

Project 2

$$① \quad T(t) = \begin{bmatrix} \cos(t\pi/3) & 0 & -\sin(t\pi/3) & t \\ 0 & 1 & 0 & 0 \\ \sin(t\pi/3) & 0 & \cos(t\pi/3) & 2t \\ 0 & 0 & 0 & 1 \end{bmatrix} \in SE(3)$$

$$a) \quad R = \begin{bmatrix} \cos(t\pi/3) & 0 & -\sin(t\pi/3) \\ 0 & 1 & 0 \\ \sin(t\pi/3) & 0 & \cos(t\pi/3) \end{bmatrix} = \begin{bmatrix} .5 & 0 & -\sqrt{3}/2 \\ 0 & 1 & 0 \\ \sqrt{3}/2 & 0 & .5 \end{bmatrix}$$

$$\text{Axis} = \begin{bmatrix} 21-12 \\ 02-20 \\ 10-01 \end{bmatrix} = \begin{bmatrix} 0-0 \\ -\sqrt{3}/2 - \sqrt{3}/2 \\ 0-0 \end{bmatrix} = \begin{bmatrix} 0 \\ -\sqrt{3} \\ 0 \end{bmatrix} \frac{1}{2\sin\theta} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$1 + 2\cos\theta = \text{Trace}(R) \rightarrow \theta = \arccos\left(\frac{\text{Trace}(R) - 1}{2}\right) \rightarrow$$

$$\theta = \arccos\left(\frac{\cos(\pi/3) + 1 + \cos(\pi/3) - 1}{2}\right) = \frac{\pi}{3}$$

$$(\text{axis}, \text{angle}) = \left(\begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}, \theta \right) = \left(\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \frac{\pi}{3} \right)$$

$$b) \quad q = \exp\left\{\frac{\theta}{2}(u_x i + u_y j + u_z k)\right\} = \cos\left(\frac{\theta}{2}\right) + (u_x i + u_y j + u_z k)\sin\left(\frac{\theta}{2}\right) \\ = \cos\left(\frac{\pi/3}{2}\right) + (0, -1, 0)\sin\left(\frac{\pi/3}{2}\right) = \left[\frac{\sqrt{3}}{2}, 0, -.5, 0\right]$$

$$q^{-1} = \exp\left\{-\frac{\theta}{2}(u_x i + u_y j + u_z k)\right\} = \cos\left(\frac{\theta}{2}\right) - (u_x i + u_y j + u_z k)\sin\left(\frac{\theta}{2}\right) \\ = \cos\left(\frac{\pi/3}{2}\right) - (0, -1, 0)\sin\left(\frac{\pi/3}{2}\right) = \left[\frac{\sqrt{3}}{2}, 0, .5, 0\right]$$

$$c) P(t) = \begin{bmatrix} t \\ 0 \\ 2t \end{bmatrix}$$

$$\text{Angular } \hat{W}(t) = \dot{R}(t) R^T(t) = \begin{bmatrix} (-1/3)\pi \sin(t\pi/3) & 0 & (-1/3)\pi \cos(t\pi/3) \\ 0 & 0 & 0 \\ (1/3)\pi \cos(t\pi/3) & 0 & (-1/3)\pi \sin(t\pi/3) \\ \cos(t\pi/3) & 0 & \sin(t\pi/3) \\ 0 & 1 & 0 \\ -\sin(t\pi/3) & 0 & \cos(t\pi/3) \end{bmatrix}$$

$$\hat{W}(1) = \begin{bmatrix} 0 & 0 & -\pi/3 \\ 0 & 0 & 0 \\ \pi/3 & 0 & 0 \end{bmatrix}$$

$$\text{Linear } \dot{P}(1) - \dot{R}(1) R^T(1) P(1) = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 0 & 0 & -\pi/3 \\ 0 & 0 & 0 \\ \pi/3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 + 2\pi/3 \\ 0 \\ 2 - \pi/3 \end{bmatrix}$$

