# MATHS

## SHUATS

## B-tech CS

### Syllabus

1. Differential Calculus I
2. Differential Calculus II
3. Linear Algebra
4. Multiple integrals
5. Vector Calculus

# MATRIX

Set of Rows & Column.

$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \end{bmatrix}^{m} \rightarrow 1$ row

$1 \times 5$

$\rightarrow$ 1 by 5.

one Column $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$ rows

$5 \times 1$

**Notify** $[\quad]$ or $(\quad)$

$m \times p \quad p \times n$

$m$ → rows, $p$ → Column, $p$ → rows, $n$ → Column

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{1n} \\ a_{21} & & & \\ a_{m1} & a_{m2} & a_{m3} & a_{mn} \end{bmatrix}$$

$a_{ij} \Rightarrow$ $i \rightarrow 1, 2, 3 \dots m$

$j \rightarrow 1, 2, 3 \dots n$.

## Various types of Matrices.

① Row matrix → If a Matrix has one row & any number of Columns called row matrix.

ex $\begin{bmatrix} 5 & 6 & 8 \end{bmatrix}$

② Column matrix → a matrix has only one Column & any number of rows called Column matrix.

ex $\begin{bmatrix} 5 \\ 6 \\ 8 \end{bmatrix}$

③ Zero matrix or Null matrix → If all the elements are Zero, that matrix is called null matrix.

ex $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

④ Square Matrix → a matrix in which the number of rows equal to no. of Columns such matrix are called square matrix.

Ex

$$\begin{bmatrix} a & b & c \\ p & q & r \\ s & t & u \end{bmatrix}$$

⑤ Diagonal Matrix → A Sq matrix is called diagonal matrix if all its non-diagonal elements are zero.

Ex

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

⑥ Unit or Identity matrix → A square matrix is called unit matrix if all the diagonal elements are unity & non diagonal elements are zero called unity or identity matrix.

Ex

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

⑦ Triangular Matrix (Echelon form)

→ a square matrix, all of whose elements below the leading diagonal are zero is called a upper triangular matrix.

→ a square matrix, all of whose elements above the leading diagonal are zero is called lower triangular matrix.

Ex

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 4 & 1 \\ 0 & 0 & 6 \end{bmatrix} \quad \begin{bmatrix} 2 & 0 & 0 \\ 4 & 1 & 0 \\ 5 & 6 & 7 \end{bmatrix}$$

upper triangular

lower triangular

## Transpose of a Matrix.

When we interchange the rows & the corresponding columns, the new matrix is obtained is called transpose of a matrix. It is denoted by $A'$ or $A^T$

ex
$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 5 \\ 6 & 7 & 8 \end{bmatrix} \qquad A' = \begin{bmatrix} 2 & 1 & 6 \\ 3 & 0 & 7 \\ 4 & 5 & 8 \end{bmatrix}$$

**Symmetric Matrix** → a sq matrix will be called Symmetric matrix, If for all values of i & j

$$a_{ij} = a_{ji}$$

ex
$$A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \qquad A' = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$$

$$\boxed{A = A'}$$

**Skew-Symmetric Matrix** → a sq matrix is called Skew Symmetric, if ① $a_{ij} = -a_{ji}$ or $A' = -A$
② all digonal elements are zero.

ex
$$\begin{bmatrix} 0 & -h & -g \\ h & 0 & -f \\ g & f & 0 \end{bmatrix}$$

**Orthogonal Matrix** → a square matrix A is called an orthogonal matrix, if the product of the matrix A and the transpose of matrix i.e $A'$ is Identity matrix.

i.e $\boxed{A.A' = I}$

## Conjugate of a Matrix :

$$A = \begin{bmatrix} 1+i & 2-3i & 4 \\ 7+2i & -i & 3-2i \end{bmatrix} \qquad \bar{A} = \begin{bmatrix} 1-i & 2+3i & 4 \\ 7-2i & i & 3+2i \end{bmatrix}$$

~~Involuntary Matrix ⇒~~

## Equal Matrices

Two matrices are said to be equal, if

1. They are of same order.
2. The elements in the corresponding positions are equal.

## Singular Matrix,

The determinant of $|A| = 0$ then $A$ is singular Matrix.

ex $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \Rightarrow |A| = 6 - 6 = 0$.

## Multiplication of Matrix,

If
$$A = \begin{array}{c} \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix} \end{array} \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array} \; 3\times 3 \quad \& \quad B = \begin{array}{cc} C_1 & C_2 \\ \hline \end{array}\begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 2 & -1 \end{bmatrix} \; 3\times 2$$

fin AB

$$AB = \begin{bmatrix} 0\times1 + 1\times(-1) + (2\times2) & 0 + 0 + -2 \\ 1 - 2 + 6 & -2 + 0 - 3 \\ 2 - 3 + 8 & -4 + 0 - 4 \end{bmatrix}$$

$$\begin{bmatrix} R_1C_1 & R_1C_2 \\ R_2C_1 & R_2C_2 \\ R_3C_1 & R_3C_2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & -2 \\ 5 & -5 \\ 7 & -8 \end{bmatrix} \; 3\times 2$$

$[A]_{3\times 3}\ [B]_{3\times 2} \rightarrow$ Possible

$[B]_{3\times 2}\ [A]_{3\times 3} \rightarrow$ Not Possible

Not Same

$$A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

Q. Find the adj & inverse of the following Matrix.

$$A = \begin{bmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$\Rightarrow$

| | | | | |
|---|---|---|---|---|
| 2 | 5 | 3 | 2 | 5 |
| 3 | 1 | 2 | 3 | 1 |
| 1 | 2 | 1 | 1 | 2 |
| 2 | 5 | 3 | 2 | 5 |
| 3 | 1 | 2 | 3 | 1 |

$$\begin{bmatrix} -3 & 1 & 7 \\ -1 & -1 & 5 \\ 5 & 1 & -13 \end{bmatrix}$$

$|A| = 2(-3) - 5(1) + 3(5)$
$= -6 - 5 + 15$
$= -11 + 15$
$= 4$

$$A^{-1} = \frac{1}{4}\begin{bmatrix} -3 & 1 & 7 \\ -1 & -1 & 5 \\ 5 & 1 & -13 \end{bmatrix}$$

