

# Path Planning for Multiple Robots with Variable Formation Using Probabilistic Roadmap

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## Abstract

The objective of the project is to implement and analyze Probabilistic Road Map (PRM) algorithm for motion planning of multiple robots with variable formation. By variable formation we mean that the robots can change the scale and orientation of the formation while maintaining a given shape. The system of robots hence have five degrees of freedom, i.e., three for translation, scale, and orientation about the  $Z$ -axis. The PRM algorithm is implemented as it can handle high dimensional configuration space. Additional connectivity, expansion, and smoothing techniques are implemented to improve upon the solutions generated. Examples of different numbers of robots in various shapes are considered to analyze the efficacy of the algorithm. The algorithm is implemented using Python 2.7 and Klamp't motion planning framework.

## I. INTRODUCTION

Teams of robots often maintain a desired shape while performing tasks such as exploration, coverage, and surveillance [1]. These formations can have the flexibility in the scale and orientation of the formation while maintaining the given shape. These additional degrees of freedom allows us to navigate through narrow passages in the environment (see Fig. 1). In this project we study an analyze the Probabilistic Roadmap path planning method to perform path planning in a static environment with obstacles.

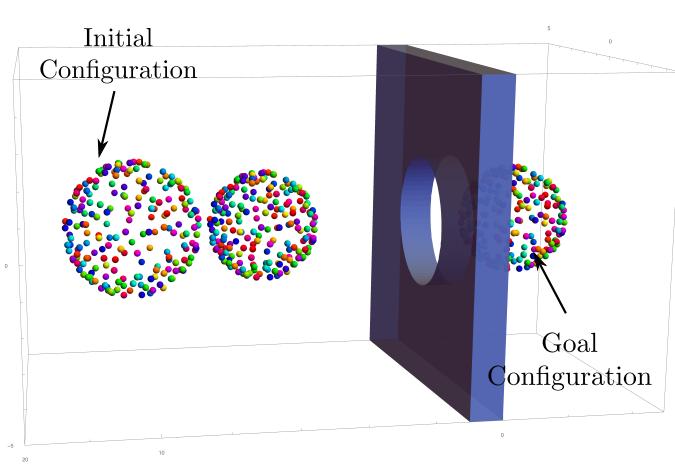


Fig. 1. An example of a scenario where robots need to change their scale in order to pass through a narrow opening. We need to ensure that the scale parameter is computed such that the robots do not collide with themselves.

Several efficient path planning techniques exist for low dimensional configuration space. Roadmap methods in  $\mathbb{R}^2$  include visibility graph, which can give optimal paths, Voronoi roadmap which keep the path far away from the obstacles (along the medial axis). Cell decomposition methods divide the free

configuration space of the robots into cells. Thereafter, a connectivity graph is created to with the cells. Given query points, a path is found in the connectivity graph. These methods can solve 2D configuration spaces efficiently. However, the as the dimension increases, the number of cells increases exponentially and both the space and time complexity increases. Potential field methods is another class of motion planning technique wherein potential functions dictate the motion of the robots. The goal position has an attractive potential and the obstacles (potentially other robots as well) have repulsive potential. The forces on the robot are then computed based on the potential function. The method suffers from getting stuck at local minima and designing potential navigation functions may not be trivial.

All the algorithms discussed above do not perform well when the dimension of the configuration space increases. In general the motion planning problem is PSPACE-hard. Owing to this difficult, in the past decade there have been significant advances in sampling based techniques. One such method is Probabilistic Roadmap (PRM) [2] composed of a learning phase and a query phase. The learning phase builds a roadmap and in the query phase shortest path in computed in the roadmap. The PRM has been shown to be effective in various environments with high dimensional configurations of robots. There have been several improvements and variants to PRM. One notable improvement is PRM\* [3], which gives asymptotically optimal solutions. Other variants are visibility-PRM, Obstacle based-PRM etc.

In this document, we first describe the problem in Section II. The PRM method is described in Section III.

