

# Lecture 5: Model-Free Control

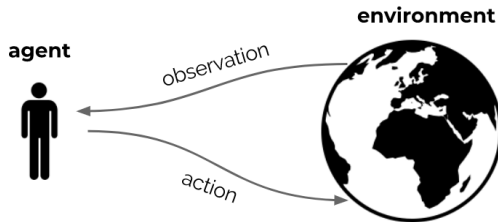
Hado van Hasselt

January 22, 2020, UCL

# Background

Sutton & Barto 2018, Chapter 6

# Recap

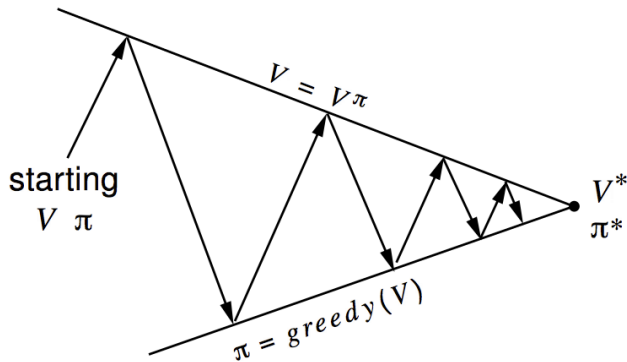


- ▶ Reinforcement learning is the science of learning to make decisions
- ▶ Agents can learn a **policy**, **value function** and/or a **model**
- ▶ The general problem involves taking into account **time** and **consequences**
- ▶ Decisions affect the **reward**, the **agent state**, and **environment state**

# Model-Free Control

- ▶ Previous lecture: **Model-free prediction**  
**Estimate** the value function of an unknown MDP
- ▶ This lecture: **Model-free control**  
**Optimise** the value function of an unknown MDP

## Generalized Policy Iteration (Refresher)

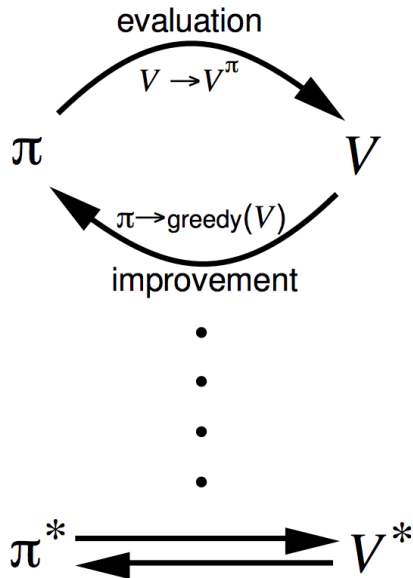


- **Policy evaluation**

Estimate  $v_\pi(s)$  for all  $s$

- **Policy improvement**

Generate  $\pi'$  such that  $v_{\pi'}(s) \geq v_\pi(s)$  for all  $s$



## Recap: Model-Free Policy Evaluation

$$v_{n+1}(S_t) = v_n(S_t) + \alpha (G_t - v_n(S_t))$$

► Variants:

$$\begin{aligned} G_t^{\text{MC}} &= R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots \\ &= R_{t+1} + \gamma G_{t+1}^{\text{MC}} \end{aligned} \quad (\text{MC})$$

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$$G_t^{(1)} = R_{t+1} + \gamma v_t(S_{t+1}) \quad (\text{TD}(0), \text{ or one-step TD})$$

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## Recap: Model-Free Policy Evaluation

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$$G_t^\lambda = R_{t+1} + \gamma[(1 - \lambda)v_t(S_{t+1}) + \lambda G_{t+1}^\lambda] \quad (\text{TD}(\lambda))$$

## Recap: Model-Free Policy Evaluation

$$v_{n+1}(S_t) = v_n(S_t) + \alpha (G_t - v_n(S_t))$$

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$$G_t^\lambda = R_{t+1} + \gamma [(1 - \lambda) v_t(S_{t+1}) + \lambda G_{t+1}^\lambda] \quad (\text{TD}(\lambda))$$

► In all cases, for given  $\pi$  goal is estimating  $v_\pi$ , data is generated to  $\pi$

