# Assignment 7

Updated 12/4/2020
Due: Friday December 11, 2020 at 11:59pm
MAE 107 Computational Methods for Engineers
Professor Michael Tolley

## Homework Submission

- Homework submission is through Gradescope. Please leave extra time for submission, especially if you have not used Gradescope previously.
- Assignments are graded based on correctness, how well you organize your homework (i.e. it should be easy to understand your thinking and easy to find your responses), and how well you follow the submission instructions below.
- You will not receive credit if you just give an answer. Your solution must demonstrate how you got the answer.
- If you ever think a problem is stated incorrectly, not enough information is given, or it is impossible to solve, don't panic! Simply make a reasonable assumption that will allow you to solve the problem (but clearly state what this assumption is), or indicate why it is not possible to solve the problem.
- Please attend office hours (listed in the syllabus) if you have any questions regarding the assignment.
- You are welcome to discuss the assignment with other members of the class, but everything you submit should be your own (i.e. you wrote it, typed it, or generated the plot in Matlab).

## Problem 1 (25 points): Numerical Differentiation

Use a Taylor series expansion to derive a centered finite difference approximation to the third derivative that is second order accurate. To do this, you will have to use four different expansions for the points $x_{i-2}$, $x_{i-1}$, $x_{i+1}$, and $x_{i+2}$. In each case, the expansion will be around the point $x_i$. The interval $\Delta x$ will be used in each case of i - 1 and i + 1, and $2\Delta x$ will be used in each case of i - 2 and i + 2. The four equations must then be combined in a way to eliminate the first and second derivatives. Carry enough terms along in each expansion to evaluate the first term that will be truncated to determine the order of the approximation.

## Problem 2 (25 points): Gauss Quadrature

Follow the approach described in class to develop a three-point Gauss Quadrature formula. The truncation error associated with this formula should be proportional to $f^{(6)}(\xi)$, i.e. it should perfectly approximate $f(x) = 1, x, x^2, x^3, x^4, x^5$. Once you have found a system of 6 equations that can be solved to find the six unknowns, you can use Matlab to solve the system of equations.

## Problem 3 (25 points): Numerical Integration

A force is applied to a block on a surface at an angle $\theta$ from the horizontal, dragging the block from $x_0 = 0$ ft to $x_n = 30$ ft. The magnitude (lbf) and angle (radians) of the force (as a function of the position $x$) are given by:

$$F(x) = 1.6x - 0.045x2$$

$$\theta(x) = 0.8 + 0.125x - 0.009x^2 + 0.0002x^3$$

Calculate the work done against friction (by the horizontal component of the force) by numerically solving the integral:

$$W = \int_{x_0}^{x_n} F(x) \cos\left[\theta(x)\right] dx$$

Using 4, 8, and 16-segment versions of the following rules:

a) Trapezoidal

b) Simpson's 1/3 rule

c) 2-point Gauss quadrature

## Problem 4 (25 points): Ordinary Differential Equations

Solve the following problem over the interval from $x = 0$ to 1 using a step size of 0.25 where $y(0) = 1$ using the methods indicated below. Display all your results on the same graph.

$$\frac{dy}{dx} = (1 + 2x)\sqrt{y}$$

a) Analytically (show your work)

b) Euler's method

c) Heun's method (1 iteration)

d) Heun's method (10 iterations)

e) Fourth-order Runge-Kutta