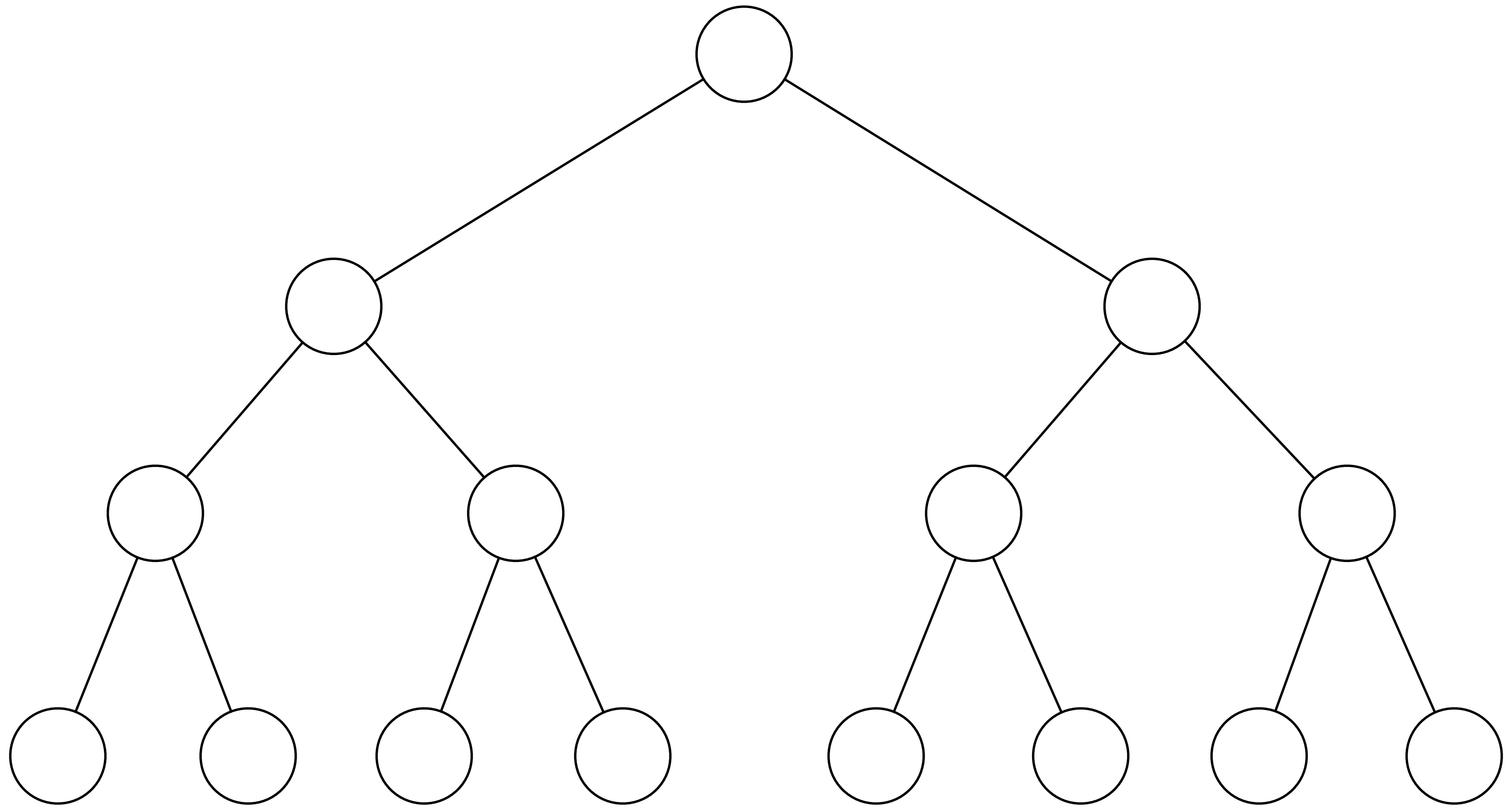
THE UNIVERSITY OF VERMONT
COLLEGE OF ENGINEERING &
MATHEMATICAL SCIENCES

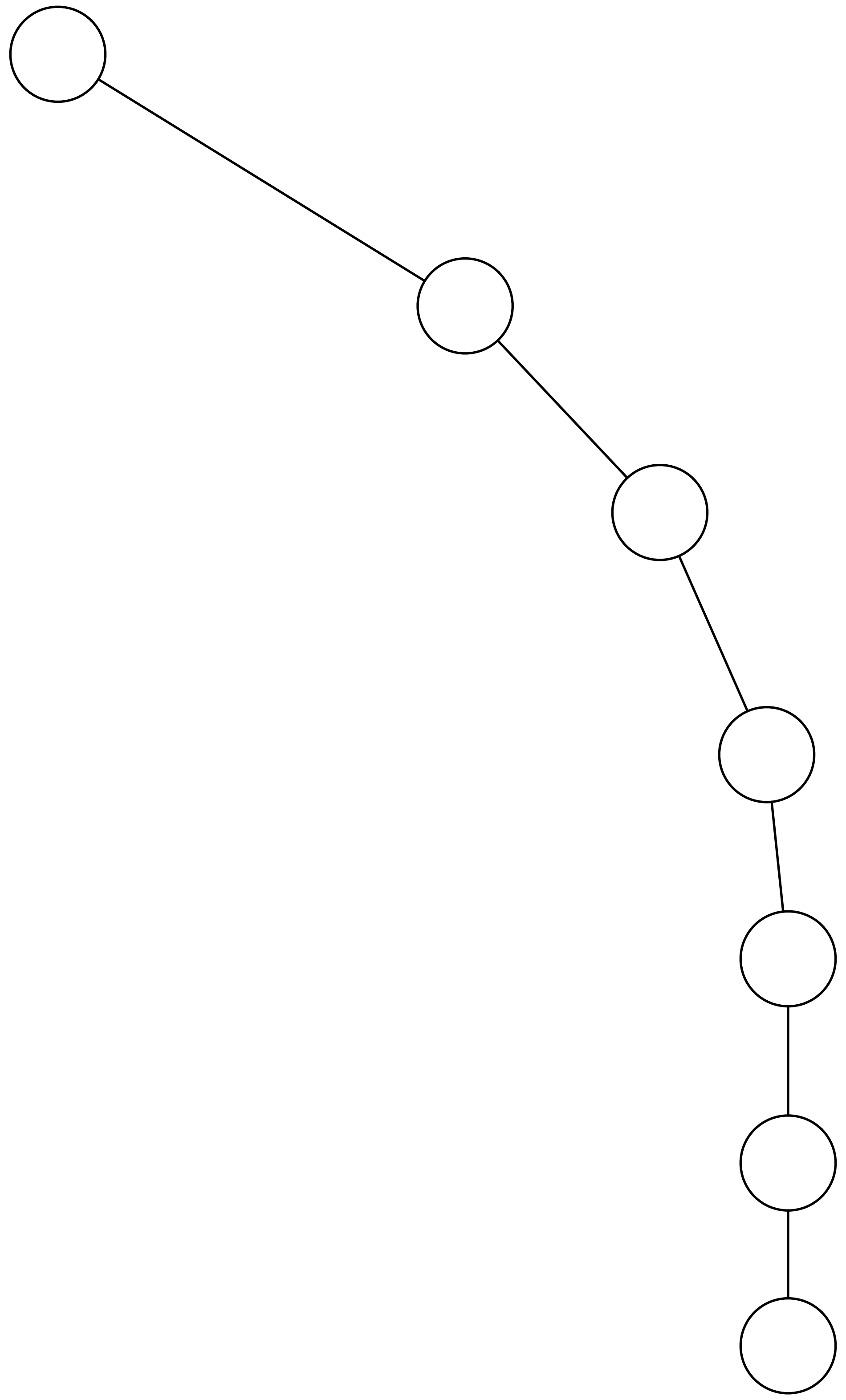
AVL Tree

CS 124 / Department of Computer Science

Complexity of search



Complete or perfect tree?
$O(\log N)$



Pathological tree?
$O(h) = O(N - 1) = O(N)$

AVL Tree

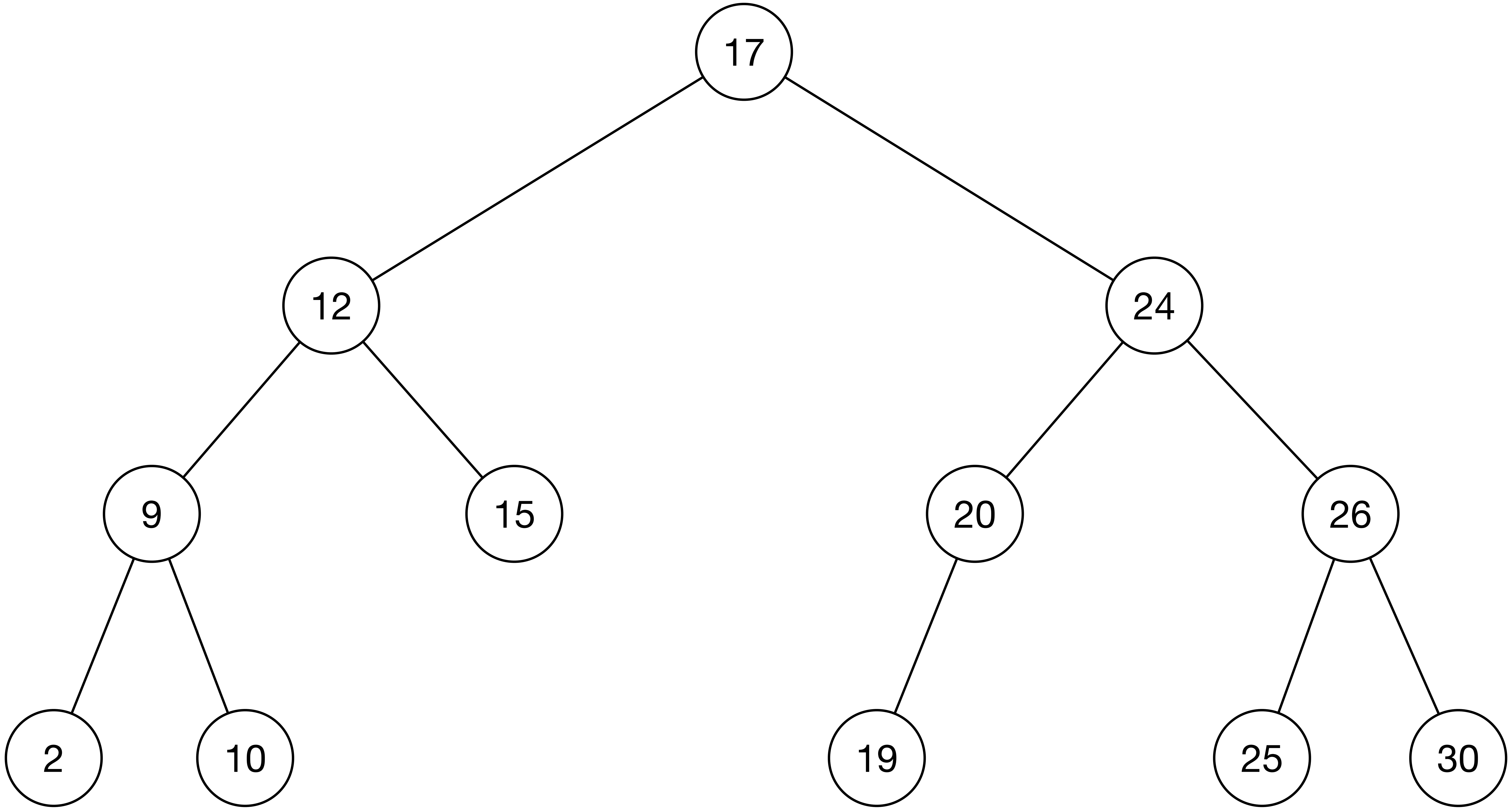
- An AVL tree is a binary search tree with one additional property:

  For each node in the tree, the height of left and right subtrees can differ by at most 1.

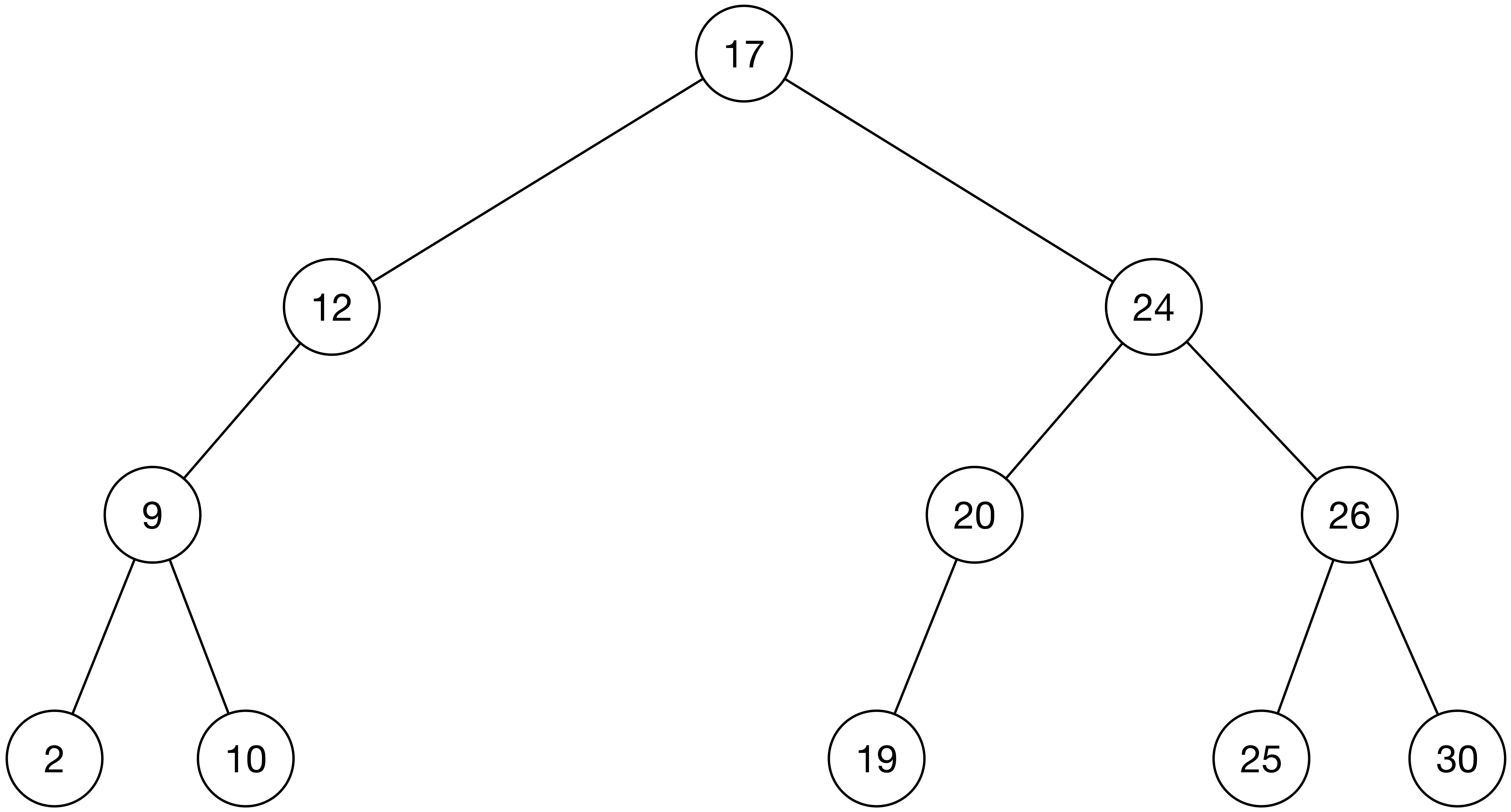
- Why "AVL"? Named after its inventors, Adelson-Velsky and Landis (1962).

