Ayush Gagrani Homework 6 11/19/20

1. a) General:

$$f_3(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} f(x_1) +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} f(x_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} f(x_2)$$

Actual: $(x_0, f(x_0)) = (27.5, 6.430) \longrightarrow (x_3, f(x_3)) = (51.5, 6.261)$

$$f_3(x) = \frac{6.430(x-35.5)(x-43.5)(x-51.5)}{(27.5-35.5)(27.5-43.5)(27.5-51.5)} + \frac{6.490(x-27.5)(x-43.5)(x-51.5)}{(35.5-27.5)(35.5-43.5)(35.5-51.5)} +$$

$$\frac{6.371(x-35.5)(x-27.5)(x-51.5)}{(43.5-35.5)(43.5-27.5)(43.5-51.5)} + \frac{6.261(x-35.5)(x-43.5)(x-27.5)}{(51.5-35.5)(51.5-43.5)(51.5-27.5)}$$

b)

i	x_i	$f(x_i)$
0	27.5	6.430
1	35.5	6.490
2	43.5	6.371
3	51.5	6.261

1) $a_1(27.5)^2 + b_1(27.5) + c_1 = 6.430$

2) $a_1(35.5)^2 + b_1(35.5) + c_1 = 6.490$

3) $a_2(35.5)^2 + b_2(35.5) + c_2 = 6.490$

4) $a_2(43.5)^2 + b_2(43.5) + c_2 = 6.371$

5) $a_3(43.5)^2 + b_3(43.5) + c_3 = 6.371$

6) $a_3(51.5)^2 + b_3(51.5) + c_3 = 6.261$

7) $2a_1(35.5) + b_1 = 2a_2(35.5) + b_2$

8) $2a_2(43.5) + b_2 = 2a_3(43.5) + b_3$

9) $a_1 = 0$

From Matlab: $a_1 = 0$, $b_1 = 0.0075$, $c_1 = 6.2237$; $a_2 = -0.0009$, $b_2 = 0.0580$, $c_2 = 5.5945$; $a_3 = -0.0045$, $b_3 = 0.4157$, $c_3 = -3.1120$

4. a) $T(t) = T_0 e^{-ct}$

$\text{error} = (T(t) - T_0 e^{-ct}) \Rightarrow e = \sum_{i=1}^{n} (T(t_i) - T_0 e^{-ct_i})^2$

b) $\ln(T(t)) = \ln T_0 - ct$. let $\ln(T(t)) = y_m$; $\ln T_0 = a_0$, $-c = a_1$

$y_m = a_0 + a_1 t$

$e = \sum_{i=1}^{n} (y_i - y_m)^2$

$= \sum_{i=1}^{n} (y_i - a_0 - a_1 t_i)^2$

$\frac{\partial e}{\partial a_0} = \sum_{i=1}^{n} (2(y_i - a_0 - a_1 t)(-1) = 0$

① $\sum_{i=1}^{n} y_i = a_0 n + a_1 \sum_{i=1}^{n} t_i$

$\frac{\partial e}{\partial a_1} = \sum_{i=1}^{n} 2(y_i - a_0 - a_1 t_i)(-t_i) = 0$

② $\sum_{i=1}^{n} t_i y_i = a_0 \sum_{i=1}^{n} t_i + a_1 \sum_{i=1}^{n} t_i^2$

c)

$$\begin{bmatrix} n & \sum_{i=1}^{n} t_i \\ \sum_{i=1}^{n} t_i & \sum_{i=1}^{n} t_i^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} t_i y_i \end{bmatrix}$$