# Model-Free Policy Iteration Using Action-Value Function

- ▶ Greedy policy improvement over  $v(s)$  requires model of MDP

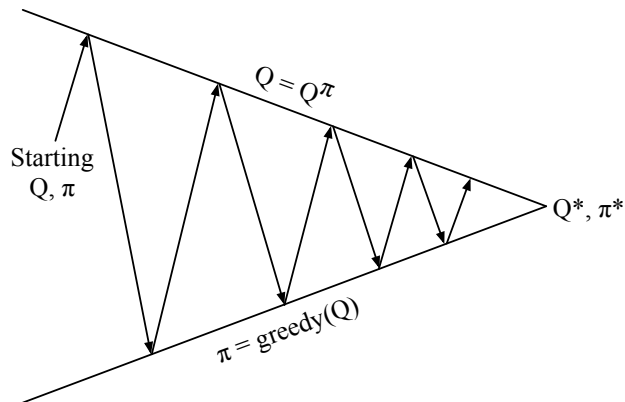
$$\pi'(s) = \operatorname{argmax}_a \mathbb{E} [R_{t+1} + \gamma v(S_{t+1}) \mid S_t = s, A_t = a]$$

- ▶ Greedy policy improvement over  $q(s, a)$  is **model-free**

$$\pi'(s) = \operatorname{argmax}_a q(s, a)$$

- ▶ This makes action values convenient

## Generalised Policy Iteration with Action-Value Function

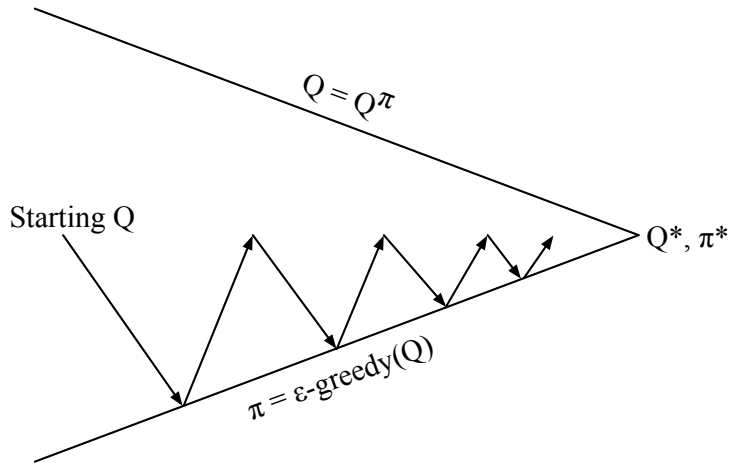


**Policy evaluation** Monte-Carlo policy evaluation,  $q \approx q_\pi$

**Policy improvement** Greedy policy improvement? No exploration!

(Can't do "for all  $s, a$ ", because we sample by interacting)

## Monte-Carlo Generalized Policy Iteration



**Every episode:**

Policy evaluation Monte-Carlo policy evaluation,  $\mathbf{q} \approx \mathbf{q}_\pi$

Policy improvement  $\epsilon$ -greedy policy improvement

## Model-free control

Repeat:

- ▶ Sample episode  $1, \dots, k, \dots$ , using  $\pi$ :  $\{S_1, A_1, R_2, \dots, S_T\} \sim \pi$
- ▶ For each state  $S_t$  and action  $A_t$  in the episode,

$$N(S_t, A_t) \leftarrow N(S_t, A_t) + 1$$

$$q(S_t, A_t) \leftarrow q(S_t, A_t) + \frac{1}{N(S_t, A_t)} (G_t - q(S_t, A_t))$$

- ▶ Improve policy based on new action-value function

$$\epsilon \leftarrow 1/k$$

$$\pi \leftarrow \epsilon\text{-greedy}(q)$$

(Generalises the  $\epsilon$ -greedy bandit algorithm)

# GLIE

## Definition

### **Greedy in the Limit with Infinite Exploration (GLIE)**

- ▶ All state-action pairs are explored infinitely many times,

$$\forall s, a \quad \lim_{t \rightarrow \infty} N_t(s, a) = \infty$$

- ▶ The policy converges to a greedy policy,

$$\lim_{t \rightarrow \infty} \pi_t(a|s) = \mathcal{I}(a = \operatorname{argmax}_{a'} q_t(s, a'))$$