- With this property, AVL trees are "self-balancing." This preserves the $O(\log N)$ time for search, insertion and deletion, by avoiding pathological structures and substructures.

AVL Tree Example



AVL Tree Counter-example



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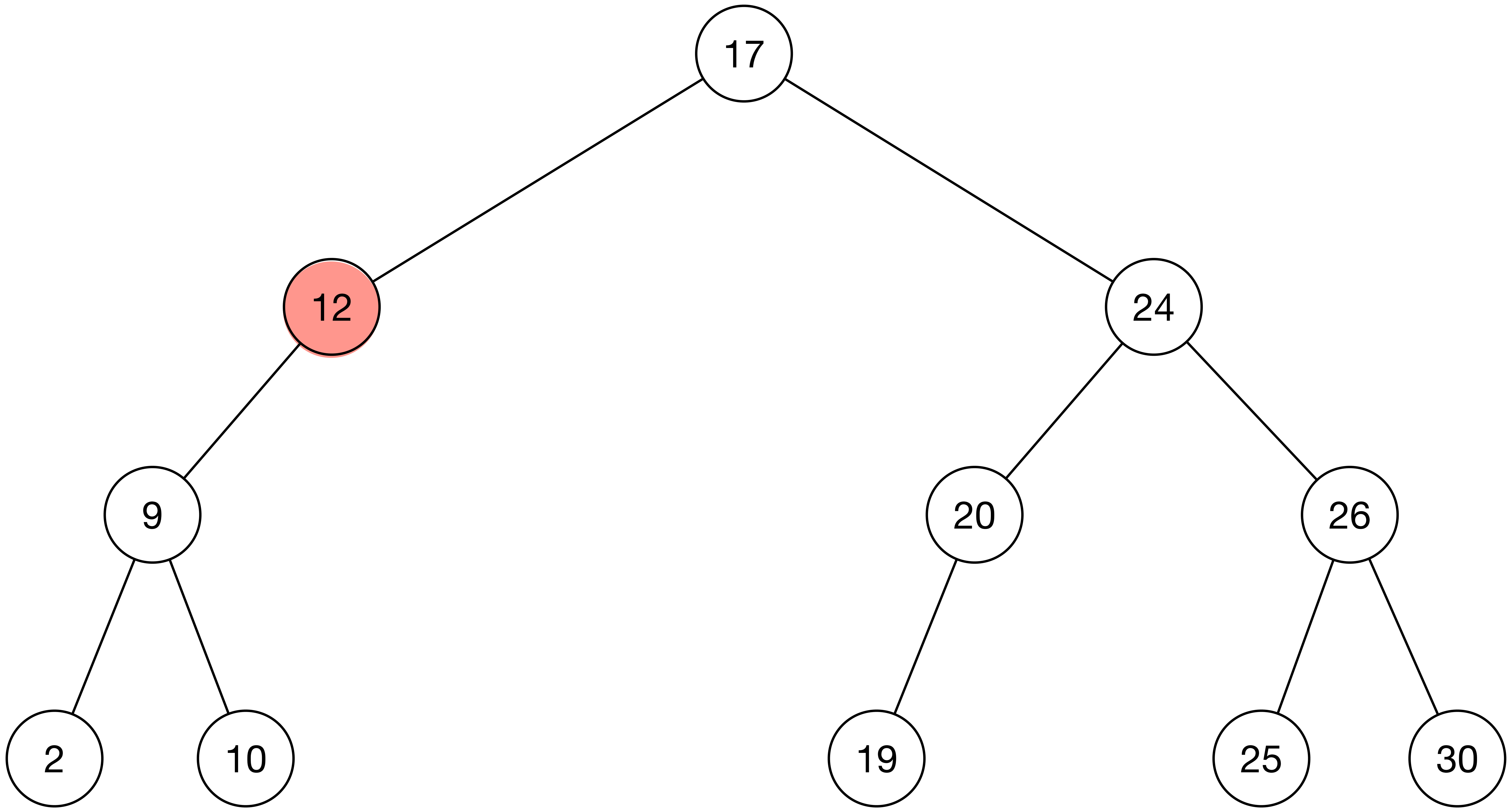
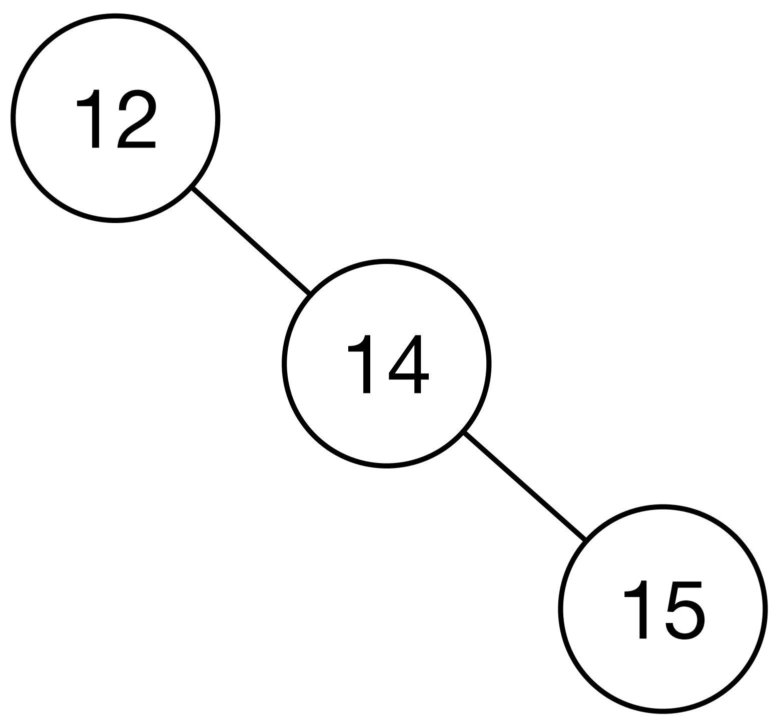
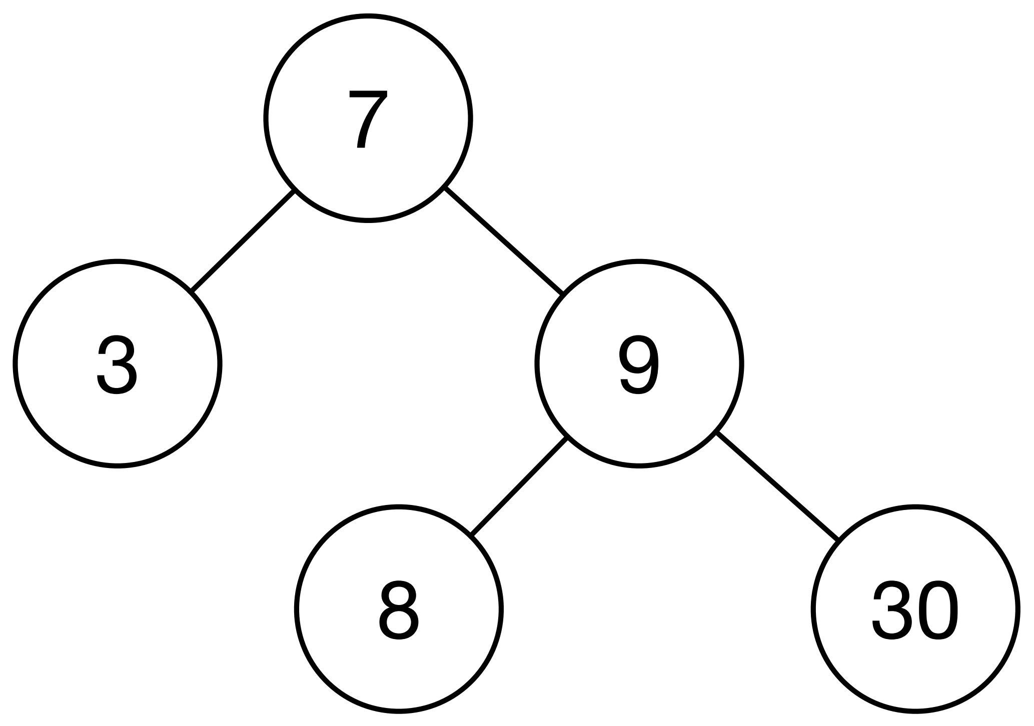
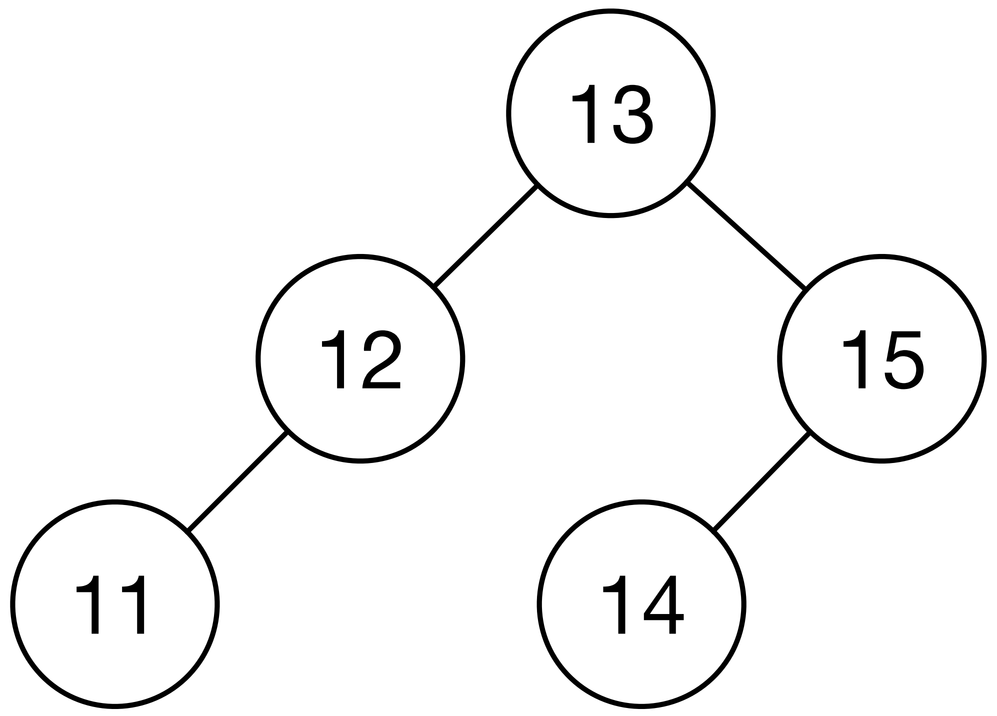
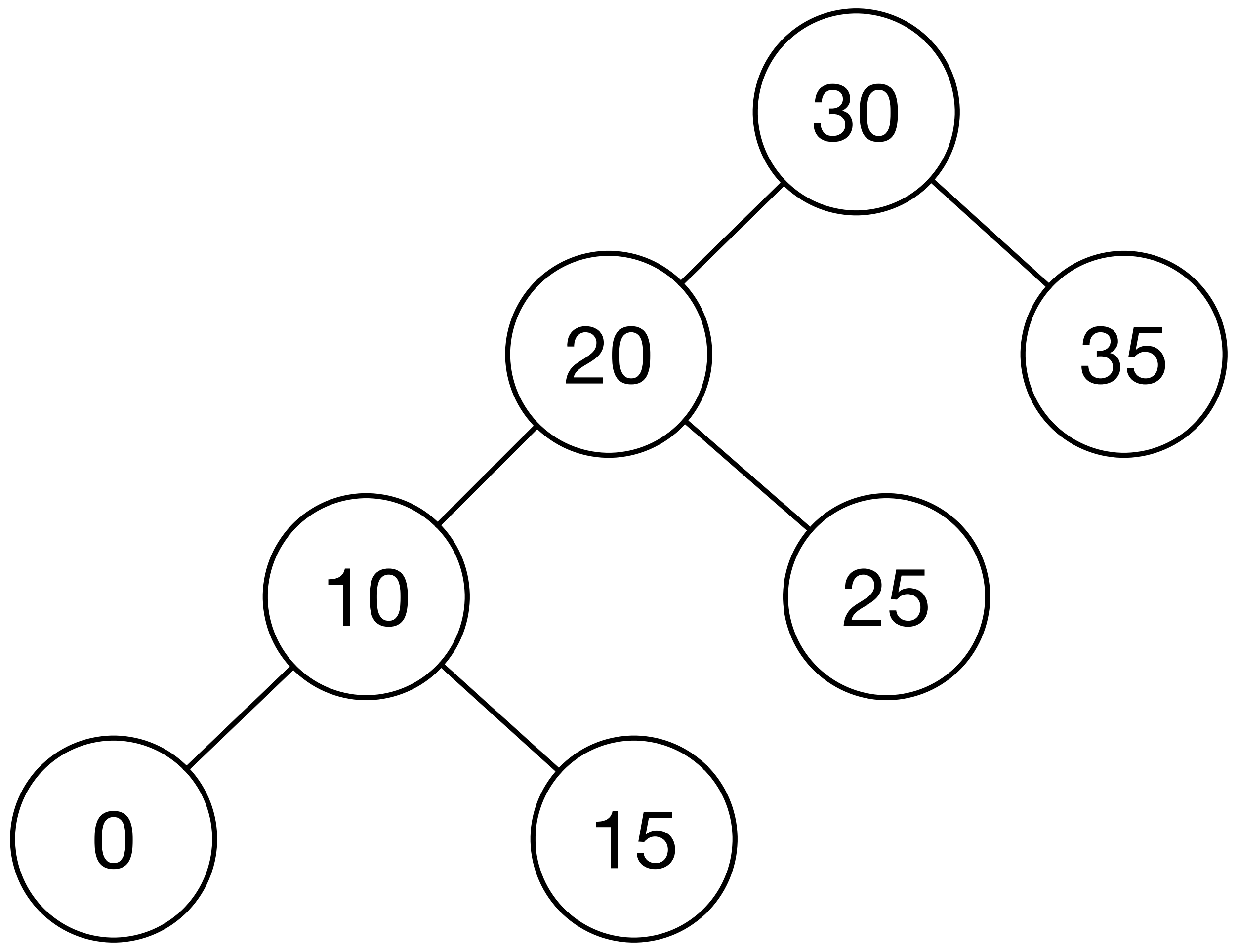


