B+ Tree

Anup Halder Joy - 2005005
Afzal Hossan - 2005021
Asif Karim - 2005024

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Agenda

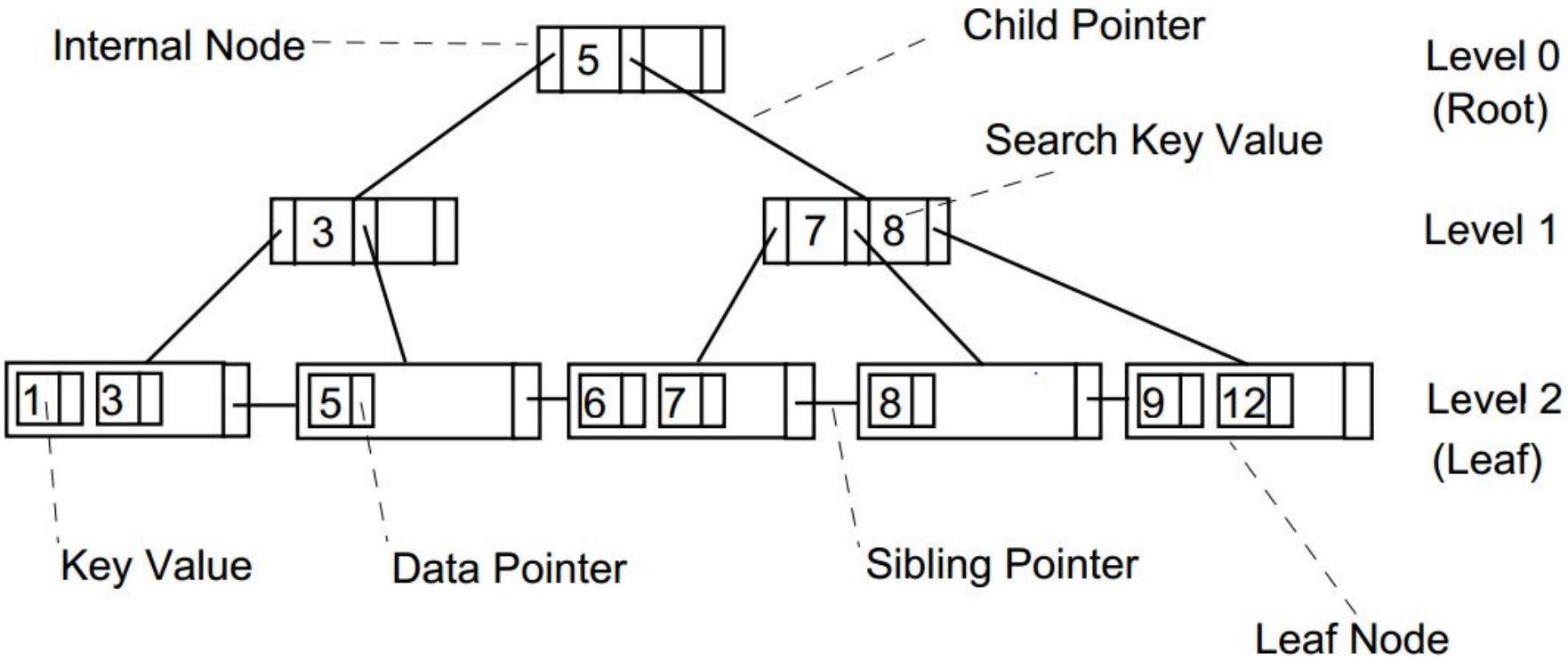


Figure: B+ Tree

WHAT is a B+ Tree?

What is a B+ Tree?

Definition of B+ Tree

A B+ tree is an m-ary tree with a variable but often large number of children per node. A B+ tree consists of a root, internal nodes and leaves. [3].

A B+ Tree is a self-balancing tree data structure that maintains sorted data and allows searches, sequential access, insertions, and deletions