$$= \begin{bmatrix} -3/4 & \frac{1}{4} & 7/4 \\ -1/4 & -1/4 & \frac{5}{4} \\ \frac{5}{4} & \frac{1}{4} & \frac{-13}{4} \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 9 & 3 \\ 1 & 4 & 2 \end{bmatrix}$$

| | | | |
|---|---|---|---|
| 1 | 2 | 1 | 1 |
| 9 | 3 | 1 | 9 |
| 4 | 2 | 1 | 4 |
| 1 | 2 | 1 | 1 |
| 9 | 3 | 1 | 9 |

$$\begin{bmatrix} 6 & 6 & -15 \\ 1 & 0 & -1 \\ -5 & -3 & 8 \end{bmatrix}$$

$|B| = 1(6) - 1(-1) + 2(-5)$
$= 6 + 1 - 10 = 7 - 10$
$= -3$

$$B^{-1} = \begin{bmatrix} -2 & -2 & -5 \\ -1/3 & 0 & 1/3 \\ 5/3 & 1 & -8/3 \end{bmatrix}$$

# Elementary Transformations

Any one of the following operations on a Matrix is called elementary transformation

① Interchanging any rows (or Columns). This transformation is indicated by $R_{ij}$, if $i^{th}$ & $j^{th}$ rows are $\boxed{\text{interchanged}}$

② Multiplication of the elements of any row $R_i$ (or Column) by a non zero scalar quantity $K$ is denoted by
$\boxed{K \cdot R_i}$

③ Addition of constant multiplication of the elements of any row $R_j$ to the corresponding elements of any other row $R_i$ denoted by $\boxed{R_i + K \cdot R_j}$

Gauss-

Finding the inverse of the matrix (only by elementary row transformation).

$$A = I \cdot A$$
$$I = A^{-1} A$$

Calculate the inverse of the Matrix.

1. $$\begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$R_2 \to R_2 - 5R_1$$

$$\begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix} A$$

$R_2 \longrightarrow \frac{R_2}{-3}$

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5/3 & -1/3 \end{bmatrix} A$$

$R_1 \longrightarrow R_1 + R_2 \times (-2)$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7/3 & 2/3 \\ 5/3 & -1/3 \end{bmatrix} A$$

$\therefore A^{-1} = \begin{bmatrix} -7/3 & 2/3 \\ 5/3 & -1/3 \end{bmatrix}$

2. $$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_1 \longrightarrow \frac{R_1}{3}$

$$\begin{bmatrix} 1 & -1 & 4/3 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \longrightarrow R_2 + R_1 \times (-2)$

$$\begin{bmatrix} 1 & -1 & 4/3 \\ 0 & -1 & 4/3 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & 0 & 0 \\ -2/3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Ye a wrong hai

real wrong.

$R_2 \longrightarrow \frac{R_2}{-1}$

$$\begin{bmatrix} 1 & -1 & 4/3 \\ 0 & 1 & -4/3 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & 0 & 0 \\ 2/3 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_1 \longrightarrow R_1 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -4/3 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ \frac{2}{3} & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 + \frac{4}{3} R_3$

$-\frac{4}{3} + \frac{4}{3} \times 1$

$-\frac{1}{3} + \frac{4}{3} \times 0$

1 0 0

0 $\frac{1}{3}$

$\frac{-11}{7} \quad \frac{7}{3}$

$0 - 2(0)$

$1 - 3(0)$

$0 - 2(\frac{-1}{7})$

$\frac{2}{7}$

$-3 - 3(1)$

$-3 - 3$

$1 - 2(\frac{2}{7})$

$1 - \frac{4}{7} \quad \frac{7-4}{7}$

$0 - 1 - 5(\frac{-1}{7})$

$\frac{-1}{7}$

$\frac{-5}{7}$

$-3 + 5(\frac{2}{7})$

$-3 + \frac{10}{7}$

$\frac{-21+10}{7}$

---

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix} \quad \text{find } A^{-1}. \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \quad A = I \cdot A$

$I = A^{-1} \cdot A \qquad R_1 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 0 & 1 \\ 2 & -3 & -1 & 0 & 1 & 0 \\ 3 & 1 & 2 & 1 & 0 & 0 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 0 & 1 \\ 0 & -7 & -3 & 0 & 1 & -2 \\ 0 & -5 & -1 & 1 & 0 & -3 \end{array}\right]$$

$R_2 \to \frac{-1}{7} R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc:ccc} 1 & 2 & 1 & 0 & 0 & 1 \\ 0 & 1 & \frac{3}{7} & 0 & \frac{-1}{7} & \frac{2}{7} \\ 0 & -5 & -1 & 1 & 0 & -3 \end{array}\right]$$

$R_1 \to R_1 - 2R_2$

$R_3 \to R_3 + 5R_2$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & \frac{1}{7} & 0 & \frac{2}{7} & \frac{3}{7} \\ 0 & 1 & \frac{3}{7} & 0 & \frac{-1}{7} & \frac{2}{7} \\ 0 & 0 & \frac{8}{7} & 1 & \frac{-5}{7} & \frac{-11}{7} \end{array}\right]$$

$R_3 \longrightarrow \frac{7}{8} R_3$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & \frac{1}{7} & 0 & \frac{2}{7} & \frac{3}{7} \\ 0 & 1 & \frac{3}{7} & 0 & \frac{-1}{7} & \frac{2}{7} \\ 0 & 0 & 1 & \frac{7}{8} & \frac{-5}{8} & \frac{-11}{8} \end{array}\right]$$

$R_1 \longrightarrow R_1 - \frac{1}{7} R_3$

$R_2 \to R_2 - \frac{3}{7} R_3$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & -\frac{1}{8} & \frac{7}{8} & \frac{5}{8} \\ 0 & 1 & 0 & -\frac{3}{8} & +\frac{1}{8} & \frac{7}{8} \\ 0 & 0 & 1 & \frac{7}{8} & -\frac{5}{8} & \frac{-11}{8} \end{array}\right]$$

# Rank of Matrix.

**Submatrix** → It is a matrix obtained by deleting some rows & some column from a Matrix.

**Rank of a Matrix** → The rank of a matrix is the order of largest square sub matrix, whose determinant is not equal to zero. Thus, if $r$ is said to be the rank of a matrix A, if there exist atleast one square submatrix of A of order $r$ whose determinant is not equal to zero.

