MAE 234 Simulation Assignment 2

Control of the yaw dynamics of a car

You are going to try and design a control system for the steering of a "bicycle" car model that will make a "real" car with nonlinear tires have the yaw dynamics of a linear model reference system. The car schematic is shown in Figure 1.

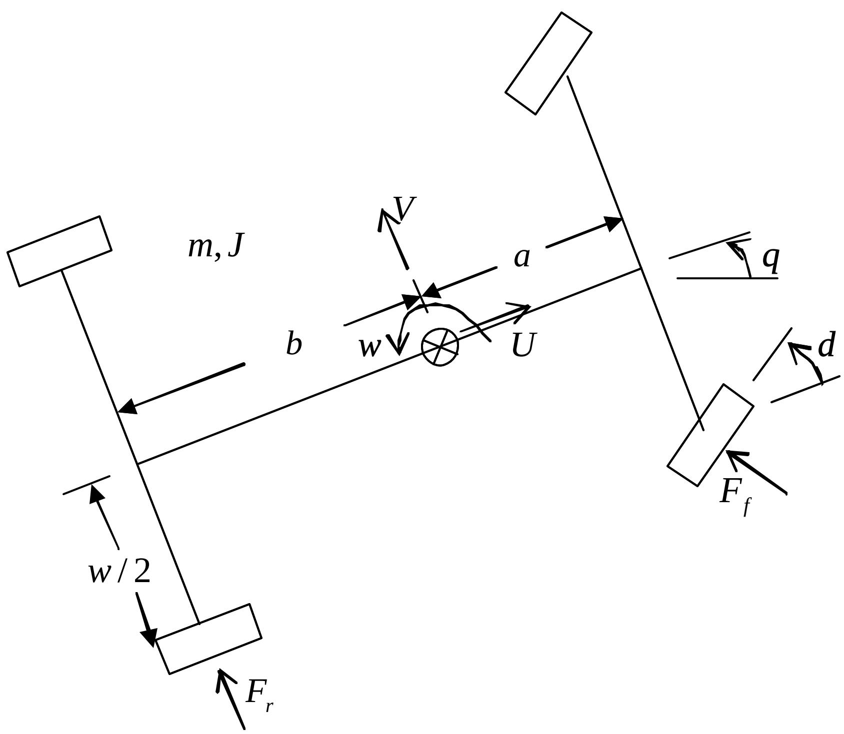

Figure 1 Vehicle schematic with body fixed coordinates

For small steer angles the general equations of motion are,

$$F_f + F_r = m(\dot{V} + Uw)$$
$$F_f a - F_r b = J\dot{w} \qquad (1)$$

We will assume that the forward speed U is constant for this simulation.

The slip angles are at the front and rear are,

$$a_f = d - \frac{(V + aw)}{U}$$
$$a_r = \frac{bw - V}{U} \qquad (2)$$

For a linear car the contact patch forces are given by,

$$F_f = C_f a_f$$
$$F_r = C_r a_r \qquad (3)$$

and for a more realistic car the forces are related to the slip angles through some nonlinear model such as the Dugoff tire model presented in class.

Your assignment is to derive and demonstrate a controller for the steering d such that the "real" car with nonlinear tires has the same yaw dynamics as a linear car set up to be neutral steer or understeer or oversteer, i.e. anything you desire.

A. Design the controller by starting with the linear transfer function relating the yaw rate to the steering angle. Derive the transfer function,

$$\frac{w}{d}(s) = \frac{\frac{aC_f}{J}s + \frac{(a+b)C_f C_r}{mJU}}{s^2 + \left(\frac{C_f + C_r}{mU} + \frac{a^2 C_f + b^2 C_r}{JU}\right)s + \frac{(a+b)^2 C_f C_r}{mJU^2} + \frac{(bC_r - aC_f)}{J}} \qquad (4)$$

and propose a control setup as indicated in the block diagram of Figure 2.

