

Test 1 MAS q16

SET B

NAME

Mark-10

ID:

1. By using elementary row operations, find the solutions if they exist for the following.

$$x + 2y + 3z = 14, \quad 3x + y + 2z = 11, \quad 2x + 3y + z = 11$$

2. Find the rank of the following matrix by reducing to normal form

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

3. Examine the following system of vectors for linear dependence. If dependent find the relation between them

$$X_1 = (3, 1, -4), \quad X_2 = (2, 2, -3), \quad X_3 = (0, -4, 1)$$

Test 1 MAS 416 SET A

NAME

Mark-10

ID: 1

1. By using elementary row operations, find the solutions if they exist for the following.

$$2x + y + z = 8, \quad -x + 2y + z = 4, \quad 3x + y - 4z = 0$$

2. Find the rank of the following matrix by reducing to normal form

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 1 & 1 & -2 & 1 \\ 1 & -1 & 4 & 1 \\ -2 & 2 & 8 & 0 \end{bmatrix}$$

3. Examine the following system of vectors for linear dependence. If dependent find the relation between them

$$X_1 = (1, -1, 1), \quad X_2 = (2, 1, 1), \quad X_3 = (3, 0, 2)$$

Set-A Test-2

1. Find  $\frac{d^4y}{dx^4}$  if  $y = x^5 + 7x^2 - 3x + 8$

2. If  $u = \tan^{-1} \left( \frac{x^3 + y^3}{\sqrt{x} + \sqrt{y}} \right)$  find

the value of  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$

3. If  $u = \sin^{-1} \frac{x}{y}$  find  $\frac{\partial u}{\partial x}$

4. If  $y = a \cos(\log x) + b \sin(\log x)$

show that  $x^2 y_{n+2} + (2n+1)x y_{n+1} +$

$(n^2+1)y_n = 0$

5. Find the  $n$ th derivative of  $\cos^2 x$

Set-B Test-2

1. If  $u = \tan^{-1} y/x$  find  $\frac{\partial u}{\partial y}$
2. Find the nth derivative of  $\sin^2 x$

3. If  $u = \sin^{-1} \frac{(x+y)}{\sqrt{x} + \sqrt{y}}$  find the value of  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$

4. If  $y = x^8 + 7x^4 - 4x + 18$   
find  $\frac{d^3 y}{dx^3}$

5. If  $y = \cos(m \log x)$   
show that  $x^2 y_{n+2} + (2n+1)x y_{n+1} + (m^2 + n^2) y_n = 0$

Ans.

### Set A

1. State Green's theorem and evaluate  $\oint_C (x^2 + xy) dx + (x^2 + y^2) dy$  where  $C$  is the square formed by the lines  $y = \pm 1$ ,  $x = \pm 1$
2. Find a unit normal vector to the surface  $z^2 = x^2 + y^2$  at the point  $(1, 0, -1)$
3. Prove that  $(y^2 - z^2 + 3xyz - 2x)\mathbf{i} + (3xz + 2xy)\mathbf{j} + (3xy - 2xz + 2z)\mathbf{k}$  is solenoidal
4. Suppose  $F(x, y, z) = x^3\mathbf{i} + y\mathbf{j} + z\mathbf{k}$  is the force field. Find the work done by  $\vec{F}$  along the line from  $(1, 3, 3)$  to  $(3, 5, 7)$
5. State Gauss's theorem of Divergence. Using it evaluate  $\iiint_V \vec{F} \cdot \vec{n} ds$  where  $\vec{F} = 4xz\mathbf{i} - y\mathbf{j} + yz\mathbf{k}$  and  $S$  is the surface of the cube bounded by  $x=0, x=1, y=0, y=1, z=0, z=1$



### Sol. B

1. State Gauss theorem of divergence, using it find  $\iint_S \vec{F} \cdot \vec{n} \, ds$  where  
 $\vec{F} = (2xz + 3z)\vec{i} - (xz + y)\vec{j} + (y^2 + 2z)\vec{k}$   
and  $S$  is the surface of the sphere having Centre  $(3, -1, 2)$  and radius 3.
2. Evaluate  $\text{grad } \phi$  if  $\phi = \log(x^2 + y^2 + z^2)$
3. Prove that  
 $(y^2 - z^2 + 3xyz - 2x)\vec{i} + (3xz + 2xy)\vec{j} + (3xy - 2xz + 2z)\vec{k}$  is irrotational
4. If  $\vec{A} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$   
Evaluate  $\int_C \vec{A} \cdot d\vec{r}$  from  $(0, 0, 0)$  to  $(1, 1, 1)$   
along the curve  $C$   $x=t, y=t^2, z=t^3$

1. A Field  $F$  is termed as irrotational if \_\_\_\_\_
2. If  $x = r \cos \theta$ ,  $y = r \sin \theta$ , then the value of  $\frac{\partial(x, y)}{\partial(r, \theta)}$  is \_\_\_\_\_
3. Stokes's theorem relates \_\_\_\_\_ to \_\_\_\_\_
4. The  $n^{\text{th}}$  derivative of  $\cos(ax+b)$  is \_\_\_\_\_
5. If  $z$  is homogeneous function of  $(x, y)$  of order  $n$  then  $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} =$  \_\_\_\_\_
6.  $\int_0^a \int_0^x xy \, dy \, dx =$  \_\_\_\_\_
7. The eigen values of  $\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$  are \_\_\_\_\_
8. The  $5^{\text{th}}$  derivative of  $a^5$  is \_\_\_\_\_
9.  $\int_{-1}^1 \int_{-2}^2 \int_{-3}^3 dx \, dy \, dz =$  \_\_\_\_\_
10. Normal to the surface  $xyz^2 = 4$  at  $(-1, -1, 2)$  is \_\_\_\_\_

1) The  $m^{\text{th}}$  derivative of  $x^m$  is \_\_\_\_\_

2) The eigen values of  $\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$  are \_\_\_\_\_

3) The  $n^{\text{th}}$  derivative of  $\log(ax+b)$  is \_\_\_\_\_

4) If  $u = x^2$   $v = y^2$  then  $\frac{\partial(u,v)}{\partial(x,y)}$  is \_\_\_\_\_

5) If  $z$  is a homogeneous function of  $x, y$  of degree  $n$  and  $z = f(u)$  then  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} =$  \_\_\_\_\_

6) Gauss theorem of divergence relates \_\_\_\_\_ to \_\_\_\_\_

7)  $\int_0^1 \int_0^x e^x dx dy =$  \_\_\_\_\_

8)  $\int_{-1}^1 \int_{-2}^2 \int_{-3}^3 dx dy dz =$  \_\_\_\_\_

9)  $\nabla$  is called a solenoidal vector function if \_\_\_\_\_

10) The eigen values of  $A$  is  $1, 2, 3$  then the eigen values of  $A^2$  is \_\_\_\_\_