It is denoted by $r$

$r(A)$ or $\rho(A)$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 6 & 3 \\ 3 & 13 & 14 \end{bmatrix}_{3\times3}.$$

$$|A| = 1\begin{vmatrix} 6 & 3 \\ 13 & 14 \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ 3 & 14 \end{vmatrix} + 2\begin{vmatrix} 2 & 6 \\ 3 & 13 \end{vmatrix}$$

$$|A| = 1(84-39) + 1(28-9) + 2(26-18)$$

$$= 1(45) + 1(19) + 2(18)$$

$$45 + 19 + 36 = 100 \qquad |A| \neq 0$$

$r(A) = 3$ $\quad\because$ rank of the matrix = order of the matrix.

**Q:** $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$

$= B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}_{2\times2} = |B| = 4-4 = 0, \qquad |B| = 0$

$r(A) = 2$

$C = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = |C| = 10-12 = -2 \qquad |C| \neq 0.$

# Normal form of a Matrix

Every non Zero matrix of order are Can be reduced ~~the form~~ to the form by Sequence of elementary transformation where, $I_r$ is the identity matrix of order are which is Called normal form of the matrix.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$$

The other possible normal forms are $[I_r]$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} I_3 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} I_3 & 0 \end{bmatrix}$$

## Rank by Normal form.

By the application of any elementary row & Column transformation. A matrix of order m by n. ~~Kind~~ Can be reduced to one of the ~~about norma~~ above normal form. then the rank of the given matrix is 'r'.

**Method**

① $1^{st}$ element in the first row is 1
② Convert all elements in its row & Column to Zero.

② $2^{nd}$ element in the second row is 1, Convert all elements in its row & Column to zero.

Q Find the rank of Matrix by normal form :-

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 1 & 1 & 5 \\ -1 & 0 & -3 \end{bmatrix}$$

= $R_2 \rightarrow R_2 + R_1$
$R_3 \rightarrow R_3 - R_1$
$R_4 \rightarrow R_4 + R_1$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \\ 0 & 1 & -2 \end{bmatrix}$$

$C_2 \rightarrow C_2 - C_1$
$C_3 \rightarrow C_3 - C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \\ 0 & 1 & -2 \end{bmatrix}$$

$R_2 \rightarrow \frac{1}{2} R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \\ 0 & 1 & -2 \end{bmatrix}$$

$R_4 \rightarrow R_4 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -2 \end{bmatrix}$$

$R_3 \rightarrow \frac{1}{4} R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -2 \end{bmatrix} \quad R_4 \to R_4 + 2R_3$$

$$\begin{bmatrix} \boxed{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} \end{bmatrix} = \begin{bmatrix} I_3 \\ 0 \end{bmatrix} \quad \underline{r(A) = 3}$$

Reduce to normal form & find the rank of the matrix.

$$A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$$

$$C_1 \to \frac{C_1}{2}$$

$$A = \begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 1 & 3 & 7 & 5 \\ 1 & 5 & 11 & 6 \end{bmatrix}$$

$$R_3 \to R_3 - R_1$$
$$R_4 \to R_4 - R_1$$

$$A = \begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 0 & 4 & 4 & 1 \\ 0 & 6 & 8 & 2 \end{bmatrix}$$

$C_2 \to C_2 + C_1$
$C_3 \to C_3 - 3C_1$
$C_4 \to C_4 - 4C_1$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 4 & 1 \\ 0 & 4 & 4 & 1 \\ 0 & 6 & 8 & 2 \end{bmatrix} \qquad C_2 \leftrightarrow C_4$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 3 \\ 0 & 1 & 4 & 4 \\ 0 & 2 & 8 & 6 \end{bmatrix} \qquad \cancel{R_3 \to R_3 - R_2}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{aligned} R_3 &\to R_3 - R_2 \\ R_4 &\to R_4 - 2R_2 \end{aligned}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{aligned} C_3 &\to C_3 - 4C_2 \\ C_4 &\to C_4 - 3C_2 \end{aligned}$$

$$A = \begin{bmatrix} \boxed{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} & 0 \end{bmatrix} \qquad C_4 \leftrightarrow C_3$$

$$A = \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$$

rank = 3

Q $\begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & 2 & 5 \\ 0 & 1 & 2 & 5 \end{bmatrix}$ $R_2 \to R_2 - R_1$, $R_3 \to R_3 - 3R_1$

$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 5 \\ 0 & 1 & 2 & 5 \end{bmatrix}$ $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$, $C_4 \to C_4 + C_1$

$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 \to R_3 - R_2$

$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $C_3 \to C_3 - 2C_2$, $C_4 \to C_4 - 5C_2$

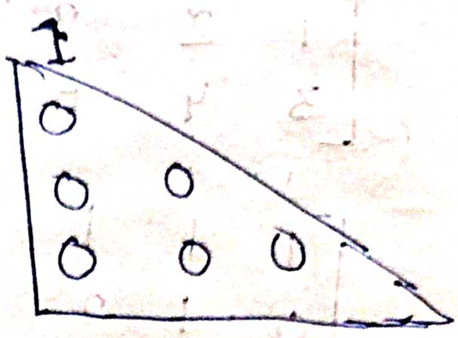
$= \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$, rank = 2

## Rank by Echelon form.

Reduced the given matrix to Echelon form by using only row-transformation, then the number of non-zeros rows is the rank of that matrix.

Reduce the matoin to ekhelon form & find the rank.