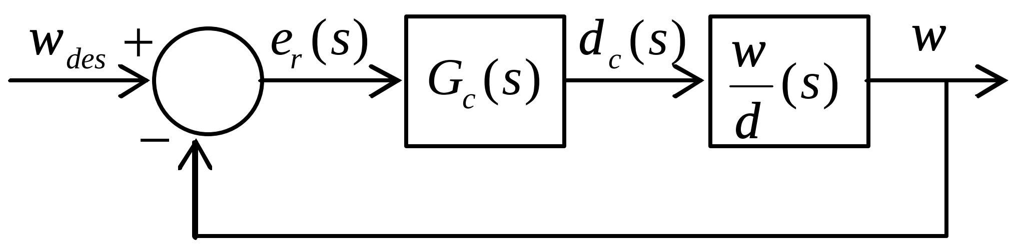

Figure 2 Block diagram of the yaw rate control system

The closed loop behavior is indicated by,

$$\frac{w}{w_{des}}(s) = \frac{G_c \frac{w}{d}(s)}{1 + G_c \frac{w}{d}(s)} = \frac{G_c N(s)}{D(s) + G_c N(s)} \qquad (5)$$

where $N(s), D(s)$ are the numerator and denominator of the transfer function from Eq. (4).

Convince yourself that a PI controller has a chance of working for this system, i. e.,

$$d_c(s) = K_P e_r + \frac{K_I}{s} e_r \qquad (6)$$

This would be realized in a simulation by breaking the control action into 2 parts.

$$d_P = K_P e_r$$
$$\dot{d}_I = K_I e_r \text{ (an additional state equation)} \tag{7}$$
$$d_c = d_P + d_I$$

To accomplish the goals of this assignment:

Using MATLAB or ACSL construct a simulation model the bicycle car model with Dugoff tires. This will be the "real" sustem.

Construct a simulation model of the bicycle car model that is linear. The yaw rate output of this model is the desired yaw rate of the real car.

Use your controller to operate on the error between the desired yaw rate and the actual yaw rate. Experiment with the gains K_P, K_I and determine if this control action will work. Make sure to look at the controlled steering angle d_c to determine if it appears practical.

To use the Dugoff tire model you need the normal force at the front and rear. Use,

$$N_f = \frac{b}{a+b} mg$$
$$N_r = \frac{a}{a+b} mg \tag{8}$$

Parameters for the vehicle are listed below. Some parameters are common to both the real vehicle and the model reference vehicle.

$m = 3000/2.2 \text{ Kg}$, common to all vehicle models
$L = 2.84m$, common to all vehicle models
$\frac{a}{b} = 0.85$, common to all vehicle models $b = \frac{L}{1+\frac{a}{b}}$ $a = L - b$
$J = 0.4mab \text{ Kg-m\^{}2}$, common to all vehicle models
$C_f = 50000 \text{ N/rad}$, common to all vehicle models
Road friction, $m = 0.85$, needed in the Dugoff model for the "real" vehicle
$C_r = K_u \frac{a}{b} C_f$ where K_u is an understeer coefficient. If $K_u > 1$ a linear car will be understeer. If $K_u = 1$ a linear car will be neutral steer. And if $K_u < 1$ a linear car will be over steer and will have a critical speed.

Both C_f and C_r are needed for use in the Dugoff model for the "real" car. Use $K_u = 1.2, 0.8$
For the model reference car, determine $C_{f_{MR}}$ and $C_{r_{MR}}$ by using $K_{u_{MR}} = 1.1$ and maybe some others of your choosing
The forward speed is constant with $U = 50, 60, 70 mph$
Input to the model reference vehicle: Step input with $d_0 = 2^0, 4^0, 8^0, 10^0$ Harmonic input at 1 Hz, $d = d_0 \sin w_{in} t$
The fore/aft slip is needed for the Dugoff model. Use $s_f = 0$ and $s_r = 0$

Table 1 Parameters for the simulation

Some outputs that should be included are the vehicle trajectory and lateral acceleration. To get the trajectory you first need the yaw angle of the vehicle. Get this from,

$$\dot{q} = w \,. \tag{9}$$

To get the trajectory write the horizontal and vertical velocity components of the vehicle c. g. as,

$$\begin{aligned} \dot{x} &= U \cos q - V \sin q \\ \dot{y} &= U \sin q + V \cos q \end{aligned} \tag{10}$$

To get the lateral acceleration,

$$a_{lat} = \frac{\dot{V} + U w}{g} \tag{11}$$

This will be the lateral acceleration in g's.