Assignment 3

Due: Thursday October 22, 2020 at 11:59pm
MAE 107 Computational Methods for Engineers
Professor Michael Tolley

Homework Submission

- Homework submission is through Gradescope. Please leave extra time for submission, especially if you have not used Gradescope previously.
- Assignments are graded based on correctness, how well you organize your homework (i.e. it should be easy to understand your thinking and easy to find your responses), and how well you follow the submission instructions below.
- You will not receive credit if you just give an answer. Your solution must demonstrate how you got the answer.
- If you ever think a problem is stated incorrectly, not enough information is given, or it is impossible to solve, don't panic! Simply make a reasonable assumption that will allow you to solve the problem (but clearly state what this assumption is), or indicate why it is not possible to solve the problem.
- Please attend office hours (listed in the syllabus) if you have any questions regarding the assignment.
- You are welcome to discuss the assignment with other members of the class, but everything you submit should be your own (i.e. you wrote it, typed it, or generated the plot in Matlab).

Problem 1: Error of Finite Differences

Use forward and backward difference approximations and a centered difference approximation to estimate the first derivative of the function:

$$f(x) = 27x^3 - 4x^2 + 6x - 12$$

Evaluate the derivative at $x = 3$ using a step size of $h = 0.1$. Calculate the true error E_t for each case using the true value of the derivative. Interpret your results on the basis of the remainder term of the Taylor series expansion.

Problem 2: Second Finite Difference

For the same function from Problem 1, use your forward and backward difference approximations for the first derivative to approximate the second derivative at $x = 3$ using a step size of $h = 0.1$. Calculate the true error using the true value of the second derivative. Hint: once you know the forward and backward first derivatives, you can find the second derivative (centered) using:

$$f'' = \frac{f'_{forward} - f'_{backward}}{h}$$

Problem 3: Condition Number

Evaluate and interpret the condition numbers for the following:

a) $f(x) = 5$ at $x = 2$

b) $f(x) = e^{-x} + 10$ at $x = 5$

c) $f(x) = tan(x)$ at $x = 0.9999\frac{\pi}{2}$

Problem 4: Root Finding

You decide to buy a Tesla Model 3 that costs $39,190 (including delivery) with zero down payment. Your monthly payments are $587 for 72 months. Use the Bisection method and the False Position method to find the annual percentage rate (APR) are you paying to within $\epsilon_a = 0.01\%$. The formula relating the amount of the loan A, the amount of the payments P, the number of periods n, and periodic interest rate i is

$$P = A\frac{i(1+i)^n}{(1+i)^n - 1} \tag{1}$$

Note: the APR is just 12 times the monthly interest rate. For each method (Bisection and False Point), publish your code, and report the following:

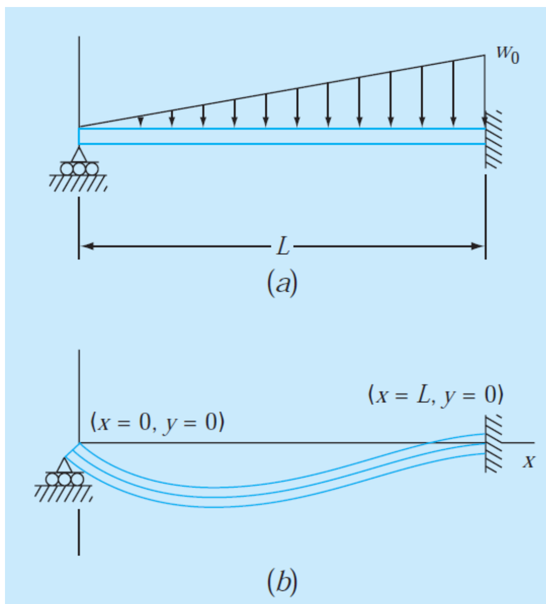
a) Your calculated APR to within $\epsilon_a = 0.01\%$.

b) The number of iterations of the method required.

c) The total number of evaluations of the interest function.

Problem 5: Minimum Finding

The following figure shows a uniform beam of length L, modulus E, moment of inertia I, subject to a linearly increasing distributed load w_0:



The equation for the resulting elastic curve is

$$y = \frac{w_0}{120EIL}(-x^5 + 2L^2x^3 - L^4x)$$

Use the Bisection Method to determine the point of maximum deflection (i.e. the value of x where $dy/dx = 0$). Then substitute this value into the beam equation to determine the value of the maximum deflection. Use the following parameter values in your computation: $L = 750$ cm, $E = 55,000$ kN/cm^2, $I = 25,000$ cm^4, and $w_0 = 2.3$ kN/cm.

Problem 6: Modified False Position

Develop a Matlab program that implements the modified false position method discussed in lecture (where endpoint values are halved if used more than once consecutively). Test the program by determining the root of the function:

$$f(x) = x^{10} - 1$$

Between $x = 0$ and $x = 1.3$ with $\epsilon_s = 0.001\%$. Plot the true and approximate percent relative errors versus number of iterations on semilog axes. Interpret your results. Below you will find pseudocode for the False Position method that you can use as a guide in developing your program.

```
FUNCTION ModFalsePos(xl, xu, es, imax, xr, iter, ea)
 iter = 0
 fl = f(xl)
 fu = f(xu)
 DO
   xrold = xr
   xr = xu - fu * (xl - xu) / (fl - fu)
   fr = f(xr)
   iter = iter + 1
   IF xr <> 0 THEN
     ea = Abs((xr - xrold) / xr) * 100
   END IF
   test = fl * fr
   IF test < 0 THEN
     xu = xr
     fu = f(xu)
     iu = 0
     il = il + 1
     If il ≥ 2 THEN fl = fl / 2
   ELSE IF test > 0 THEN
     xl = xr
     fl = f(xl)
     il = 0
     iu = iu + 1
     IF iu ≥ 2 THEN fu = fu / 2
   ELSE
     ea = 0
   END IF
   IF ea < es OR iter ≥ imax THEN EXIT
 END DO
 ModFalsePos = xr
END ModFalsePos
```