Which of these are AVL trees?









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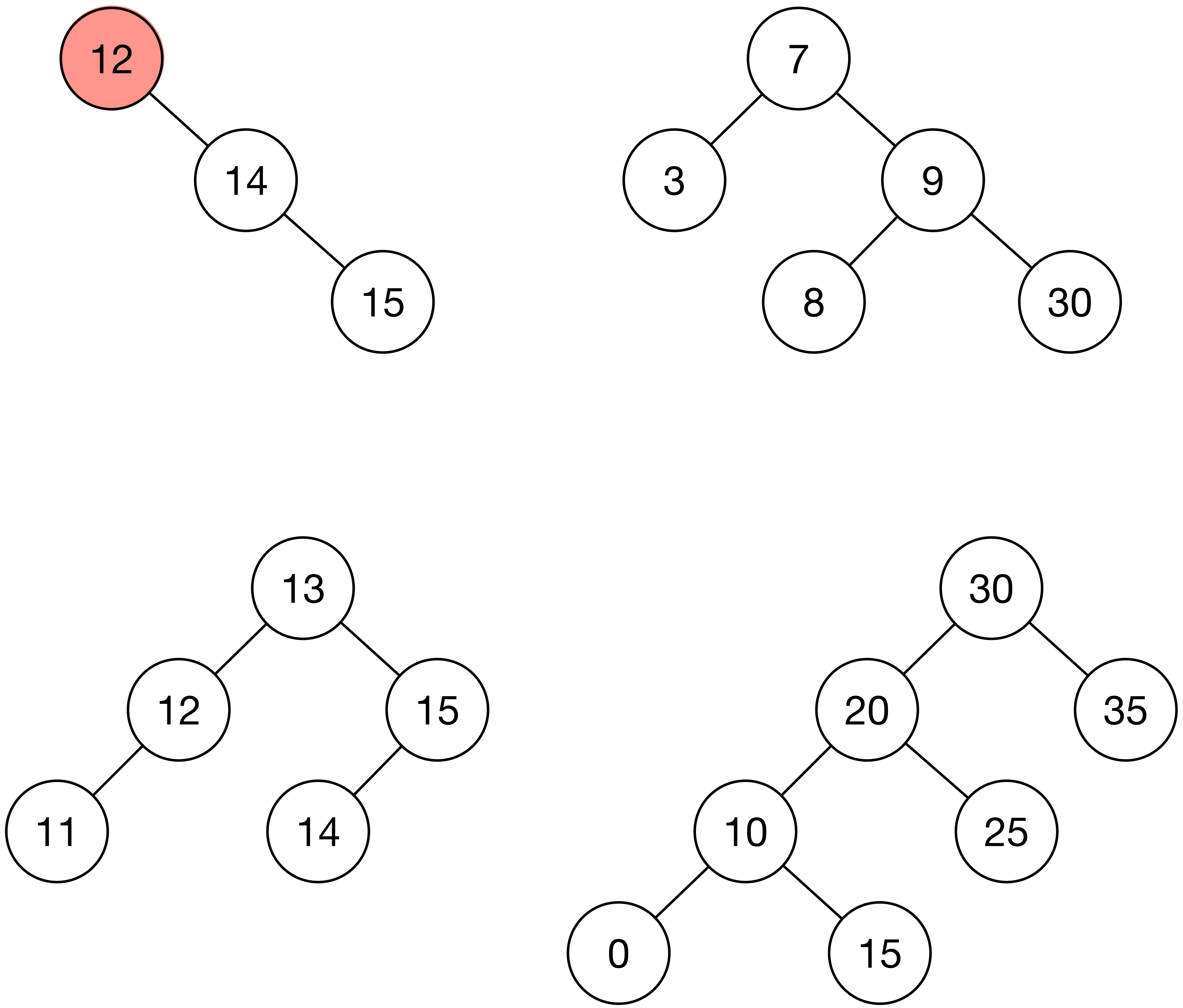

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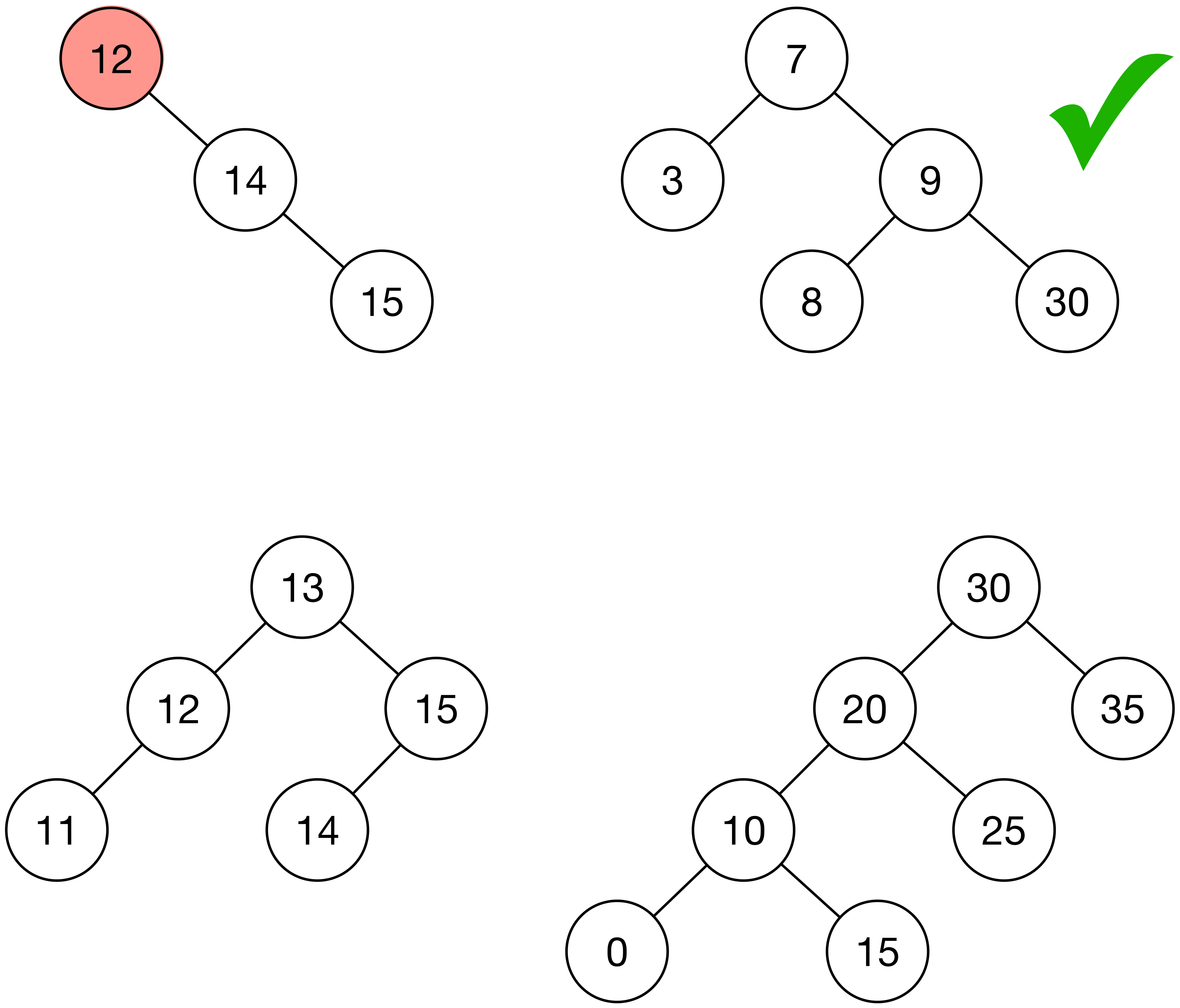

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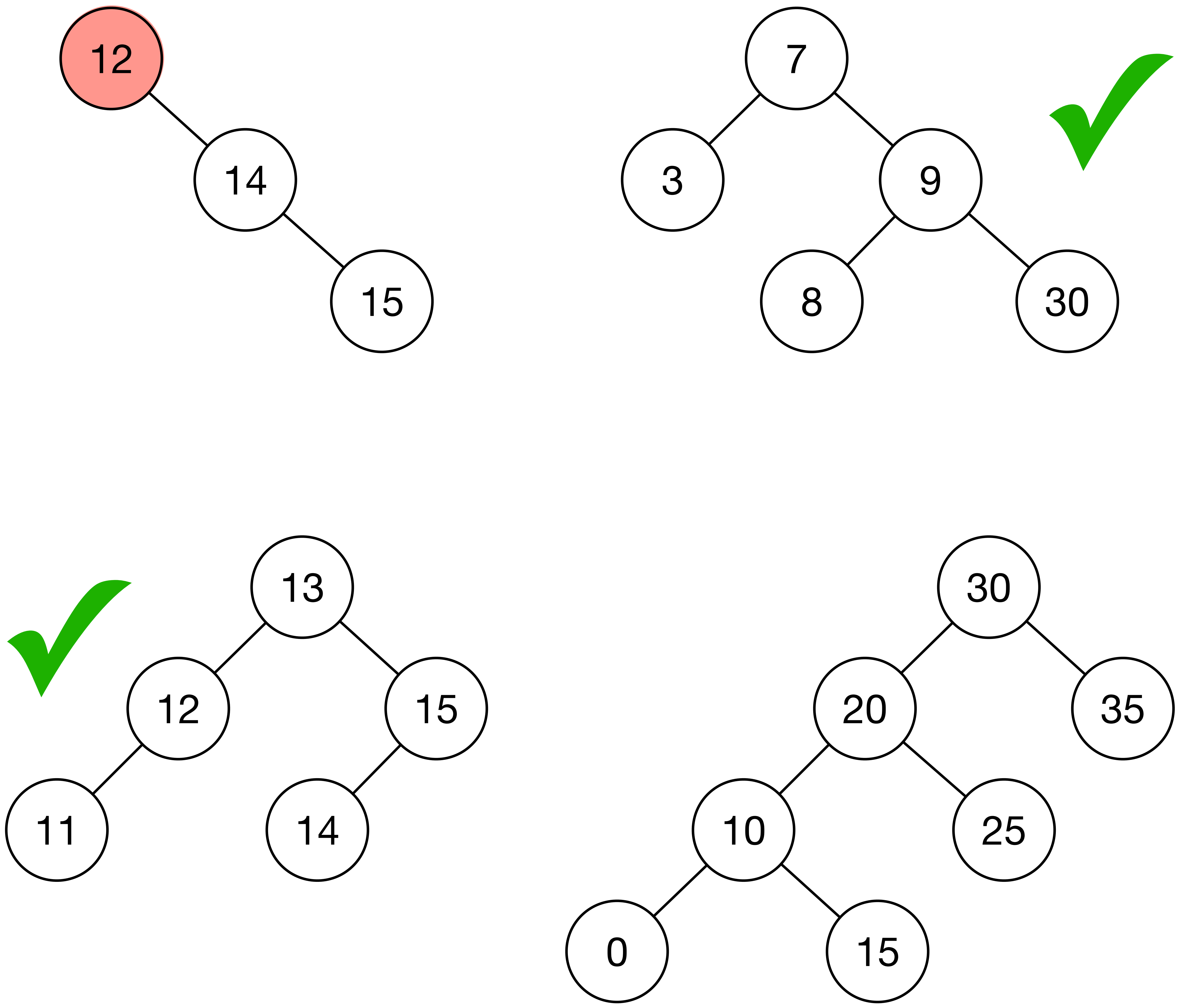

