Volume double integration

$A = \iint z \, dx \, dy$.

Use double integral is find the Volume under this Surface

$\therefore \quad z = x^2 + 3x^2y$ over the rectangle .

$-1 \leq x \leq 3 \qquad 0 \leq y \leq 2$.

$$\int_{-1}^{3} \int_{0}^{2} x^2 + 3x^2 y \, dy \, dx$$

$$\cancel{\int_{0}^{3} \left[\frac{x^3}{3} + 3 \cdot y \cdot \frac{x^3}{3} \right]_0^2 dy}$$

$$\int_{-1}^{3} \left[x^2 \cdot y + 3 \cdot x^2 \cdot \frac{y^2}{2} \right]_0^2 dx .$$

$$\int_{-1}^{3} x^2 \cdot [2] + 3x^2 \left[\frac{4}{2} \right] dx$$

$$\int_{-1}^{3} 2x^2 + 6x^2 \, dx .$$

$$\left[2 \cdot \frac{x^3}{3} + 6 \cdot \frac{x^3}{3} \right]_{-1}^{3} \qquad -1 \; -3$$

$$\frac{2}{3} \left[x^3 + 3x^3 \right]_{-1}^{3}$$

$$\frac{2}{3} \left[(27 + 3 \cdot 27) - (-1 + 3(-1)) \right]$$

$$\frac{2}{3} \left[(27 + 81) - (-4) \right]$$

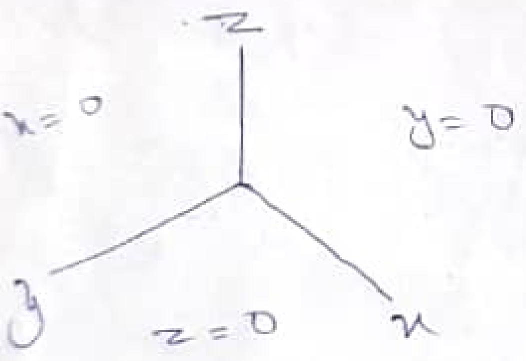
$\frac{2}{3}\cdot[108+4]$

$\frac{2}{3}\times 112$

$= \frac{224}{3}$ Ans

3 Find the volume of tetrahedron bounded by coordinate planes & $z = 4-4x-2y$.

$\Rightarrow \quad V = \iint z\,dx\,dy$.



$z = 4-4x-2y$

At $z = 0$

$0 = 4-4x-2y$

$2y = 4-4x$

$y = \frac{4-4x}{2}$

$y = 2(1-x)$

$z=0,\ y=0$

$2-2x=0$

$2 = 2x \Rightarrow \boxed{x=1}$

$$\int_0^1\int_0^{(2-2x)}(4-4x-2y)\,dx\,dy.$$

$$\int_0^1\int_0^{2-2x}(4-4x-2y)\,dy\,dx \Rightarrow$$

$$\int_0^1\left[\cancel{4y} - x\cdot 4y - 2\cdot\frac{y^2}{2}\right]_0^{2-2x} dx$$

$$\int_0^1 \left[4\cdot(\cancel{2-2x})^{0} - 4x(2-2x) - \cancel{2}\cdot\frac{(2-2x)^2}{\cancel{2}}\right] dx$$

$$\int_0^1 (\cancel{8-8x})^{0} - 8x + 8x^2 - (4 + 4x^2 - 8x)\, dx$$

$$0 - \cancel{8x}$$

$$\int_0^1 \cancel{8}\left[(8 - 16x) + 8x^2 - 4 - 4x^2 + \cancel{8x}\right] dx$$

$$\times\left\{\int_0^1 4 - 8x + 4x^2 \Rightarrow 4 - 8 + 4 \quad 8 - 8 = 0\right\}$$

2) $\int_0^1 (4x^2 - 4)\, dx$

Evaluate of double Integral in polar Co-ordinate

Evaluate $\int_0^\pi \int_0^{a(1-\cos\theta)} r^2 \sin\theta \, dr\, d\theta$.

