Ayush Gaggar — Assignment 2 — 10/15/20

1.a) Machine Epsilon is a convergence criteria used as an upper bound for a given error. Machine Epsilon ($\mathcal{E}$) is specific to a given computer's processor, and is dependent on the number of significant digit bits the processor chip can store. In other words, the computed error should never be more than the computer's $\mathcal{E}$.

b) $\mathcal{E}$ for a 64-bit processor: $\mathcal{E} = 2^{1-53} = 2^{-52} = 2.204 \times 10^{-16}$

c) See Matlab code on page 3 (page 1 of MATLAB published code)

d) See comments on Matlab Code on page 3

e) Yes, my computer has a 64-bit processor, as my machine epsilon was equal to 2.204×10^{-16}

2. Approximate $\sin(0.3\pi)$ to 8 significant digits. **Answer: 7 iterations**

$\mathcal{E}_s = (0.5 \times 10^{2-8})\,\% = 5 \times 10^{-7}\,\%$; Continue iterating until $|\mathcal{E}_a| \le \mathcal{E}_s$

$$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

True Value: $\sin(0.3\pi) = 0.80901699$; See attached Matlab Code on pages 3 & 4

Iteration	Approx by ML	$\mathcal{E}_t = \frac{\text{true}-\text{approx}}{\text{true}} \times 100\%$	$\mathcal{E}_a = \frac{\text{current}-\text{prev}}{\text{current}}$
1	$(0.3\pi)^1/1!$ $= 0.942477796$	$\frac{0.8090 - 0.9425}{0.8090} \times 100\%$ $= 16.4967\%$	N/A
2	0.80294955	0.7500%	17.377%
3	0.80914645	0.0160%	0.7659%
4	0.80901539	$1.9825 \times 10^{-4}\,\%$	0.0162%
5	0.80901701	$1.6047 \times 10^{-6}\,\%$	$1.9986 \times 10^{-4}\,\%$
6	0.80901699	$9.1507 \times 10^{-9}\,\%$	$1.6139 \times 10^{-6}\,\%$
7	0.80901699	$3.8727 \times 10^{-11}\,\%$	$9.1894 \times 10^{-9}\,\%$

4. $\tanh(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$; $\dfrac{d}{dx}(\tanh(x)) = 1 - \tanh^2(x)$; $\dfrac{d^2}{dx^2}(\tanh(x)) = -2\tanh(x)(1-\tanh^2(x))$

a) 0^{th} term $= \tanh(0) = \tanh(0) = 0 \longrightarrow P_0(x) = 0x^0$

1^{st} term $= \tanh(0) + (1-\tanh^2(0))(x-0) = \tanh(0) + x(1-\tanh^2(0)) = x$

$\hookrightarrow P_1(x) = 0x^0 + x$

2nd term $= \dfrac{-2\tanh(0)(1-\tanh^2(0))(x-0)^2}{2!} = -x^2\tanh(0)(1-\tanh^2(0)) = 0$

$\hookrightarrow P_2(x) = 0x^0 + x^1 + 0x^2$

$P_0(x) = 0x^0$

$P_1(x) = 0x^0 + 1x^1$

$P_2(x) = 0x^0 + 1x^1 + 0x^2$

b) See MATLAB code on pages 5 & 6