Engineering Data Analysis with Matlab

Anke Scherb

Engineering Risk Analysis Group
Technische Universität München

Phone: 089 289 23013
E-mail: anke.scherb@tum.de

Engineering Data Analysis with Matlab

Today’s lecture

- Matlab functions
- Random variables
- Distribution fitting

Matlab – Functions

- M-files that provide output as a function of input (e.g. data variables)
- Called by other Matlab programs or from command window
- Types:
  - Built-in functions, e.g. sin, cos, log, …
  - User-defined functions

Matlab – Functions

- The definition of new functions must be stored in a file with name *function_name.m*
- Function syntax, first executable line of the function:

```
function [output1 output2 …] = function_name(input1, input2, …)
Matlab statements
```

- Declare a function named by “function_name” that accepts inputs (input1, input2,…) and returns outputs (output1, output2, …)

Matlab – Functions

Example: Function *fun* stored in file *fun.m*

```
function y = fun(x)
y = x^2 - 1;
```

Call function in main program

```
…
b = 1.5;
f_b = fun(b);
…
```

Example: fun.m

Matlab – Functions

Function with two outputs - Example: Function *stat* stored in file *stat.m*

```
function [mean, stdev] = stat(data)
mean = sum(data)/length(data);
var = sum((data-mean).^2)/(length(data) - 1);
stdev = sqrt(var);
```

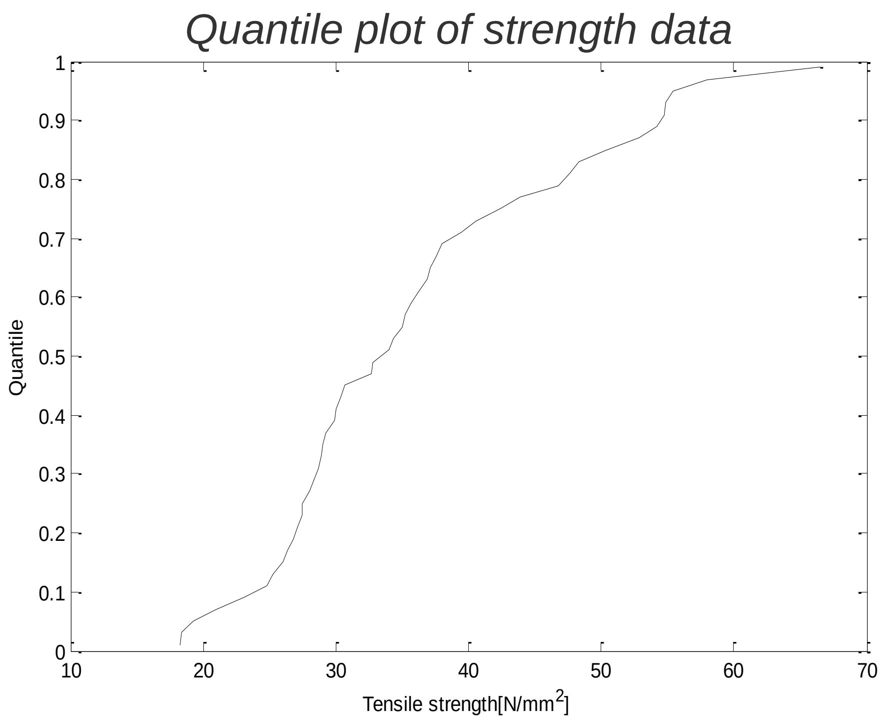
Note:

- All variables within the body of a function are local variables
- A function does not necessarily need to provide an output, e.g. plotting functions

Example: stat.m

Quantile plot – Plots the values below which a certain fraction of the samples fall

- Sort samples in ascending order $x_1, x_2, \ldots, x_n$
- Plot the pairs x_i and $(i - 0.5)/n$



Example: [qtlplot.m]

Matlab – Function handles

- Callable association to a Matlab function; a MATLAB data type that stores an association to a function.

```
function_handle = @function_name
```

Function handles can be used for passing a function to another function

```
h = @sin;
h(pi)
```

Matlab – Anonymous Functions

- Simple functions that are not saved in separate files but is associated with a variable whose data type is function_handle

```
function_handle = @(input1, input2, …) expression
```