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 6 \end{bmatrix}$$



$=$ $R_2 \to R_2 - R_1$

$R_3 \to R_3 - 2R_1$

$R_4 \to R_4 - 3R_1$

$$= \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$R_4 \to R_4 - R_2 \Rightarrow$

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3 non Zeroes row. rank = 3

Q $A = \begin{bmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$ $\boxed{\text{rank} = 4}$

$R_1 \leftrightarrow R_3$

$$A = \begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & 2 & 2 & 1 \\ 3 & -2 & 0 & -1 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$

$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & 2 & 2 & 1 \\ 0 & 4 & 9 & -7 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$

$R_2 \to \frac{R_2}{2}$

$$\begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & 1 & 1 & \frac{1}{2} \\ 0 & 4 & 9 & -7 \\ 0 & 1 & 2 & -1 \end{bmatrix}$$

$R_3 \to R_3 - 4R_2$

$R_4 \to R_4 - R_2$

$$\begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & 1 & 1 & \frac{1}{2} \\ 0 & 0 & 5 & -9 \\ 0 & 0 & 1 & -\frac{1}{2} \end{bmatrix}$$

$R_3 \to \frac{R_3}{5}$

$$\begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & 1 & 1 & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{9}{5} \\ 0 & 0 & 1 & -\frac{1}{2} \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$\begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & 1 & 1 & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{9}{5} \\ 0 & 0 & 0 & \frac{13}{10} \end{bmatrix}$$

rank = 4

Q. 
$$A = \begin{bmatrix} 3 & -2 & 0 & -1 & -7 \\ 0 & 2 & 2 & 1 & -5 \\ 1 & -2 & -3 & -2 & 1 \\ 0 & 1 & 2 & 1 & -6 \end{bmatrix}$$

$R_3 \leftrightarrow R_1$

$$A = \begin{bmatrix} 1 & -2 & -3 & -2 & 1 \\ 0 & 2 & 2 & 1 & -5 \\ 3 & -2 & 0 & -1 & -7 \\ 0 & 1 & 2 & 1 & -6 \end{bmatrix}$$

$R_3 \to R_3 - 3R_1 \longrightarrow$
$$\begin{bmatrix} 1 & -2 & -3 & -2 & 1 \\ 0 & 2 & 2 & 1 & -5 \\ 0 & 4 & 9 & 5 & -10 \\ 0 & 1 & 2 & 1 & -6 \end{bmatrix}$$

$R_2 \to \frac{R_2}{2}$;

$$A = \begin{bmatrix} 1 & -2 & -3 & -2 & 1 \\ 0 & 1 & 1 & 1/2 & -5/2 \\ 0 & 4 & 9 & 5 & -10 \\ 0 & 1 & 2 & 1 & -6 \end{bmatrix}$$

$5 - 4 \times \frac{1}{2}$

$-10 - \left(-\frac{1}{2} \times \frac{5}{2}\right)$

$R_3 \to R_3 - 4R_2$

$R_4 \to R_4 - R_2$

$$A = \begin{bmatrix} 1 & -2 & -3 & -2 & 1 \\ 0 & 1 & 1 & 1/2 & -5/2 \\ 0 & 0 & 5 & 3 & 0 \\ 0 & 0 & 1 & -1/2 & -\frac{7}{2} \end{bmatrix}$$

$-\frac{1}{2} - \frac{3}{5}$

$\frac{-5-6}{10}$

$R_3 \to \frac{R_3}{5}$

$$A = \begin{bmatrix} 1 & -2 & -3 & -2 & 1 \\ 0 & 1 & 1 & 1/2 & -5/2 \\ 0 & 0 & 1 & 3/5 & 0 \\ 0 & 0 & 1 & -1/2 & -7/2 \end{bmatrix}$$

$-\frac{7}{2} - 0$

$R_4 \to R_4 - R_3$

$$A = \begin{bmatrix} 1 & -2 & -3 & -2 & 1 \\ 0 & 1 & 1 & 1/2 & -5/2 \\ 0 & 0 & 1 & 3/5 & 0 \\ 0 & 0 & 0 & -11/10 & -7/2 \end{bmatrix}$$

rank = 4

Q.1 $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ -3 & -6 & -9 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 + 3R_1$

$\longrightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

rank = 1

Q.2 $\begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 6 \end{bmatrix}$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - 2R_1$
$R_4 \to R_4 - 3R_1$

$\longrightarrow \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

$R_4 \to R_4 - R_2$

$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

rank = 3

Q $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ $\rightarrow$ 1 0 0 0.

$R_2 \to R_2 - 2R_1$

$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -3 \\ 1 & 3 & 4 & 5 \end{bmatrix}$

$R_3 \to R_3 - R_1$

$\longrightarrow \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & 1 & 1 & 3 \end{bmatrix}$

$R_3 \to R_3 + R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

rank = 2

$R(A) = 2$

$6 - 10$

$-1 + 15$

$10 \quad -2 - (-2)$

Q $$\begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$$

$-2 - (-1 \times 2)$ $\quad -2 + 2$ $\quad 1 + 6 \quad 2 + (-4)$

$-1 - (-4), 14 \quad -11 \quad 14 - 14. \quad 2 - 4$

$2 - 6 \quad -4 - (-2 \times 2)$

$= \begin{matrix} R_2 \to R_2 + 2R_1 \\ R_3 \to R_3 + 3R_1 \\ R_4 \to R_4 + 5R_1 \end{matrix} \to \begin{bmatrix} -1 & 2 & 3 & -2 \\ 0 & -1 & 7 & -2 \\ 0 & -2 & 14 & -4 \\ 0 & -2 & 14 & -4 \end{bmatrix}$

$= \begin{bmatrix} -1 & 2 & 3 & -2 \\ 0 & -1 & 7 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R_3 \to R_3 - 2R_2$

$R_4 \to R_4 - R_3$

$5 + 9$

$14 - 14$

$R_4 \to R_4 + R_3$

rank = 2

$r(A) = 2.$

$14 - 14$

$$\begin{bmatrix} -1 & 2 & 3 & -2 \\ 0 & -1 & 7 & -2 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(1) $$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$$

(2) $$\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$$

(3) $$\begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$$

(1) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$\longrightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

~~$R(A) = 3$~~ $R_3 - R_2$

$R(A) = 2.$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(2) $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$

$R_2 \to R_2 + R_1$

$R_3 \to R_3 - 2R_1$

$\longrightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 3 \\ 0 & -3 & -5 \end{bmatrix}$

