Assignment 5

Due: Thursday November 12, 2020 at 11:59pm
MAE 107 Computational Methods for Engineers
Professor Michael Tolley

Homework Submission

- Homework submission is through Gradescope. Please leave extra time for submission, especially if you have not used Gradescope previously.
- Assignments are graded based on correctness, how well you organize your homework (i.e. it should be easy to understand your thinking and easy to find your responses), and how well you follow the submission instructions below.
- You will not receive credit if you just give an answer. Your solution must demonstrate how you got the answer.
- If you ever think a problem is stated incorrectly, not enough information is given, or it is impossible to solve, don't panic! Simply make a reasonable assumption that will allow you to solve the problem (but clearly state what this assumption is), or indicate why it is not possible to solve the problem.
- Please attend office hours (listed in the syllabus) if you have any questions regarding the assignment.
- You are welcome to discuss the assignment with other members of the class, but everything you submit should be your own (i.e. you wrote it, typed it, or generated the plot in Matlab).

Problem 1 (25 points): 2D Optimization, Steepest Ascent Method

By hand, perform one iteration of the steepest ascent method to locate the maximum of

$$f(x, y) = 4.5x + 3y + x^2 - x^4 - 5xy - 2y^2$$

using initial guesses x = 0 and y = 0. Employ bisection to find the optimal step size in the gradient search direction.

Problem 2 (25 points): 2D Optimization, Newton's Method

Consider the following function,

$$f(x, y) = \cos(x)\cos(y), \quad x \in [-1, 4], \, y \in [-1, 4],$$

and use the following Matlab code to generate its surface and contour plots over its domain,

```
[X,Y]=meshgrid([-1:0.1:4]);
fxy=cos(X).*cos(Y);

figure(1)
surf(X,Y,fxy)
xlabel('x')
ylabel('y')

figure(2)
contourf(X,Y,fxy)
colorbar;
xlabel('x')
ylabel('y')
```

The following Matlab code contains the basic blocks of the Newton's Method optimization algorithm for the function f, for 10 iterations, starting from $z_0 = (x_0, y_0)$,

```
clear all; clc;
%% Find the optima of the following function.
f=@(x,y) cos(x)*cos(y);
grad_f=@(x,y) [ ; ]; % 2 X 1 gradient vector
Hessian_f=@(x,y) [ , ; , ]; % 2 X 2 Hessian matrix

contourf(X,Y,fxy)
colorbar;
```

```
xlabel('x')
ylabel('y')
hold all

%% Starting Guess
x0= ;
y0= ;
z0=[x0;y0];
plot(z0(1),z0(2),'o','MarkerFaceColor','c')
%% Newton's method
for i=1:10
    z1 = ; % insert code for Newton's method update
    z0 = z1;
    pause
    plot(z0(1),z0(2),'o','MarkerFaceColor','c')
end
z1
```

Fill the previous code with the appropriate formulas of the gradient and the Hessian of f, then use the following points as starting points of this algorithm: $(x_0, y_0) = (3, 1), (3.5, 3.5), (1, 1)$. Report the points (x, y) to which each of the previous points converges after 10 iterations. Using the surface and contour plots of f, identify the type of these convergence points as max, min or saddle. (Note: the previous code contains several plotting functions which animate the algorithm at each step to see convergence. Attach the figure containing the contour plot and the sequence of estimates for each case).

Problem 3 (25 points): Constrained Optimization

Your company makes two types of products, A and B. These products are produced during a 40-hour work week and then shipped out at the end of the week. They require 20 and 5 kg of raw material per kg of product, respectively, and your company has access to 9500 kg of raw material per week. Only one product can be created at a time with production times for each of 0.04 and 0.12 hr, respectively. Your plant can only store 550 kg of total product per week. Finally, your company makes profits of \$45 and \$20 on each unit of A and B, respectively. Each unit of product is equivalent to a kg.

a) Set up the linear programming problem to maximize profit.

b) Solve the linear programming problem graphically (i.e. how much of A and B should you produce each week?). Submit your graph as well as your conclusion.

c) Solve the problem using the "Simplex LP" solver in Excel. For a tutorial on how to solve a constrained optimization problem in Excel, including installing the *Solver* add-in, please see the end of Lecture 9. After solving, submit screenshots of two versions of your spreadsheet:

i Showing values

ii Showing equations.

Note: to toggle showing equations in Excel, press CTRL + ‘ (the key above TAB)

d) Which of the following options will raise profits the most: increasing raw material, storage, or production time.

Problem 4 (25 points): Interpolation with Newton Polynomials

Given the following data:

x	1.6	2.0	2.5	3.2	4.0	4.5
$f(x)$	2	8	14	15	8	2

a) Calculate $f(2.8)$ using Newton's interpolating polynomials of order 1 through 3. Choose the sequence of the points for your estimates to attain the best possible accuracy.

b) Use the equation discussed in lecture to estimate the error for each prediction.