- ▶ For example,  $\epsilon$ -greedy with  $\epsilon_k = \frac{1}{k}$

# GLIE

## Greedy in the Limit with Infinite Exploration (GLIE)

### Definition

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### Theorem

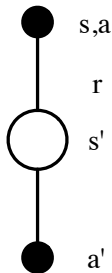
*GLIE Model-free control converges to the optimal action-value function,*  
 $q(s, a) \rightarrow q_*(s, a)$



## MC vs. TD Control

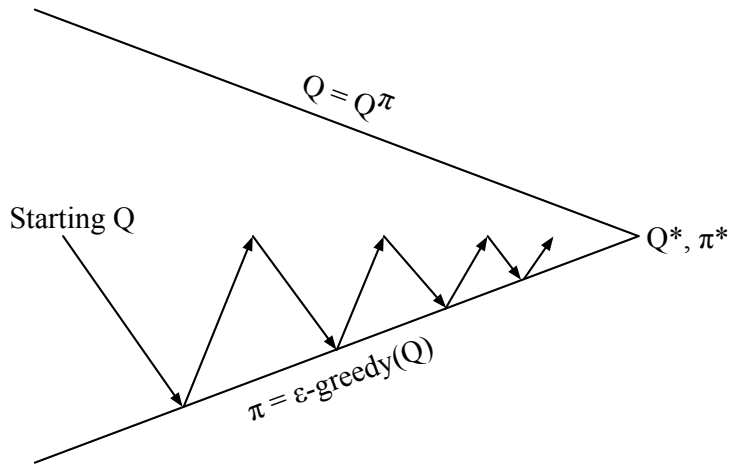
- ▶ Temporal-difference (TD) learning has several advantages over Monte-Carlo (MC)
  - ▶ Lower variance
  - ▶ Online
  - ▶ Can learn from incomplete sequences
- ▶ Natural idea: use TD instead of MC for control
  - ▶ Apply TD to  $q(s, a)$
  - ▶ Use, e.g.,  $\epsilon$ -greedy policy improvement
  - ▶ Update every time-step

## Updating Action-Value Functions with Sarsa



$$q(s, a) \leftarrow q(s, a) + \alpha (r + \gamma q(s', a') - q(s, a))$$

# Sarsa



Every **time-step**:

Policy evaluation **Sarsa**,  $q \approx q_\pi$

Policy improvement  $\epsilon$ -greedy policy improvement

## Tabular Sarsa

Initialize  $Q(s, a)$  arbitrarily

Repeat (for each episode):

    Initialize  $s$

    Choose  $a$  from  $s$  using policy derived from  $Q$  (e.g.,  $\epsilon$ -greedy)

    Repeat (for each step of episode):

        Take action  $a$ , observe  $r, s'$

        Choose  $a'$  from  $s'$  using policy derived from  $Q$  (e.g.,  $\epsilon$ -greedy)

$Q(s, a) \leftarrow Q(s, a) + \alpha[r + \gamma Q(s', a') - Q(s, a)]$

$s \leftarrow s'; a \leftarrow a';$

    until  $s$  is terminal

## Updating Action-Value Functions with Sarsa

$$q(s, a) \leftarrow q(s, a) + \alpha (r + \gamma q(s', a') - q(s, a))$$

### Theorem

*Tabular Sarsa converges to the optimal action-value function,  $q(s, a) \rightarrow q_*(s, a)$ , if the policy is GLIE*

# Dynamic programming

- We discussed several dynamic programming algorithms

$$v_{k+1}(s) = \mathbb{E}[R_{t+1} + \gamma v_k(S_{t+1}) \mid S_t = s, A_t \sim \pi(S_t)] \quad (\text{policy evaluation})$$

$$v_{k+1}(s) = \max_a \mathbb{E}[R_{t+1} + \gamma v_k(S_{t+1}) \mid S_t = s, A_t = a] \quad (\text{value iteration})$$

$$q_{k+1}(s, a) = \mathbb{E}[R_{t+1} + \gamma q_k(S_{t+1}, A_{t+1}) \mid S_t = s, A_t = a] \quad (\text{policy evaluation})$$

$$q_{k+1}(s, a) = \mathbb{E}\left[R_{t+1} + \gamma \max_{a'} q_k(S_{t+1}, a') \mid S_t = s, A_t = a\right] \quad (\text{value iteration})$$