## II. PROBLEM DESCRIPTION

Consider a team of  $n$  robots. These robots are arranged in a desired shape given by  $\mathbf{S} = (\mathbf{s}_j^\top), j = 1, \dots, n$  with respect to a local coordinate system attached to the system of robots. The configuration of the system of robots is given by a five dimensional vector  $\mathbf{q} = (x, y, z, \alpha, \theta)^\top$ , where  $\mathbf{q}_t = (x, y, z)^\top$  defines the location in the  $\mathbb{R}^3$  space,  $\alpha$  is the scale parameter for the robots, and  $\theta$  is the orientation of the formation about the  $Z$ -axis. The location of the individual robots in the world coordinate system can now be written as  $\mathbf{p}_j = \mathbf{q}_t + \mathbf{R}\alpha\mathbf{s}_j$ , where  $\mathbf{R}$  is the rotation matrix corresponding to a rotation by an angle  $\theta$  about the  $Z$ -axis. Given an initial and goal configuration, the task is then to compute an obstacle-free path in the environment.

We assume the following:

- 1) The robots are holonomic, i.e., they can move in any direction.
- 2) The position of the individual robots, and thereby the position and orientation of the robotic system can be obtained exactly.
- 3) The environment in 2D or 3D is static and known, i.e., the location of the obstacles are provided exactly.
- 4) There is no error in the motion of the robots, i.e., the robots move exactly as commanded.

## III. PROBABILISTIC ROADMAP

We use Probabilistic Roadmap (PRM), a sampling based technique, for creating a *roadmap* in the configuration space of the robot. The roadmap here refers to an undirected graph in the configuration space where in the nodes represent sampled configurations and the edges represent collision-free paths which the system of robots can take. The weight of the edges represent the cost of travelling through the two nodes of the edge.

The PRM and its improvement PRM\* have the following characteristics which make it suitable for our problem:

- 1) Probabilistically complete: probability of finding a solution (if one exist) approaches to one as the running time approaches to infinity. The algorithm will generate a roadmap such that path between any two configuration points can be determined if it exists, given sufficient time for running the algorithm.

- 2) Asymptotically optimal: the probability of find the optimal path (if one exist) approaches to one as the running time approaches infinity. The algorithm will find the most optimal path for the query if sufficient time is given for running building the roadmap.
- 3) The PRM method can handle high-dimensional configuration space. In our problem we have a five dimensional configuration space.
- 4) Once the roadmap is created, the multiple queries can be performed on the same environment, which is computationally very efficient.

There are few caveats to the PRM method. The probabilistically complete and asymptotically optimal features require building an large sized graph with algorithm running for a long time. This also creates slower query decisions. However, in practice fewer samples points may be enough for generating good paths.

#### A. Overall Algorithm

The PRM method is composed of a *learning* phase and a *query* phase. In the learning phase a roadmap is built. The roadmap is an undirected graph  $G = (V, E)$ , where  $V$  is the set of sampled vertices and  $E$  is the set of edges representing collision-free paths. The query phase of the method is used to solve individual path planning problems for the same environment in which the roadmap was built. Given a start/initial configuration,  $\mathbf{q}_i$ , and a goal configuration,  $\mathbf{q}_g$ , the query phase first connects these nodes to the roadmap and thereafter finds the shortest path in the roadmap between the connected configurations.

We shall now describe the two phases and their components specific to our problem.

#### B. Distance Metric

The PRM method requires a distance metric  $d(\mathbf{q}_1, \mathbf{q}_2)$  to be defined for two configurations  $\mathbf{q}_1$  and  $\mathbf{q}_2$ . The metric is straight forward for Euclidean space and is given by the Euclidean distance. However, we have a five-dimensional non-Euclidean configuration space. Moreover, there are multiple robots having different motion. We consider the sum of the distance travelled by the robots as the distance metric.

1) *Traslation of Formation:* First let us consider the translation of the system of robots in formation. The translation distance metric  $d_t(\mathbf{q}_1, \mathbf{q}_2)$  is then given by:

$$d_t(\mathbf{q}_1, \mathbf{q}_2) = \sum_{i=1}^n \|\mathbf{q}_{t1} - \mathbf{q}_{t2}\|_2 = n\|\mathbf{q}_{t1} - \mathbf{q}_{t2}\|_2 \quad (1)$$

where,  $\mathbf{q}_{t1}$  and  $\mathbf{q}_{t2}$  represent the translation part of the configuration, i.e,  $(x, y, z)$ .