~~$R_2 \to R_2$~~

$R_2 \to \frac{R_2}{2}.$

$-5 + 3 \times \frac{3}{2}$

$-5 + \frac{9}{2}$

$\frac{-10+9}{2}$

$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & \frac{3}{2} \\ 0 & -3 & -5 \end{bmatrix}$

$R_3 \to R_3 + 3R_2 \longrightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & \frac{-1}{2} \end{bmatrix}$

$R(A) = 3$

③ $\begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$

$C_2 \leftrightarrow C_1$

$\begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & 4 & 2 & 6 \\ 1 & 2 & 3 & 1 \end{bmatrix}$

$R_3 \rightarrow R_3 - R_1$

$\begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & 4 & 2 & 6 \\ 0 & 2 & 1 & 3 \end{bmatrix}$

$R_2 \rightarrow \frac{R_2}{4}$

$\begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & 1 & 1/2 & 3 \\ 0 & 2 & 1 & 3 \end{bmatrix}$

$R_3 \rightarrow R_3 - 2R_2$

$\begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & 1 & 1/2 & 3 \\ 0 & 0 & 0 & -3 \end{bmatrix}$

$R(A) = 3$

## Elementary transformation.

1. **Interchanging** : The interchange of $i^{th}$ rows & $j^{th}$ rows (columns) denoted by
$R_i \leftrightarrow R_j$ or $C_i \leftrightarrow C_j$

2. **Scaling** the multiplication of the $i^{th}$ row or (column) by non zero scalar $K$ denoted by $R_j \rightarrow KR_j$, $C_i \rightarrow KC_i$

3. **Combining** : the addition to (or subtraction from the elements of the $i^{th}$ row (or column) of $K$ times the elements of $j^{th}$ row (or column) denoted by

$R_i \rightarrow R_i \pm KR_j$ | $R_i \rightarrow R_i \pm KR_j$

$C_i \rightarrow C_i \pm KC_j$ | $C_i \rightarrow C_i \pm KC_j$

Linear dependence, Consistency of linear System of equation.

→ A n tuple is set of n similar things. If the place of every members of the set is fixed then it is called ordered set. (1, 2) (1, 2, 3)

→ Any ordered ~~n type of~~ n-tuple of number is called n-Vector

(x, y, z)

3-vectors

→ Vectors (Matrices), $x_1, x_2, x_3 \ldots\ldots x_n$ are said to be dependent if

(I) All the vectors are of same order.

(II) n scalar, $\lambda_1 \lambda_2 \ldots\ldots \lambda_n$ (non all Zero)

(x, y, z)

$x_1 = (1, 2, 3)$

$x_2 = (4, 5, 6)$

$x_1 \, x_2 \, (1, 2, 3)_{1\times3}$

Exy. such that

$\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 \ldots\ldots + \lambda_n x_n = 0$

Otherwise, they are linearly independent.

Q. If in a set of vectors, any vector of the set is combination of the remaining vectors, then the vectors are dependent vectors.

1. Examine the following vectors for linear dependence and find the relation if it exist.

$x_1 = (1, 2, 4)$ , $x_2 = (2, -1, 3)$ , $x_3 = (0, 1, 2)$

$x_4 = (-3, 7, 2)$

$\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + \lambda_4 x_4 = 0$

$\lambda_1(1, 2, 4) + \lambda_2(2, -1, 3) + \lambda_3(0, 1, 2) + \lambda_4(-3, 7, 2)$

$$\lambda_1 + 2\lambda_2 + 0\lambda_3 - 3\lambda_4 = 0$$
$$2\lambda_1 - \lambda_2 + \lambda_3 + 7\lambda_4 = 0$$
$$4\lambda_1 + 3\lambda_2 + 2\lambda_3 + 2\lambda_4 = 0$$

$$\begin{bmatrix} 1 & 2 & 0 & -3 \\ 2 & -1 & 1 & 7 \\ 4 & 3 & 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 & -3 \\ 2 & -1 & 1 & 7 \\ 4 & 3 & 2 & 2 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2 - (-12)$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 4R_1$

$$\begin{bmatrix} 1 & 2 & 0 & -3 \\ 0 & -5 & 1 & 1 \\ 0 & -5 & 2 & 14 \end{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & 0 & -3 \\ 0 & -5 & 1 & 1 \\ 0 & 0 & 1 & 13 \end{bmatrix}$$

$\lambda_1 + 2\lambda_2 - 3\lambda_4 = 0$

$-5\lambda_2 + \lambda_3 + \lambda_4 = 0$

$\lambda_3 + 13\lambda_4 = 0$

Let $\lambda_3 = k$

$k + 13\lambda_4 = 0$

$13\lambda_4 = -k$

$\lambda_4 = \frac{-k}{13}$

Q

$$\begin{bmatrix} 1 & 2 & 0 & -3 \\ 0 & -5 & 1 & 13 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\lambda_1 + 2\lambda_2 + 0\lambda_3 - 3\lambda_4 = 0 \rightarrow \lambda_1 + 2\lambda_2 - 3\lambda_4 = 0$

$0\lambda_1 - 5\lambda_2 + \lambda_3 + 13\lambda_4 = 0 \rightarrow -5\lambda_2 + \lambda_3 + 13\lambda_4 = 0$

$0\lambda_1 + 0\lambda_2 + 1\lambda_3 + 1\lambda_4 = 0 \rightarrow \lambda_3 + \lambda_4 = 0$

Let $\lambda_4 = t$

$\lambda_3 + t = 0$

$\lambda_3 = -t$

$-5\lambda_2 + \lambda_3 + 13\lambda_4 = 0$

$\cancel{-5\lambda_2 - t + t = 0}$ $\quad -5\lambda_2 - t + 13t = 0$

$\cancel{\lambda_2 = 5}$ $\quad -5\lambda_2 + 12t = 0$

$\lambda_2 = \frac{+12t}{+5} = \frac{12}{5}t$

$\lambda_1 + 2 \times \frac{12}{5}t - 3t = 0$

$\lambda_1 + \frac{24}{5}t - 3t = 0$

$\lambda_1 = 3t - \frac{24}{5}t$

$\lambda_1 = \frac{15t - 24t}{5} = \frac{-9}{5}t$

$\lambda_1 = \frac{-9}{5}t$

$\lambda_2 = \frac{12}{5}t$

$\lambda_3 = -t$

$\lambda_4 = t$

$\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + \lambda_4 x_4 = 0$

$-\frac{9}{5}t\, x_1 + \frac{12}{5}t\, x_2 - t x_3 + t x_4 = 0$

$-\frac{9}{5}x_1 + \frac{12}{5}x_2 - x_3 + x_4 = 0$

$-9x_1 + 12x_2 - 5x_3 + 5x_4 = 0$

Q/ Show that row vector of the Matrix

$$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$ are linearly independent.