Solution $\int_0^\pi \sin\theta \, d\theta \int_0^{a(1-\cos\theta)} r^2 \, dr$

$\int_0^\pi \sin\theta \, d\theta \left[\frac{r^3}{3}\right]_0^{a(1-\cos\theta)}$

$\int_0^\pi \sin\theta \, d\theta \ \frac{1}{3}\left[a^3 (1-\cos\theta)^3\right]$

$\frac{a^3}{3} \int_0^\pi (1-\cos\theta)^3 \sin\theta \, d\theta$

Let $1-\cos\theta = t$

$\sin\theta \, d\theta = dt$

$\theta = 0 \quad , t = 0$

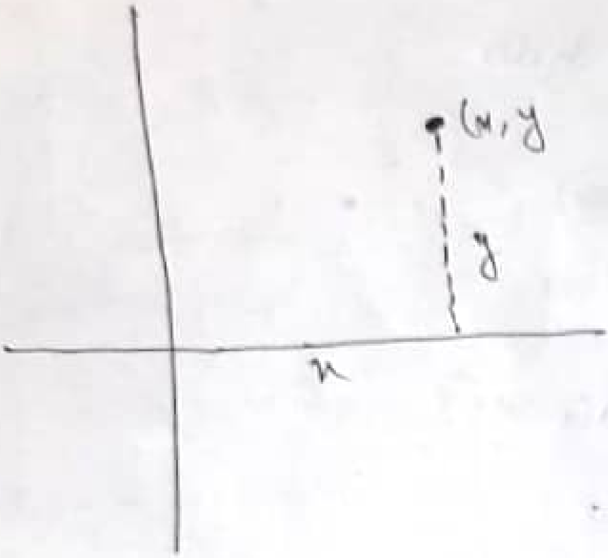
$\theta = \pi \quad , t = 2$

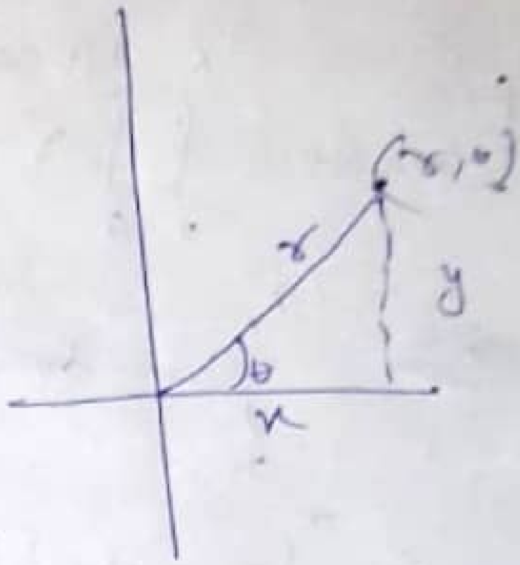
$\frac{a^3}{3} \int_0^2 t^3 \, dt \Rightarrow \frac{a^3}{3} \left[\frac{t^4}{4}\right]_0^2$

$\frac{a^3}{3} \left[\frac{2^4}{4}\right] \Rightarrow \frac{a^3}{3} \times \frac{16}{16} \cdot \frac{a^3}{12} \times 16^4 \, 3$

$\frac{4a^3}{3}$ Ans

Integrating by Changing into ~~its~~ polar Co-ordinate





$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$y = r\sin\theta$

$x = r\cos\theta$

$dy\,dx = r \cdot dr\,d\theta$

Q.