Example:

```
fun = @(x) x^2 - 1;

fzero(fun,0.5)
```

Note: Matlab build-in functions:

- **fzero (fun, x0)** finds a root of fun near x0

Example: fzero.m

Matlab – Structures

- Collection of data of different types that represent a single idea or “object”

Example:

```
data.name = ‘strength’;
data.values = [37.1 36.9 40.5 46.8 37.6];
data.mean = mean(data.values);
data.std = std(data.values);
```

- Note: Structures can also be created using the `struct()` function

Example: struct_.m

Random variables

Mathematical tool for modeling uncertain quantities

- Random variables map possible outcomes of an experiment to the real numbers
- A random variable is represented by a capital letter, e.g. X
- An outcome of a random variable is represented by a lower case letter, e.g. x

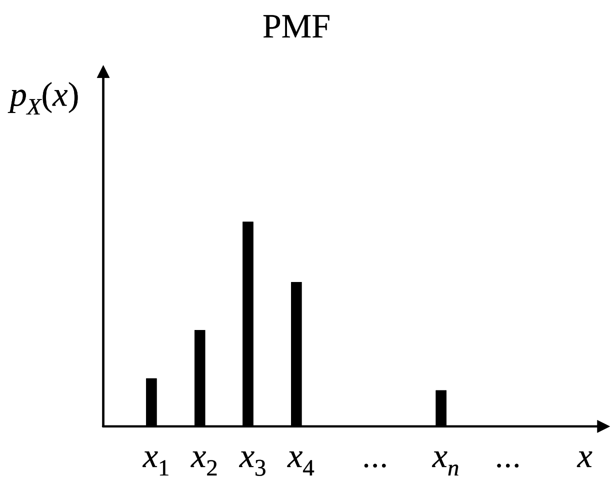
Discrete random variables

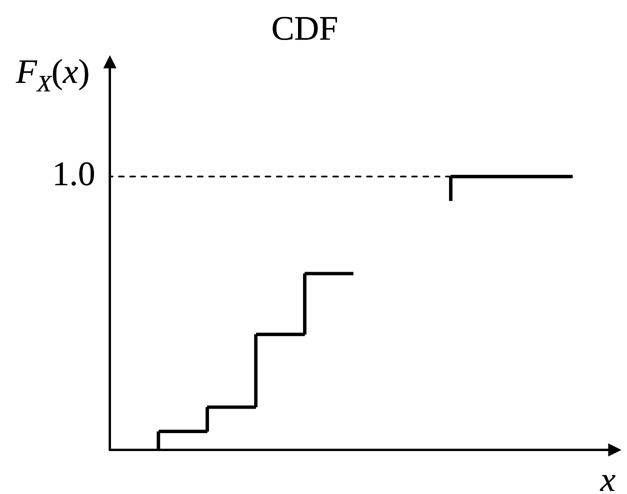
Random variables with discrete number of outcomes

Probability Mass Function (PMF) – Function that describes the probability that a discrete random variable X takes a specific value x