# TD learning

- Analogous model-free TD algorithms

$$v_{t+1}(S_t) = v_t(S_t) + \alpha_t (R_{t+1} + \gamma v_t(S_{t+1}) - v_t(S_t)) \quad (\text{TD})$$

$$q_{t+1}(s, a) = q_t(S_t, A_t) + \alpha_t (R_{t+1} + \gamma q_t(S_{t+1}, A_{t+1}) - q_t(S_t, A_t)) \quad (\text{Sarsa})$$

$$q_{t+1}(s, a) = q_t(S_t, A_t) + \alpha_t \left( R_{t+1} + \gamma \max_{a'} q_t(S_{t+1}, a') - q_t(S_t, A_t) \right) \quad (\text{Q-learning})$$

- Note, no trivial analogous version of value iteration

$$v_{k+1}(s) = \max_a \mathbb{E} [R_{t+1} + \gamma v_k(S_{t+1}) \mid S_t = s, A_t = a]$$

Can you explain why?

# On and Off-Policy Learning

- ▶ **On-policy** learning
  - ▶ “Learn on the job”
  - ▶ Learn about policy  $\pi$  from experience sampled from  $\pi$
- ▶ **Off-policy** learning
  - ▶ “Look over someone’s shoulder”
  - ▶ Learn about policy  $\pi$  from experience sampled from  $\mu$



# Off-Policy Learning

- ▶ Evaluate target policy  $\pi(a|s)$  to compute  $v_\pi(s)$  or  $q_\pi(s, a)$
- ▶ While using behaviour policy  $\mu(a|s)$  to generate actions
- ▶ Why is this important?
  - ▶ Learn from observing humans or other agents (e.g., from logged data)
  - ▶ Re-use experience from old policies (e.g., from your own past experience)
  - ▶ Learn about **greedy** policy while following **exploratory** policy
  - ▶ Learn about **multiple** policies while following **one** policy
- ▶ **Q-learning** estimates the value of the **greedy** policy

$$q_{t+1}(s, a) = q_t(S_t, A_t) + \alpha_t \left( R_{t+1} + \gamma \max_{a'} q_t(S_{t+1}, a') - q_t(S_t, A_t) \right)$$

But just being greedy would not explore

# Q-Learning Control Algorithm

## Theorem

*Q-learning control converges to the optimal action-value function,  $q \rightarrow q^*$ , as long as we take each action in each state infinitely often.*

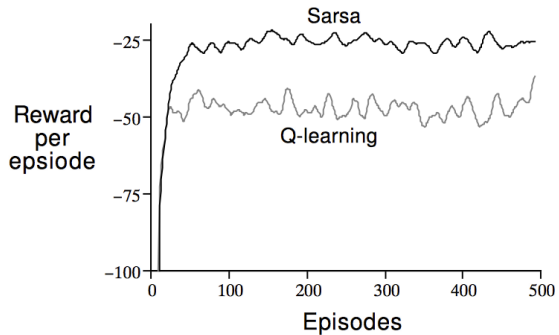
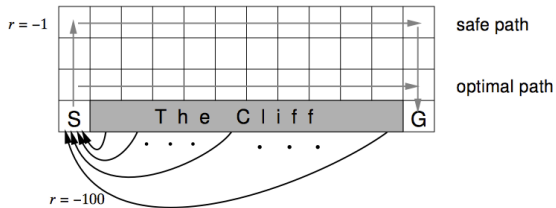
Note: no need for greedy behaviour!