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} (x^2+y^2)^{3/2}\,dy\,dx$$

Limit

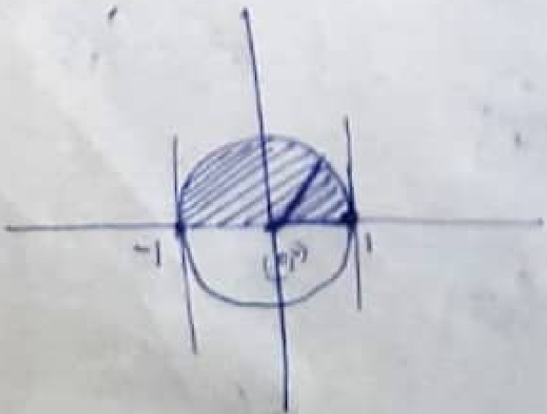
y varies from $y = 0$ to $y = \sqrt{1-x^2}$

$y = 0$

$y^2 = 1-x^2$

$x^2+y^2 = 1$

$x = -1$



$$\int_0^{\pi}\int_0^{1} r^3 \cdot r \, dr \, d\theta$$

$x = r\cos\theta$

$y = r\sin\theta$

$(x^2+y^2)^{3/2}$

$(r^2\cos^2\theta + r^2\sin^2\theta)^{3/2}$

$(r^2)^{3/2} = r^3$

$$\int_0^{\pi}\left[\frac{r^5}{5}\right]_0^1 d\theta$$

$$\int_0^{\pi}\frac{1}{5}\, d\theta$$

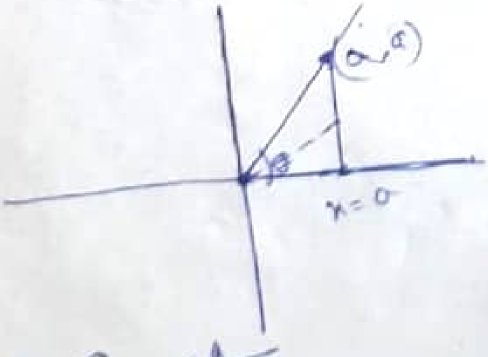
$$\frac{1}{5}\int_0^{\pi} d\theta = \frac{1}{5}[\theta]_0^{\pi} = \frac{1}{5}[\pi - 0] = \frac{1}{5}\pi \quad \text{Ans}$$

Q. $$\int_0^{a}\int_0^{x}\frac{x}{x^2+y^2}\, dy\, dx$$

$y = 0$, $y = x$

y varies from 0 to x.

$x = 0$ to a, $x = a$



$$\frac{x}{x^2+y^2}$$

$$= \cancel{\frac{r\cos\theta}{r^2}} = \cancel{\frac{r}{r^2}} = \frac{1}{r}$$

$x = r\cos\theta$

$a = r\cos\theta$

$r = \frac{a}{\cos\theta}$

$$\cancel{\int_0^{\pi/4}\int_0^{\frac{a}{\cos\theta}} \frac{1}{r} \cdot r \, dr\, d\theta} \qquad \cancel{\int_0^{\pi/4}\int_0^{\frac{a}{\cos\theta}} dr\, d\theta} = \cancel{\int_0^{\pi/4}\left[r\right]_0^{\frac{a}{\cos\theta}} d\theta}$$

$$\cancel{\int_0^{\pi/4}\left[\theta\right]_0^{\frac{a}{\cos\theta}} d\theta} \Rightarrow$$

$$\Rightarrow \int_0^{\pi/4} \left(\frac{a}{\cos\theta} - 0\right) d\theta$$

$$\int_0^{\pi/4} \frac{a}{\cos\theta}\, d\theta$$

$$= a \int_0^{\pi/4} \frac{1}{\cos\theta}\, d\theta$$

$$\Rightarrow a.$$

$$\int_0^{\pi/4} \int_0^{\frac{a}{\cos\theta}} \frac{r\cos\theta \cdot r\, dr\, d\theta}{r^2}$$