Sol$^n$ $x_1 = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$, $x_2 = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$, $x_3 = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$

$$\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 = 0$$

$$\begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} \lambda_1 + \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix} \lambda_2 + \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} \lambda_3 = 0$$

$\lambda_1 - \lambda_2 + 0\lambda_3 = 0$

$2\lambda_1 + 3\lambda_2 - 2\lambda_3 = 0$

$-2\lambda_1 + 0\lambda_2 + 1\lambda_3 = 0$

$\cancel{\lambda_1 - \lambda_2 = 0}$

$\cancel{2\lambda_1 + 3\lambda_2 - 2\lambda_3 = 0}$

$\cancel{-2\lambda_1 + \lambda_3 = 0}$

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -2 \\ -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$3 - (-1 \times 2)$

$3 + 2$

$0 + 2 \times -1$

$1 + 0 \times 2$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 + 2R_1$

$$\cancel{\begin{bmatrix} 1 & -1 & 0 \\ 0 & 5 & -2 \\ 0 & 3 & -1 \end{bmatrix}} \qquad \begin{bmatrix} 1 & -1 & 0 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\cancel{R_3 \to R_3}$ $\qquad R_2 \to \frac{R_2}{5}$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -2/5 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 + 2R_2$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -2/5 \\ 0 & 0 & 1/5 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$1 + \left(-\frac{2}{5}\right)$

$\frac{5-2}{5}$

$1 + \left(-\frac{4}{5}\right)$

$\frac{5-4}{5}$

$1 + \left(\frac{-4}{5}\right)$

$\frac{5-4}{5}$

$3/_2 \lambda_3 = 0$

$\frac{1}{5}\lambda_3 = 0$

$\lambda_3 = 0$

$\lambda_3 = 0$

$\lambda_2 - 2/5 \lambda_3 = 0$

$0\lambda_1 + 0\lambda_2 + \frac{9}{5}\lambda_3 = 0$

$\lambda_3 = 0$

$\lambda_1 - \lambda_2 + 0\lambda_3 = 0$

$\lambda_2 = 0$

$\lambda_2 - \frac{2}{5}\lambda_3$

$\lambda_1 - \lambda_2 = 0$

$\frac{1}{5}\lambda_3 = 0$

$\lambda_1 = 0$

$\lambda_3 = 0$

$\lambda_1 - \lambda_2 \; 0\lambda_3 = 0$

$\lambda_2 - 0 = 0$

$\boxed{\lambda_2 = 0}$ $\lambda_2 - \frac{2}{5}\lambda_3 = 0.$

$\lambda_1 - \lambda_2 = 0$

$\boxed{\lambda_3 = \frac{1}{5}}$ $\boxed{\lambda_1 = \lambda_2}$

$\lambda_3 - \frac{1}{5}$ $\lambda_3 = 0$ $\lambda_1 - \lambda_2 = 0$

$\frac{1}{5}\lambda_3 = 0$

# Linearly dependence and independence of Vectors by rank method.

① If the rank of the matrix of given Vectors is equal to number of Vectors then they are linearly independent.

② If the rank of the matrix is less than number of Vectors, they are linearly dependent.

eg.

$X = [1, 2, -3, 4]$

$Y = [3, -1, 2, 1]$

$Z = [1, -5, 8, -7]$

-1 - 6

2 + 9

1 - 12

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 3 & -1 & 2 & 1 \\ 1 & -5 & 8 & -7 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 3R_1$

$R_3 \rightarrow R_3 - R_1$.

-7 + 7

-11 - (-11)

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -7 & 11 & -11 \\ 0 & -7 & 11 & -11 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -7 & 11 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R(A) = 2$.

no. of Vectors = 3

$2 < 3$

Here Rank of Matrix less than the no. of Vectors

So, they are linearly dependent.

# Solution of System of Equation

$$\begin{bmatrix} a_{11} + a_{12} + a_{13} + \cdots + a_{1n} \\ a_{21} + a_{22} + a_{23} + \cdots + a_{2n} \\ \cdots \\ \cdots \\ \cdots \\ a_{m1} + a_{m2} + a_{m3} + a_{m4} + \cdots + a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_m \end{bmatrix}$$

$$AX = B$$

$$\underbrace{[A\ B]}_{\text{Augmented Matrix}} = C$$

$$2x_1 + 2x_2 + 3x_3 = 4$$
$$3x_1 + 2x_2 + x_3 = 5$$
$$4x_1 + 5x_2 + x_3 = 6$$

$$\boxed{AX = B}$$

$$\Rightarrow \left[\begin{array}{ccc:c} 2 & 2 & 3 & 4 \\ 3 & 2 & 1 & 5 \\ 4 & 5 & 1 & 6 \end{array}\right]$$

Augmented Matrix

If $r(A) = r[A B] = r = n$, where, $n$ = no. of variables, then the System has Unique Solution.

If $r(A) = r[A.B] = r < n$, it has infinite Solution.

If $r(A) \neq r[A B]$, the System is inconsistent i.e. ~~It has many Solution~~. No Solution.

Use the ~~list of~~ rank to Check the Consistency of the following Systems of Equations & if Consistent find the Solution:

$$x + y + z = 1$$
$$x + 2y + 3z = 4$$
$$x + 3y + 5z = 7$$
$$x + 4y + 7z = 10.$$

$$[A \; B] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 4 & 7 \end{bmatrix}$$

$\underline{\underline{Sol^n}}$

$$[A \; B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 5 & 7 \\ 1 & 4 & 7 & 10 \end{array}\right]$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$R_4 \to R_4 - R_1$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 4 & 6 \\ 0 & 3 & 6 & 9 \end{array}\right]$$

$R_3 \to R_3 - 2R_2$

$R_4 \to R_4 - 3R_2$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} r(A) = 2 \\ r[A\,B] = 2 \\ n = 3 \end{array}$$

$\longleftarrow [r(A)] \longrightarrow$

$\longleftarrow r[AB] \longrightarrow$

The System has infinite solution.

