Assignment 2

1) Find the nth derivative of

    a) $\dfrac{x+1}{x^2-4}$ b) $\cos^2 x$ c) $e^a \cos x$

2) If $y = \sin(m \sin^{-1} x)$ prove that

$$(1-x^2) y_{n+2} - (2n+1) x \cdot y_{n+1} + (m^2 - n^2) y_n = 0$$

3) If $y = \log(x + \sqrt{1+x^2})$. Prove that

$$(1+x^2) y_{n+2} + (2n+1) x \, y_{n+1} + n^2 y_n = 0$$

4) If $x = r \cos \theta$, $y = r \sin \theta$ find

    1) $\left(\dfrac{\partial y}{\partial \theta}\right)_r$ 2) $\left(\dfrac{\partial r}{\partial x}\right)_y$.

5) If $u = \log(\tan x + \tan y)$ prove that

$$\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} = 2.$$

6. If $v = \dfrac{x^3 y^3}{x^3 + y^3}$, show that $x \cdot \dfrac{\partial v}{\partial x} + y \cdot \dfrac{\partial v}{\partial y} = 3v$

7. If $u = \log \dfrac{x^3 + y^3}{x^2 + y^2}$, prove that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 1$$

8- If $u = \tan^{-1} \dfrac{\sqrt{x^3 + y^3}}{\sqrt{x} + \sqrt{y}}$, find the value of

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$$

9) If $u = f(e^{y-z}, e^{z-x}, e^{x-y})$, then

prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$

10) If $w = f(x,y)$ $x = r\cos\theta,$ $y = r\sin\theta$

Show that-

$$\left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2 =$$

$$\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2$$