

# Practice Exam 1

## Tables Provided on Exam 1

Table 1.5.1: Laws of propositional logic.

Idempotent laws:	$p \vee p = p$	$p \wedge p = p$
Associative laws:	$(p \vee q) \vee r = p \vee (q \vee r)$	$(p \wedge q) \wedge r = p \wedge (q \wedge r)$
Commutative laws:	$p \vee q = q \vee p$	$p \wedge q = q \wedge p$
Distributive laws:	$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$
Identity laws:	$p \vee F = p$	$p \wedge T = p$
Domination laws:	$p \wedge F = F$	$p \vee T = T$
Double negation law:	$\neg\neg p = p$	
Complement laws:	$p \wedge \neg p = F$ $\neg T = F$	$p \vee \neg p = T$ $\neg F = T$
De Morgan's laws:	$\neg(p \vee q) = \neg p \wedge \neg q$	$\neg(p \wedge q) = \neg p \vee \neg q$
Absorption laws:	$p \vee (p \wedge q) = p$	$p \wedge (p \vee q) = p$
Conditional identities:	$p \rightarrow q = \neg p \vee q$	$p \leftrightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$

Table 1.12.1: Rules of inference known to be valid arguments.

Rule of inference	Name
$\frac{p \quad p \rightarrow q}{\therefore q}$	Modus ponens
$\frac{\neg q \quad p \rightarrow q}{\therefore \neg p}$	Modus tollens
$\frac{p}{\therefore p \vee q}$	Addition
$\frac{p \wedge q}{\therefore p}$	Simplification

Rule of inference	Name
$\frac{p \quad q}{\therefore p \wedge q}$	Conjunction
$\frac{p \rightarrow q \quad q \rightarrow r}{\therefore p \rightarrow r}$	Hypothetical syllogism
$\frac{p \vee q \quad \neg p}{\therefore q}$	Disjunctive syllogism
$\frac{p \vee q \quad \neg p \vee r}{\therefore q \vee r}$	Resolution

Table 1.13.1: Rules of inference for quantified statements

Rule of Inference	Name
c is an element (arbitrary or particular) $\forall x P(x)$ $\therefore P(c)$	Universal instantiation
c is an arbitrary element $P(c)$ ____ $\therefore \forall x P(x)$	Universal generalization
$\exists x P(x)$ $\therefore (c \text{ is a particular element}) \wedge P(c)$	Existential instantiation*
c is an element (arbitrary or particular) $P(c)$ ____ $\therefore \exists x P(x)$	Existential generalization

Table 3.6.1: Set identities.

Name	Identities	
Idempotent laws	$A \cup A = A$	$A \cap A = A$
Associative laws	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Commutative laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Distributive laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity laws	$A \cup \emptyset = A$	$A \cap U = A$
Domination laws	$A \cap \emptyset = \emptyset$	$A \cup U = U$
Double Complement law	$\overline{\overline{A}} = A$	
Complement laws	$A \cap \overline{A} = \emptyset$ $\overline{\overline{U}} = U$	$A \cup \overline{A} = U$ $\overline{\emptyset} = U$
De Morgan's laws	$\overline{A \cup B} = \overline{A} \cap \overline{B}$	$\overline{A \cap B} = \overline{A} \cup \overline{B}$
Absorption laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$

# Set Theory

Given:

$$A = \{ 1, \{2\}, \{\{3, 4\}\} \}$$

For each of the following statements, state whether they are **true** or **false**.

- a.  $1 \in A$
- b.  $1 \subseteq A$
- c.  $\{2\} \in A$
- d.  $\{2\} \subseteq A$
- e.  $\{3, 4\} \in A$
- f.  $\{3, 4\} \subseteq A$
- g.  $\{\{3,4\}\} \in A$
- h.  $\{\{3,4\}\} \subseteq A$
- i.  $\emptyset \in A$
- j.  $\emptyset \subseteq A$

1.2 Let  $A = \{1, 2, 3, 4\}$ . Select the statement that is **false**.

- a.  $\emptyset \in P(A)$
- b.  $\emptyset \subseteq P(A)$
- c.  $\{2, 3\} \in A$
- d.  $\{2, 3\} \subseteq A$

# Functions

Choose the property for which the function satisfies if well defined.

- a. Neither one-to-one, nor onto
- b. One-to-one, but not onto
- c. Onto, but not one-to-one
- d. one-to-one and onto
- e. not well defined

Given a function whose domain is the set of all integers and whose target is the set of all positive integers:

a)

$$f(x) = 2x + 1$$

b)

$$f(x) = |x| + 1$$

c)

$$f(x) = x^2 + 1$$

d)

$$f(x) = \{(x \geq 0 : 2x + 1) \wedge (x \leq 0 : -2x)\}$$

e)

$$f(x) = \{(x \geq 0 : 2x + 1) \wedge (x < 0 : -2x + 2)\}$$

# Proofs

## 4.1 Direct Proof

Prove that the product of two odd integers is an odd integer.

## 4.2 Proof by Contrapositive

Prove that if  $n^2$  is even, then  $n$  is even.

Consider the contraposition of the proposition which is  $\sim q \rightarrow \sim p$ . Show that if  $n$  is odd, then  $n^2$  is odd.

## 4.2 Proof by Contradiction

Prove by contradiction that if  $3n+5$  is odd, then  $n$  is even.

# Number Systems Conversion

## 5.1 Decimal to 8-bit Two's Complement

$$(-43)_{10} =$$

## 5.2 Binary to Hexadecimal

$$(110011100)_2 =$$

# Coding

```
  a
 b b
c  c
d    d
e      e
d      d
 c     c
  b    b
   a
```

Make a hollowed-out diamond made up of in-order alphabet letters. An example is below when `n=5`.

# Challenge Question

For this question, you have to create the following Barn Door shape with `n=10`:

```
#####  
# $      $ #  
# $      $ #  
# $ $ $  #  
# $ $ $  #  
# $ $ $  #  
# $ $ $  #  
# $      $ #  
# $      $ #  
# $      $ #  
#####
```