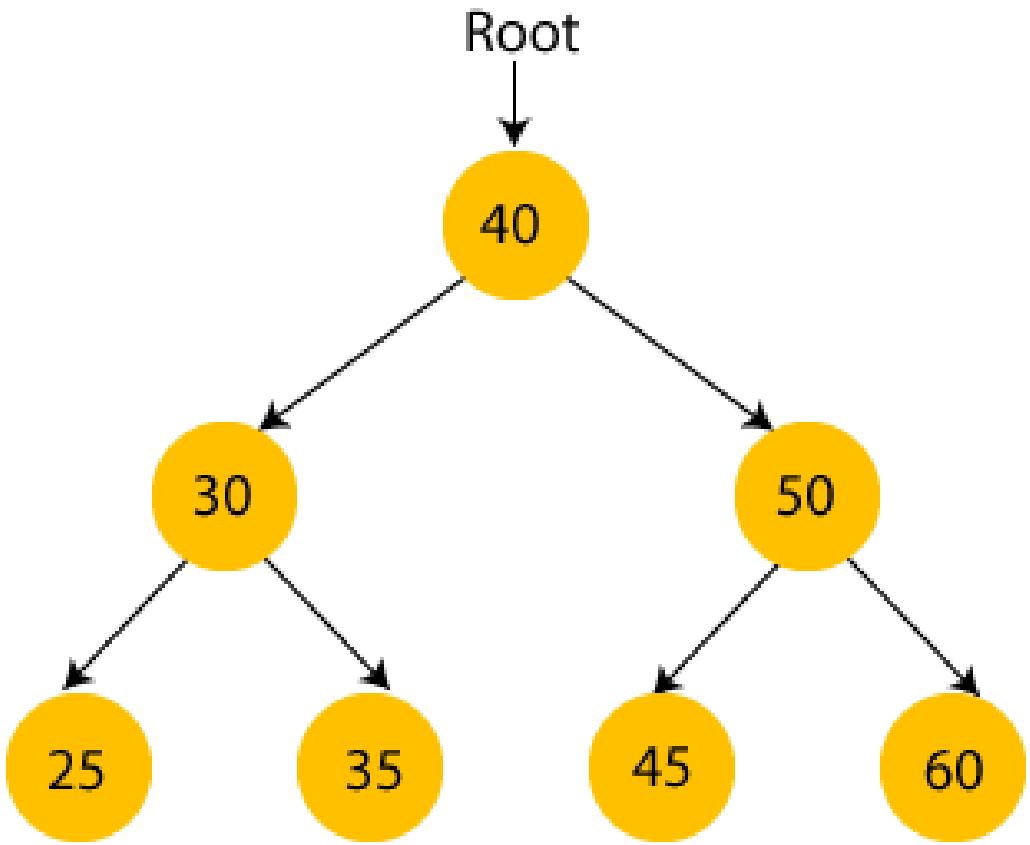
Inventors

Rudolf Bayer

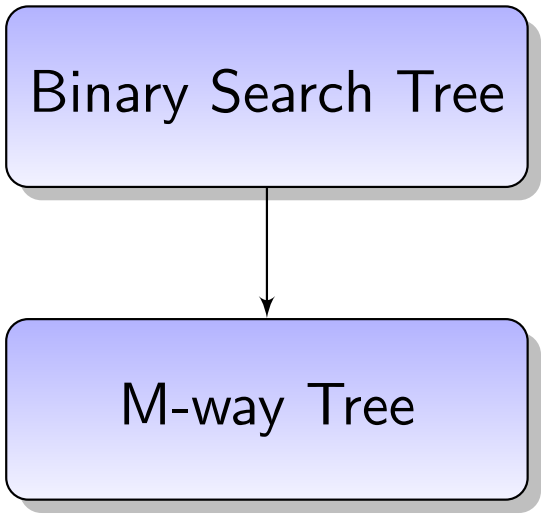
Edward M. McCreight

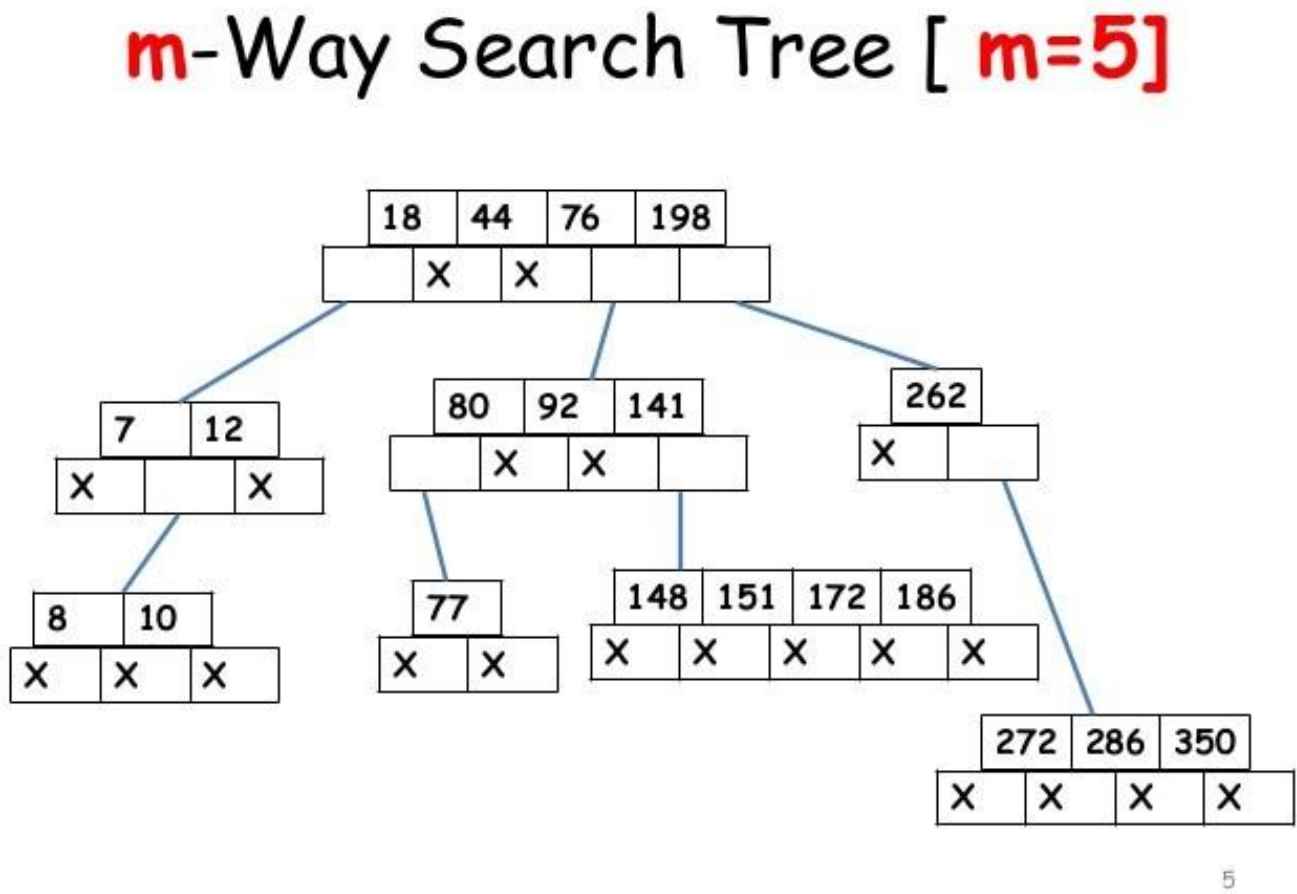
Evolution of B+ Tree

Binary Search Tree

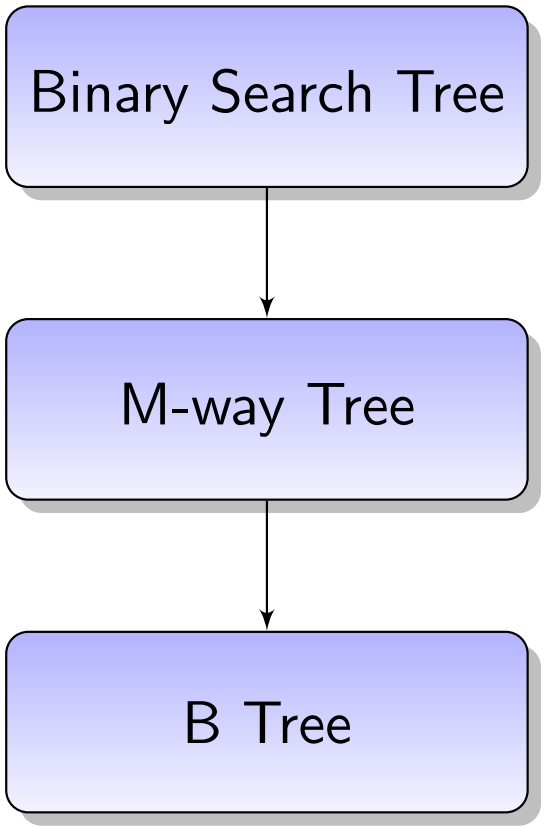


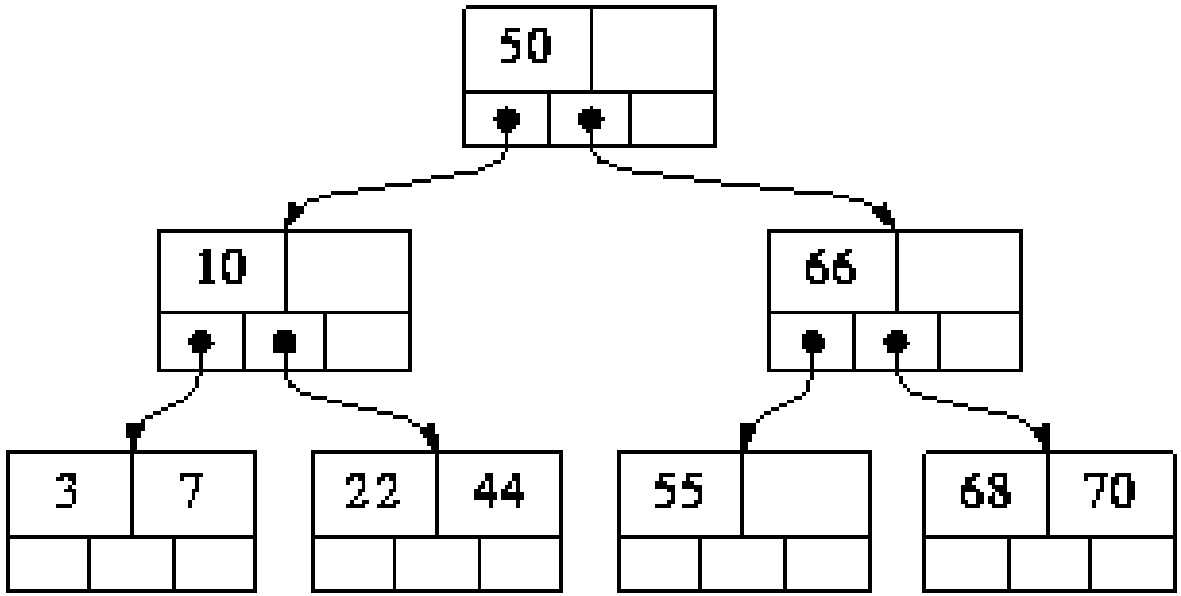
Evolution of B+ Tree



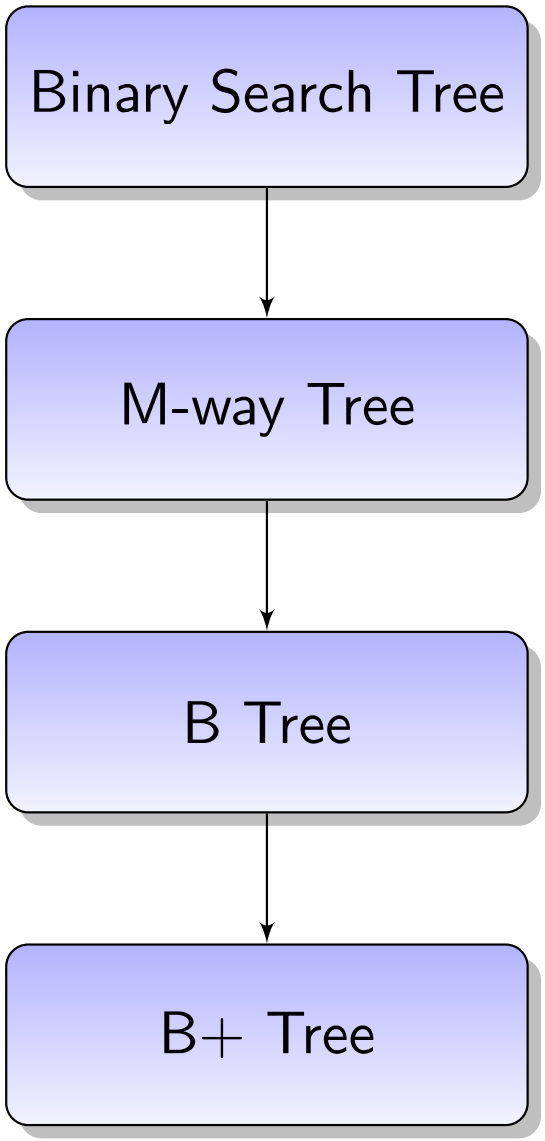


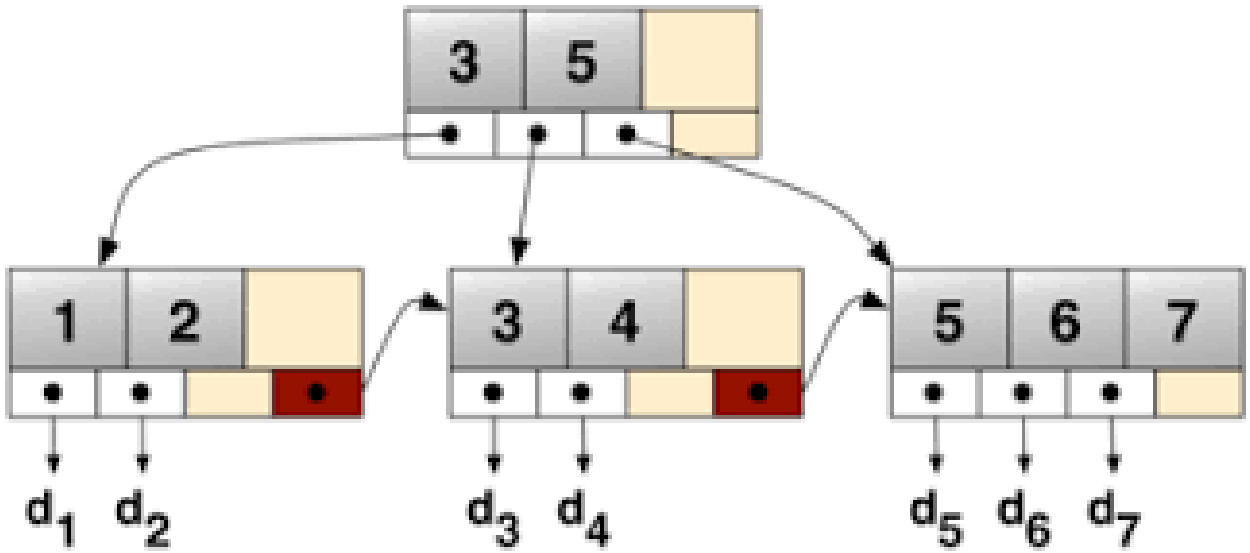
Evolution of B+ Tree





Evolution of B+ Tree





Structure of Internal Node

‘

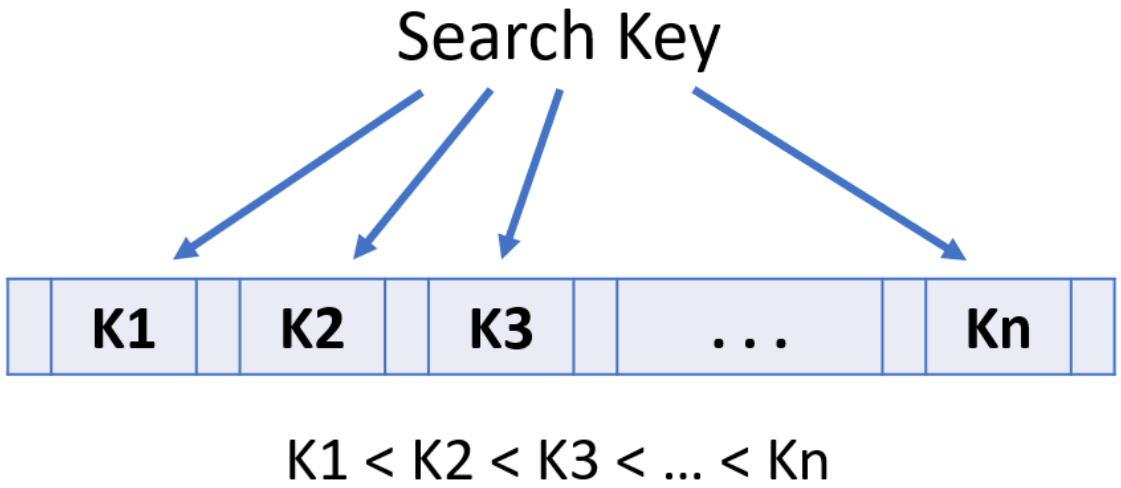


Figure: Internal Node Structure

Structure of Leaf Node

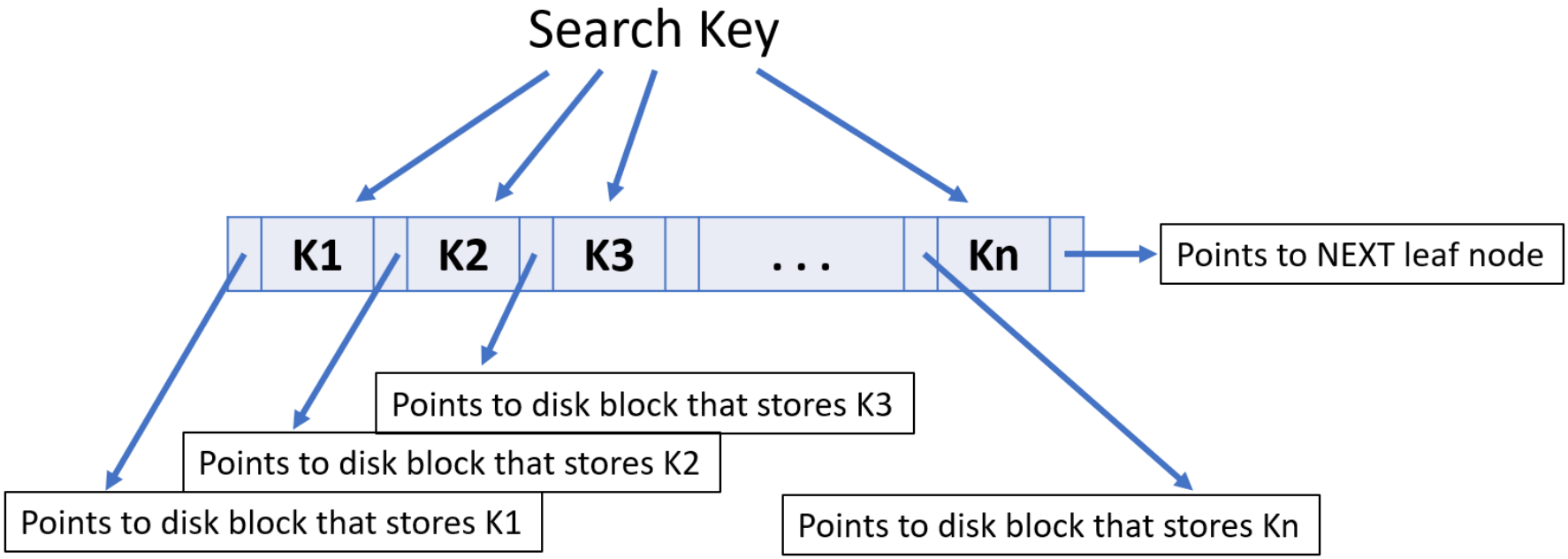


Figure: Leaf Node Structure

Node Bounds

Node Type	**Min #Keys**	**Max #Keys**	**Min #Child**	**Max #Child**
Root Node	1	$M-1$	2[3]	M
Internal Node	$\lceil \frac{M}{2} \rceil - 1$	$M-1$	$\lceil \frac{M}{2} \rceil$	M
Leaf Node	$\lceil \frac{M}{2} \rceil - 1$	$M-1$	0	0

M = Order of B+ Tree

Operations on B+ Tree

HOW do We Operate on B+ Tree?