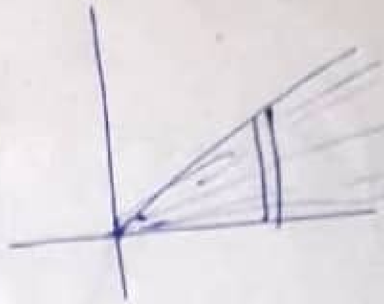
$$\int_0^{\pi/4} \int_0^{\frac{a}{\cos\theta}} \cos\theta\, dr\, d\theta$$

$$\int_0^{\pi/4} \cos\theta \left[r\right]_0^{\frac{a}{\cos\theta}} \Rightarrow \int_0^{\pi/4} \cancel{\cos\theta} \times \frac{a}{\cancel{\cos\theta}}\, d\theta$$

$$a \left[\theta\right]_0^{\pi/4} \qquad a\frac{\pi}{4} \quad \boxed{A} \leftarrow$$

Changing the order of integration

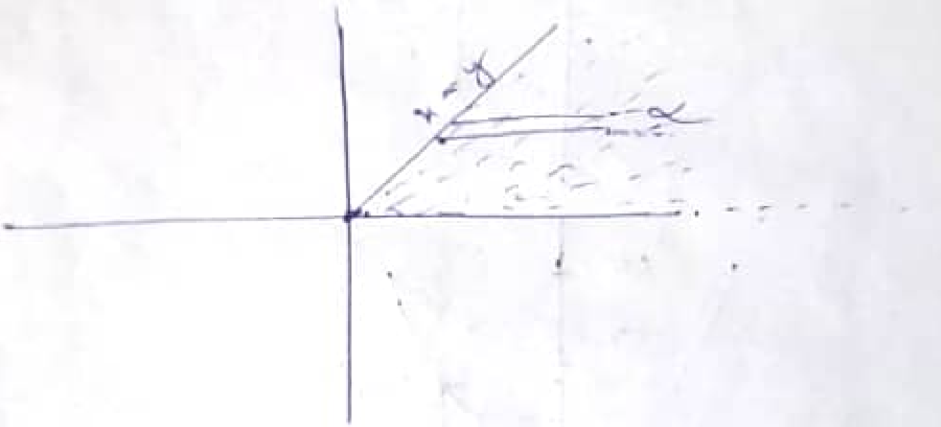
$$\int_{0}^{\infty}\int_{y=0}^{y=x} e^{-xy}\, y\,dy\,dx$$