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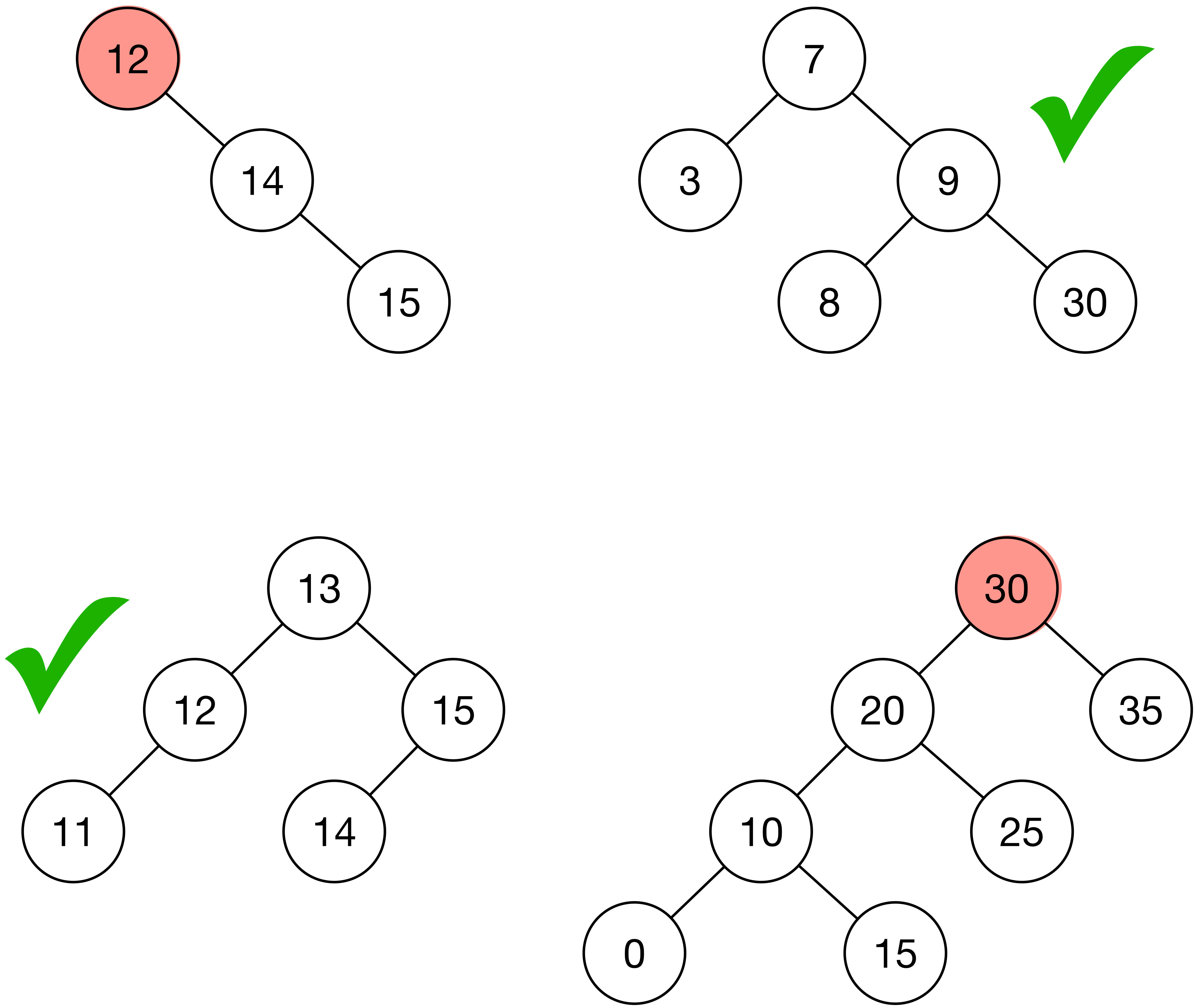
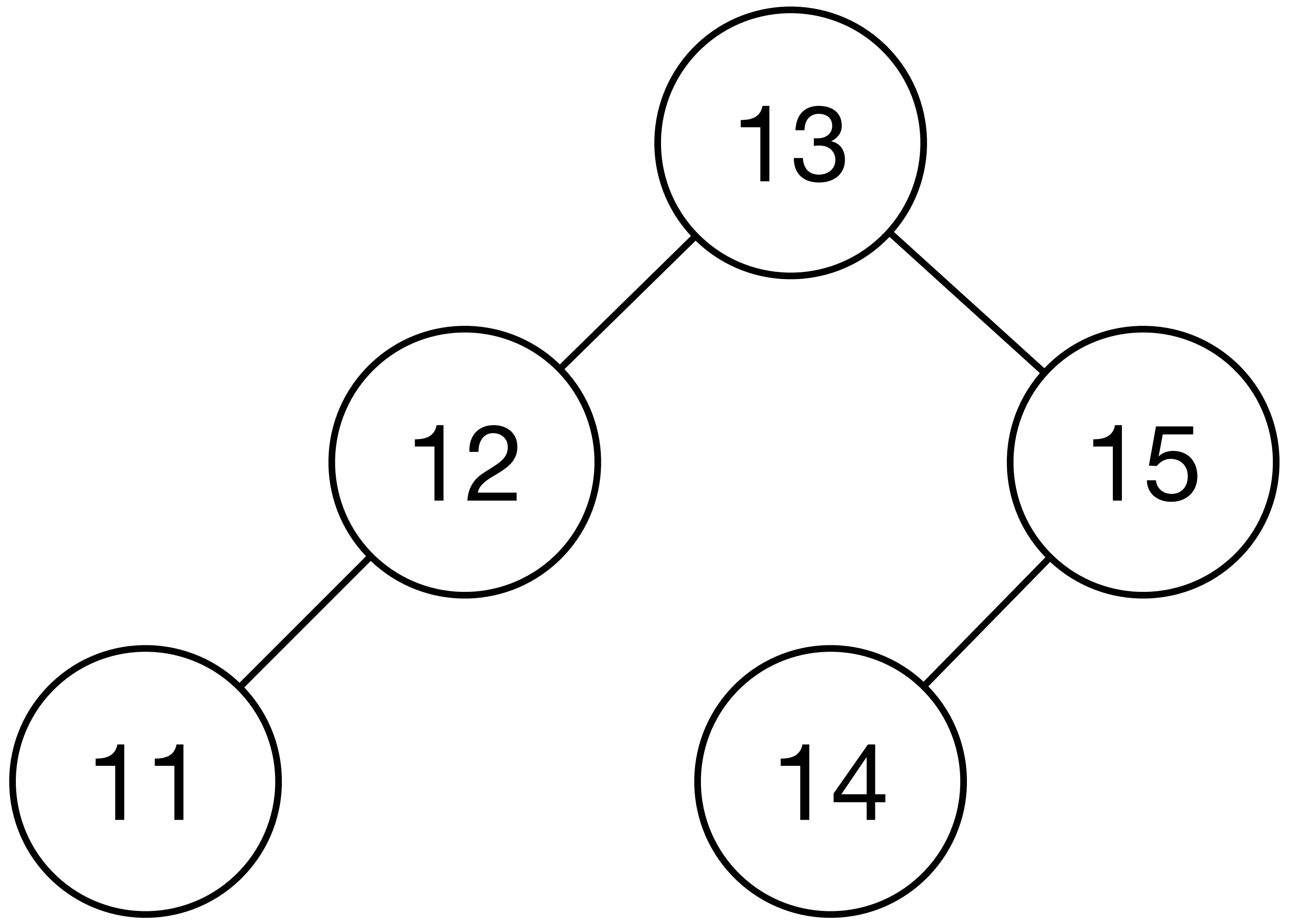


Self-balancing: Preserving the AVL property

- What does it mean to be "self-balancing"?
- Recall we have methods for handling insertions and deletions for binary search trees that preserve the properties of a binary search tree.
- Similarly, AVL trees have methods for handling insertions and deletions that preserve its BST properties *and* keep it balanced, preserving the AVL property.

Self-balancing: Preserving the AVL property



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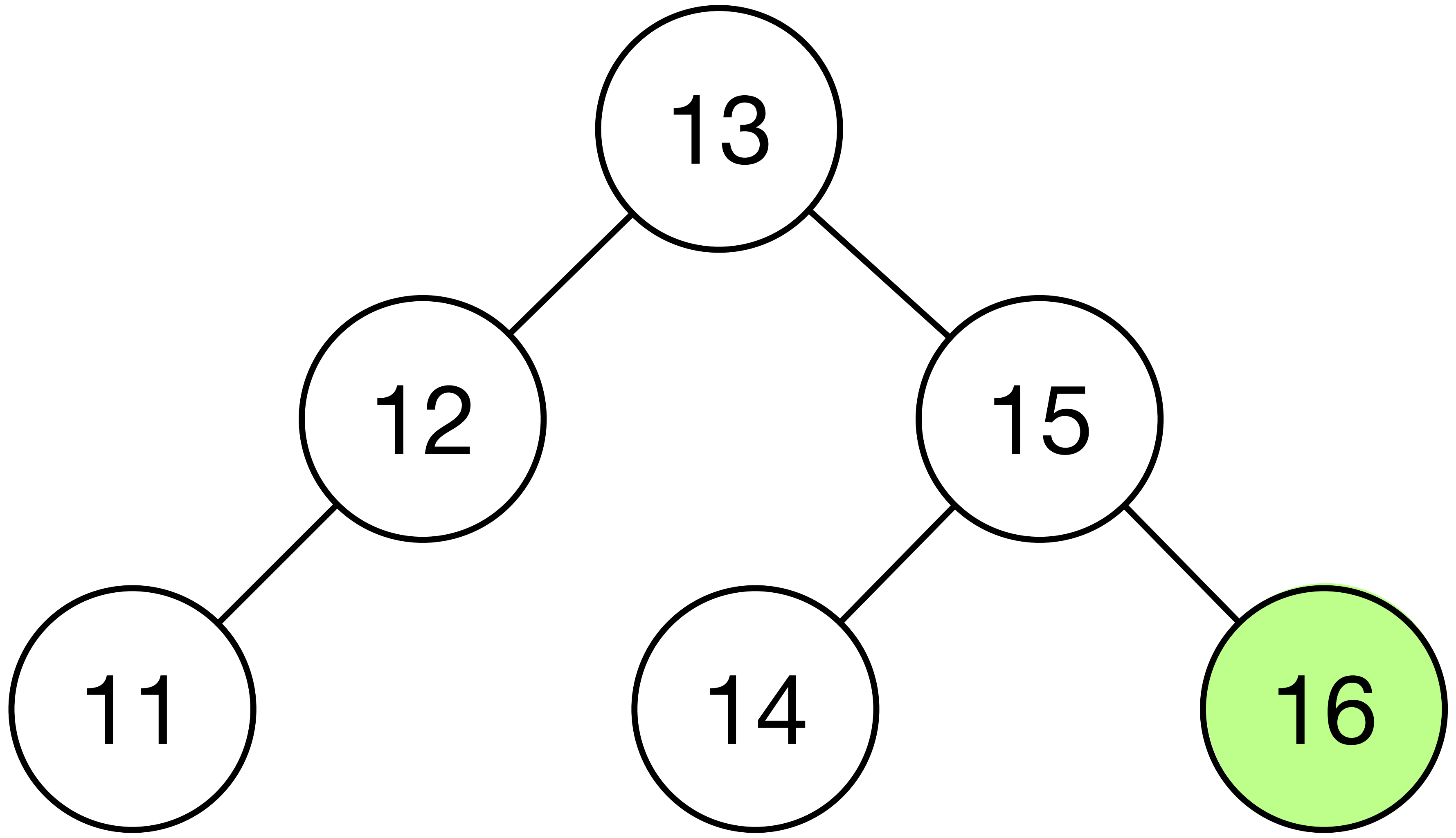

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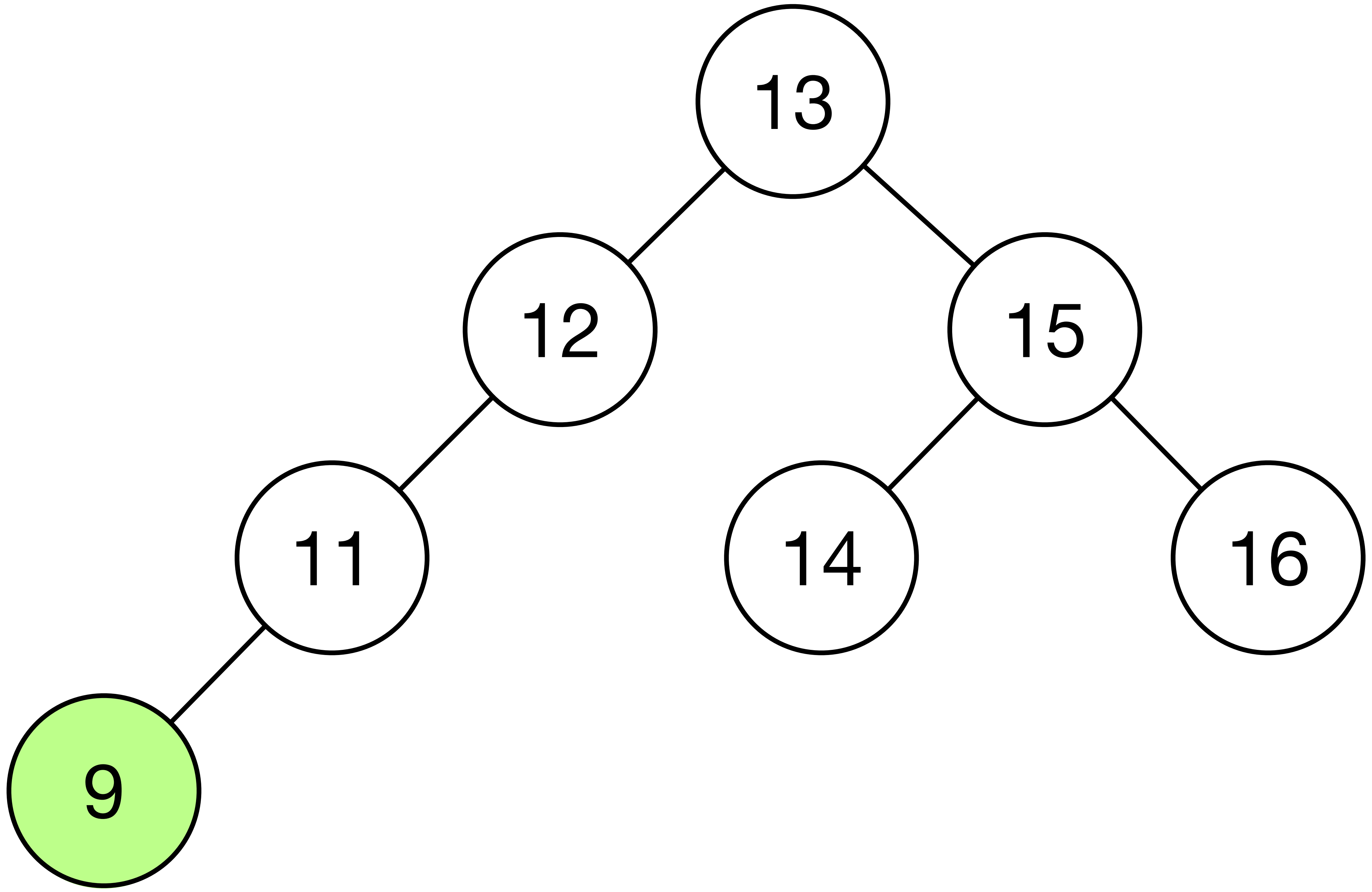