Insertion

B+ Tree Insertion

How to insert

Since every element is inserted into the leaf node, go to the appropriate leaf node. Insert the key into the leaf node.

Case I:

If the leaf is not full, insert the key into the leaf node in increasing order.

B+ Tree Insertion

How to insert

Since every element is inserted into the leaf node, go to the appropriate leaf node. Insert the key into the leaf node.

Case I:

If the leaf is not full, insert the key into the leaf node in increasing order.

Case II:

1. If the leaf is full, insert the key into the leaf node in increasing order and balance the tree in the following way.
2. Break the node at $\frac{m}{2}$ th position.
3. Add $\frac{m}{2}$ th key to the parent node as well.
4. If the parent node is already full, follow steps 2 to 3.

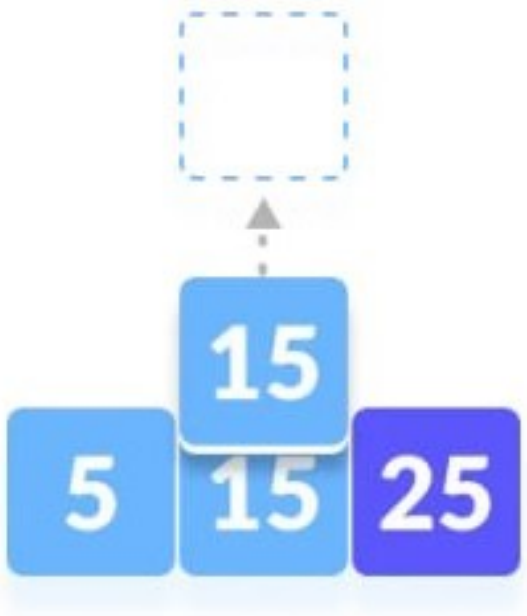
B+ Tree Insertion Animation : Insert 5

Order = 3

B+ Tree Insertion Animation : Insert 15

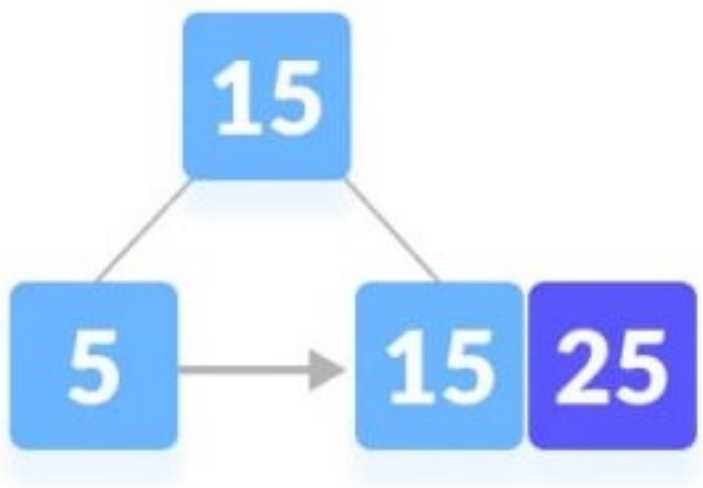
