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1. $f(x,y) = 4.5x + 3y + x^2 - x^4 - 5xy - 2y^2$; $x=0, y=0$

$\frac{\partial f}{\partial x} = 4.5 + 2x - 4x^3 - 5y \Rightarrow \frac{\partial f}{\partial x}\Big|_{\substack{y=0\\x=0}} = 4.5$

$\frac{\partial f}{\partial y} = 3 - 5x - 4y \Rightarrow \frac{\partial f}{\partial y}\Big|_{\substack{y=0\\x=0}} = 3$. Expressing $f(x,y)$ as a function of h

$g(h) = f(0 + 4.5h, 0 + 3h)$

$= \frac{81}{4}h + 9h + \frac{81}{4}h^2 - (4.5)^4 h^4 - 15h(\frac{9}{2}h) - 2(9h^2)$

$= \frac{117}{4}h + \frac{81}{4}h^2 - \frac{135}{2}h^2 - \frac{72}{4}h^2 - (\frac{9}{2})^4 h^4$

$= -(\frac{9}{2})^4 h^4 - \frac{261}{4}h^2 + \frac{117}{4}h$

$g'(h^*) = 0 = -\frac{9^4}{4}h^3 - \frac{261}{2}h + \frac{117}{4} = 0$

$-6561h^3 - 512h + 117 = 0$

Bisection method: Between $h=0$ & $h=1/2$

$g'(0.25) = -6561(\frac{1}{4})^3 - 512(\frac{1}{4}) + 117 = -113.5156 < 0$

$g'(1/8) = -6561(\frac{1}{8})^3 - 512(\frac{1}{8}) + 117 = 40.1855 > 0$

$g'(\frac{1}{2}(\frac{1}{8} + \frac{1}{4})) = -6561(\frac{3}{16})^3 - 512(\frac{3}{16}) + 117 = -22.2488 < 0$

$g'(5/32) = 11.9718 > 0$ → After some iterations by hand, see exact

$g'(\frac{1}{2}(\frac{5}{32} + \frac{3}{16})) = -4.3126$ root h^* (optimal step size) in Matlab

2. $f(x) = \cos(x)\cos(y)$

$\frac{\partial f}{\partial x} = -\sin(x)\cos(y)$; $\frac{\partial f}{\partial y} = -\sin(y)\cos(x)$

$\frac{\partial^2 f}{\partial x^2} = -\cos(x)\cos(y)$; $\frac{\partial^2 f}{\partial y^2} = -\cos(y)\cos(x)$

$\frac{\partial^2 f}{\partial x \partial y} = \sin(x)\sin(y)$ $\frac{\partial^2 f}{\partial y \partial x} = \sin(y)\sin(x)$

$$H = \begin{bmatrix} -\cos(x)\cos(y) & \sin(x)\sin(y) \\ \sin(x)\sin(y) & -\cos(y)\cos(x) \end{bmatrix}$$

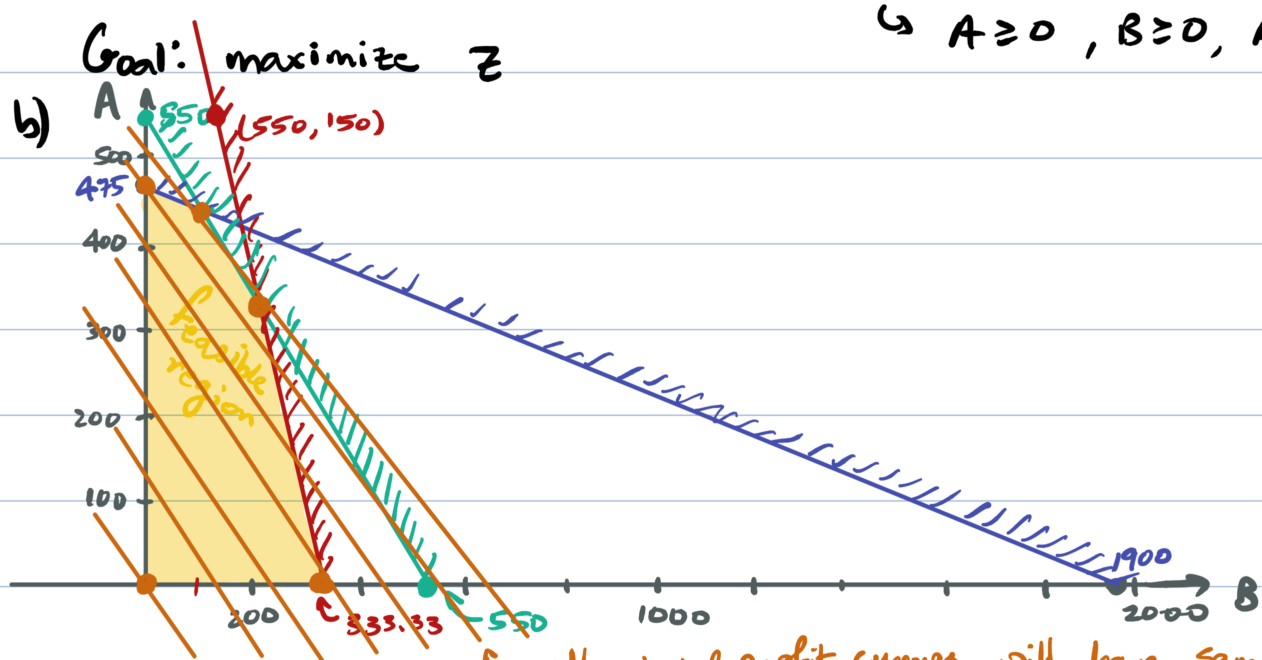
3\. a) $20A + 5B \le 9500$ ← plastic $A + B \le 550$ ← storage