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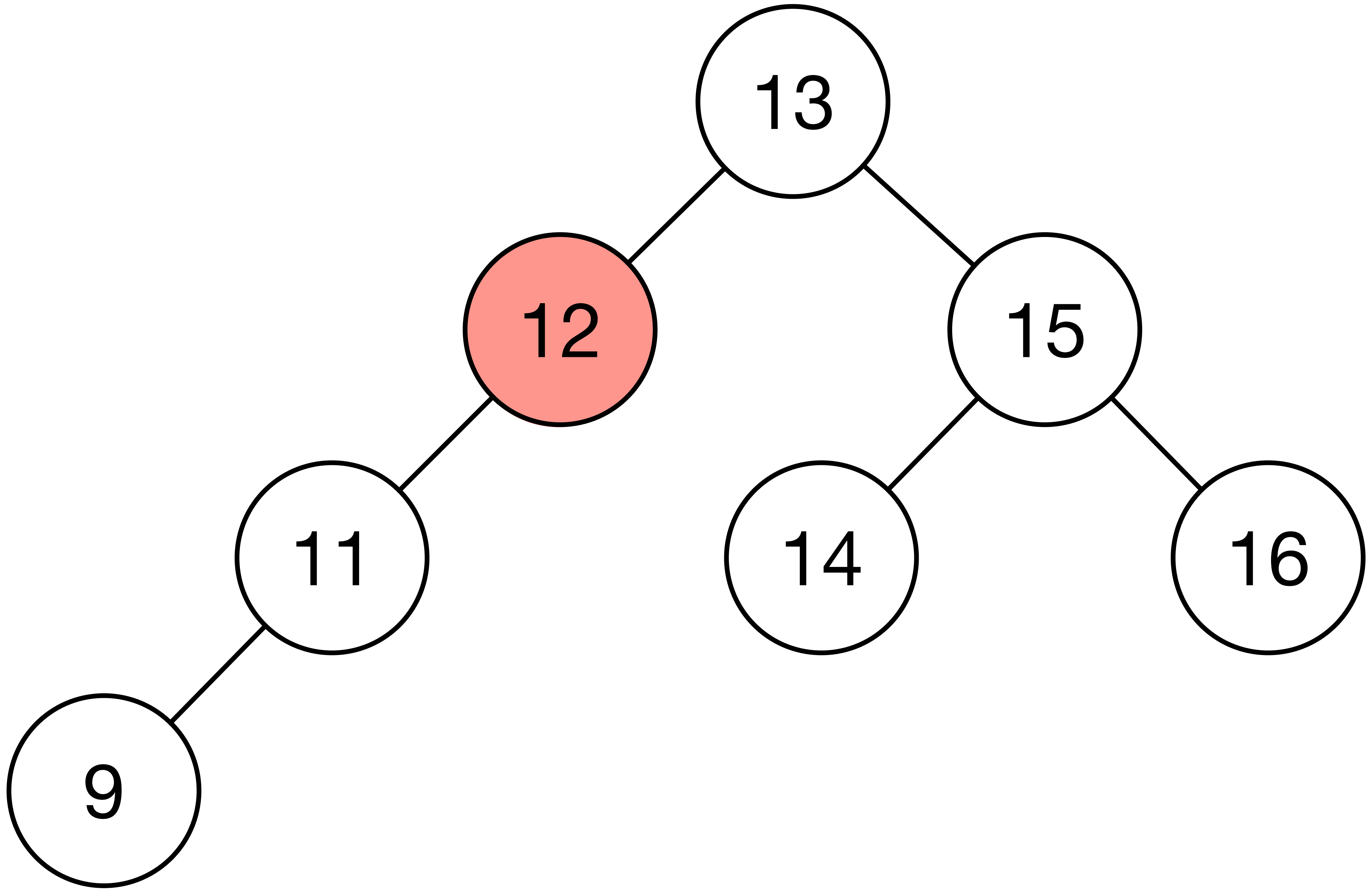

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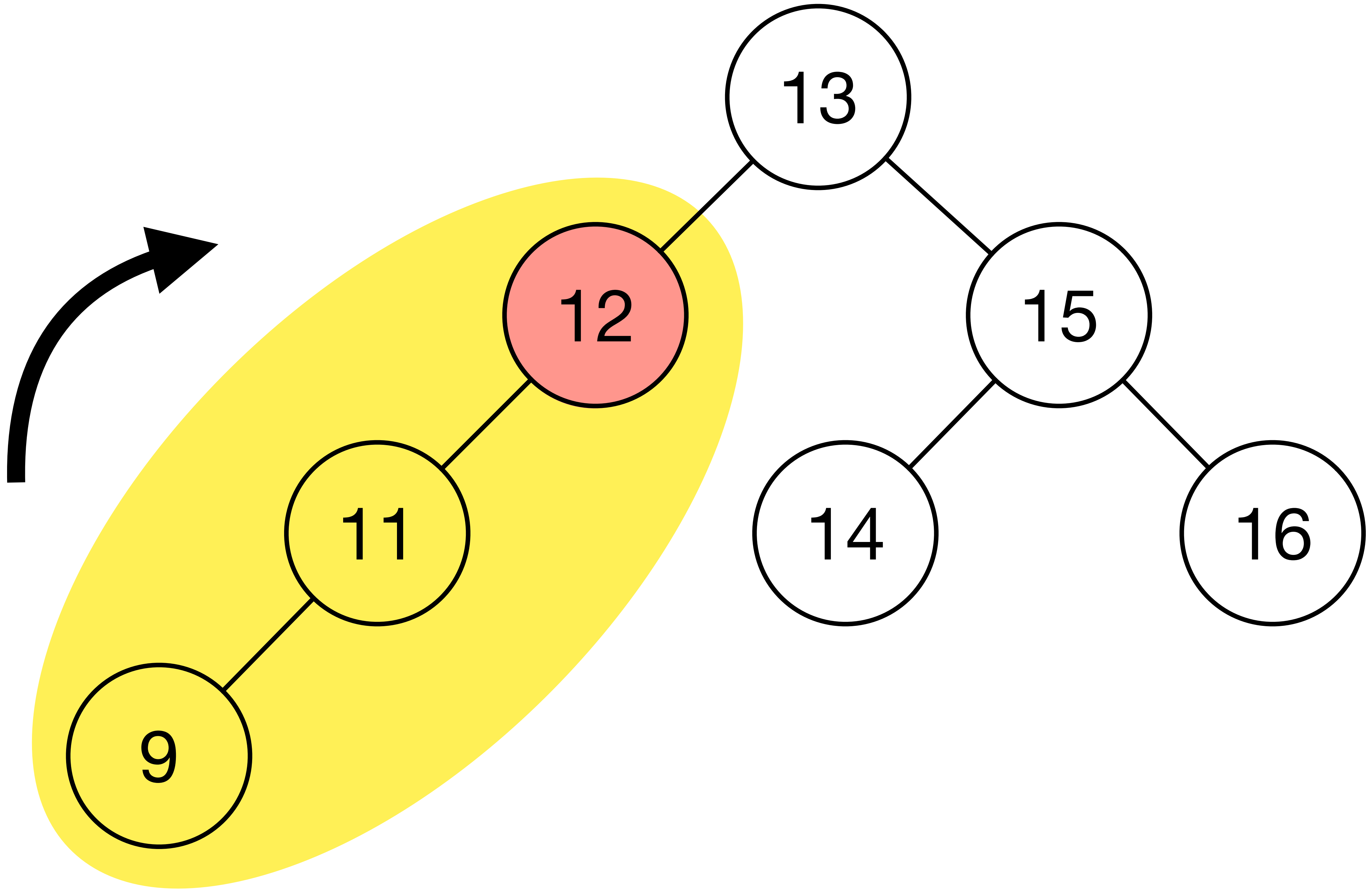

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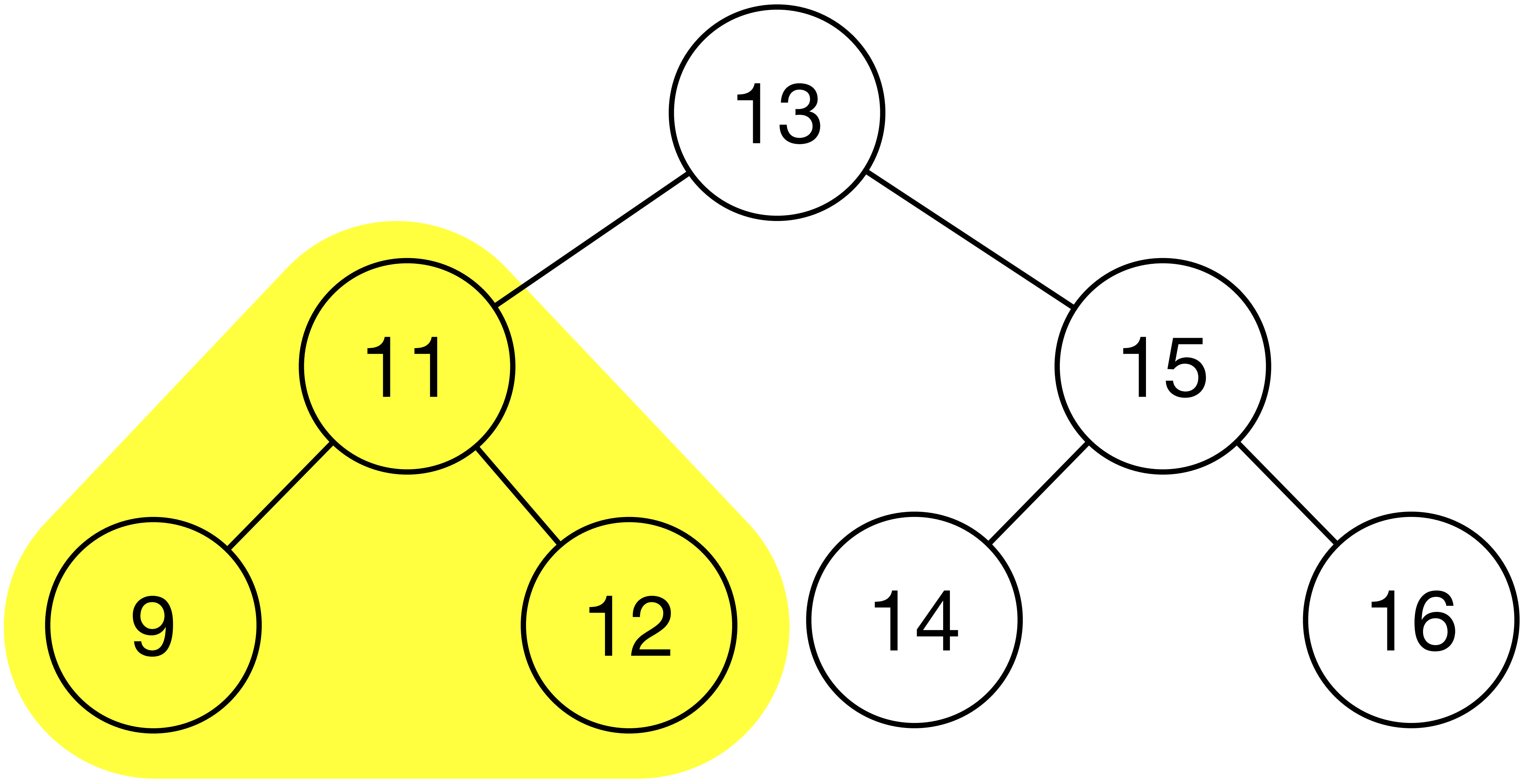