B+ Tree Insertion Animation : Insert 25

B+ Tree Insertion Animation : Insert 25

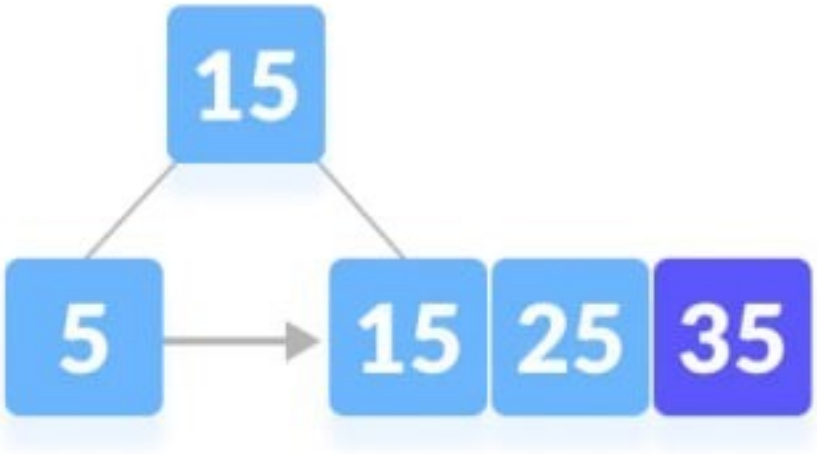


B+ Tree Insertion Animation : Insert 25

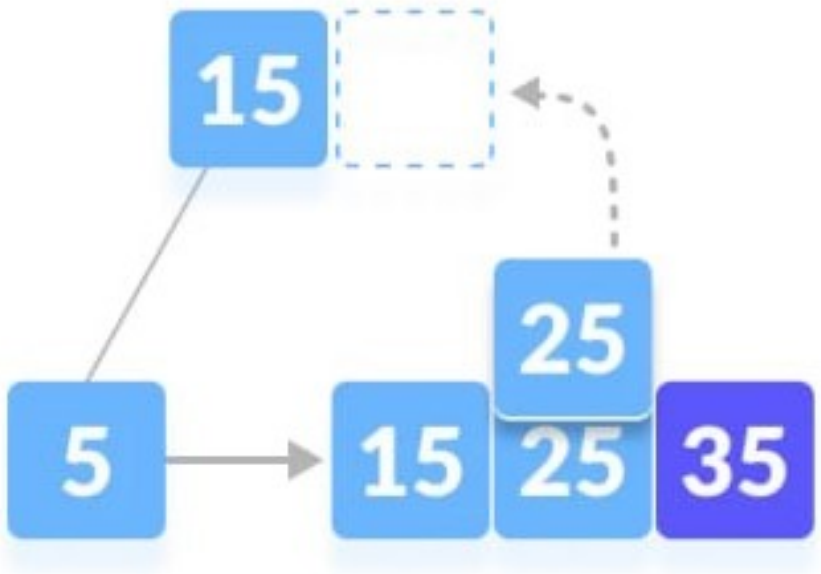
B+ Tree Insertion Animation : Insert 25



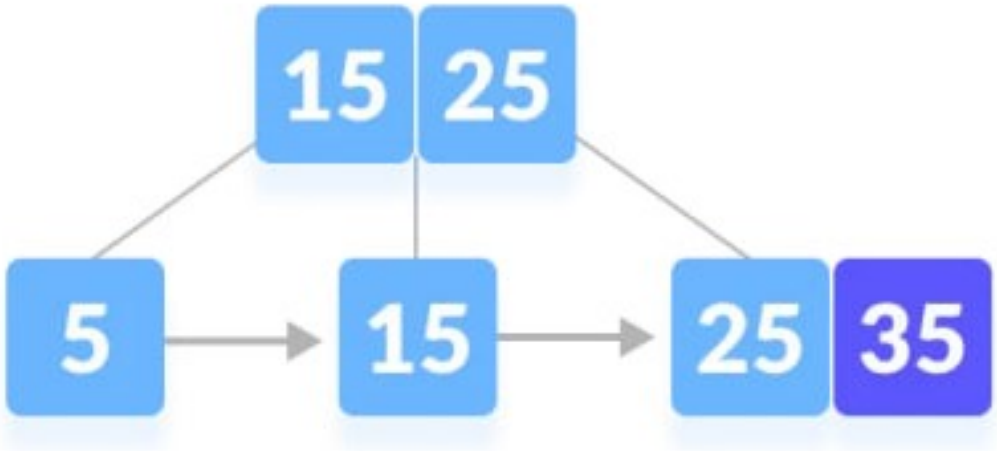
B+ Tree Insertion Animation : Insert 35



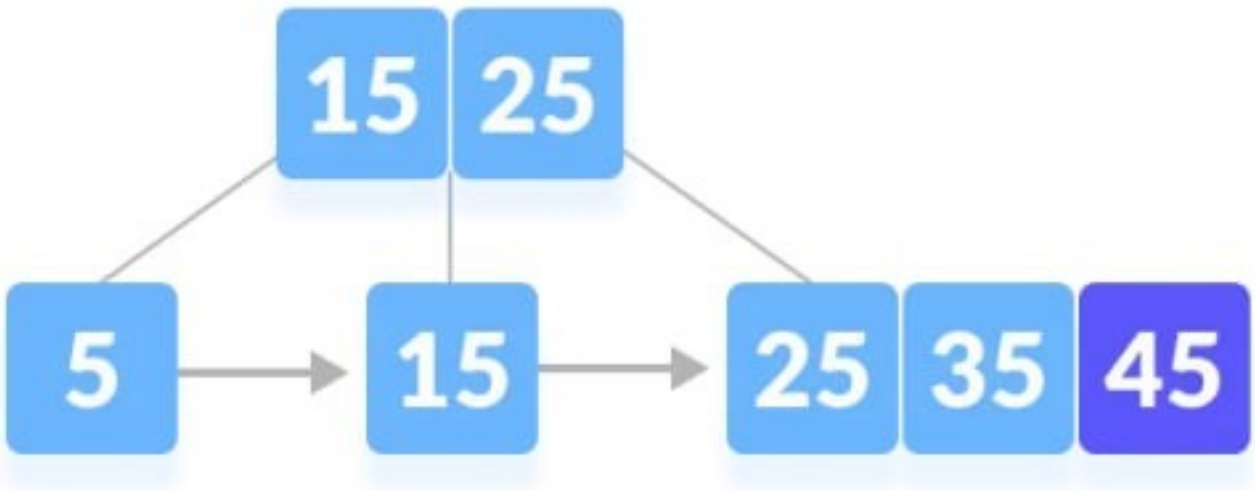
B+ Tree Insertion Animation : Insert 35



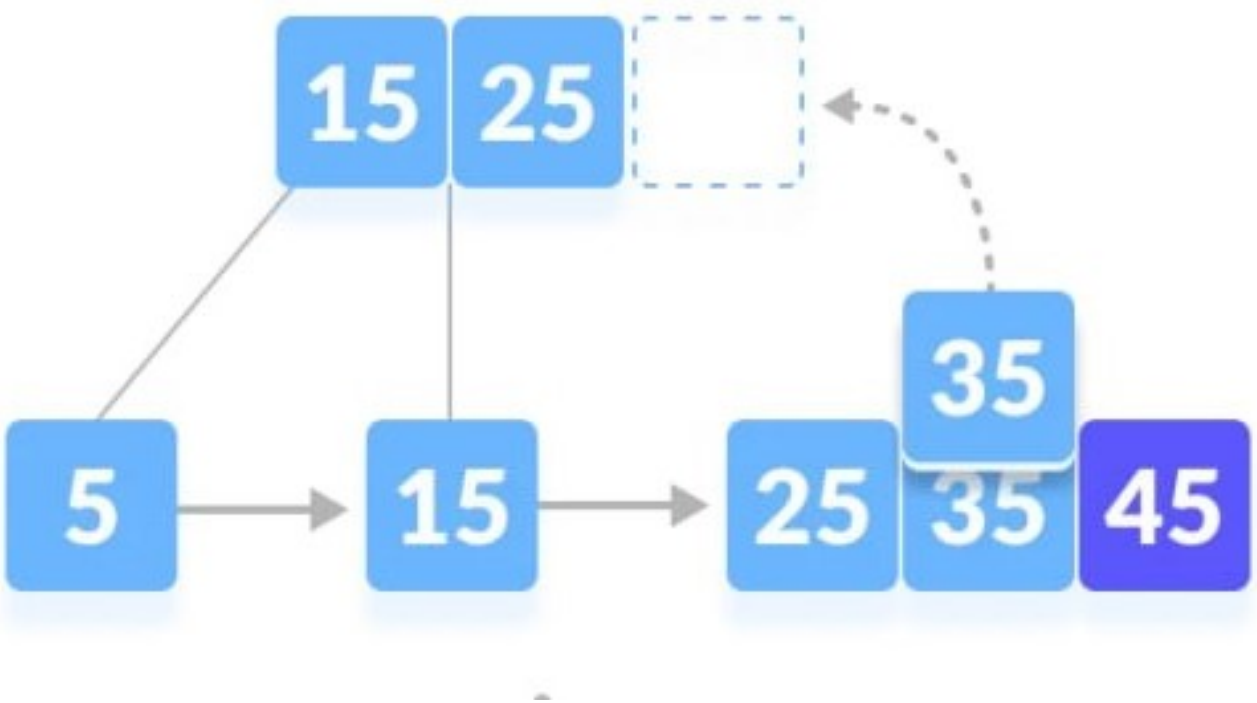
B+ Tree Insertion Animation : Insert 35



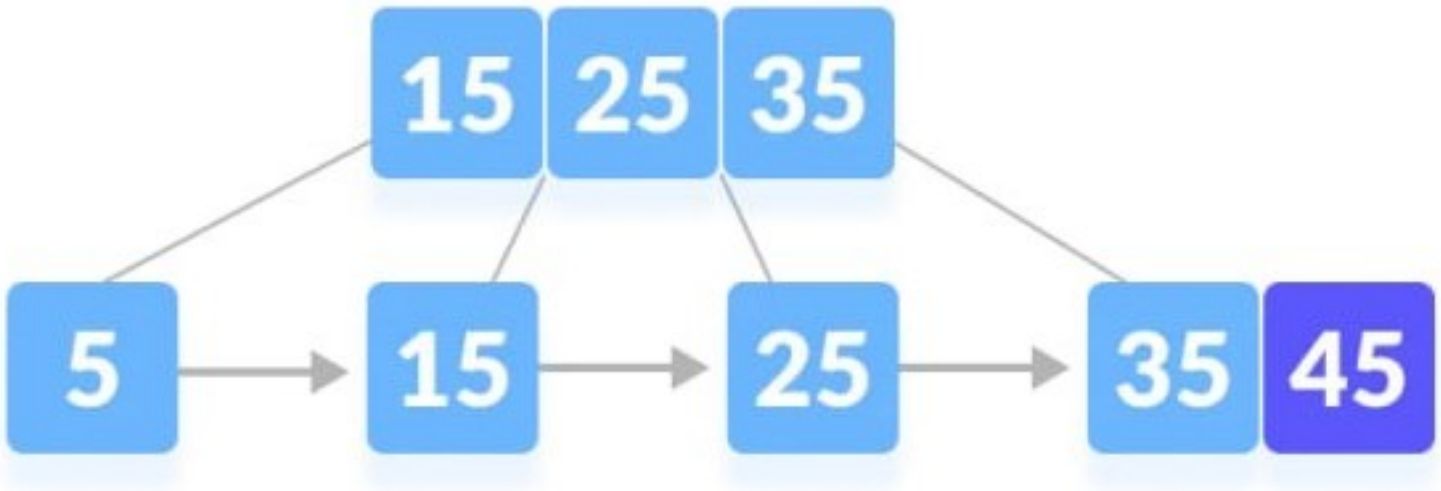
B+ Tree Insertion Animation : Insert 45



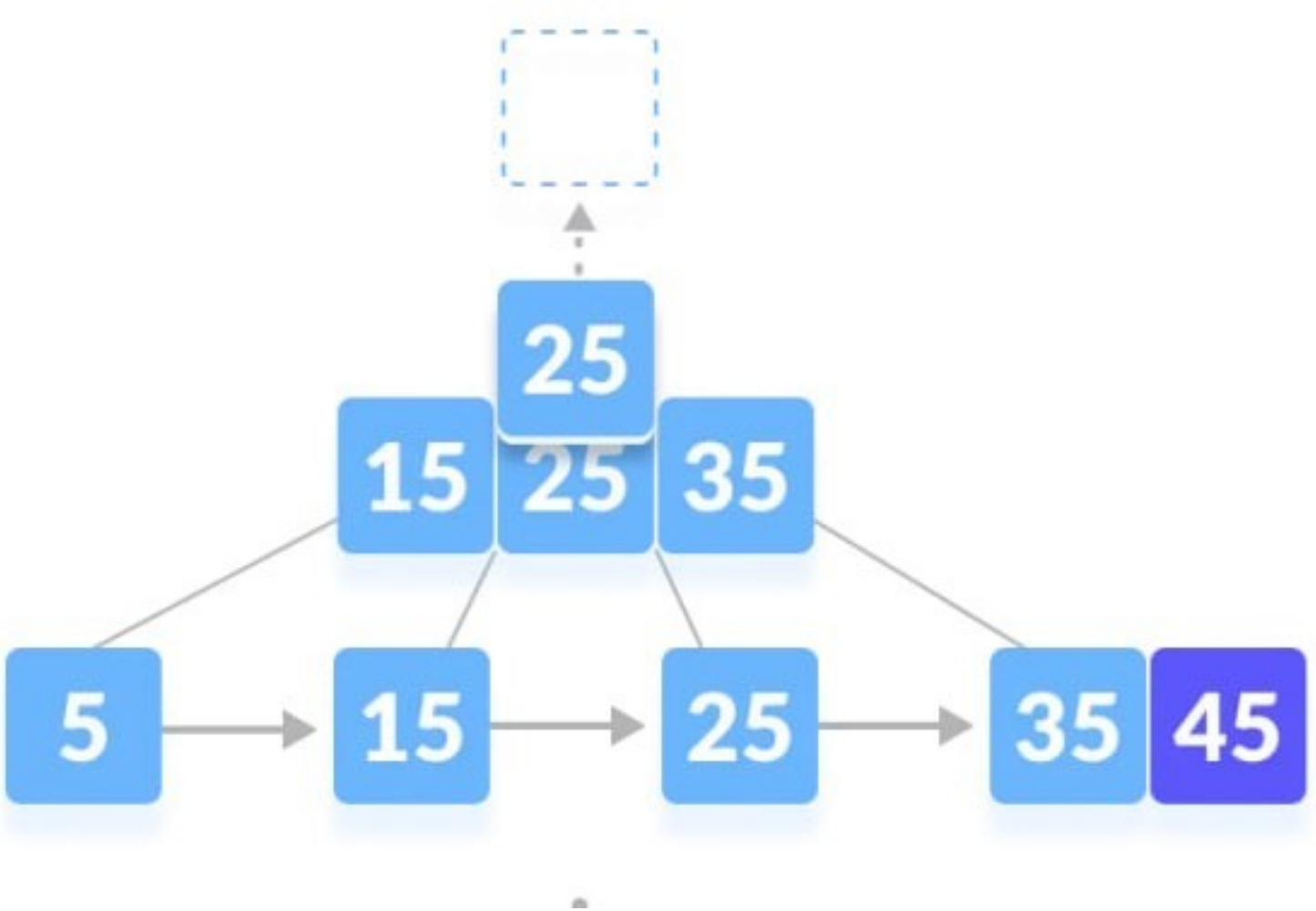
B+ Tree Insertion Animation : Insert 45



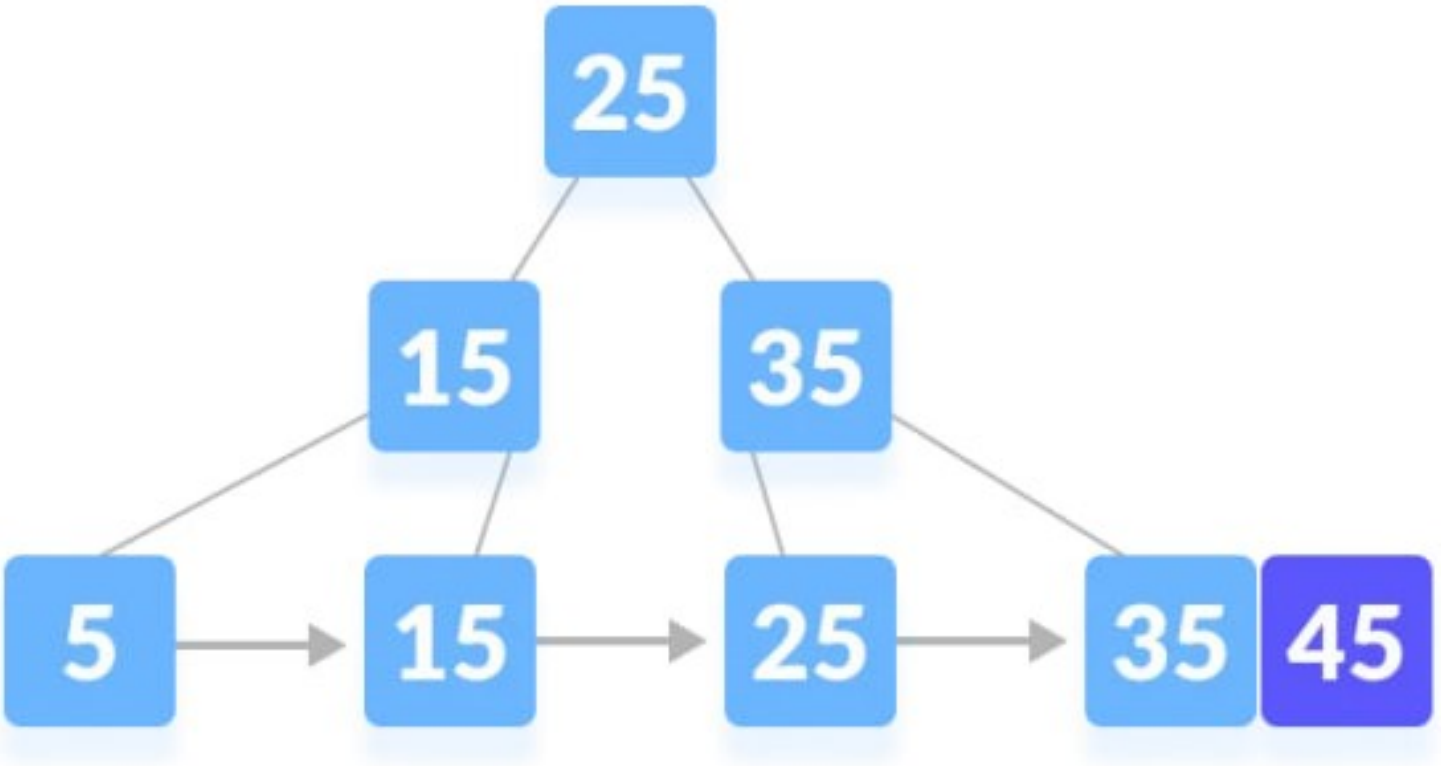
B+ Tree Insertion Animation : Insert 45



B+ Tree Insertion Animation : Insert 45



B+ Tree Insertion Animation : Insert 45



Deletion

B+ Tree Deletion

Before going through the steps below, one must know these facts about a B+ tree of degree m. A node can have -