Works for **any** policy that eventually selects all actions sufficiently often

(Does require appropriately decaying step sizes  $\sum_t \alpha_t = \infty$ ,  $\sum_t \alpha_t^2 < \infty$ ,

E.g.,  $\alpha = 1/t^\omega$ , with  $\omega \in (0.5, 1)$ )

# Cliff Walking Example



# Q-learning overestimation

- ▶ Classical Q-learning has potential issues
- ▶ Recall

$$\max_a q_t(S_{t+1}, a) = q_t(S_{t+1}, \operatorname{argmax}_a q_t(S_{t+1}, a))$$

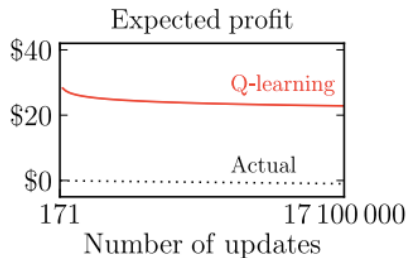
- ▶ Uses same values to **select** and to **evaluate**
- ▶ ... but values are approximate
  - ▶ **more** likely to select **overestimated values**
  - ▶ **less** likely to select **underestimated values**
- ▶ This causes upward bias

## Q-learning overestimation: roulette example

- ▶ Roulette: gambling game
- ▶ Here, 171 actions: bet \$1 on one of 170 options, or 'stop'
- ▶ 'Stop' ends the episode, with \$0
- ▶ All other actions have high variance reward, with negative expected value
- ▶ Betting actions do not end the episode, instead can bet again

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## Q-learning overestimation

- ▶ Q-learning overestimates because it uses the same values to **select** and to **evaluate**

$$\max_a q_t(S_{t+1}, a) = q_t(S_{t+1}, \operatorname{argmax}_a q_t(S_{t+1}, a))$$

- ▶ Roulette: quite likely that some actions have won, on average
  - ▶ Q-learning will update if the state actually has high value
- ▶ Solution: decouple selection from evaluation

# Double Q-learning

- ▶ **Double Q-learning:**

- ▶ Store two action-value functions:  $q$  and  $q'$

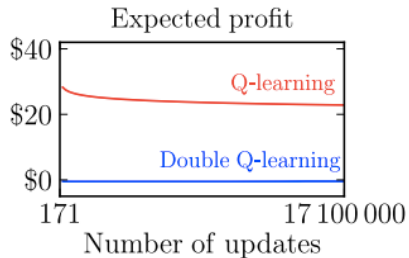
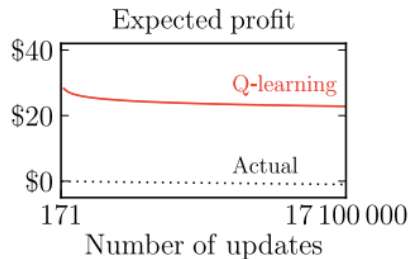
$$R_{t+1} + \gamma \mathbf{q}'_t(S_{t+1}, \operatorname{argmax}_a q_t(S_{t+1}, a)) \quad (1)$$

$$R_{t+1} + \gamma q_t(S_{t+1}, \operatorname{argmax}_a \mathbf{q}'_t(S_{t+1}, a)) \quad (2)$$

- ▶ Each  $t$ , pick  $q$  or  $q'$  (e.g., randomly) and update using (1) for  $q$  or (2) for  $q'$
    - ▶ Can use both to act (e.g., use policy based on  $(q + q')/2$ )
- ▶ Double Q-learning also converges to the optimal policy under the same conditions as Q-learning



## Roulette example

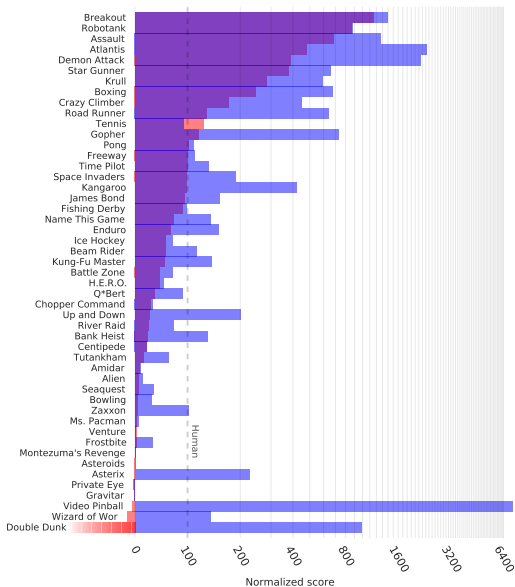


# Double DQN on Atari

DQN

Double DQN

(This used a 'target network',  
to be explained later)