$$d) \begin{bmatrix} R & P \\ 0 & I \end{bmatrix}^{-1} = \begin{bmatrix} R^T & -R^T P \\ 0 & I \end{bmatrix} \quad \begin{matrix} \hat{W}_B = (\dot{R}^T) R \\ v_B = (-\dot{R}^T P) - (R^T) R (-R^T P) \end{matrix}$$

$$\begin{aligned} \hat{W}_B &= \begin{bmatrix} (-\pi/3) \sin(t\pi/3) & 0 & (\pi/3) \cos(t\pi/3) \\ 0 & 0 & 0 \\ (-\pi/3) \cos(t\pi/3) & 0 & (-\pi/3) \sin(t\pi/3) \end{bmatrix} \begin{bmatrix} \cos(t\pi/3) & 0 & -\sin(t\pi/3) \\ 0 & 1 & 0 \\ \sin(t\pi/3) & 0 & \cos(t\pi/3) \end{bmatrix} \\ &= \begin{bmatrix} -.907 & 0 & .524 \\ 0 & 0 & 0 \\ -.524 & 0 & -.907 \end{bmatrix} \begin{bmatrix} 1/2 & 0 & -.866 \\ 0 & 1 & 0 \\ .866 & 0 & 1/2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \pi/3 \\ 0 & 0 & 0 \\ -\pi/3 & 0 & 0 \end{bmatrix} \end{aligned}$$

$$P_{new} = \begin{bmatrix} -\cos(t\pi/3) & 0 & -\sin(t\pi/3) \\ 0 & 1 & 0 \\ \sin(t\pi/3) & 0 & -\cos(t\pi/3) \end{bmatrix} \begin{bmatrix} t \\ 0 \\ 2t \end{bmatrix} = \begin{bmatrix} -t\cos(t\pi/3) - 2t\sin(t\pi/3) \\ 0 \\ t\sin(t\pi/3) - 2t\cos(t\pi/3) \end{bmatrix}$$

Project 2

$$= \begin{bmatrix} -1.2321 \\ 0 \\ -1.1340 \end{bmatrix}$$

$$P_{new} = \begin{bmatrix} -\cos(t\pi/3) + (t\pi/3)\sin(t\pi/3) - 2\sin(4\pi/3) - (2t\pi/3)\cos(t\pi/3) \\ 0 \\ \sin(t\pi/3) + (t\pi/3)\cos(4\pi/3) - 2\cos(t\pi/3) + (2t\pi/3)\sin(t\pi/3) \end{bmatrix}$$

$$= \begin{bmatrix} -2.37 \\ 0 \\ 2.203 \end{bmatrix}$$

$$V_B = P_{new} - \hat{\omega}_B P_{new} = \begin{bmatrix} -2.37 \\ 0 \\ 2.2034 \end{bmatrix} - \begin{bmatrix} 0 & 0 & \pi/3 \\ 0 & 0 & 0 \\ -\pi/3 & 0 & 0 \end{bmatrix} \begin{bmatrix} -1.2321 \\ 0 \\ -1.1340 \end{bmatrix}$$

$$= \begin{bmatrix} -2.23 \\ 0 \\ -1.134 \end{bmatrix}$$

e) $S_B = R^T S_W + (-R^T P)$ where $S_W = \begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix}$

$$= \begin{bmatrix} \cos(t\pi/3) & 0 & \sin(t\pi/3) \\ 0 & 1 & 0 \\ -\sin(t\pi/3) & 0 & \cos(t\pi/3) \end{bmatrix} \begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \cos(t\pi/3) & 0 & \sin(t\pi/3) \\ 0 & 1 & 0 \\ -\sin(t\pi/3) & 0 & \cos(t\pi/3) \end{bmatrix} \begin{bmatrix} + \\ 0 \\ 2t \end{bmatrix}$$

$$= \begin{bmatrix} 2.2679 \\ 0 \\ -7.9282 \end{bmatrix}$$

Project 2

$$\begin{aligned} \textcircled{2} \quad x_{t+1} &= -\frac{1}{2} |x_t| + \cos(x_t) + w_t & w_t &\sim \mathcal{N}(0, 1) \\ z_t &= x_t^2 + v_t & v_t &\sim \mathcal{N}(0, .5) \\ a) \quad x_t^{\text{nan}} &= 1 & x_0 &\sim \mathcal{N}(0, 1) \end{aligned}$$