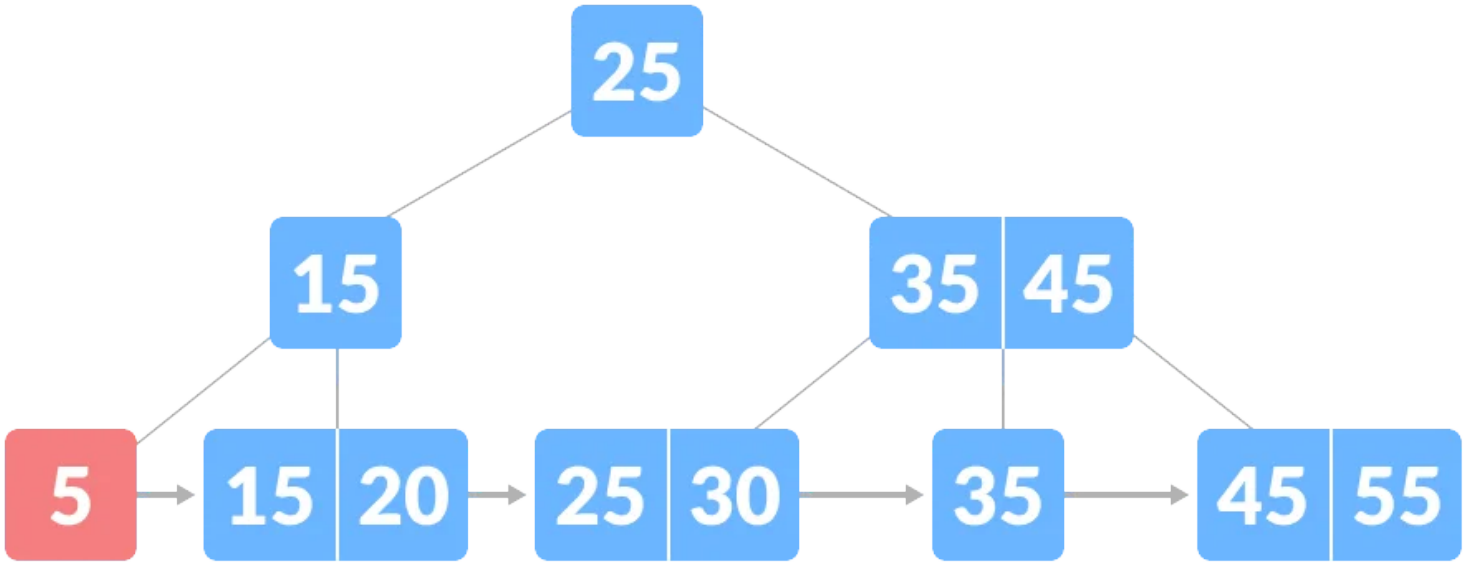
1. a maximum of m children. (i.e. 3)
2. a maximum of m - 1 keys. (i.e. 2)
3. a minimum of $\lceil m/2 \rceil$ children. (i.e. 2)
4. a minimum of $\lceil m/2 \rceil - 1$ keys (except root node). (ie 1)

* In our example m $= 3$

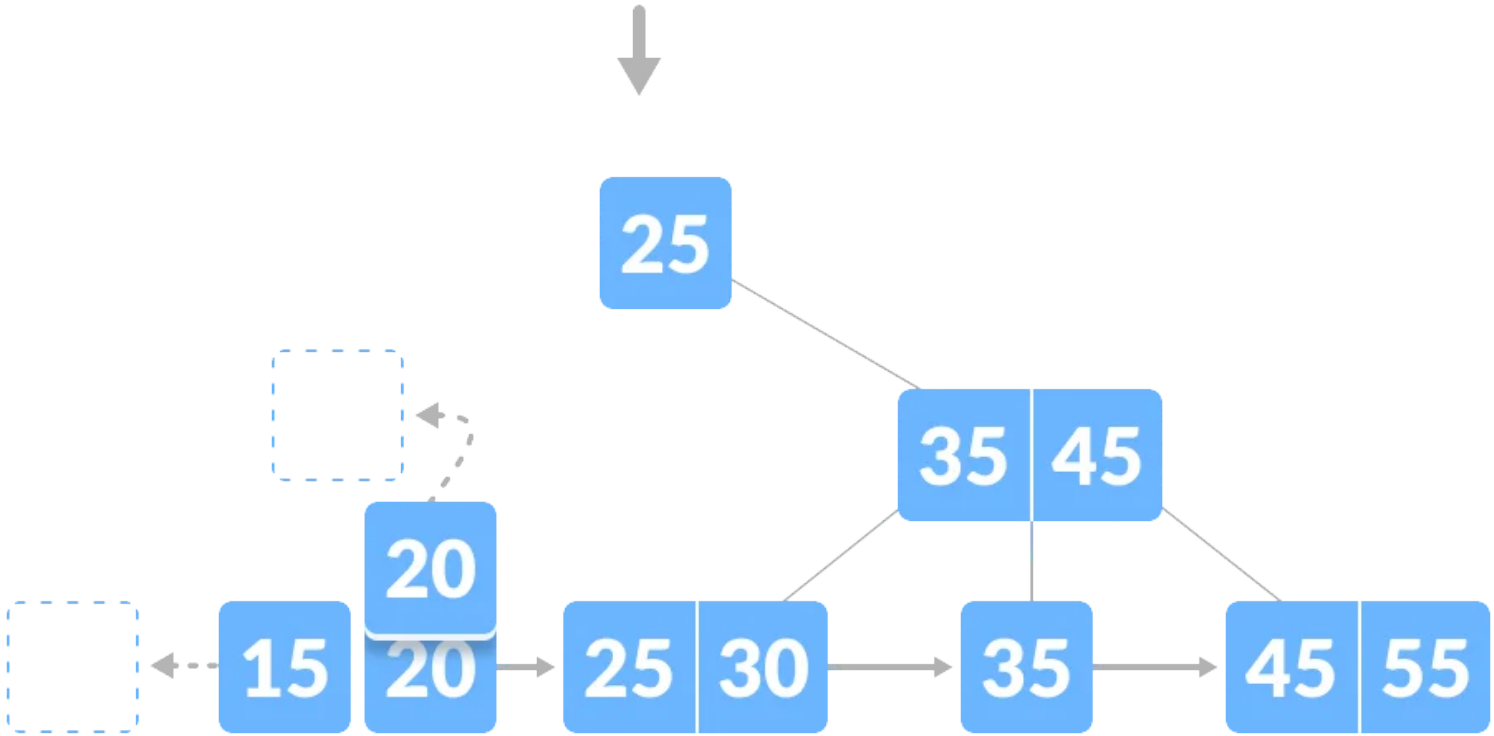
B+ Tree Deletion Animation

Case 1

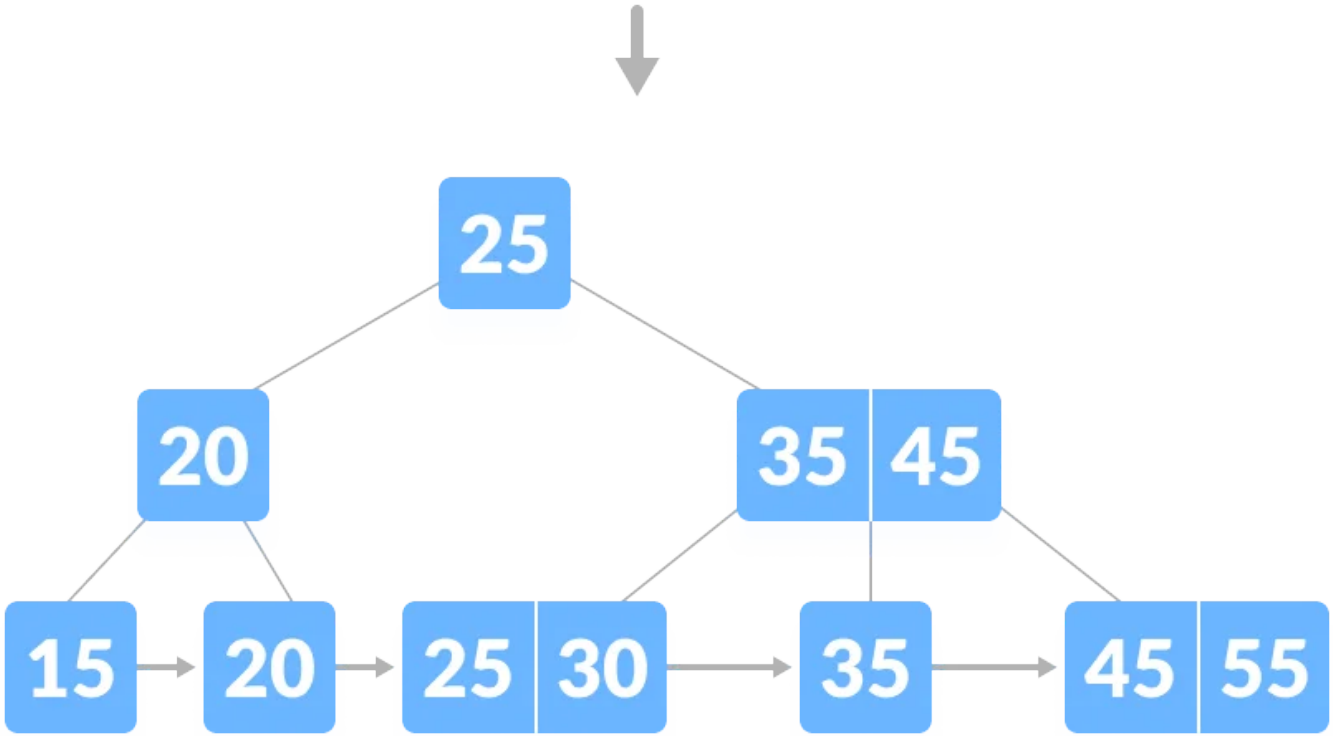
The key to be deleted is present only in the leaf node and not in the internal nodes. For instance 5.