$0.04A + 0.12B \le 40$ ← hours $45A + 20B = z =$ Profit

↳ $A \ge 0,\ B \ge 0,\ A, B \in \mathbb{Z}^+$

Goal: maximize z

b)



all potential profit curves will have same slope

To maximize z, we just need to check the 5 vertices of the feasible region.

1) $z(0,0) \rightarrow z = 0 \rightarrow$ Trivial

2) $z(0,475) \rightarrow z = 45(475) + 0 = \$21,375$

3) $z(333\frac{1}{3}, 0) \rightarrow z = 0 + 20(333\frac{1}{3}) = \$\,6666.67$

4) Intersection of blue & green:

$20A + 5B = 9500$

$A + B = 550 \Rightarrow 5B = 2750 - 5A$

$15A + 2750 = 9500 \rightarrow A = 450 \rightarrow B = 100$

$z(450, 100) \rightarrow z = 45(450) + 20(100) = \$\,22,250$

5) Intersection of red & green:

$4A + 12B = 4000$

$A + B = 550 \rightarrow 4A = 2200 - 4B$

$8B + 2200 = 4000 \rightarrow B = 225 \rightarrow A = 325$

$z(225, 225) \rightarrow z = 45(325) + 20(225) = \$\,19,125$

Maximum profit is \$22,250, when making 450 units of A & 100 units of B.

c) See Excel screenshots

d) The point (450, 100) lies on the intersection of the blue (plastic) and green (storage), so changing production time will not affect the max profit. Playing around with the Simplex LP solver, profit increases more when increasing raw material constraint as compared to increasing more storage

4. a) Use points closest & centered around $x = 2.8$

First Order: $f_1(x) = f(x_0) + \left[\frac{f(x_1) - f(x_0)}{x_1 - x_0}\right](x - x_0)$

Use points $x_0 = 2.5$, $f(x_0) = 14$ & $x_1 = 3.2$, $f(x_1) = 15$ to estimate $f(2.8)$

$$f_1(2.8) = 14 + \left(\frac{15-14}{3.2-2.5}\right)(2.8-2.5) = 14.4286$$

Second Order:

$$f_2(x) = f(x_0) + \left(\frac{f(x_1) - f(x_0)}{x_1 - x_0}\right)(x - x_0) + \left(\frac{\frac{f(x_2)-f(x_1)}{x_2 - x_1} - \frac{f(x_1)-f(x_0)}{x_1 - x_0}}{x_2 - x_0}\right) \cdot (x-x_0)(x-x_1)$$

$x_0 = 2.5$, $x_1 = 3.2$, $x_2 = 2.0$; $f(x_0) = 14$, $f(x_1) = 15$, $f(x_2) = 8$

$$f_2(2.8) = 14 + \left(\frac{15-14}{3.2-2.5}\right)(2.8-2.5) + \left(\frac{\frac{8-15}{2.0-3.2} - \frac{15-14}{3.2-2.5}}{2.0-2.5}\right)(2.8-2.5)(2.8-3.2)$$

$= 15.4857$

Third order

$x_0 = 2.5$, $[f_0] = 14$

$> [f_0, f_1] = \frac{10}{7}$

$x_1 = 3.2, [f_1] = 15$ $\quad > [f_0, f_1, f_2] = \frac{-305}{21}$

$> [f_1, f_2] = \frac{35}{6}$

$x_2 = 2.0, [f_2] = 8$ $\quad > [f_1, f_2, f_3] = \frac{-175}{24}$

$> [f_2, f_3] = 0$

$x_3 = 4.0, [f_3] = 8$

See the rest of the work in MATLAB

b) $R_n = \frac{f_{n+1}(x) - f_n(x)}{f_{n+1}(x)}$. See answers in MATLAB