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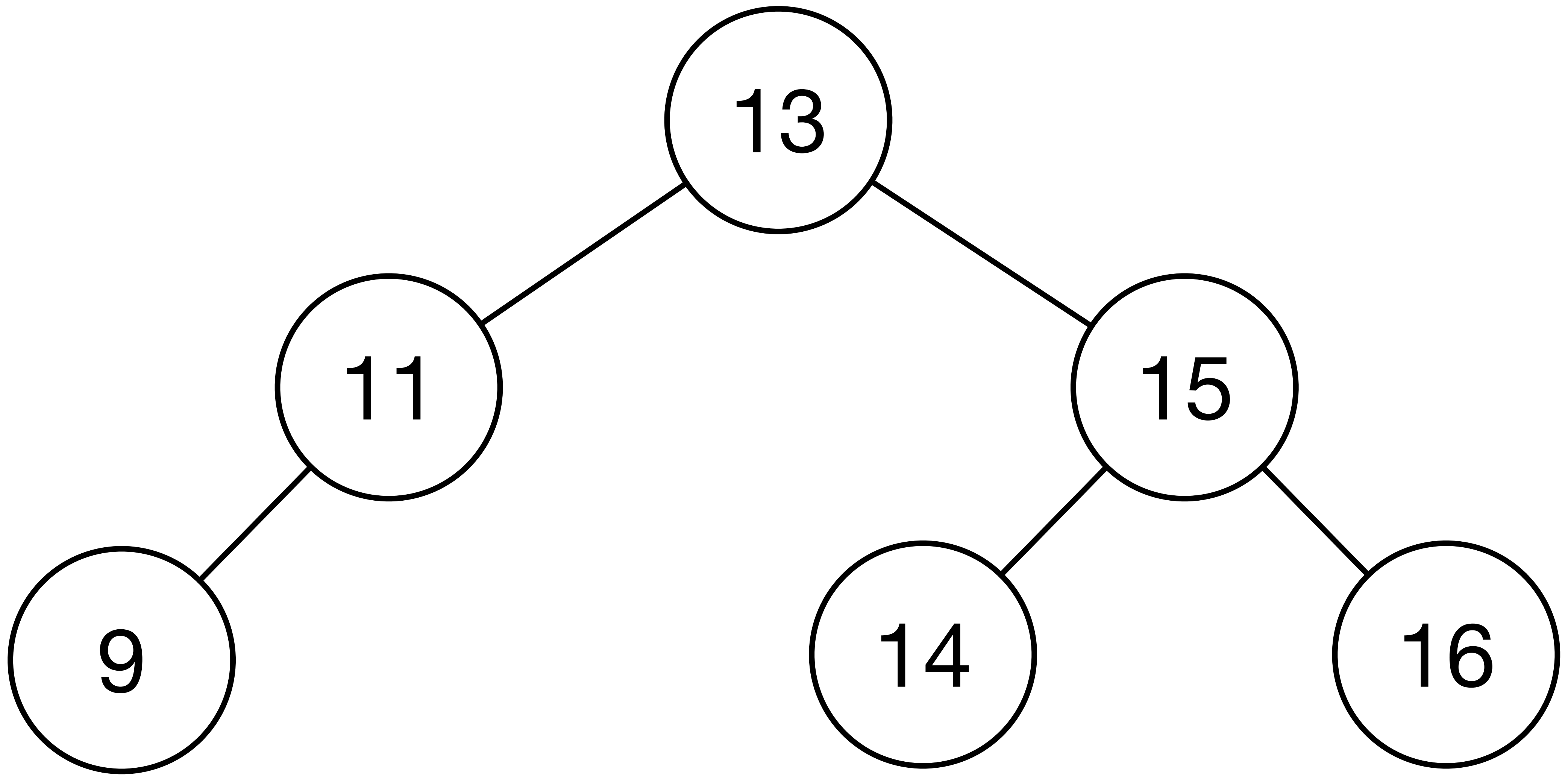

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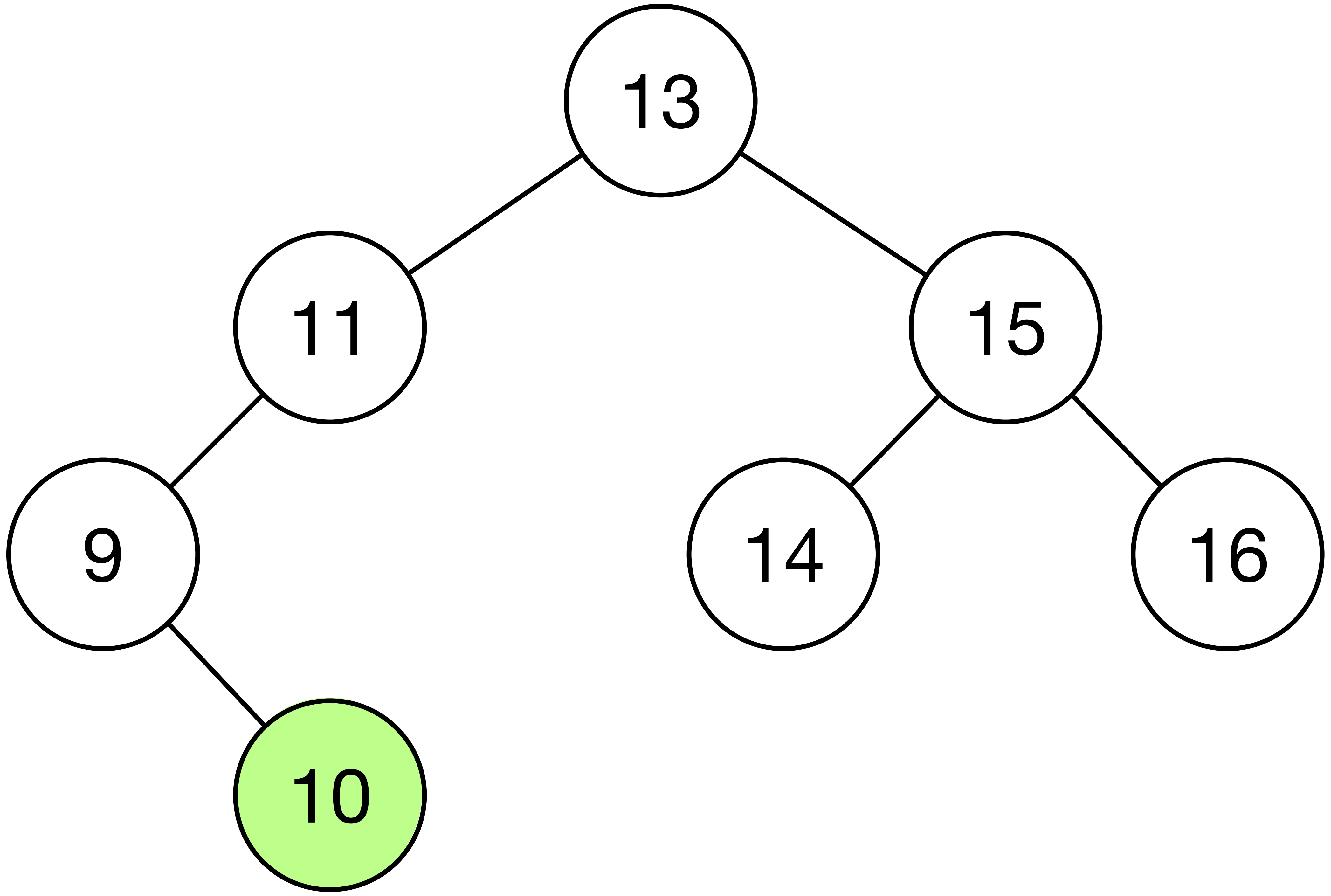

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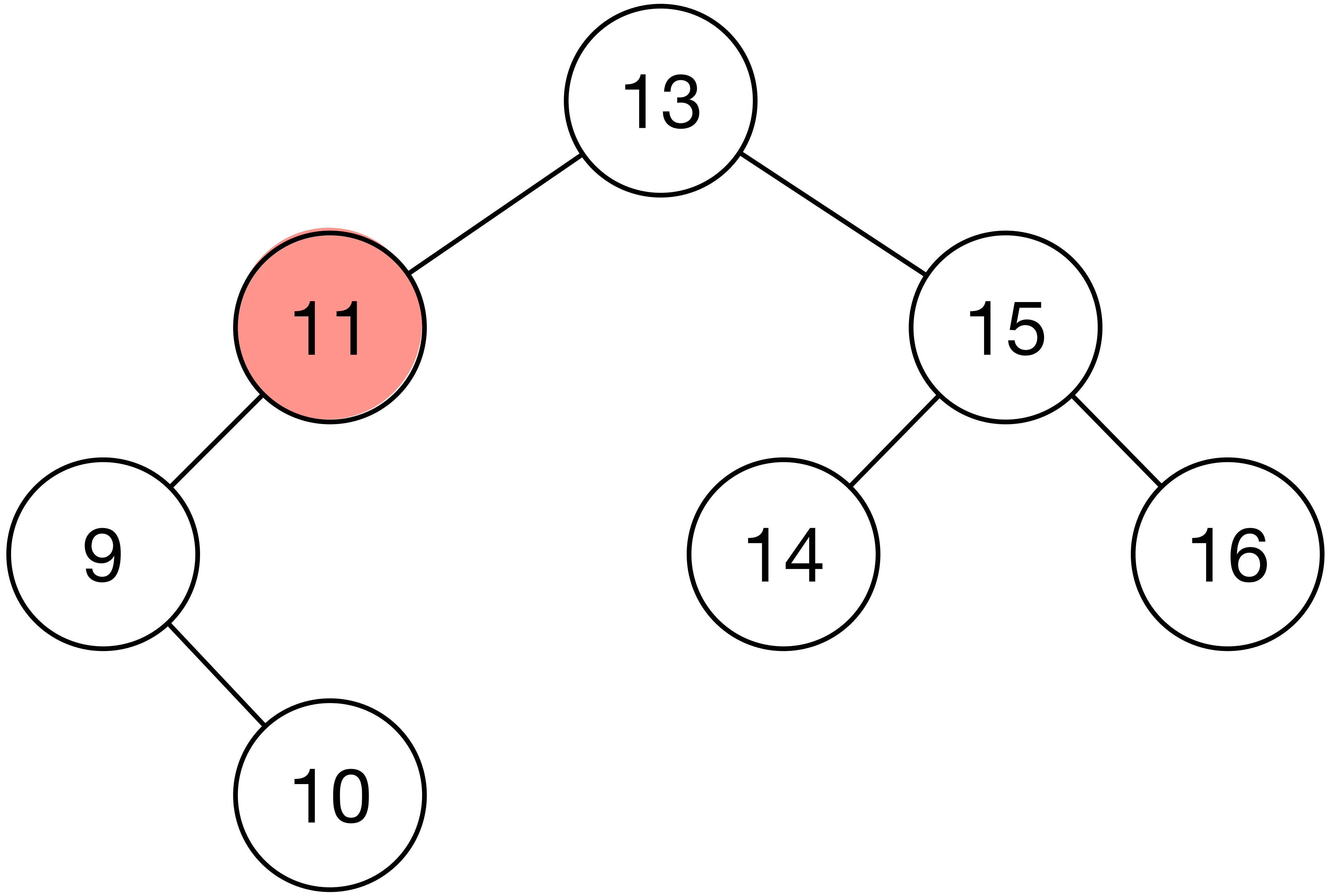

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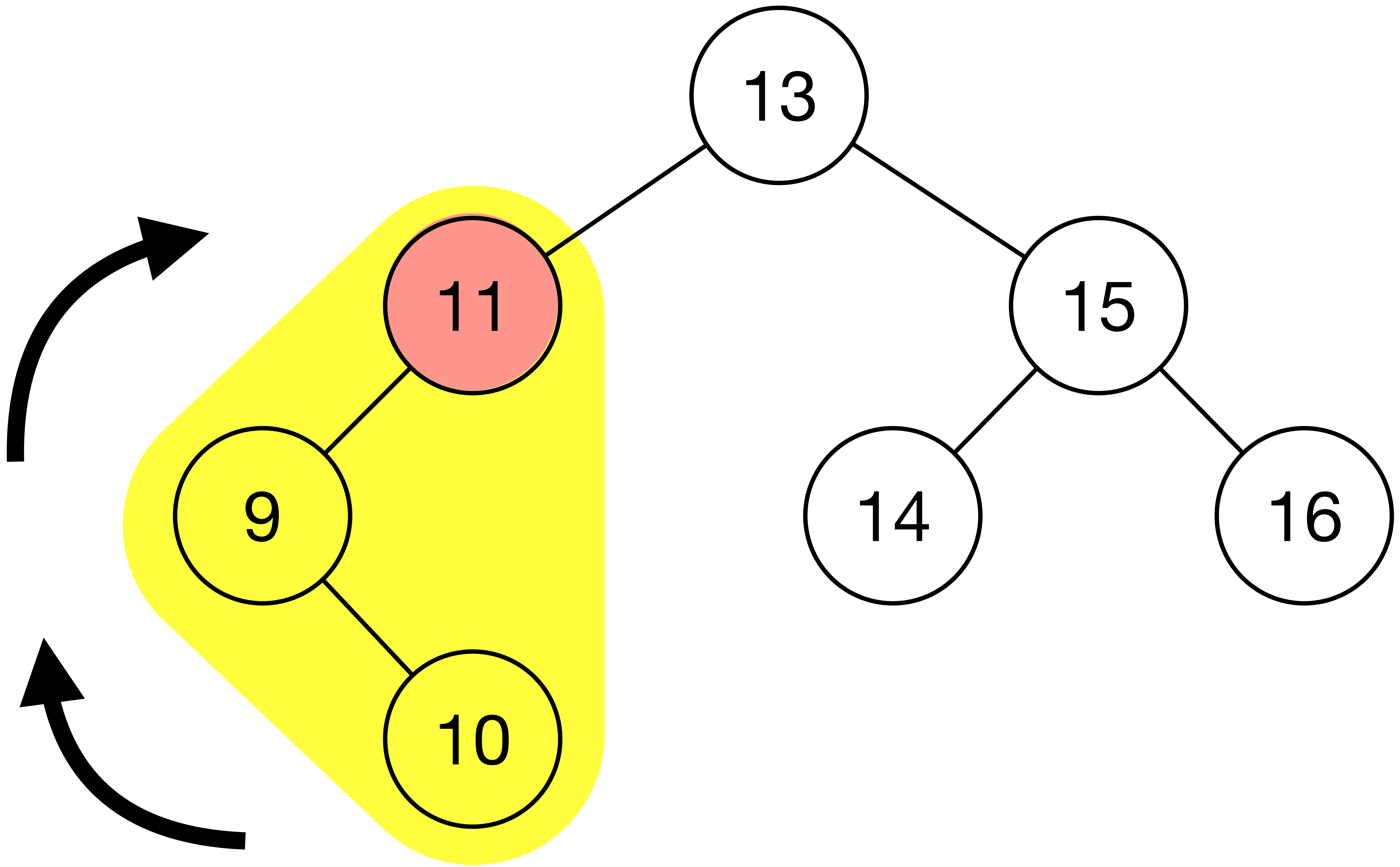