B+ Tree Deletion Animation



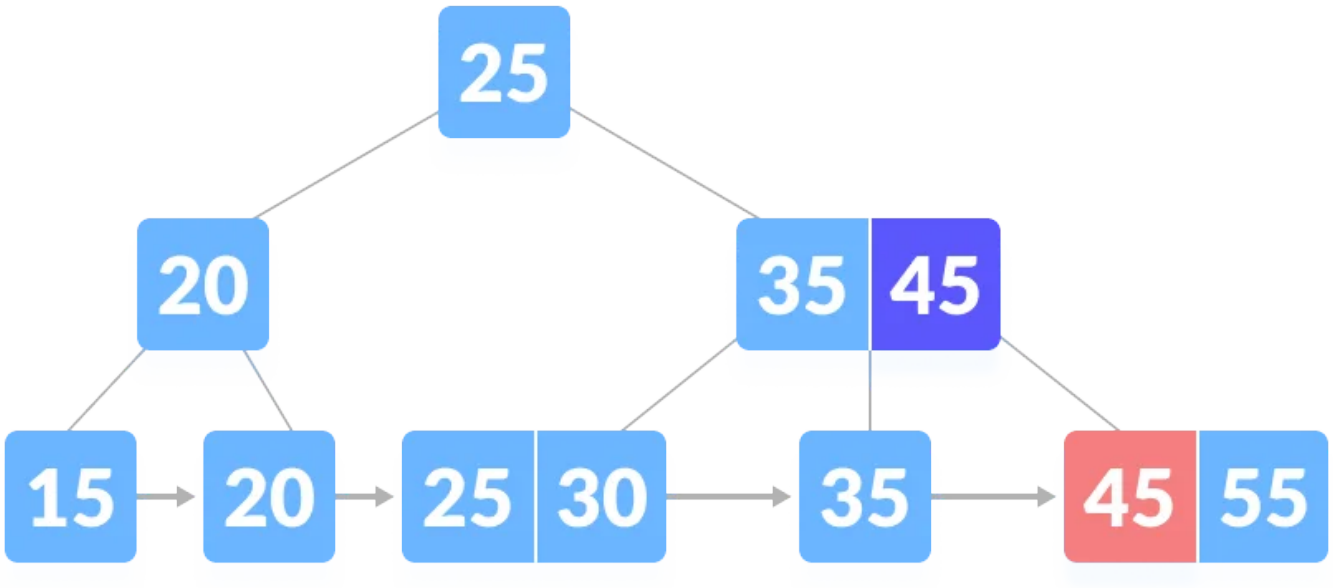
B+ Tree Deletion Animation



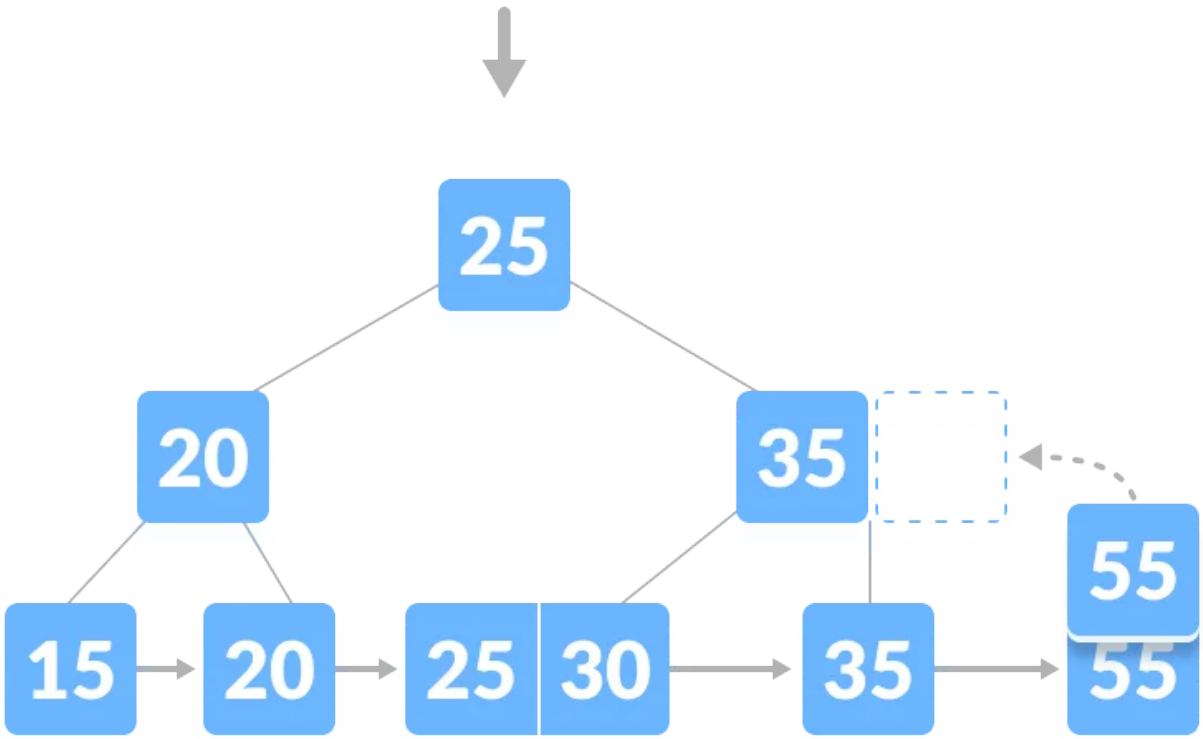
B+ Tree Deletion Animation

Case 2

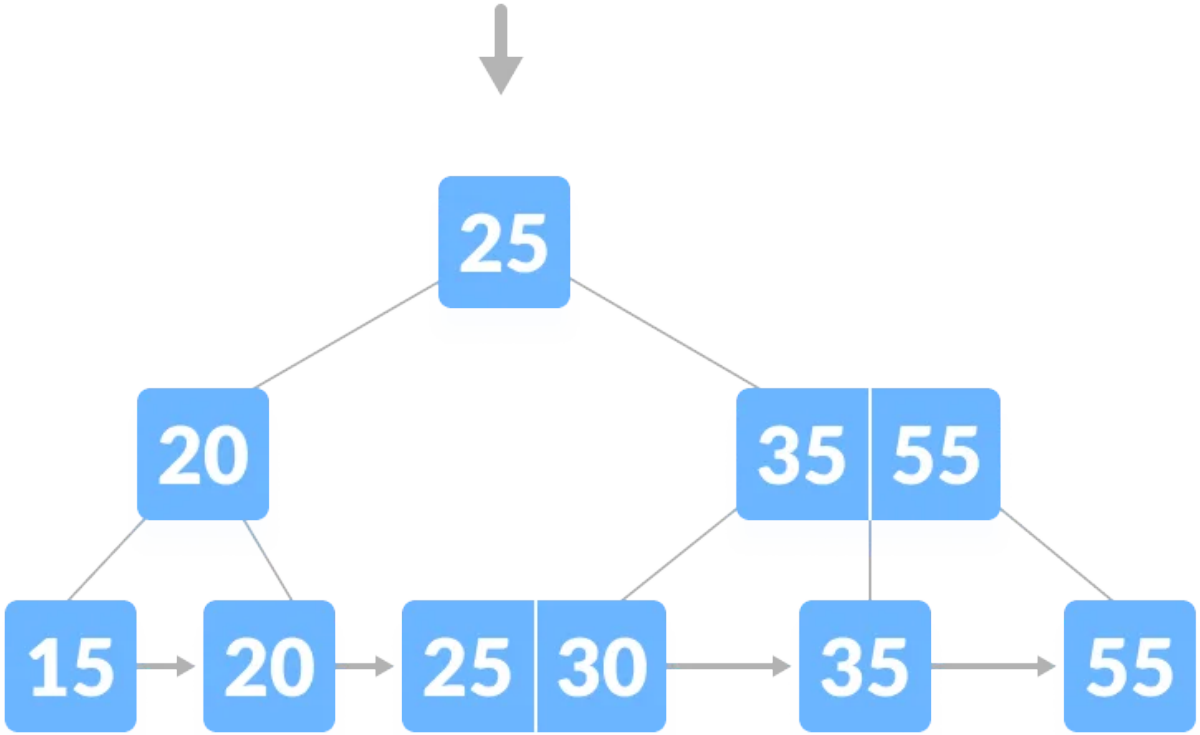
The key to be deleted is present in the internal nodes as well. For example, 45.



