Volume Integrated.

Let $\vec{F}$ be vector point function and volume V enclosed by a closed surface.

The volume integral $= \iiint_V F\,dv$

If $\vec{F} = 2z\hat{i} - x\hat{j} + y\hat{k}$,

Evaluate $\iiint_V \vec{F}\,dv$ where V is the region bounded by the surface

$x=0,\ y=0,\ x=2,\ y=4,\ z=x^2,\ z=2$

Solution $dv = dx\,dy\,dz$

$$\iiint_V \vec{F}\cdot dv = \int_0^2\int_0^4\int_{x^2}^2 (2z\hat{i} - x\hat{j} + y\hat{k})\,dx\,dy\,dz$$

$$\int_0^2\int_0^4 2\cdot\left[\frac{z^2}{2}\right]_0^2\hat{i} - \hat{j}\,x[z]_0^2 + \hat{k}\,y[z]_0^2\ dy\,dz$$

$$\int_0^2\int_0^4 2\cdot 2\hat{i} - 2x\hat{j} + 2y\hat{k}\ dy\,dz$$

$$\int_0^2 4[y]_0^4\hat{i} - 2x[y]_0^4\hat{j} + 2\hat{k}\left[\frac{y^2}{2}\right]_0^4\ dz$$

$$\int_0^2 16\hat{i} - 8x\hat{j} + 16\hat{k}\ dz$$

$$\cancel{16[x]_0^2 \hat{i} - 8\cdot\left[\frac{x^2}{2}\right]_0^2 \hat{j} + 16[z]_0^2 \hat{k}}$$

$$\cancel{32\hat{i} - 16\hat{j} + 32\hat{k}}$$

$$\int_0^2\int_0^4\int_{x^2}^2 (2z\hat{i} - x\hat{j} + y\hat{k})\, dx\, dy\, dz$$

$$\int_0^2\int_0^4 2\cdot\left[\frac{z^2}{2}\right]_{x^2}^2 \hat{i} - x[z]_{x^2}^2 \hat{j} + y[z]_{x^2}^2 \hat{k}\, dx\, dy$$

$$\int_0^2\int_0^4 2\left[2 - \frac{x^4}{2}\right]\hat{i} - x\left[2 - x^2\right]\hat{j} + y\left[2 - x^2\right]\hat{k}\, dx\, y$$

$$\int_0^2\int_0^4 (4 - x^4)\hat{i} - (2x + x^3)\hat{j} + (2y - x^2 y)\hat{k}\, dx\, dy$$

$$\int_0^2 \left[4[y]_0^4 - x^4\cdot[y]_0^4\right]\hat{i} - \left[2x\cdot[y]_0^4 + x^3[y]_0^4\right]\hat{j} +$$

$$\left[2\left[\frac{y^2}{2}\right]_0^4 - x^2\left[\frac{y^2}{2}\right]_0^4 \hat{k}\right] dx$$

$$\int_0^2 (16 - 4x^4)\hat{i} - 8x\hat{j} + 4x^3\hat{j} + (16 - 8x^2)\hat{k}\, dx$$

$$\left[16[x]_0^2 - 4\left[\frac{x^5}{5}\right]_0^2\right]\hat{i} - \left[8\cdot\left[\frac{x^2}{2}\right]_0^2 + 4\left[\frac{x^4}{4}\right]_0^2\right]\hat{j} +$$

$$\left[16[x]_0^2 - 8\cdot\left[\frac{x^3}{3}\right]_0^2\right]\hat{k}$$

$$\left(32 - 4\cdot\frac{32}{5}\right)\hat{i} - (16 - 16)\hat{j} + \left(32 - \frac{64}{3}\right)\hat{k}$$

$$\left(32 - 4\cdot\frac{32}{5}\right)\hat{i} + \left(32 - \frac{64}{3}\right)\hat{k}$$

$$\left(32 - \frac{128}{5}\right)\hat{i} + \left(\frac{96 - 64}{3}\right)\hat{k}$$

$$\left(\frac{160 - 128}{5}\right)\hat{i} + \left(\frac{96-64}{3}\right)\hat{k}$$

$$\frac{32}{5}\hat{i} + \frac{32}{3}\hat{k}$$

$$\cancel{\frac{96\hat{i} + 32\hat{k}}{15}} \qquad \frac{96\hat{i} + 160\hat{k}}{15}$$

$$\frac{32}{15}\left[3\hat{i} + 5\hat{k}\right]$$ Ans