## Double learning

- ▶ The idea of double Q-learning can be generalised to other updates
  - ▶ E.g., if you are (soft-) greedy (e.g.,  $\epsilon$ -greedy), then Sarsa can also overestimate
  - ▶ The same solution can be used
  - ▶  $\implies$  double Sarsa

# Off-policy learning

- ▶ Recall: off-policy learning means learning about one policy  $\pi$  from experience generated according to a different policy  $\mu$
- ▶ Q-learning is one example, but there are other options
- ▶ Fortunately, there are general tools to help with this
- ▶ Caveat: you can't expect to learn about things you **never** do

## Importance sampling corrections

- ▶ Goal: given some function  $f$  with random inputs  $X$ , and a distribution  $d'$ , estimate the expectation of  $f(X)$  under a different (target) distribution  $d$
- ▶ Solution: weight the data by the ration  $d/d'$

$$\begin{aligned}\mathbb{E}_{x \sim d}[f(x)] &= \sum d(x)f(x) \\ &= \sum d'(x) \frac{d(x)}{d'(x)} f(x) \\ &= \mathbb{E}_{x \sim d'} \left[ \frac{d(x)}{d'(x)} f(x) \right]\end{aligned}$$

- ▶ Intuition:
  - ▶ scale up events that are rare under  $d'$ , but common under  $d$
  - ▶ scale down events that are common under  $d'$ , but rare under  $d$

## Importance sampling corrections

- ▶ Example: estimate one-step reward
- ▶ Behaviour is  $\mu(a|s)$

$$\begin{aligned}\mathbb{E}[R_{t+1} \mid S_t = s, A_t \sim \pi] &= \sum_a \pi(a|s) r(s, a) \\ &= \sum_a \mu(a|s) \frac{\pi(a|s)}{\mu(a|s)} r(s, a) \\ &= \mathbb{E} \left[ \frac{\pi(A_t|S_t)}{\mu(A_t|S_t)} R_{t+1} \mid S_t = s, A_t \sim \mu \right]\end{aligned}$$

- ▶ Ergo, when following policy  $\mu$ , can use  $\frac{\pi(A_t|S_t)}{\mu(A_t|S_t)} R_{t+1}$  as unbiased sample

# Importance Sampling for Off-Policy Monte-Carlo

- ▶ Goal: estimate  $v_\pi$
- ▶ Data: trajectory  $\tau_t = \{S_t, A_t, R_{t+1}, S_{t+1}, \dots\}$  generated with  $\mu$
- ▶ Solution: use return  $G(\tau_t) = G_t = R_{t+1} + \gamma R_{t+2} + \dots$ , and correct:

$$\frac{p(\tau_t|\pi)}{p(\tau_t|\mu)} G(\tau_t)$$

- ▶ But what are  $p(\tau_t|\pi)$  and  $p(\tau_t|\mu)$ ?
- ▶ Seems unwieldy...?

# Importance Sampling for Off-Policy Monte-Carlo

- ▶ Goal: estimate  $v_\pi$
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$$\frac{p(\tau_t|\pi)}{p(\tau_t|\mu)} G(\tau_t) = \frac{p(A_t|S_t, \pi)p(R_{t+1}, S_{t+1}|S_t, A_t)p(A_{t+1}|S_{t+1}, \pi) \cdots}{p(A_t|S_t, \mu)p(R_{t+1}, S_{t+1}|S_t, A_t)p(A_{t+1}|S_{t+1}, \mu) \cdots} G_t$$



# Importance Sampling for Off-Policy Monte-Carlo

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# Importance Sampling for Off-Policy Monte-Carlo

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# Importance Sampling for Off-Policy Monte-Carlo

