

CSCI046 Homework 2: Runtime Analysis

DUE: Thursday, 6 February beginning of class

Name: _____

Grading note: Each of these problems contains several subproblems, and there are typically more subproblems than the point value of the problem. You will lose 1 point for each incorrect subproblem. If this would result in a negative score, then you get zero for the problem.

Problem 1. (3 points) Simplify the following expressions:

1. $O\left((n^2 + n \log n)(n^3 + \log n)\right)$

2. $\Omega\left((3.45n + n)(\log n^2)\right)$

3. $\Theta\left(n(1 + \log n) + n^{3.2} + \log 2^n\right)$

Problem 2. (3 points) Complete each equation below by adding the symbol O if $f = O(g)$, Ω if $f = \Omega(g)$, or Θ if $f = \Theta(g)$. The first row is completed for you as an example.

$f(n)$		$g(n)$
1	=	$O(n)$
$3n \log n$	=	n^2
1	=	$1/n$
$\log_2 n$	=	$\log_3 n$
n^{42}	=	42^n
$5 \cdot 10^{30}$	=	$\log n$
$\log n$	=	$\log(n^2)$
2^n	=	3^n
$n!$	=	n^2
$\log n$	=	$\log n^2$

Problem 4. (2 points) Answer the questions below based on the following python code:

```
1 for i in range(n**2):
2     print('a')
3     for j in range(n/2):
4         print('b')
5         for k in range(int(math.sqrt(n))):
6             print('c')
7             print('b')
8         print('b')
9 for i in range(n):
10     print('b')
11 for i in range(100):
12     print('d')
13     print('a')
```

1. What is the asymptotic number of times that the letter **a** will be printed? (Use Θ notation.)
2. What is the asymptotic number of times that the letter **b** will be printed? (Use Θ notation.)
3. What is the asymptotic number of times that the letter **c** will be printed? (Use Θ notation.)
4. What is the asymptotic number of times that the letter **d** will be printed? (Use Θ notation.)

Problem 5. (0.5 points extra credit) Prove the following: $\log(n!) = \theta(n \log n)$,