$$p_X(x) = \Pr(X = x)$$

CDF of discrete random variable: $F_X(x) = \sum_{x_i \leq x} p_x(x_i)$





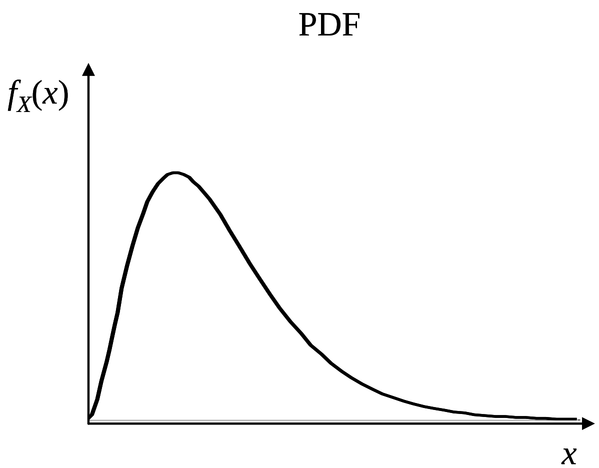
Continuous random variables

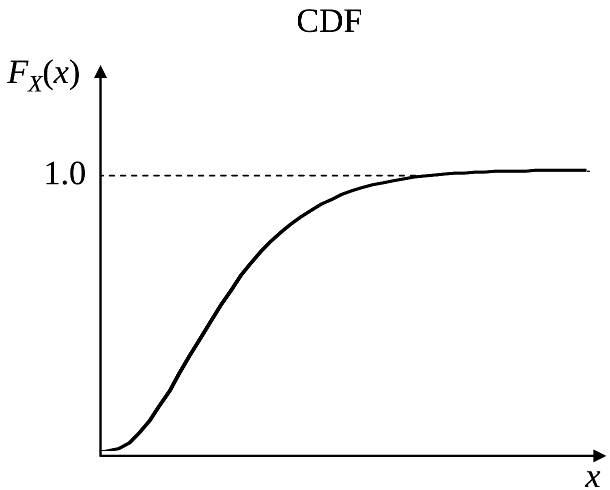
Random variables which can take any value within one or more intervals

Probability Density Function (PDF) – Function whose integral over an interval gives the probability that X takes a value within this interval

$$f_X(x)dx = \Pr(x < X \leq x + dx)$$

CDF of continuous random variable: $F_X(x) = \int_{-\infty}^{x} f_x(t)dt$





Matlab – Random variables

Probability Density Function (PDF) – Function whose integral over an interval gives the probability that X takes a value within this interval

$$f_X(x)dx = \Pr(x < X \leq x + dx)$$

```
pdf('name',x,par1,par2,…)
```

```
namepdf(x,par1,par2,…)
```

Note: If **`'name'`** is a distribution that describes a discrete random variable then **`pdf`** returns the PMF

Matlab – Random variables

Cumulative distribution function (CDF) – Function that describes the probability that a random variable X is smaller than or equal to an outcome x

$$F_X(x) = \Pr(X \leq x)$$

```
cdf('name',x,par1,par2,…)
```

```
namecdf(x,par1,par2,…)
```

Notes:

- The CDF is a non-decreasing function
- The CDF has limits $F_X(-\infty) = 0,\ F_X(\infty) = 1$