- ▶ Goal: estimate  $v_\pi$
- ▶ Data: trajectory  $\tau_t = \{S_t, A_t, R_{t+1}, S_{t+1}, \dots\}$  generated with  $\mu$
- ▶ Solution: use return  $G(\tau_t) = G_t = R_{t+1} + \gamma R_{t+2} + \dots$ , and correct:

$$\begin{aligned}\frac{p(\tau_t|\pi)}{p(\tau_t|\mu)} G(\tau_t) &= \frac{p(A_t|S_t, \pi)p(R_{t+1}, S_{t+1}|S_t, A_t)p(A_{t+1}|S_{t+1}, \pi) \cdots}{p(A_t|S_t, \mu)p(R_{t+1}, S_{t+1}|S_t, A_t)p(A_{t+1}|S_{t+1}, \mu) \cdots} G_t \\&= \frac{p(A_t|S_t, \pi)\cancel{p(R_{t+1}, S_{t+1}|S_t, A_t})p(A_{t+1}|S_{t+1}, \pi) \cdots}{p(A_t|S_t, \mu)\cancel{p(R_{t+1}, S_{t+1}|S_t, A_t})p(A_{t+1}|S_{t+1}, \mu) \cdots} G_t \\&= \frac{p(A_t|S_t, \pi)p(A_{t+1}|S_{t+1}, \pi) \cdots}{p(A_t|S_t, \mu)p(A_{t+1}|S_{t+1}, \mu) \cdots} G_t \\&= \frac{\pi(A_t|S_t)}{\mu(A_t|S_t)} \frac{\pi(A_{t+1}|S_{t+1})}{\mu(A_{t+1}|S_{t+1})} \cdots G_t\end{aligned}$$

## Importance Sampling for Off-Policy TD Updates

- ▶ Use TD targets generated from  $\mu$  to evaluate  $\pi$
- ▶ Weight TD target  $r + \gamma v(s')$  by importance sampling
- ▶ Only need a single importance sampling correction

$$v(S_t) \leftarrow v(S_t) + \alpha \left( \frac{\pi(\mathbf{A}_t | \mathbf{S}_t)}{\mu(\mathbf{A}_t | \mathbf{S}_t)} (R_{t+1} + \gamma v(\mathbf{S}_{t+1})) - v(S_t) \right)$$

- ▶ Much lower variance than Monte-Carlo importance sampling
- ▶ Policies only need to be similar over a single step

## Expected Sarsa

- ▶ We now consider off-policy learning of action-values  $q(s, a)$
- ▶ No importance sampling is required
- ▶ Next action may be chosen using behaviour policy  $A_{t+1} \sim \mu(\cdot|S_{t+1})$
- ▶ But we consider probabilities under  $\pi(\cdot|S_t)$
- ▶ Update  $q(S_t, A_t)$  towards value of alternative action

$$q(S_t, A_t) \leftarrow q(S_t, A_t) + \alpha \left( \mathbf{R}_{t+1} + \gamma \sum_a \pi(a|S_{t+1}) \mathbf{q}(S_{t+1}, a) - q(S_t, A_t) \right)$$

- ▶ Called **Expected Sarsa** (sometimes called 'General Q-learning')
- ▶ Q-learning is a special case with greedy target policy  $\pi$

# Off-Policy Control with Q-Learning

- ▶ We want behaviour and target policies to **improve**
- ▶ E.g., the target policy  $\pi$  is **greedy** w.r.t.  $q(s, a)$

$$\pi(S_{t+1}) = \operatorname{argmax}_{a'} q(S_{t+1}, a')$$

- ▶ The behaviour policy  $\mu$  is e.g.  **$\epsilon$ -greedy** w.r.t.  $q(s, a)$
- ▶ The Q-learning target is:

$$\begin{aligned} R_{t+1} + \gamma \sum_a \pi^{\text{greedy}}(a|S_{t+1}) q(S_{t+1}, a) \\ = R_{t+1} + \gamma \max_a q(S_{t+1}, a) \end{aligned}$$

## On-Policy Control with Sarsa

- ▶ In Sarsa, the target and behaviour policies are the same

$$target = R_{t+1} + \gamma q(S_{t+1}, A_{t+1})$$

- ▶ Then, for convergence, we need the addition requirement that  $\pi$  becomes greedy
- ▶ For instance,  $\epsilon$ -greedy, or softmax with decreasing exploration

## Questions?

*The only stupid question is the one you were afraid to ask but never did.*  
-Rich Sutton

For questions that arise outside of class, please use Moodle