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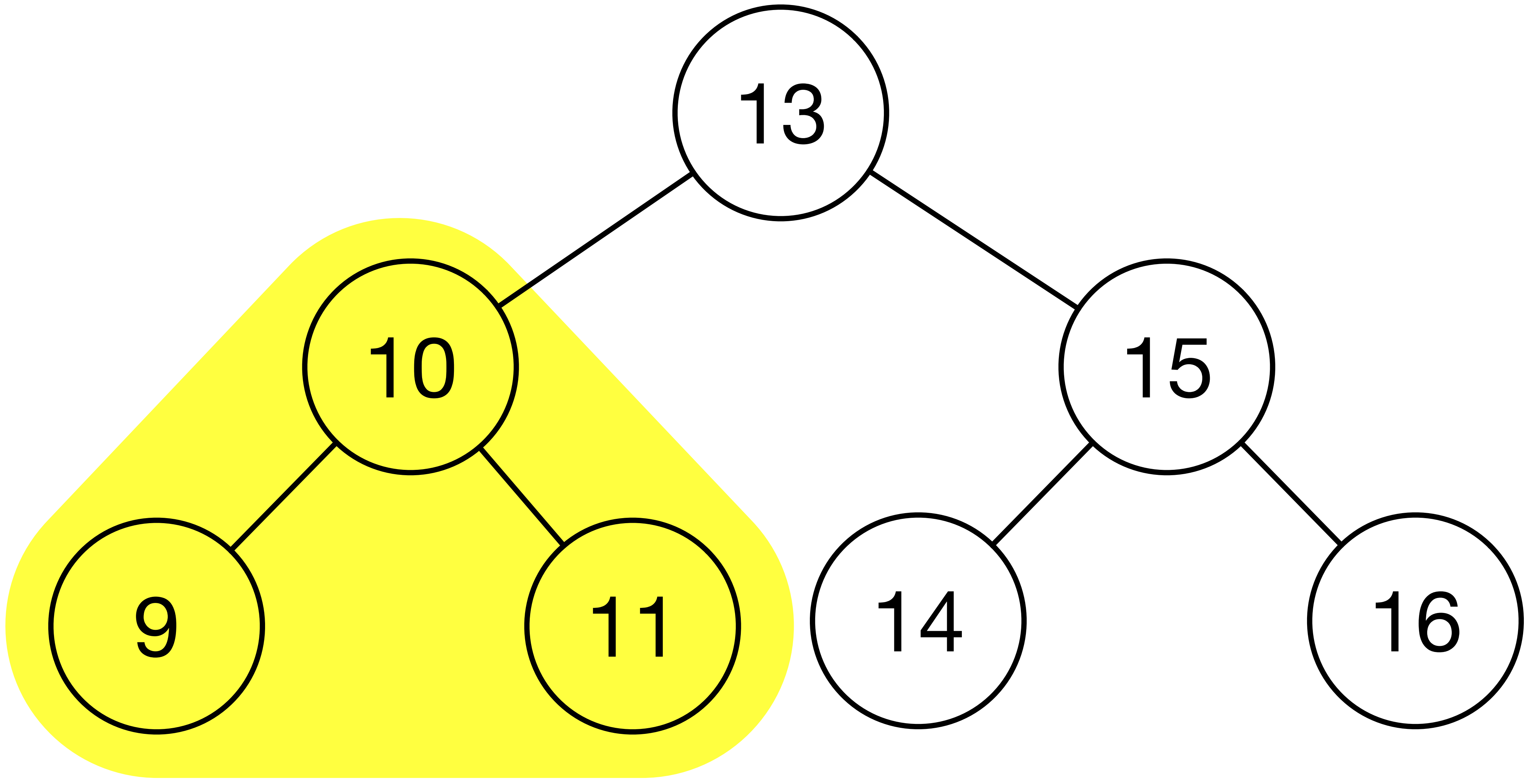


AVL: General Algorithm

- Perform the insertion or deletion you would as for a BST.
- Check to see if the AVL property holds -- that is, for each node in the tree, the height of its left and right subtrees differ by at most one.
- If the tree does not have the AVL property, find the lowest node in the tree that has subtrees differing in height by more than one -- this is the "problem node"
- Perform necessary rotation(s).

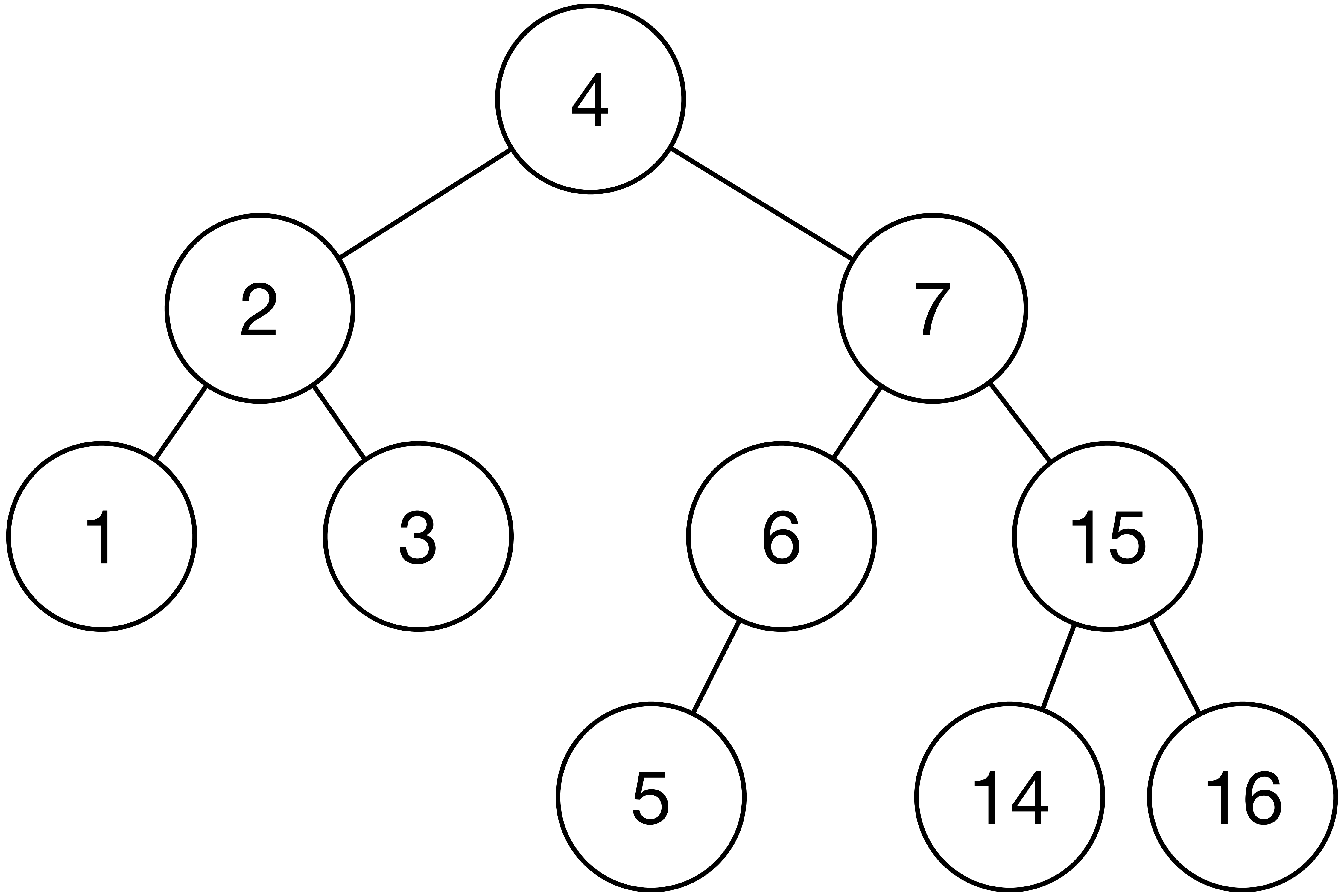
AVL: General Algorithm

Determining the necessary rotation

- Examine the path from the "problem node" to its most distant leaf.
- "Zig-zig": If the path goes in the same direction for the first two generations (*i.e.*, both left children or both right children), do a single rotation.
- "Zig-zag": If the path follows different directions (*i.e.*, a left child then a right child, or *vice versa*), do a double rotation.
- If there are multiple leaves that are farthest from the problem node and both paths are possible, either rotation will work to make the tree AVL again.

Self-balancing: Preserving the AVL property

Another example



Self-balancing: Preserving the AVL property

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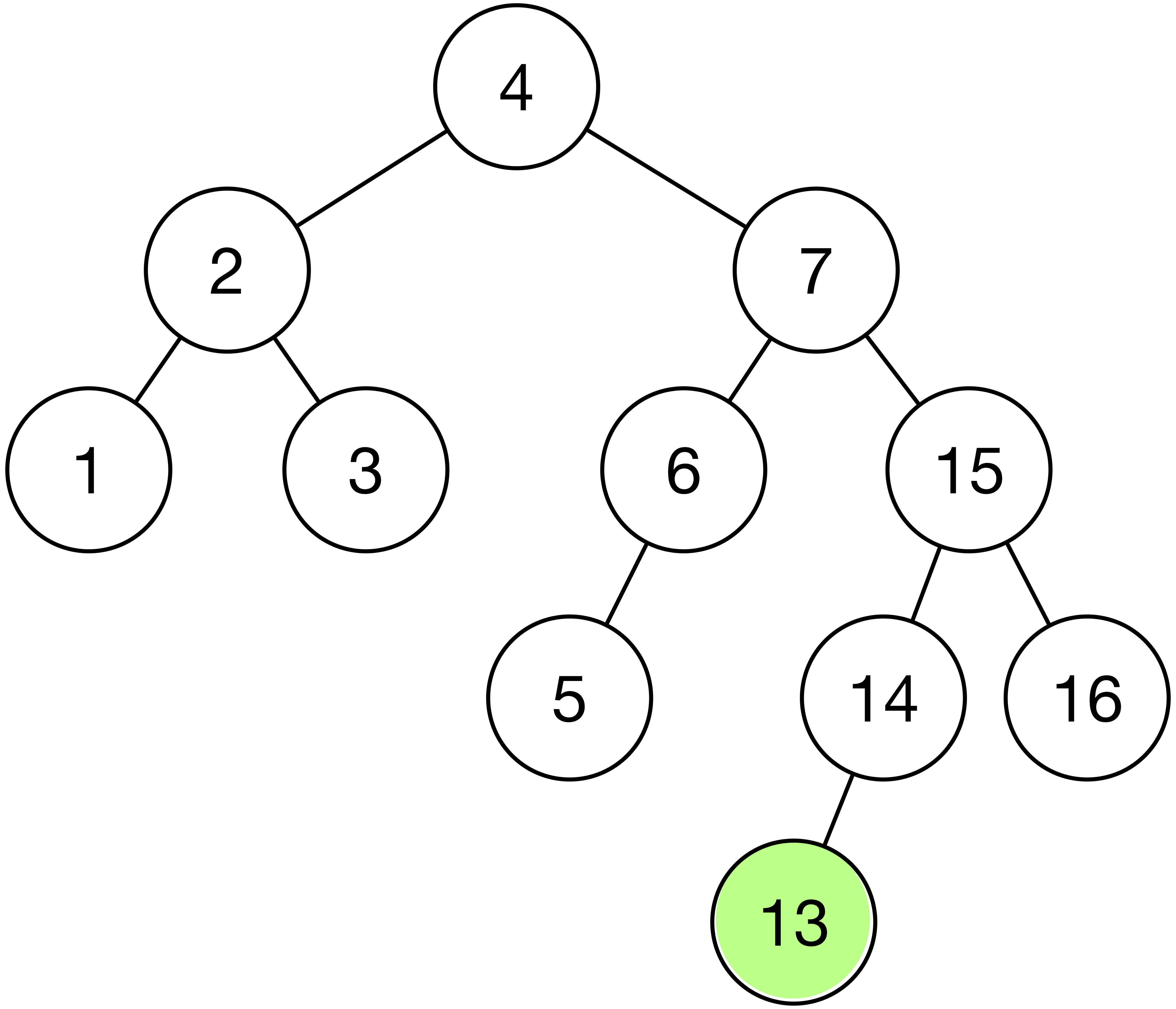