Motion $A_t = -\frac{1}{2} |x_t| + \cos(x_t)$ $da/dx = -\frac{1}{2} - \sin(x_t)$
 $Q_t = 1$

Obs. $H_t = x_t^2$ $dh/dx = 2x_t \rightarrow x_t = 1 \rightarrow = 2$
 $R_t = .5$

Predict $u_{t+1|t} = -\frac{1}{2} |u_t| + \cos(u_t) \approx 1$
 $\Sigma_{t+1|t} = (-\frac{1}{2} - \sin(x_t))(1)(-\frac{1}{2} - \sin(x_t)) + (1)(1)(1) \approx 1.886$

Kalman Gain $K_{t+1|t} = (1.886)(2)^T / ((2)(1.886)(2)^T + (.5)(.5)(.5)^T)^{-1} \approx .449$

Update $u_{t+1|t+1} = (1) + (.449)(1 - 1) = 1$
 $\Sigma_{t+1|t+1} = (1 - (.449)(2))(1.886) = .0377$

b) $x_{t+1}^0 = 1$ $\lambda = 0, d = 1$

$$x_{t+1}^1 = 1 + \sqrt{1+0}(1) = 2$$

$$x_{t+1}^2 = 1 - \sqrt{1+0}(1) = 0$$

$$W_i^m = [0, 1/2, 1/2] \text{ and } W_i^c = [2, 1/2, 1/2]$$

Predict $u_{t+1|t} = 0(-\frac{1}{2}(1) + \cos(1)) + 1/2(-\frac{1}{2}(2) + \cos(2)) + 1/2(-\frac{1}{2}(0) + \cos(0))$
 $= .1919$

$$\begin{aligned} \Sigma_{t+1|t} &= 2((-1/2(1) + \cos(1)) - .1919)^2 + 1 \\ &\quad + (1/2)((-1/2(2) + \cos(2)) - .1919)^2 + 1 \\ &\quad + (1/2)((-1/2(0) + \cos(0)) - .1919)^2 + 1 = 3.7764 \end{aligned}$$

$$x_{++11+}^0 = .1919$$

$$x_{++11+}^1 = .1919 + \sqrt{1+0} \sqrt{3.77} = 2.13$$

$$x_{++11+}^2 = .1919 - \sqrt{1+0} \sqrt{3.77} = -1.75$$

$$m_{++11+} = 0(.1919^2) + \frac{1}{2}(2.13^2) + \frac{1}{2}(-1.75^2) = 3.7997$$

$$s_{++11+} = 2((.1919)^2 - 3.7997)^2 + .5$$

Kalman
Gain

$$\frac{1}{2}((2.13)^2 - 3.7997)^2 + .5$$

$$\frac{1}{2}((-1.75)^2 - 3.7997)^2 + .5 = 30.3619$$

$$c_{++11+} = 2((.1919 - .1919)((.1919)^2 - 3.7997))$$

$$\frac{1}{2}((2.13 - .1919)((2.13)^2 - 3.7997))$$

$$\frac{1}{2}((-1.75 - .1919)((-1.75)^2 - 3.7997)) = 1.43$$

$$k_{++11+} = (1.43)(30.3619)^{-1} = .047$$

Update

$$u_{++11++1} = (.1919) + (.047)((1) - (3.7997)) = .06$$

$$z_{++11++1} = (3.7764) - (.047)(30.3619)(.047)^T = 3.71$$