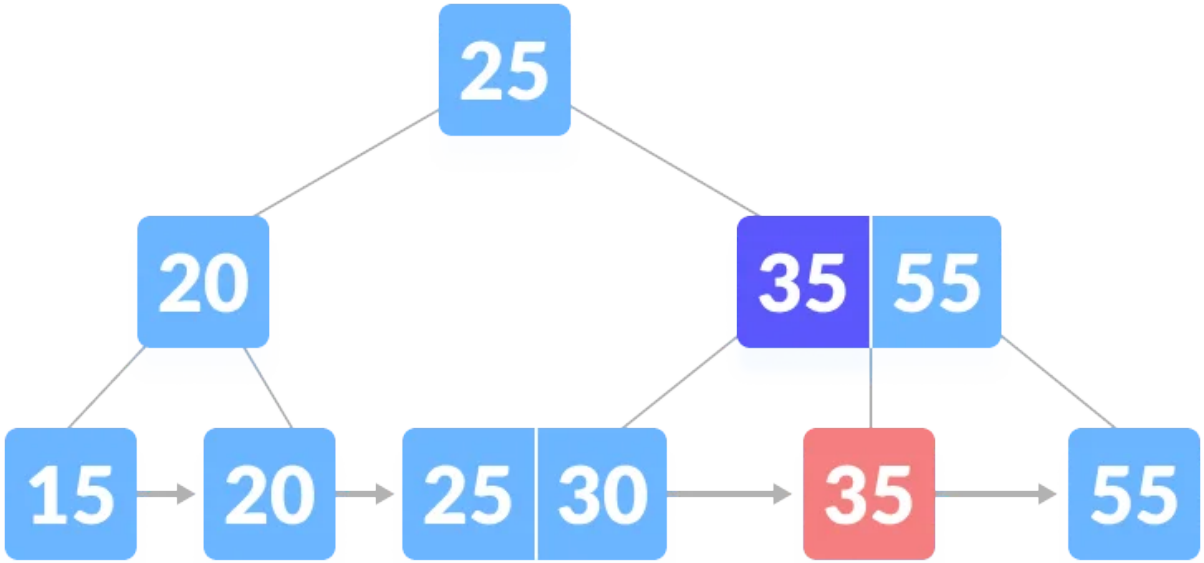
B+ Tree Deletion Animation



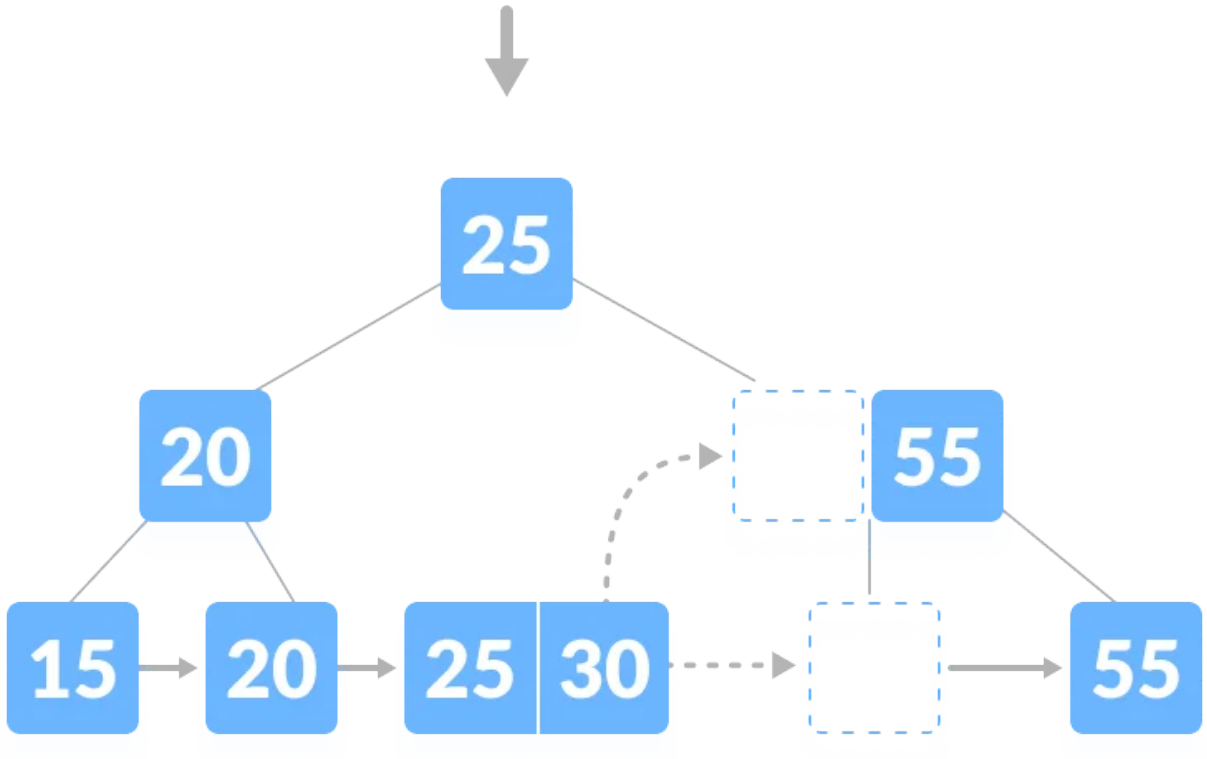
B+ Tree Deletion Animation



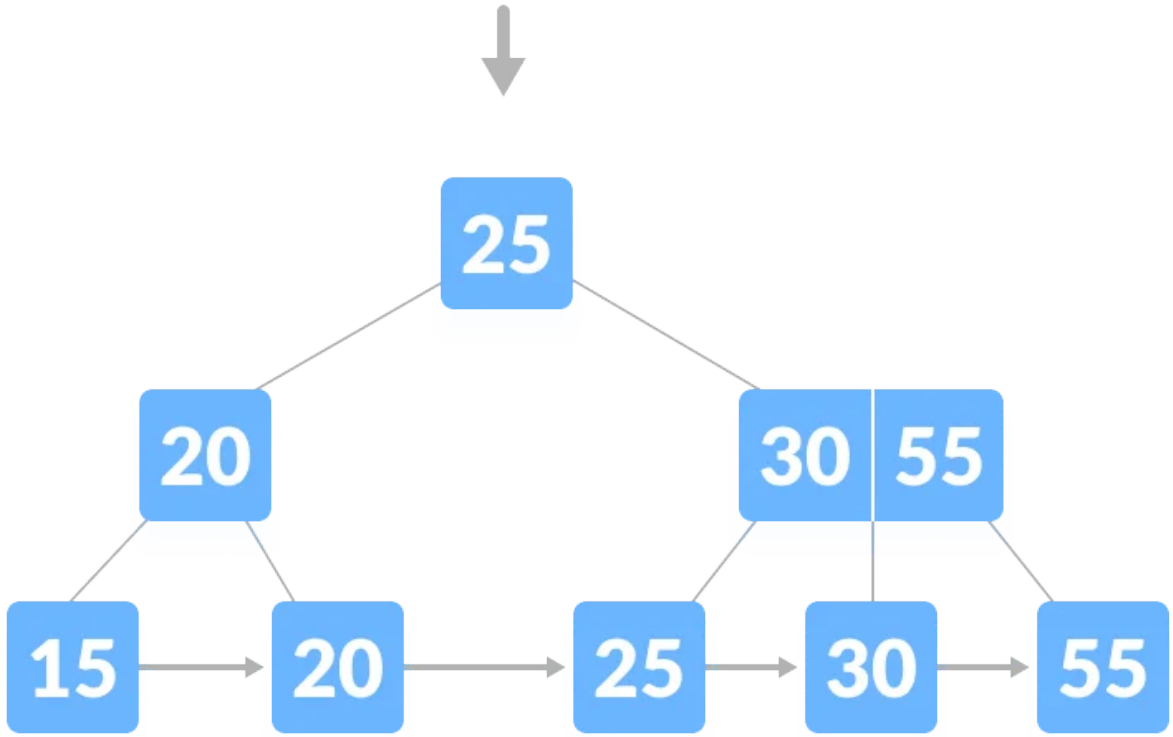
B+ Tree Deletion Animation



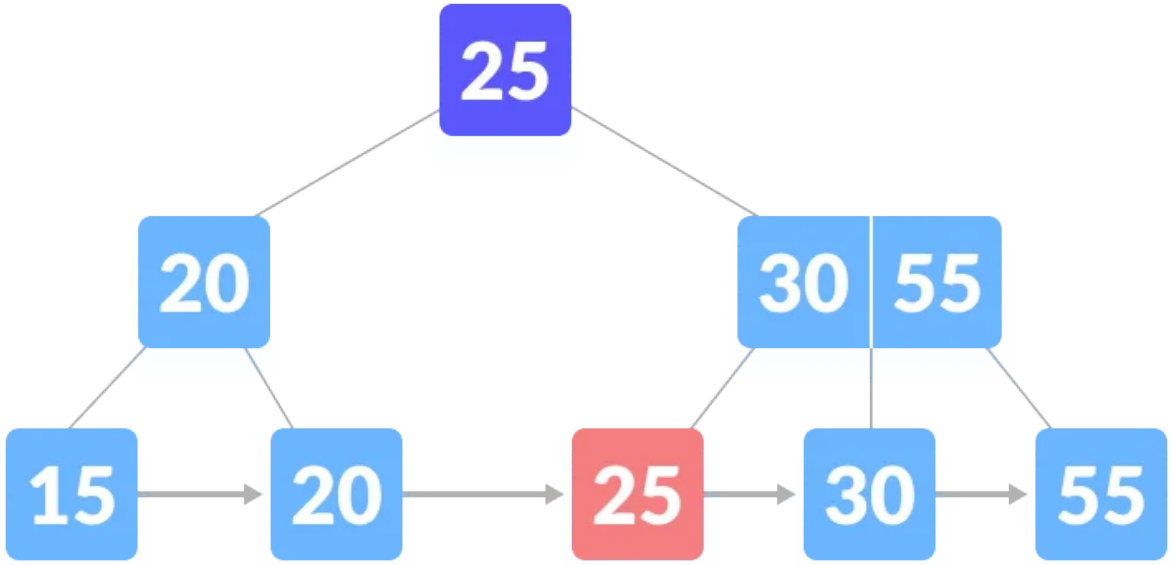
B+ Tree Deletion Animation



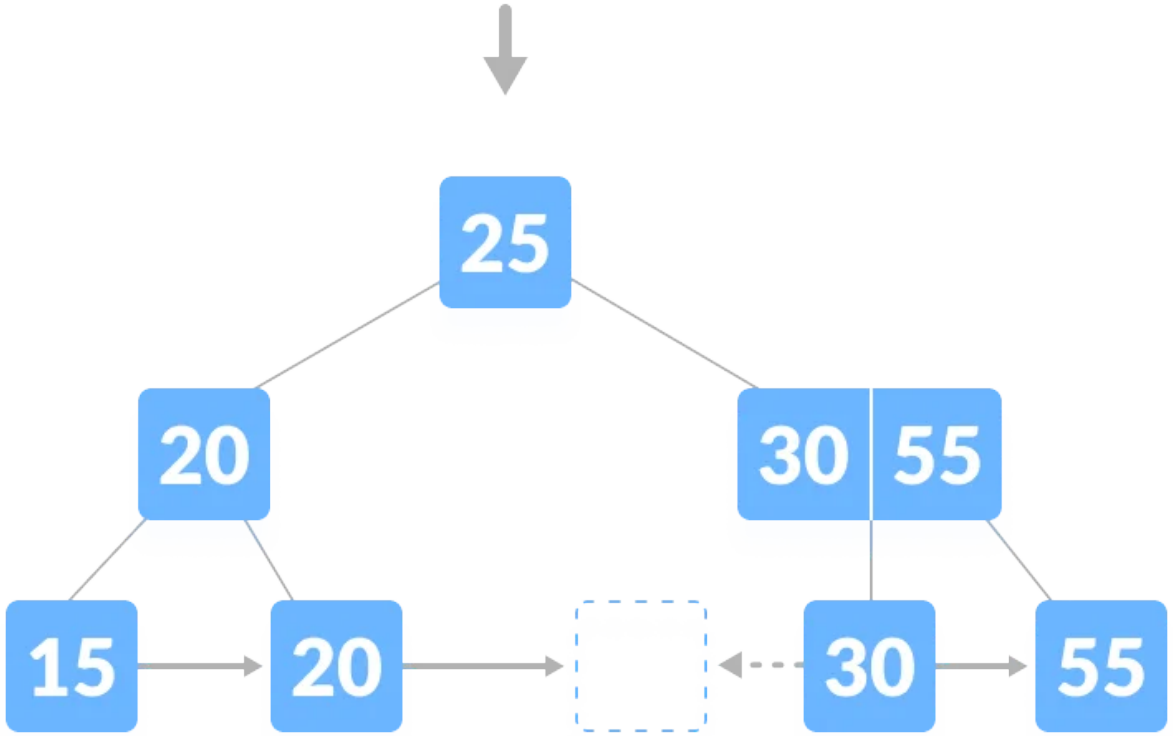
B+ Tree Deletion Animation



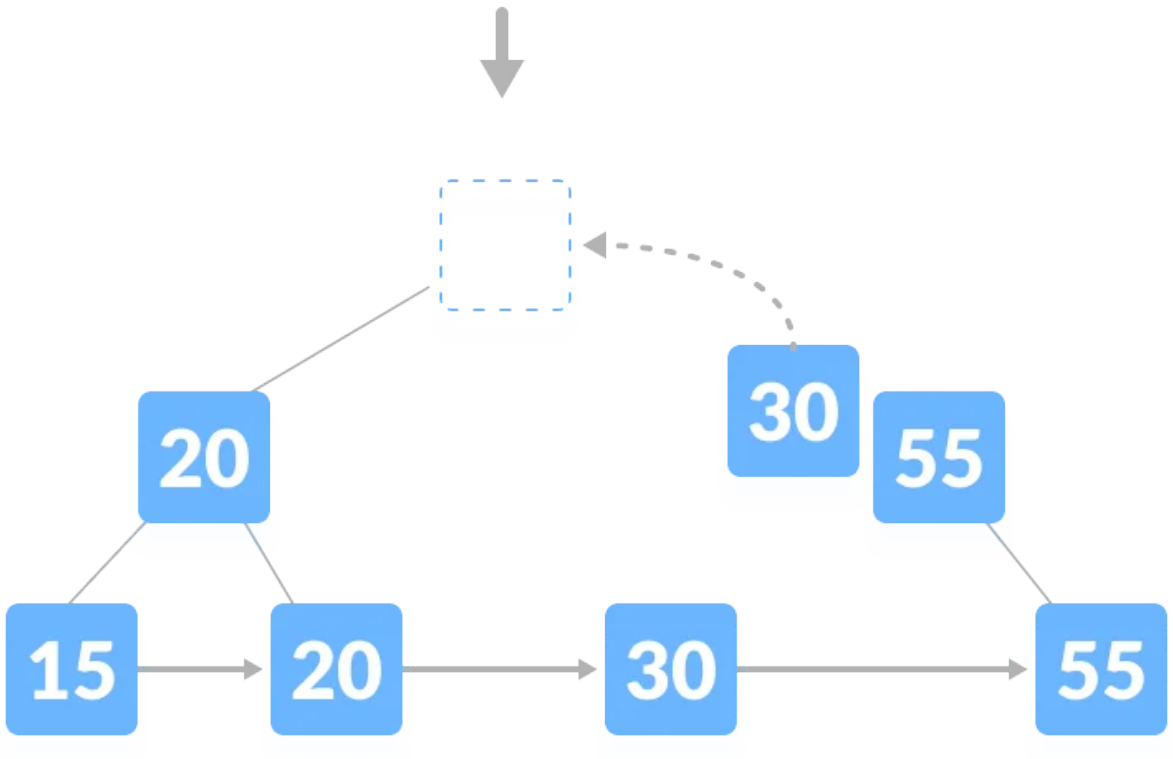
B+ Tree Deletion Animation



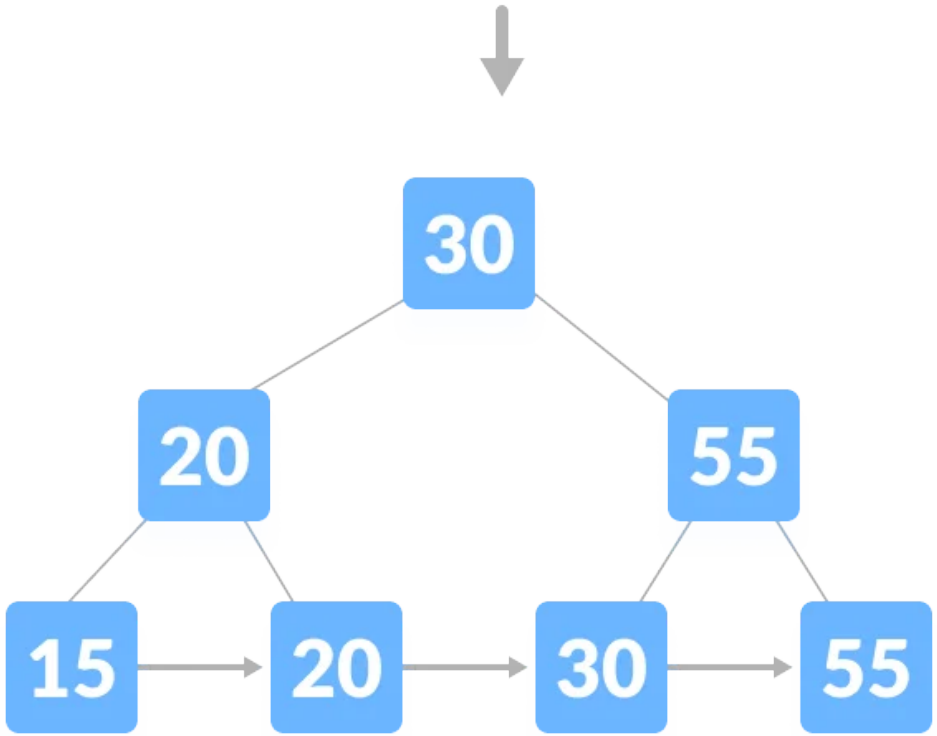
B+ Tree Deletion Animation



B+ Tree Deletion Animation



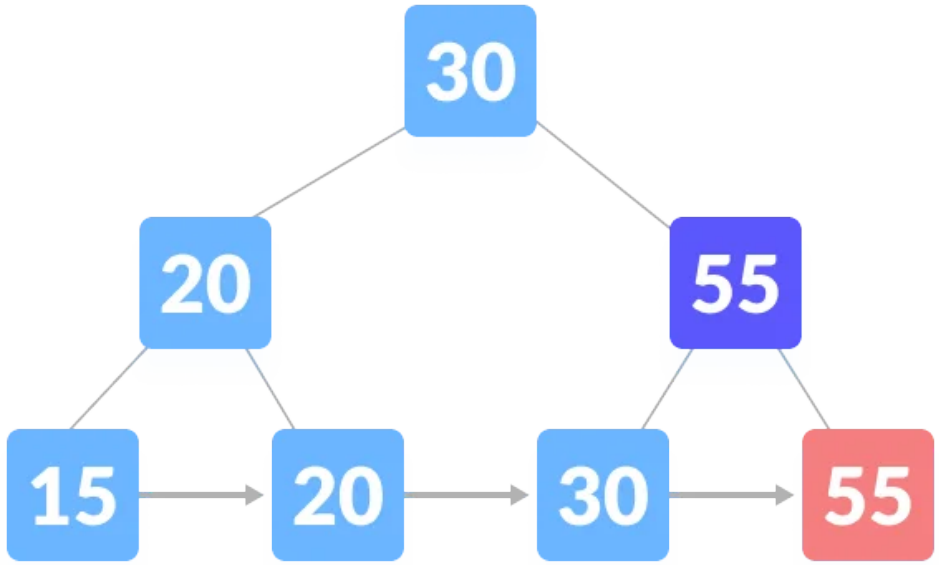
B+ Tree Deletion Animation



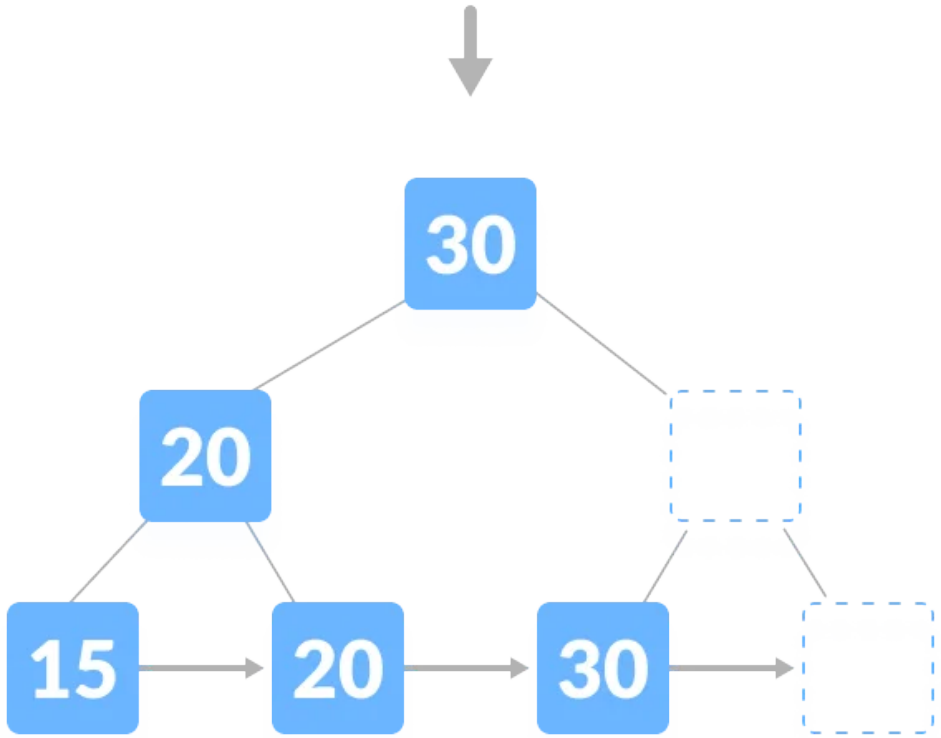
B+ Tree Deletion Animation

Case 3

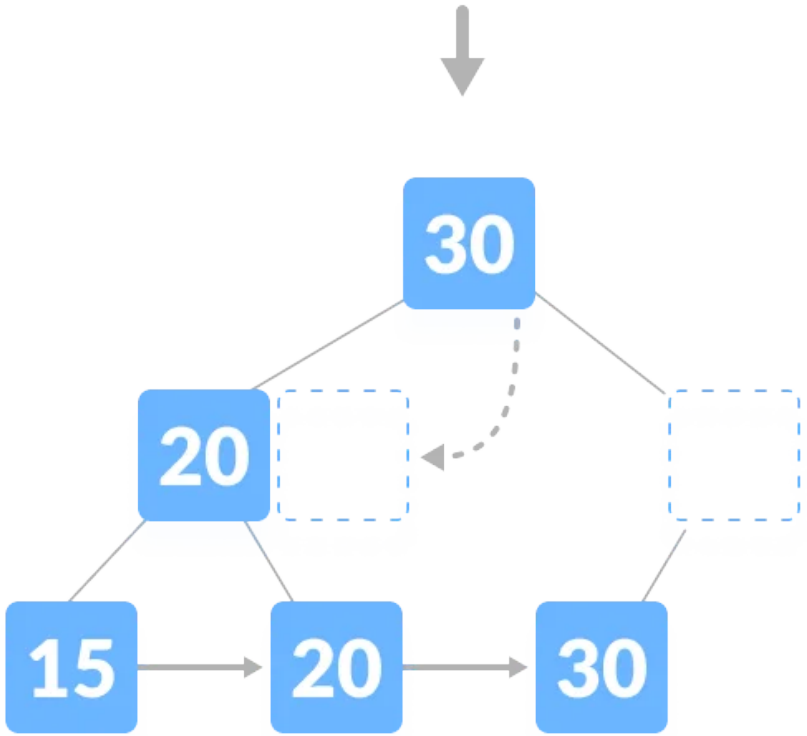
The height of the tree gets shrinked. For example deletion of 55.



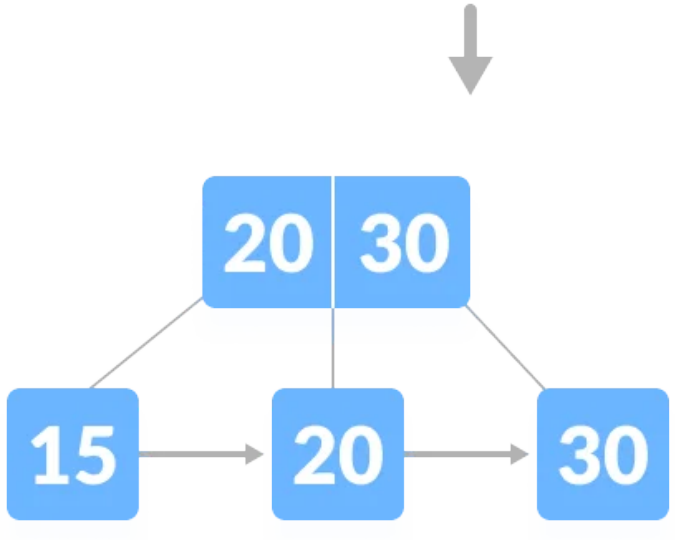
B+ Tree Deletion Animation



B+ Tree Deletion Animation



B+ Tree Deletion Animation



Importance of B+ Tree

WHY do We Actually Need B+ Tree?

Time Analysis

B+ Tree Operation	Time Complexity
Insertion	$O(\log_m n)$

m = Order of B+ Tree

n = Number of elements in B+ Tree

Time Analysis

B+ Tree Operation	Time Complexity
Insertion	$O(\log_m n)$
Deletion	$O(\log_m n)$

m = Order of B+ Tree
n = Number of elements in B+ Tree

Time Analysis

B+ Tree Operation	Time Complexity
Insertion	$O(\log_m n)$
Deletion	$O(\log_m n)$
Search	$O(\log_m n)$

m = Order of B+ Tree
n = Number of elements in B+ Tree

Advantages of B+ Trees

Advantages of B+ Trees

Ordered Structure for Sequential Access

B+ trees maintain data in sorted order, facilitating efficient sequential access to keys. [2].

Advantages of B+ Trees (Cont'd)

Concurrency Control and Performance

B+ trees supports locking and MVCC to ensure consistency and isolation among concurrent transactions [1].

Advantages of B+ Trees (Cont'd)

Optimal Disk Access and Cache Efficiency

B+ trees utilize node-based structures and node sizes that align well with the block size of storage devices, minimizing disk access and maximizing cache hits [4].

Real Life Application of B+ Tree

WHERE do We Use B+ Trees in Real Life?

Real Life Application of B+ Tree

Real Life Application of B+ Tree

Database Indexing

B+ trees enable effective retrieval of data based on indexed characteristics, resulting in faster query execution times.

Real Life Application of B+ Tree

Web Browsers

B+ trees are employed in web browsers for managing bookmarks and history data.

Real Life Application of B+ Tree

Geo-spatial Databases

B+ trees are well-suited for indexing geo-spatial data such as coordinates, shapes, and spatial relationships.

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End of Presentation

Thank You!