Description of random variables

Mean value

- Discrete random variable

$$\mu_X = \mathrm{E}[X] = \sum_{\text{all } x_i} x_i p_X(x_i)$$

- Continuous random variable

$$\mu_X = \mathrm{E}[X] = \int_{-\infty}^{\infty} x \cdot f_X(x) dx$$

Variance

$$\mathrm{Var}[X] = \mathrm{E}[(X - \mu_X)^2]$$

- Discrete random variable

$$\mathrm{Var}[X] = \sum_{\text{all } x_i} (x_i - \mu_X)^2 p_X(x_i)$$

- Continuous random variable

$$\mathrm{Var}[X] = \int_{-\infty}^{\infty} (x - \mu_X)^2 f_X(x) dx$$

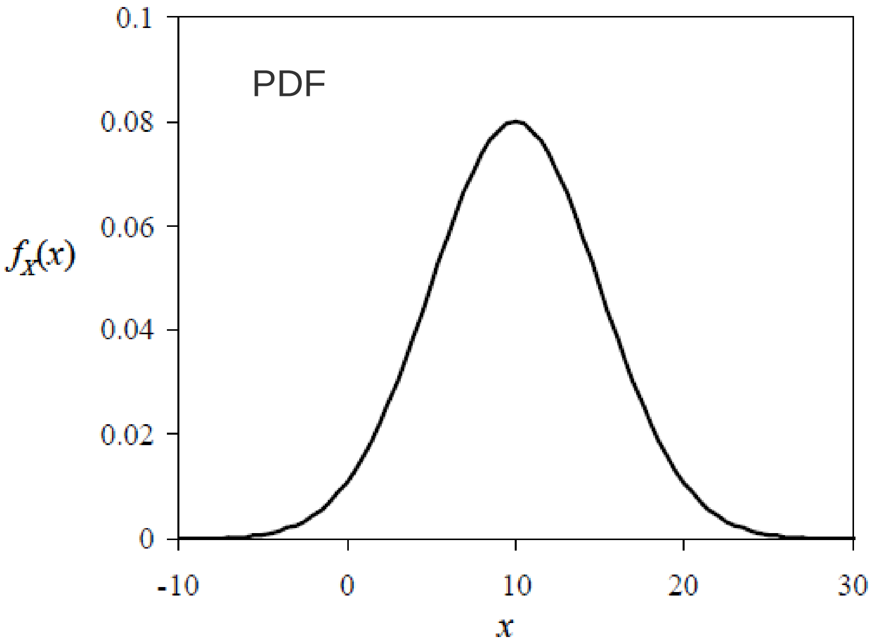
Standard deviation

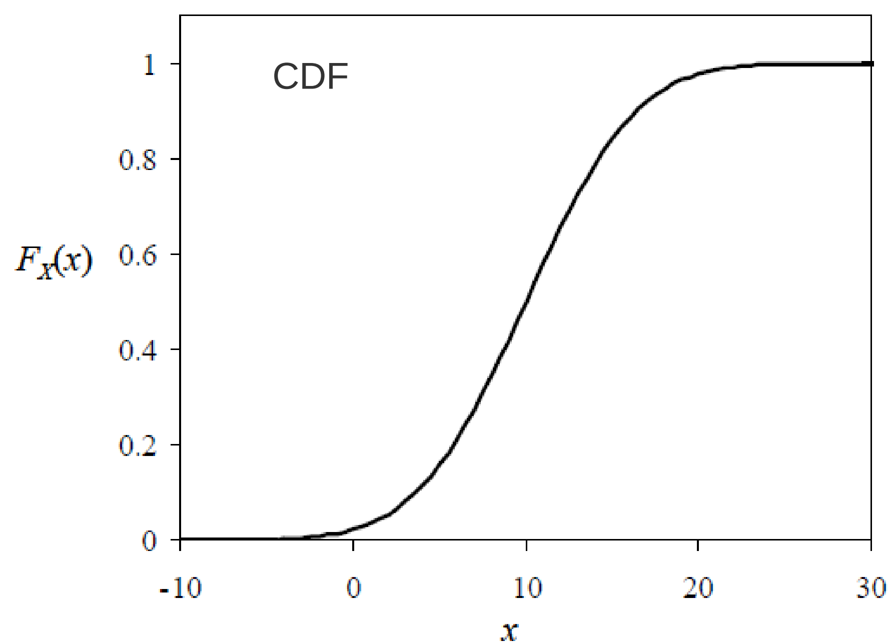
$$\sigma_X = \sqrt{\mathrm{Var}[X]}$$

Distribution types - Examples

Normal (Gaussian) distribution

- Defined by two parameters, the mean and the standard deviation: $X \sim N(\mu, \sigma)$
- PDF is symmetric around the mean value





$$f_X(x) = \frac{1}{\sigma}\varphi\left(\frac{x-\mu}{\sigma}\right) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] \qquad x \in (-\infty, +\infty)$$

$$\Phi(u) = \int_{-\infty}^{u} \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{z^2}{2}\right]dz$$