$x + y + z = 1$

$y + 2z = 3$

Let $z = K$.

$y + 2K = 3$

$y = 3 - 2K$

$x + 3 - 2K + K = 1$

$x + 3 - K = 1$

$x = K - 2$

Q. $2x + y - 2z = 2$

$x + y + z = 4$

$3x - y + z = 2$

$x + 2y + 2z = 7$

Soln

$$\begin{bmatrix} 2 & 1 & -2 & : & 2 \\ 1 & 1 & 1 & : & 4 \\ 3 & -1 & 1 & : & 2 \\ 1 & 2 & 2 & : & 7 \end{bmatrix}$$

$R_2 \leftrightarrow R_1$.

$$\begin{bmatrix} 1 & 1 & 1 & : & 4 \\ 2 & 1 & -2 & : & 2 \\ 3 & -1 & 1 & : & 2 \\ 1 & 2 & 2 & : & 7 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & : & 4 \\ 0 & -1 & -4 & : & -6 \\ 0 & -4 & -2 & : & -10 \\ 0 & 1 & 1 & : & 3 \end{bmatrix}$$

$-2 + 16$

$-10 + 24$
$14$

$3 + 6$
$= 3$

$R_3 \to R_3 - 4R_2$

$R_4 \to R_4 + R_2$

$$\begin{bmatrix} 1 & 1 & 1 & : & 4 \\ 0 & -1 & -4 & : & -6 \\ 0 & 0 & 14 & : & 14 \\ 0 & 0 & -3 & : & -3 \end{bmatrix}$$

$R_3 \to \frac{R_3}{14}$

$-3+3$

$-3+0$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 4 \\ 0 & -1 & -4 & -6 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -3 & -3 \end{array}\right]$$

$R_4 \rightarrow R_4 + 3R_3$.

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 4 \\ 0 & -1 & -4 & -6 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R[A\ B] = 3$

$R[A] = 3$

$n = 3$

It has <u>unique Solution</u>.

$x + y + z = 4$

$-y - 4z = -6$

$\boxed{z = 1}$

$-y - 4 = -6$

$-y = -6 + 4 = -2$

$\boxed{y = 2}$

$x + 2 + 1 = 4$

$\boxed{x = 1}$

~~Let $z = k$~~

$k = 1$

$-y - 4k = -6$

$-y - 4 = -6$

$-y = -6 + 4$

$-y = -2$

$y = 2$

$x + y + z = 4$

$x + 2 + 1 = 4$

$x = 4 - 3 = 1$

$x = 1$

$y = 2$

$z = 1$

$k = 1$

Investigate for what values of $\lambda$ & $\mu$ the System of equations

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu \text{ , has}$$

(i) No Solution.

(ii) Unique Solution

(iii) Infinite Solution.

Sol:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$C = [A \; B]$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$$

$R_2 \to R_1 - R_1$

$R_3 \to R_3 \to R_1$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & (\lambda-1) & (\mu-6) \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & \mu \\ 0 & 0 & (\lambda-3) & (\mu-10) \end{array}\right]$$

~~$R(AB) = 3$~~

~~$R(A) = 3$~~

For no Solution, $\lambda = 3$, $\mu \neq 10$.

$R(A) \neq R(C)$

$R(A) = 2$

$R(C) = 3$

For the infinite Solution, $\lambda = 3$, $\mu = 10$

$R(A) = R(C) \cancel{<} n = 3$

$R(A) = 2$

$R(C) = 2$.

For Unique Solution.

$r(A) = r(AB) = n = 3$

$\lambda - 3 \neq 0$

$\lambda \neq 3$, $\mu$ can take any Value.

Q $x + 2y + 3z = 4$

$x + 3y + 4z = 5$

$x + 3y + az = b$ has

(i) No Solution

(ii) Unique Solution

(iii) Infinite Solution.

$$= \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 3 & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ b \end{bmatrix}$$

$C = [AB]$

$$= \begin{bmatrix} 1 & 2 & 3 & : & 4 \\ 1 & 3 & 4 & : & 5 \\ 1 & 3 & a & : & b \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - R_1$

$$\left[\begin{array}{ccc:c} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & (a-3) & (b-4) \end{array}\right]$$

$R_3 \rightarrow R_3 - R_2$

$$\left[\begin{array}{ccc:c} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & (a-4) & (b-5) \end{array}\right]$$

For no Solution.

$r(A) \neq r(AB)$. , $a = 4$ but , $b \neq 5$.

for the infinite Solution.

$r(A) = r[AB] < n = 3$,

~~a-4=~~ $a = 4$ & $b = 5$.

for unique Solution

$r(A) = r[AB] = n$.

$a-4 \neq 0$ $a \neq 4$, b can take any Value.

## System of Homogeneous Equation.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + \dots\dots + a_{1n} = 0$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots\dots\dots + a_{2n} = 0$$

$$\dots\dots\dots\dots\dots\dots$$

$$\dots\dots\dots\dots\dots\dots$$

$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \dots\dots\dots + a_{mn} = 0$$

It is known as Homogeneous Equations.