limit

$y=0$ to $y=x$

$x=0$ to $x=\infty$



$$\int_{0}^{\infty}\int_{y}^{\infty} e^{-xy}\, y\,dy\,dx$$

Q $$\int_{0}^{1}\int_{x^2}^{2-x} xy\,dx\,dy$$

Change the order of integration & Evaluate

limit

$y=x^2$ to $y=2-x \Rightarrow x+y=2$

$x=0$ to $x=1$

$y = x^2$ to $y = 2 - x$

$x + y = 2$

$(0,2)$ $(2,0)$

$x = 0$ to $x = 1$

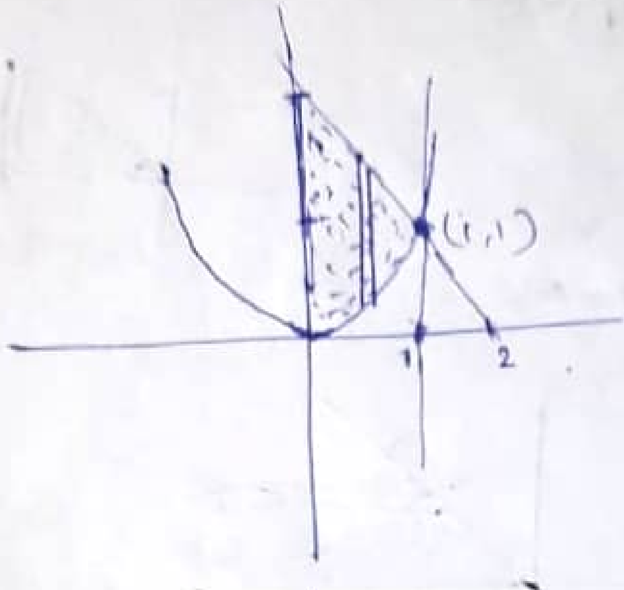
$y = x^2$

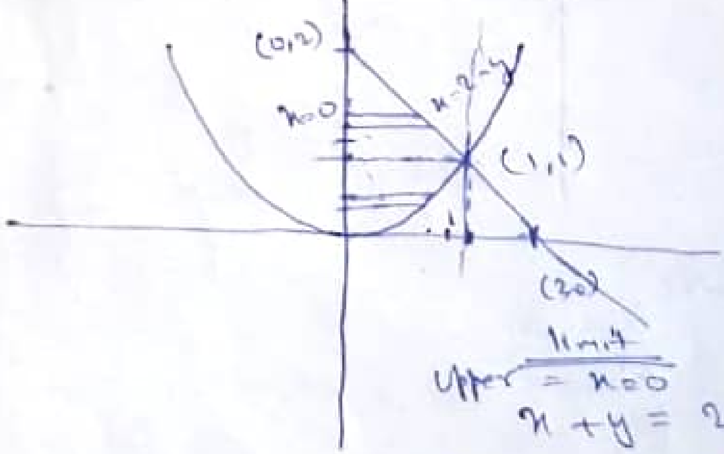
$x + y = 2$

$x + x^2 = 2$

$x^2 + x - 2 = 0$

$(1,1)$





Upper limit = $x = 0$

$x + y = 2$

lower = $x = 2 - y$

$y = x^2$

$x = \sqrt{y}$

$$\Rightarrow \int_1^2 \int_0^{2-y} xy \, dx \, dy + \int_0^1 \int_0^{\sqrt{y}} xy \, dx \, dy$$

$$\Rightarrow \int_1^2 y\left[\frac{x^2}{2}\right]_0^{2-y} dy + \int_0^1 y\left[\frac{x^2}{2}\right]_0^{\sqrt{y}} dy$$

$$\Rightarrow \int_1^2 y \frac{(2-y)^2}{2} dy + \int_0^1 y \frac{(\sqrt{y})^2}{2} dy$$

$$\begin{array}{r|l} 2 & 4, 3, 3 \\ \hline 3 & 2, 3, 3 \\ \hline & 2, 1, 1 \end{array}$$

$$\int_1^2 y \frac{[4+y^2-4y]}{2} dy + \int_0^1 y \cdot \frac{y}{2} dy$$

$$\frac{1}{2}\left[\int_1^2 (4y + y^3 - 4y^2) dy + \int_0^1 y^2 \, dy\right.$$

$$\frac{1}{2}\left\{4 \cdot \left[\frac{y^2}{2}\right]_1^2 + 4\left[\frac{y^4}{4}\right]_1^2 - 2 \cdot \left[\frac{y^3}{3}\right]_1^2 + \left[\frac{y^3}{3}\right]_0^1\right.$$

$$\frac{1}{2}\left[4 \cdot \left(\frac{4}{2} - \frac{1}{2}\right) + 4\left(\frac{16}{4} - \frac{1}{4}\right) - 2 \cdot \left(\frac{8}{3} - \frac{1}{3}\right) + \frac{1}{3}\right]$$

$$\frac{1}{2}\left[\cancel{4}^2 \cdot \frac{3}{\cancel{2}} + \cancel{4} \times \frac{15}{\cancel{4}} - 2 \cdot \frac{7}{3} + \frac{1}{3}\right]$$

$$\frac{1}{2}\left[6 + 15 - \frac{14}{3} + \frac{1}{3}\right]$$

$$\begin{array}{r} 46 \\ 18 \\ \hline 64 \\ 14 \\ \hline 50 \end{array} \qquad \begin{array}{r} 49 \\ 72 \\ \hline 121 \end{array}$$

$$\frac{1}{2}\left[\frac{18 + 45 - 14 + 1}{3}\right]$$

$$\begin{array}{r} 121 \\ 56 \\ \hline 65 \end{array}$$

$$\frac{1}{2}\left[\frac{121 - 56}{12}\right] = \frac{1}{2} \times \frac{65}{12} = \frac{65}{24}$$