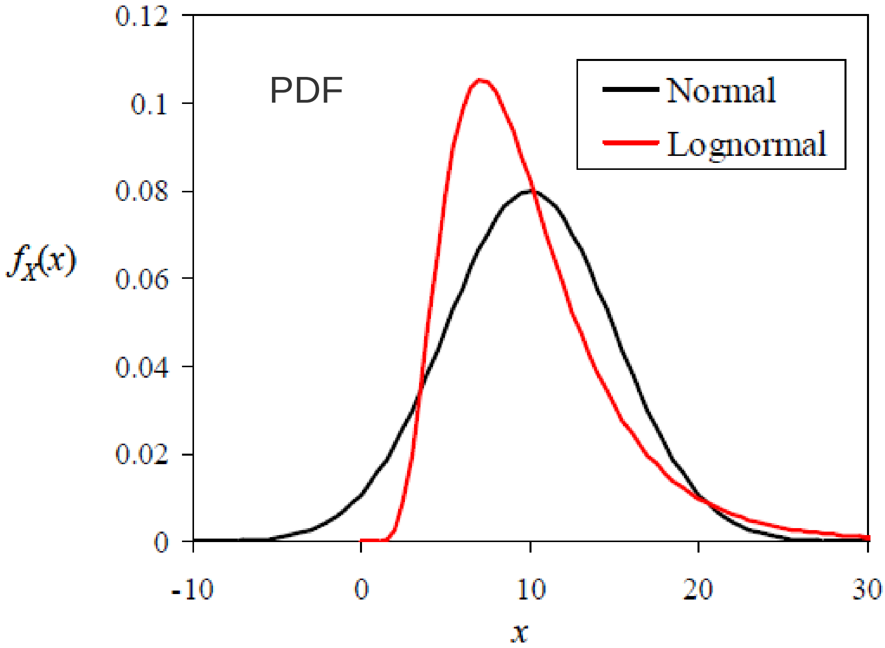
Distribution types - Examples

Lognormal distribution

- The logarithm of a lognormal random variable is normal
- Defined by two parameters
- Asymmetric distribution with strictly positive outcomes

$$f_X(x) = \frac{1}{\zeta x}\varphi\left(\frac{\ln x - \lambda}{\zeta}\right) \qquad x \in (0, +\infty)$$

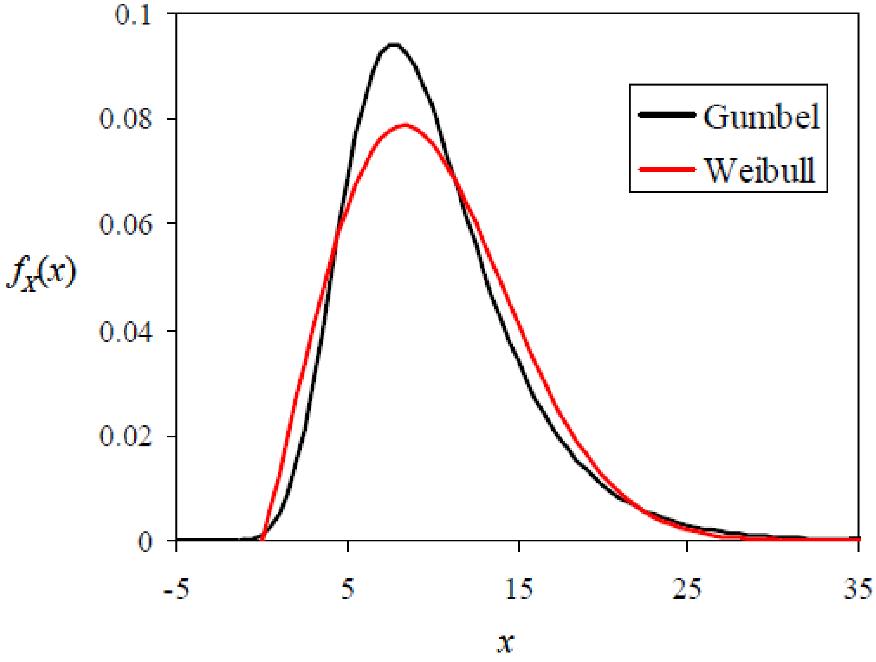
$$F_X(x) = \Phi\left(\frac{\ln x - \lambda}{\zeta}\right)$$



Distribution types - Examples

Extreme value distributions – Model maxima or minima of random variables

- Gumbel distribution: Models maxima of distributions with exponential tail
- Weibull distribution: Models minima of distributions with zero lower bound



Matlab - Distribution types

Distributions available in Matlab

`‘norm’ or ‘normal’`	Normal distribution
`‘logn’ or ‘lognormal’`	Lognormal distribution
`‘beta’ or ‘Beta’`	Beta distribution
`‘ev’ or ‘Extreme Value’`	Gumbel or Extreme value distribution
`‘wbl’ or ‘Weibull’`	Weibull distribution
…	…

Matlab - Distribution types

Distribution statistics

```
namestat(par1,par2,…)
```

Computes the mean and variance of the random variable with distribution **`name`** and parameters **`par1, par2, …`**

Matlab - Distribution objects

- Construct a distribution object using **`makedist`**
- Collect parameters and data sets related to a particular distribution type
- The CDF/PDF and statistics can be evaluated using available methods (**`cdf,pdf,mean,var,std,`**…)

Example:

```
pn = makedist('norm','mu',10,'sigma',5);
cdf(pn,x)
pdf(pn,x)
mean(pn)
var(pn)
```

Example: distrib.m

Distribution fitting

- Find the suitable distribution and its parameters to model an uncertain quantity for which experimental data (observations) are available
- Aim: predict the probability or forecast the frequency of occurrence of the value of the phenomenon; find the distribution type that fits the data best.

