

Problem 1

We know:

- * Miss rate = 0.1
- * Block size = 4 words (16 bytes)
- * Frequency of memory operations from processor = 10^9
- * Frequency of writes from processor = $0.25 * 10^9$
- * Bus can only transfer one word at a time to/from processor/memory
- * On average 50% of blocks in the cache have been modified (must be written back in the case of the write back cache)
- * Cache is write allocate

So:

$$\text{Fraction of read hits} = 0.75 * 0.9 = 0.675$$

$$\text{Fraction of read misses} = 0.75 * 0.1 = 0.075$$

$$\text{Fraction of write hits} = 0.25 * 0.9 = 0.225$$

$$\text{Fraction of write misses} = 0.25 * 0.1 = 0.025$$

2 (b) Write through cache

- On a read hit there is no memory access
- On a read miss memory must send four words to the cache
- On a write hit the cache must send a word to memory
- On a write miss memory must send four words to the cache, and then the cache must send a word to memory

Thus:

$$\text{Average words transferred} = 0.675 * 0 + 0.075 * 4 + 0.225 * 1 + 0.025 * 5 = 0.65$$

$$\text{Average bandwidth used} = 0.65 * 10^9$$

$$\text{Fraction of bandwidth used} = [0.65 * 10^9] / 10^9 = 0.65$$

2(a) Write back cache

On a read hit there is no memory access

On a read miss:

1. If replaced line is modified then cache must send four words to memory, and then memory must send four words to the cache
 2. If replaced line is clean then memory must send four words to the cache
- On a write hit there is no memory access

On a write miss:

1. If replaced line is modified then cache must send four words to memory, and then memory must send four words to the cache
2. If replaced line is clean then memory must send four words to the cache

Thus:

$$\text{Average words transferred} = 0.675 * 0 + 0.075 * (0.4 * 8 + 0.6 * 4) + 0.225 * 0 + 0.025 * (0.4 * 8 + 0.6 * 4) = 0.56$$

$$\text{Average bandwidth used} = 0.56 * 10^9$$

$$\text{Fraction of bandwidth used} = 0.56 * 10^9 / 10^9 = 0.56$$

Comparing 1 and 2 we notice that the write through cache uses more than the cache-memory bandwidth of the write back cache.

Problem 1.

Your favorite Computer is described with the following features:

- 90% of all memory accesses are found in the cache.
- Each cache block is 4 words, and the whole block is read on any miss.
- The processor sends references to its cache at the rate of 10^9 words per second.
- 25% of those references are writes.
- Assume that the memory system can support 10^9 words per second, reads or writes.
- The bus reads or writes a single word at a time (the memory system cannot read or write two words at once).
- Assume at any one time, 40% of the blocks in the cache have been modified.
- The cache uses write allocate on a write miss.

You are considering adding a peripheral to the system, and you want to know how much of the memory system bandwidth is already used.

- (a) Calculate the percentage of memory system bandwidth used assuming the cache is Write Back.
- (b) Calculate the percentage of memory system bandwidth used assuming the cache is Write Through. Be sure to state your assumptions.

Problem 2

CPU performance equation: $\text{CPU Time} = \text{IC} * \text{CPI} * \text{Clock Time}$

$\text{CPI} = \text{CPI execution} + \text{Stall Cycles Per Instruction}$

We know:

Instruction miss penalty is 100 cycles

Data read hit takes 1 cycle

Data write hit takes 2 cycles

Data miss penalty is 100 cycles for write through cache

Data miss penalty is 150 cycles or 100 cycles for write back cache

Miss rate is 2% for data cache (MRD) and 0.5% for instruction cache (MRI)

30% of cache blocks are dirty in the write back cache

25% of all instructions are loads

10% of all instructions are stores

Then: $\text{CPI execution} = 0.25 * 1 + 0.1 * 2 + 0.65 * 1 = 1.1$

Write through

$\text{Stall Cycles Per Instruction} = \text{MRI} * 100 + \text{MRD} * (0.25 * 100 + 0.1 * 100) = 1.2$

so: $\text{CPI} = 1.1 + 1.2 = 2.3$

Write back

$\text{Stall Cycles Per Instruction} = \text{MRI} * 100 + \text{MRD} * (0.25 * (0.3 * 150 + 0.7 * 100) + 0.1 * (0.3 * 150 + 0.7 * 100)) = 1.405$

so: $\text{CPI} = 1.1 + 1.405 = 2.405$

Comparing 1 and 2 we notice that the system with the write back cache is 4.6% slower.

Problem 2.

One difference between a write-through cache and a write-back cache can be in the time it takes to write. During the first cycle, we detect whether a hit will occur, and during the second (assuming a hit) we actually write the data.

Let's assume that 30% of the blocks are dirty for a write-back cache. For this question, assume that the write buffer for the write through will never stall the CPU (no penalty). Assume a cache read hit takes 1 clock cycle, the cache miss penalty is 100 clock cycles, and a block write from the cache to main memory takes 50 clock cycles. Finally, assume the instruction cache miss rate is 0.5% and the data cache miss rate is 2%. Assuming that on average 25% and 10% of instructions in the workload are loads and stores, respectively, **estimate the performance of a write-through cache with a two-cycle write versus a write-back cache with a two-cycle write.**

Problem 3.

Consider the following program and cache behaviors.