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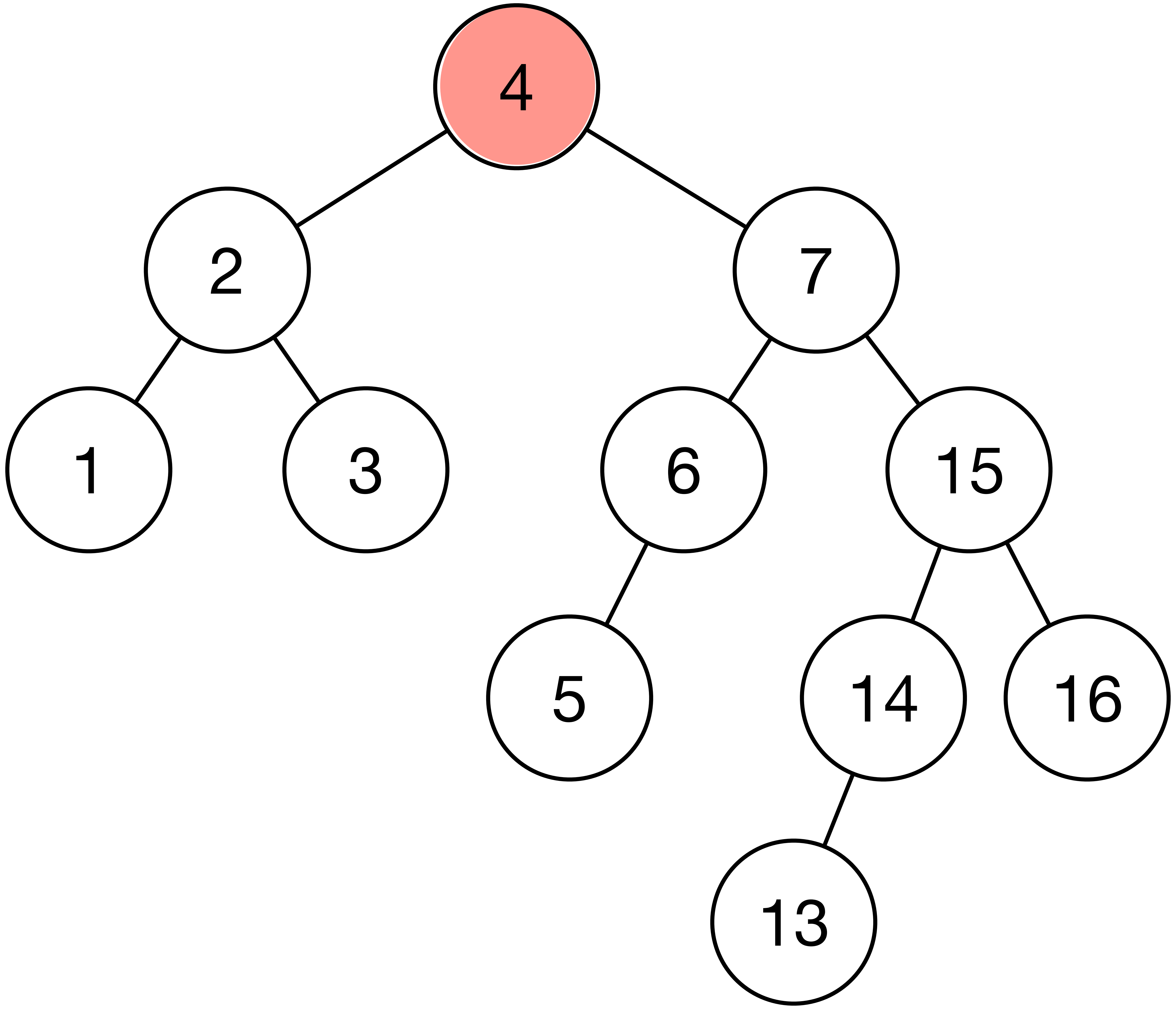

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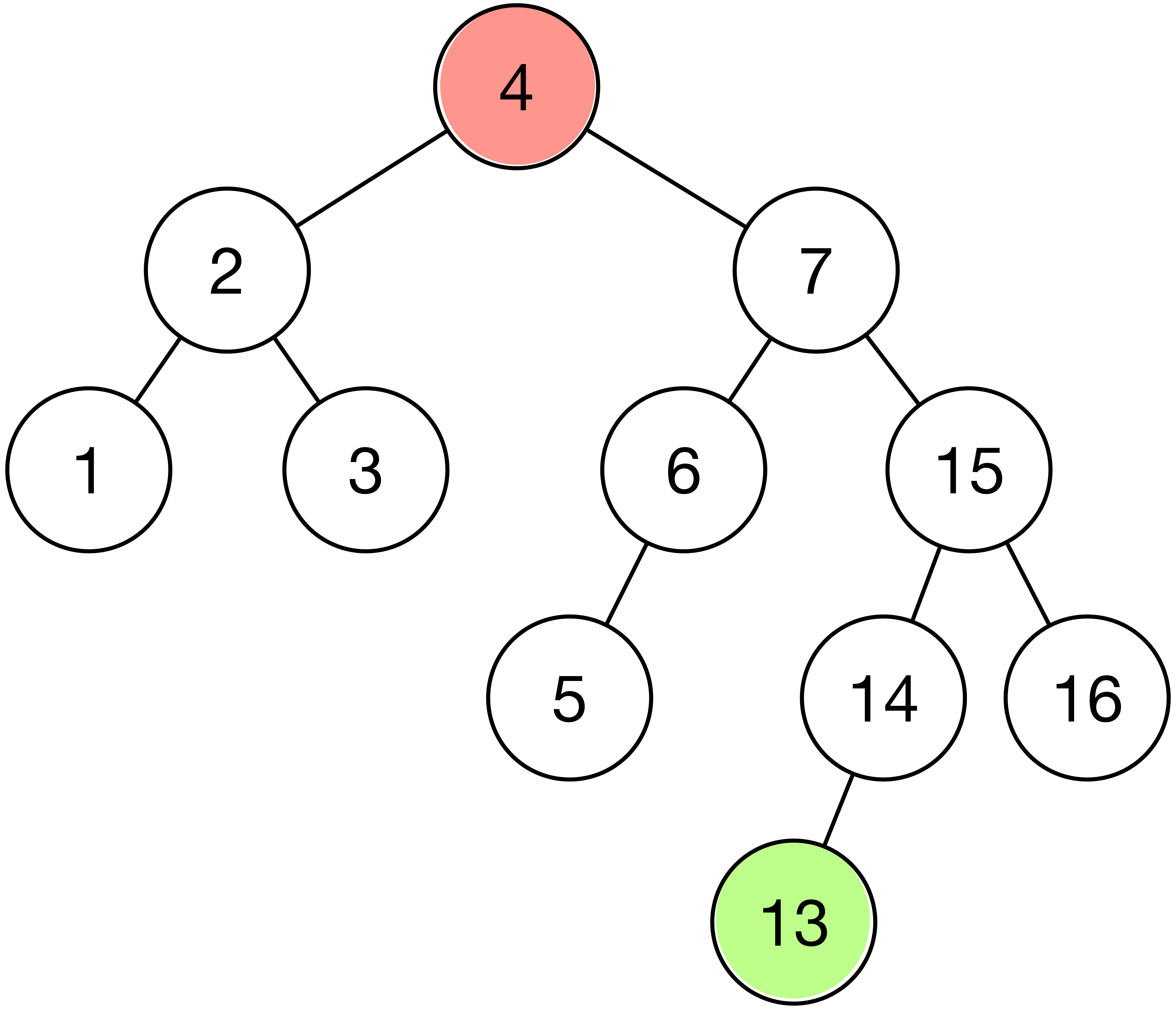

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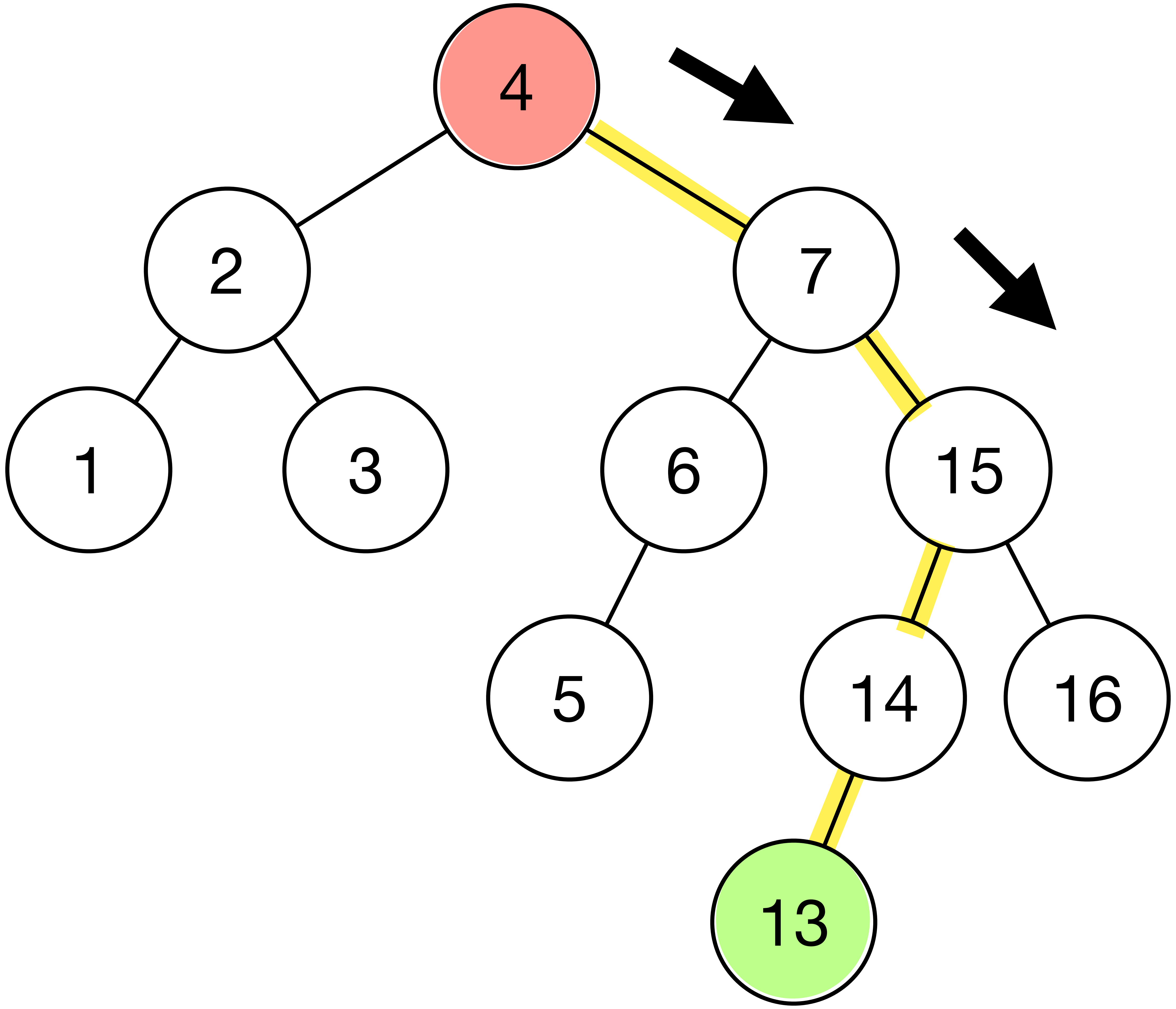

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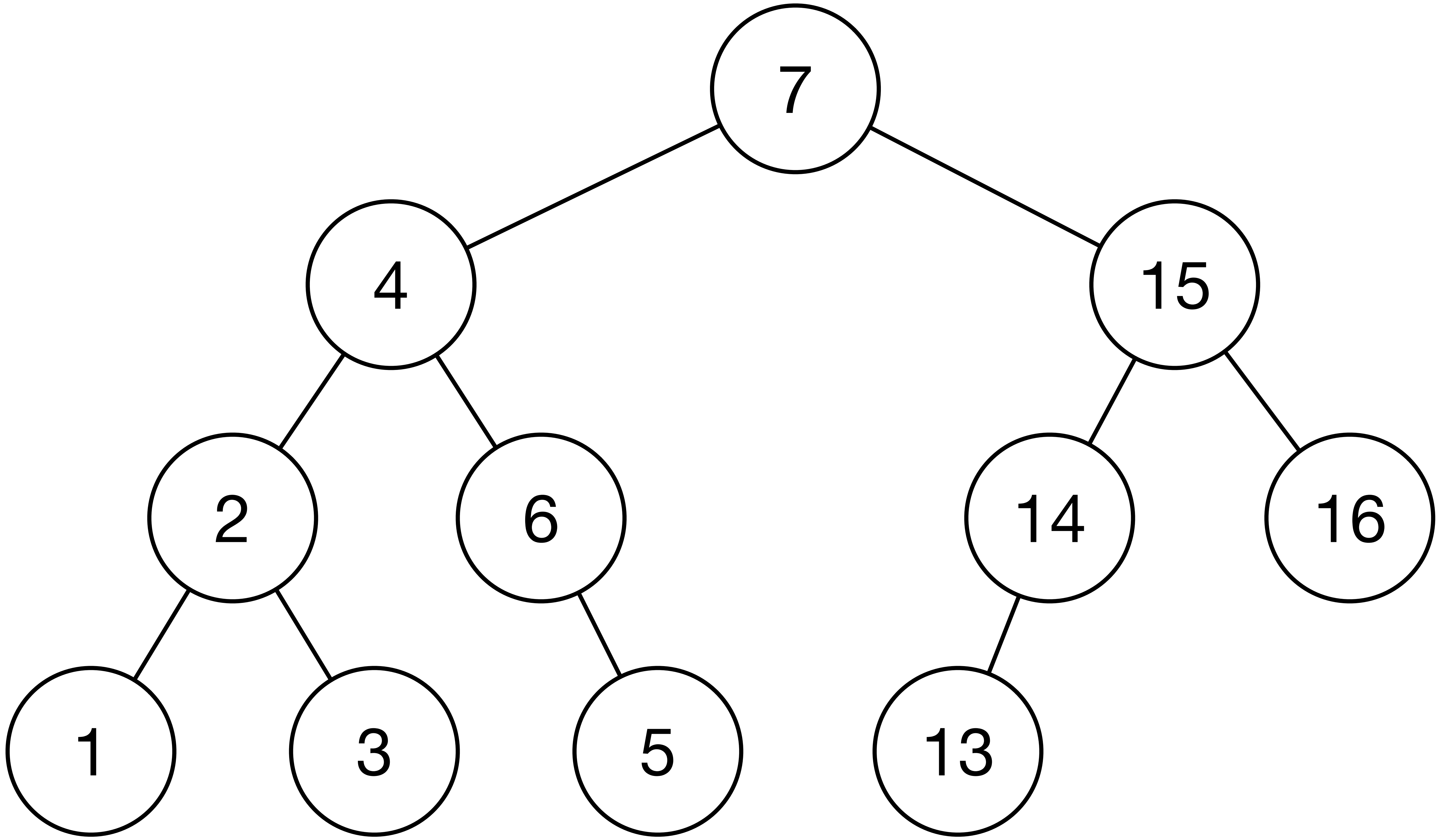

Some visualisations: a helpful tool

https://www.cs.usfca.edu/~galles/visualization/AVLtree.html