Method of moments

- Compute the sample statistics (sample mean and sample standard deviation)

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}$$

- **Idea: Find the distribution parameters by requiring that the mean and standard deviation of the distribution are equal to $\bar{x}$ and s**

Distribution fitting

Method of moments – Example

- Fitting of Timber tensile strength data to the **Weibull** distribution

Weibull CDF: $F_X(x) = 1 - \exp\left[-\left(\frac{x}{u}\right)^k\right], \qquad x \geq 0$

Mean: $\mu = u \cdot \Gamma\left(1 + \frac{1}{k}\right)$

Standard deviation: $\sigma = u \cdot \sqrt{\Gamma\left(1+\frac{2}{k}\right) - \Gamma^2\left(1+\frac{1}{k}\right)}$

$$\rightarrow \frac{\sigma}{\mu} = \frac{\sqrt{\Gamma\left(1+\frac{2}{k}\right) - \Gamma^2\left(1+\frac{1}{k}\right)}}{\Gamma\left(1+\frac{1}{k}\right)}$$

- Γ: Gamma function => Matlab built-in function ***gamma()***

Distribution fitting

Method of moments – Example

- Fitting of Timber tensile strength data to the **Weibull** distribution

Weibull CDF: $$F_X(x) = 1 - \exp\left[-\left(\frac{x}{u}\right)^k\right], \qquad x \geq 0$$

$k =$ … use Matlab built-in functions fzero and gamma to find parameter k

$$u = \frac{\mu}{\Gamma(1 + \frac{1}{k})}$$

Example: wblfit_.m

Distribution fitting

Method of moments – Example

- Fitting of Timber tensile strength data to the **Lognormal** distribution

Lognormal CDF:

$$F_X\left(x\right)=\Phi\left(\frac{\ln x-\lambda}{\zeta}\right)$$

Standard deviation:

$$\zeta=\sqrt{\ln\left(\frac{\sigma^2}{\mu^2}+1\right)}$$

Mean:

$$\lambda=\ln\mu-\frac{\zeta^2}{2}$$

Matlab - Distribution fitting

Maximum likelihood estimation – Find parameters that maximize the likelihood of the observations

```
namefit(data)
```

Estimates the parameters of the distribution **`name`** associated with **`data`**

Construct a distribution object by fitting to data using **`fitdist`**

Example:

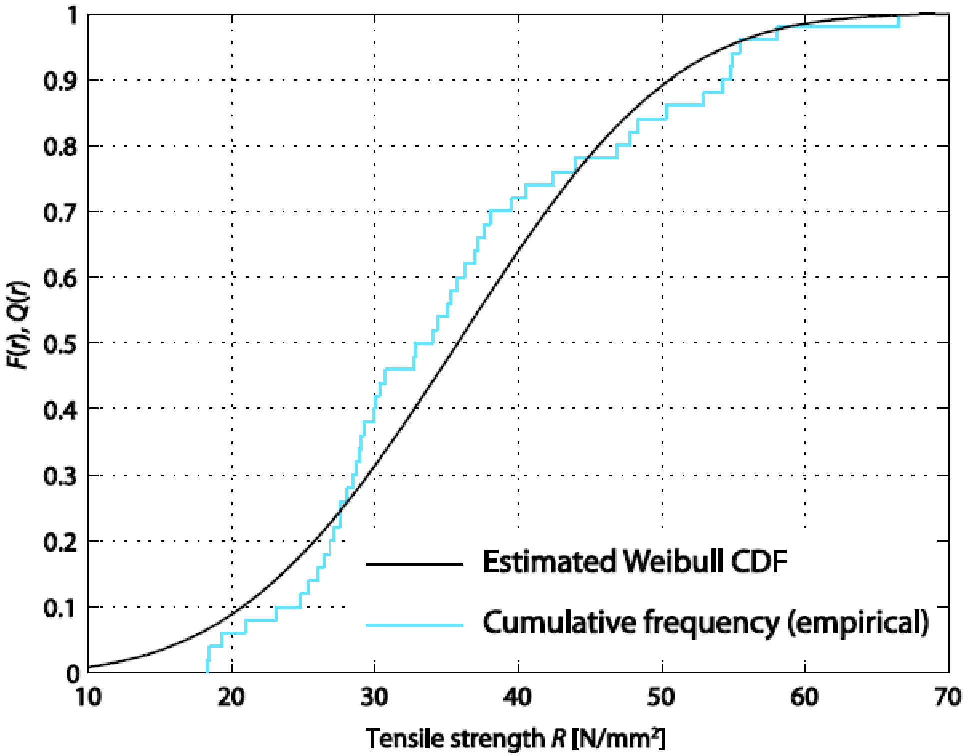
```
pn = fitdist(data,'norm');
```

Example: wblfit_.m

Graphical methods for distribution fitting

- Plot cumulative frequency diagram (empirical CDF) against CDF of fitted distribution

Tensile strength of timber material



Graphical methods for distribution fitting

Probability plot

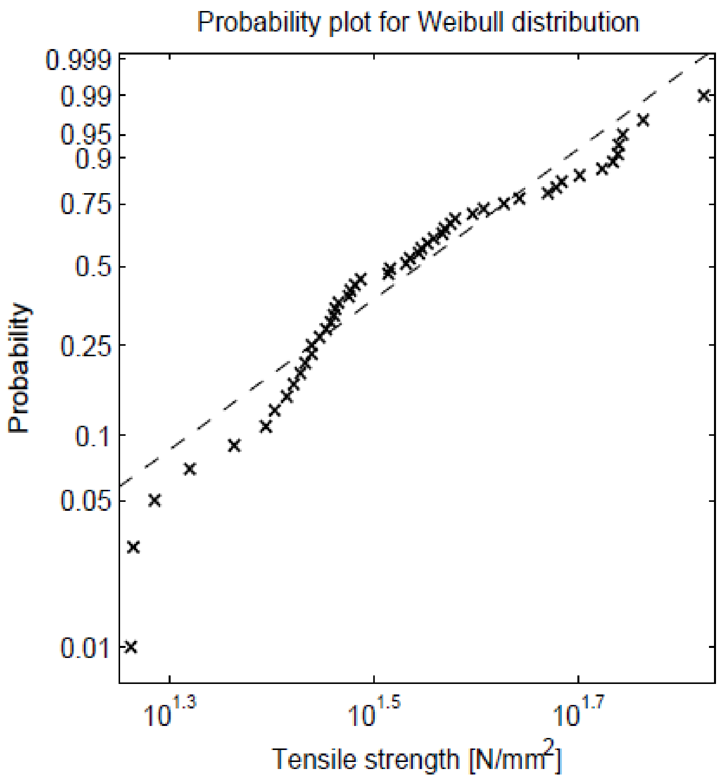
- A quantile plot with scaled axes
- The scaling is selected so that if the data follow exactly a distribution type then the plot is a straight line