Data Reads per 1K instructions	Data Writes per 1K instructions	Instruction Cache Miss Rate	Data Cache Miss Rate	Block Size (Bytes)
400	200	0.5%	2%	64

Suppose a CPU with a write-through, write allocate cache achieves a CPI of 2. What are the read and write bandwidths (measured by bytes per cycle) between RAM and the cache? (Assume each miss generates a request for one block.). *For a write-allocate policy, a write miss also makes a read request to RAM – please be sure to consider its impact on Read Bandwidth*

Problem 3

Instruction Bandwidth:

When the CPI is 2, there are, on average, 0.5 instruction accesses per cycle.

0.5 instructions read from Instruction memory per cycle

0.5% of these *instruction* accesses cause a cache **Read** miss (and subsequent memory request).

$$[0.5 \text{ instr/cycle}] \times [0.005 \text{ misses/instruction}] = \text{missed instructions/cycle}$$

Assuming each miss requests one block and each block is 128 bytes [16 words with 8 bytes (64 bits) per word], instruction accesses generate an average of

$$[0.5 \text{ instr/cycle}] \times [0.005 \text{ misses/instruction}] \times [64 \text{ bytes/miss}] = 0.16 \text{ bytes/cycle of read traffic}$$

Read Data bandwidth:

40% of instructions generate a **read** request from data memory.

$$[0.5 \text{ instr/cycle}] \times [0.4 \text{ Read Data Accesses/instruction}] = [0.2 \text{ Read Data Accesses / cycle}]$$

2% of these generate a cache miss;

$$[0.2 \text{ Read Data Accesses / cycle}] \times [0.02 \text{ misses / Read Data Access}] = 0.004 \text{ Read Misses/cycle}$$

Assuming each miss requests one block and each block is 128 bytes [16 words with 8 bytes (64 bits) per word],

$$[0.004 \text{ Read Misses/cycle}] \times [64 \text{ Bytes/block}] \times [1 \text{ block/miss}] = 0.004 \times 64 \text{ Bytes/cycle} \\ = 0.256 \text{ Bytes/cycle}$$

Write Data bandwidth:

20% of instructions generate a **write** request into data memory.

$$[0.5 \text{ instr/cycle}] \times [0.2 \text{ Write Data Accesses/instruction}] = [0.1 \text{ Write Data Accesses / cycle}]$$

All of the words written to the cache must be written into Memory:

$$[0.1 \text{ Write Data Accesses / cycle}] \times [8 \text{ bytes/word}] \times [1 \text{ word/write-through}] = 0.8 \text{ Bytes/cycle}$$

For a *Write-allocate policy*, a Write miss also makes a **read** request to RAM

$$[0.5 \text{ instr/cycle}] \times [0.2 \text{ Write Data Accesses/instruction}] \times [0.02 \text{ misses/Write Data Access}] \times [64 \text{ Bytes/miss}] \\ = 0.128 \text{ Bytes/cycle}$$

Assuming each miss requests one Word (8 bytes) since this is a write-through cache *with only 1 word written per miss into memory*,

$$[0.002 \text{ Write Misses/cycle}] \times [8 \text{ Bytes/word}] \times [1 \text{ word/miss}] = 0.016 \text{ Bytes/cycle}$$

Total Read Bandwidth

$$0.16 \text{ (Instruction memory)} + 0.256 \text{ (data memory)} + 0.128 \text{ (Write-miss in Write-through cache with Write Allocate)} \text{ Bytes/cycle} = 0.544 \text{ Bytes/cycle}$$

Total Write Bandwidth:

$$0.8 \text{ Bytes/cycle} + 0.016 \text{ Bytes/cycle} = 0.816 \text{ Bytes/cycle}$$

Problem 4.

Consider the following RISC V Instruction sequence executing in a 5-stage pipeline:

```
and    x13, x12, x11
ld     x10, 0(x13)
ld     x11, 8(x10)
add    x12, x10, x11
subi   x13, x12, 16
```

4.1 Identify all of the data hazards and their resolution with NOPs assuming no forwarding or hazard detection hardware is being used

4.2 If there is forwarding, for the first seven cycles during the execution of this code, specify which signals are asserted in each cycle by hazard detection and forwarding units in Figure below.

Mux control	Source	Explanation
ForwardA = 00	ID/EX	The first ALU operand comes from the register file.
ForwardA = 10	EX/MEM	The first ALU operand is forwarded from the prior ALU result.
ForwardA = 01	MEM/WB	The first ALU operand is forwarded from data memory or an earlier ALU result.
ForwardB = 00	ID/EX	The second ALU operand comes from the register file.
ForwardB = 10	EX/MEM	The second ALU operand is forwarded from the prior ALU result.
ForwardB = 01	MEM/WB	The second ALU operand is forwarded from data memory or an earlier ALU result.

4.1 Hazards identified:

and x13, x12, x11

ld x10, 0(x13) EX 1st

ld x11, 8(x10) MEM 1st

add x12, x10, x11 MEM 1st and 2nd.

subi x13, x12, 16 EX 1st

↓

and x13, x12, x11

Nops

Nops

ld x10, 0(x13).

Nops

Nops

ld x11, 8(x10)

Nops

Nops

add x12, x10, x11

Nops

Nops

subi x13, x12, 16

4.2

	1	2	3	4	5	6	7	8
and	IF	ID	EX	MEM	WB			
ld		IF	ID	EX	MEM	WB		
ld				IF	ID	EX	MEM	WB
add						IF	ID	EX
subi							IF	ID

1	A	B
2	x	x
3	x	x
4	0	0
5	2	x
6	x	0
7	1	0
8	x	x