Solve

1. $x_1 - x_2 + x_3 = 0$
$x_1 + 2x_2 - x_3 = 0$
$2x_1 + x_2 - 3x_3 = 0$

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & -1 \\ 2 & 1 & -3 \end{bmatrix}$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - 2R_1$

$$\sim \begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & -2 \\ 0 & 3 & -5 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\sim \begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & -2 \\ 0 & 0 & -3 \end{bmatrix}$$

$R(A) = 3$

$\boxed{r(A) = 3 = n}$

Trivial Solution
i.e $x_1 = 0, x_2 = 0,$
$x_3 = 0$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & -2 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-3x_3 = 0, \quad x_3 = 0$

$3x_2 - 2x_3 = 0$
$3x_2 = 0, \quad x_2 = 0$

$x_1 = 0$

2. $x - 2y + z - w = 0$
$x + y - 2z + 3w = 0$
$4x + y - 5z + 8w = 0$
$5x - 7y + 2z - w = 0$

$$A = \begin{bmatrix} 1 & -2 & 1 & -1 \\ 1 & 1 & -2 & 3 \\ 4 & 1 & -5 & 8 \\ 5 & -7 & 2 & -1 \end{bmatrix}$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - 4R_1$
$R_4 \to R_4 - 5R_1$

$$\sim \begin{bmatrix} 1 & -2 & 1 & -1 \\ 0 & 3 & -3 & 4 \\ 0 & 9 & -9 & 12 \\ 0 & 3 & -3 & 4 \end{bmatrix}$$

$12 - \cancel{36}$
$24$

$R_3 \to R_3 - 3R_2$
$R_4 \to R_4 - R_2$

$$\sim \begin{bmatrix} 1 & -2 & 1 & -1 \\ 0 & 3 & -3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$r(A) = 2$
$n = 4$

$r(A) = 2 < n = 4$
It has infinite Solution

$$\begin{bmatrix} 1 & -2 & 1 & -1 \\ 0 & 3 & -3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$x - 2y + z - w = 0$ Let, $w = c_1$
$3y - 3z + 4w = 0$ $z = c_2$

~~$3y = 0$~~

$$3y - 3c_2 + 4c_1 = 0$$

$$y = \frac{3c_2 - 4c_1}{3}$$

$$x - 2\left(\frac{3c_2 - 4c_1}{3}\right) + c_2 - c_1 = 0$$

$$3x - 2[3c_2 - 4c_1] + 3c_2 - 3c_1 = 0$$

$$3x = 2[3c_2 - 4c_1] - 3c_2 + 3c_1$$

$$x = \frac{6c_2 - 8c_1 - 3c_2 + 3c_1}{3}$$

$$x = \cancel{6c_2 - 5c_1 - 3c_2} \quad \frac{3c_2 - 5c_1}{3}$$

$$x = \frac{1}{3}[3c_2 - 5c_1]$$

## Eigen Values and Eigen Vectors

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & a_{mn} \\ a_{m1} & a_{m2} & a_{m3} & \cdots & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

$$AX = Y$$

$$AX = \lambda X$$

$$AX = \lambda X$$

$$AX - \lambda X = 0$$

$$(A - \lambda)X = 0$$

$$(A - \lambda I)X = 0$$

eigen Vector.

Eigen Values

Find eigen Values of

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

~~$|A - \lambda I|$~~ $A - \lambda I = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$= \begin{bmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{bmatrix}$

$|A - \lambda I|$

$= \begin{vmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{vmatrix} \Rightarrow (5-\lambda)(2-\lambda) - (1 \times 4) = 0$

$= 10 - 5\lambda - 2\lambda + \lambda^2 - 4 = 0$

$= 6 - 7\lambda + \lambda^2 = 0$

$\lambda^2 - 7\lambda + 6 = 0$

→ Characteristic equation

$\lambda^2 - 6\lambda - \lambda + 6 = 0$

$\lambda(\lambda - 6) - 1(\lambda - 6) = 0$

$\lambda = 1$ & $\lambda = 6$

$\Rightarrow$ $A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \Rightarrow \begin{bmatrix} 8-\lambda & -4 \\ 2 & 2-\lambda \end{bmatrix}$

$$|A - \lambda I| = \begin{vmatrix} 8-\lambda & -4 \\ 2 & 2-\lambda \end{vmatrix}$$

$$= (8-\lambda)(2-\lambda) - (-8)$$

$$16 - 8\lambda - 2\lambda + \lambda^2 + 8 = 0$$

$$24 - 10\lambda + \lambda^2 = 0$$

$$\lambda^2 - 10\lambda + 24 = 0$$

$$\lambda^2 - 6\lambda - 4\lambda + 24 = 0$$

$$\lambda(\lambda - 6) - 4(\lambda - 6) = 0$$

$$\boxed{\lambda = 6 \;\&\; \lambda = 4}$$

## Eigen Values and Eigen Vectors for three

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix}$$

$$2-\lambda \begin{vmatrix} 3-\lambda & 1 \\ 2 & 2-\lambda \end{vmatrix} - 2 \begin{vmatrix} 1 & 1 \\ 1 & 2-\lambda \end{vmatrix} + 1 \begin{vmatrix} 1 & 3-\lambda \\ 1 & 2 \end{vmatrix}$$

$\Rightarrow 2-\lambda[(3-\lambda)(2-\lambda)-2] - 2[2-\lambda-1] + 1[2-(3-\lambda)]$

$\Rightarrow 2-\lambda[6-3\lambda-2\lambda+\lambda^2-2] \; -2[1-\lambda] + 1[2-3+\lambda]$

$= 2-\lambda[4-5\lambda+\lambda^2] -2 + 2\lambda - 1 + \lambda$

$= 8 - \underline{10\lambda} + 2\lambda^2 - 4\lambda + 5\lambda^2 - \lambda^3 + 3\lambda - 3$

$= 5 - 11\lambda + 7\lambda^2 - \lambda^3 = 0$

$= \lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$

$\lambda - 1 = 0$

$\lambda - 1 = 0$

$\lambda = 1$

$1 - 7 + 11 - 5 = 0$

$12 - 12 = 0$

$\lambda - 1 ) \; \lambda^3 - 7\lambda^2 + 11\lambda - 5 \; ( \lambda^2 - 6\lambda + 5$

$\lambda^3 - \lambda^2$

$- \quad +$

$-6\lambda^2 + 11\lambda - 5$

$-6\lambda^2 + 6\lambda$

$+ \quad -$

$5\lambda - 5$

$5\lambda - 5$

$0$

$(\lambda-1)(\lambda^2-6\lambda+5) = 0$

$(\lambda-1)(\lambda^2-5\lambda-\lambda+5) = 0$

$(\lambda-1)[\lambda(\lambda-5)-1(\lambda-5)] = 0$

$$\boxed{\lambda = 1, \; \lambda = 5, \; \lambda = 1}$$