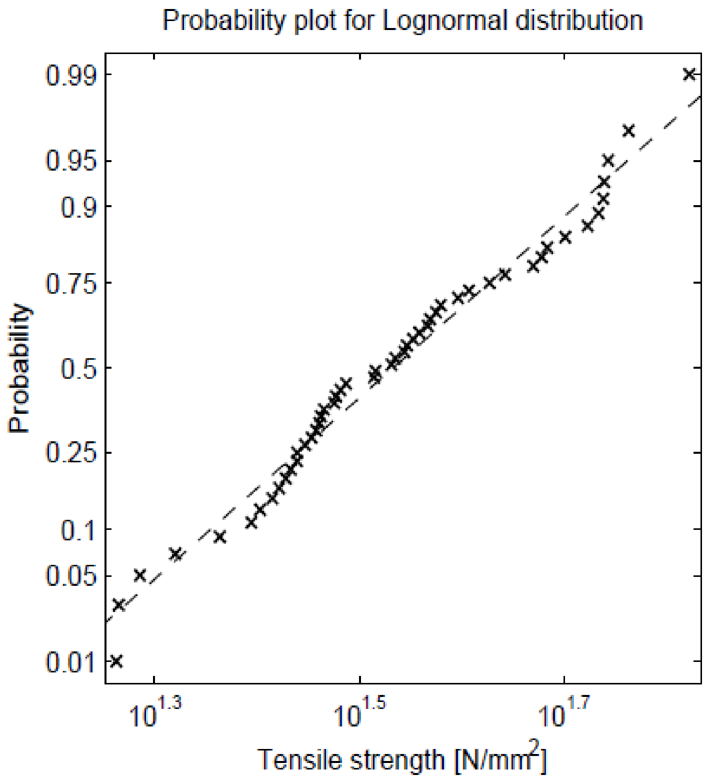
```
probplot('name',data)
```

Graphical methods for distribution fitting

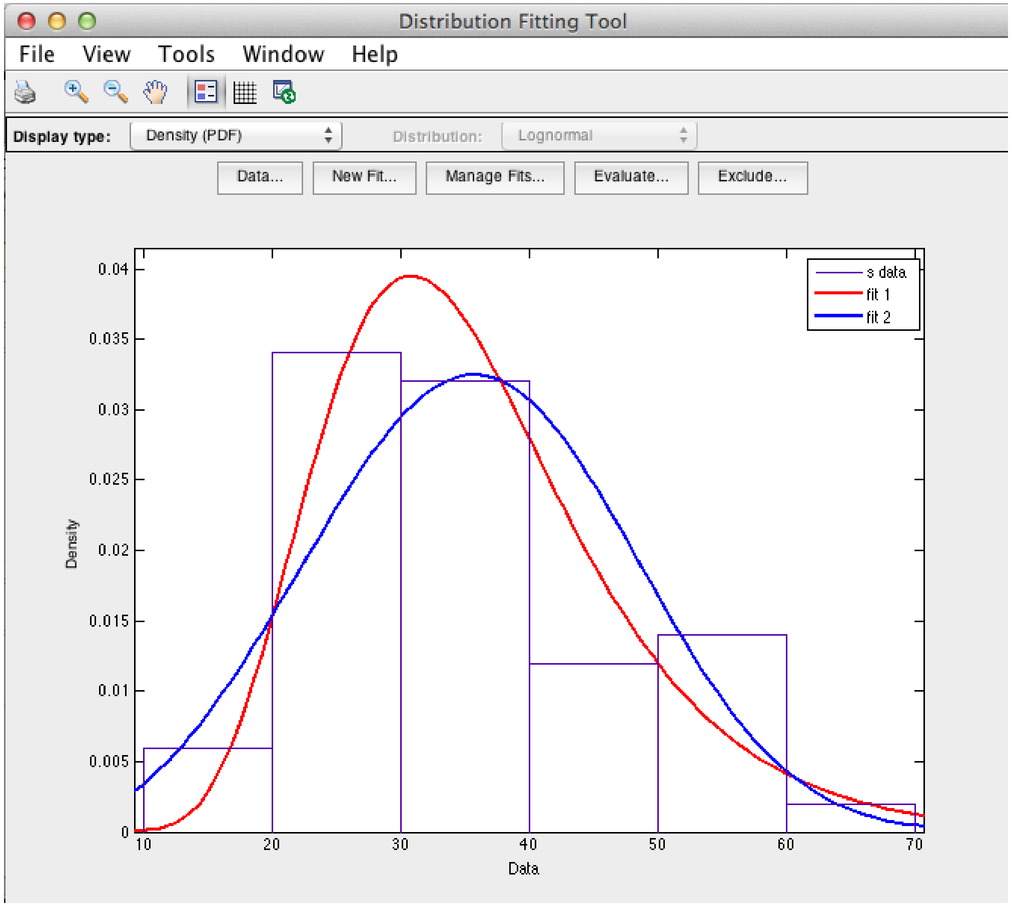
Probability plot

Tensile strength of timber material





Distribution fitting tool



Engineering Data Analysis with Matlab

Group organization for project teams

Please form groups of 2 to 3 students for your project work

⇒ Send an e-mail to anke.scherb@tum.de or tell the tutors during the tutorial **until May 9**

Project work

- Start May 16
- Deadline June 27
- Submit your team's report and Matlab codes

Engineering Data Analysis with Matlab

Project report:

- Structure your report, e.g.: introduction, approach/methods, results, conclusions
- Justify and reinforce your answers and thoughts
- Layout the report and stick to your layout chosen
- Use tables and figures to present your results
- Label all tables and figures in the report
- Always give units and label axis
- Cite correctly and consistently when using literature / internet sources