2) *Scaling:* When the formation changes the scale parameter  $\alpha$ , the motion of the individual robots is in the direction of the vector pointing to its position  $\mathbf{s}_j$  in local coordinate system of the formation. The scaling distance metric  $d_s(\mathbf{q}_1, \mathbf{q}_2)$  is then given by:

$$d_s(\mathbf{q}_1, \mathbf{q}_2) = \sum_{i=1}^n (\alpha_1 - \alpha_2) \|\mathbf{s}_i\|_2 = \mathbf{s}(\alpha_1 - \alpha_2), \text{ where } \mathbf{s} = \sum_{i=1}^n \|\mathbf{s}_i\|_2 \quad (2)$$

The constant  $\mathbf{s}$  can be precomputed.

3) *Rotation:* When the formation rotates by an angle  $\theta$  about the Z-axis, the motion of the individual robots is in a arc of radius given by the distance of its position  $\mathbf{s}_j$  in the shape in local coordinate system of the formation. The scaling distance metric  $d_\theta(\mathbf{q}_1, \mathbf{q}_2)$  is then given by:

$$d_\theta(\mathbf{q}_1, \mathbf{q}_2) = \sum_{i=1}^n \text{abs}(\theta_1 - \theta_2) \|\mathbf{s}_i\|_2 = \mathbf{s}|\theta_1 - \theta_2| \quad (3)$$

4) *Rotation with Scaling:* We first assume that the rate of rotation and rate of scaling are constants, which may not be equal. Furthermore the relative rates for the two parameters is assumed to be constant. This means that the two parameters reach their value in the goal configuration at the same time, i.e.,  $b = \frac{dr}{d\theta}$  is constant. Where  $dr$  is the rate of increase of radius  $r$  of individual robots.

The radius at any instant is given by  $r(\theta) = r_1 + b\theta$ . This gives us the following derivatives:

$$\begin{aligned} x &= dr(\theta) \cos \theta \\ y &= dr(\theta) \sin \theta \\ dx &= (b \cos \theta - \sin \theta r(\theta)) d\theta \\ dy &= (b \sin \theta + \cos \theta r(\theta)) d\theta \end{aligned} \tag{4}$$

The arc length  $l$  is then given by:

$$\begin{aligned} dl &= \sqrt{dx^2 + dy^2} \\ &= \sqrt{r(\theta)^2 + b^2} d\theta \\ l &= \int_{\theta_1}^{\theta_2} dl \end{aligned} \tag{5}$$

The above equation is difficult to integrate and involve hyperbolic functions, and hence is computationally expensive. Hence, an approximation is used by taking the average radius. The distance function  $d_{s\theta}(\mathbf{q}_1, \mathbf{q}_2)$  is given by:

$$\begin{aligned} d_{s\theta}(\mathbf{q}_1, \mathbf{q}_2) &= \sum_{i=1}^n \text{abs} \left( \frac{(r_{i1} + r_{i2})}{2} (\theta_1 - \theta_2) \right) \\ &= \sum_{i=1}^n \text{abs} \left( \|\mathbf{s}_i\| \frac{(\alpha_1 + \alpha_2)}{2} (\theta_1 - \theta_2) \right) \\ &= \frac{\mathbf{s}(\alpha_1 + \alpha_2) |\theta_1 - \theta_2|}{2} \end{aligned} \tag{6}$$

5) *Total Distance:* The total distance between two configurations  $d(\mathbf{q}_1, \mathbf{q}_2)$  is given below:

$$d(\mathbf{q}_1, \mathbf{q}_2) = d_t + d_s + d_{s\theta} \tag{7}$$

Note that the distance is now a function of number of robots. Weights can be added to each of the terms depending on the application.

### C. Local Path Planning

The PRM algorithm requires a local path planner which takes in two configurations and checks whether a path exists in free space. The local planner needs to be deterministic and computationally very efficient. We use a simple straight line motion between the two configurations. We then discretize the path into small steps and check for collisions at these steps. It is important that the number of steps be carefully chosen to ensure that no collision is missed by the planner.

### D. Learning Phase

The task of the learning phase is to build a roadmap. It is composed of two steps: (1) the construction step and (2) the expansion step. The construction step creates a reasonably connected graph whereas the expansion step takes in the result of the construction step and tries to improve connectivity by adding nodes to the neighborhood of low connectivity regions.

*1) The Construction Step:* The construction step starts with initializing an empty undirected graph  $G = (V, E)$ . A new configuration  $q_s$  is randomly sampled in the configuration space  $\mathcal{C}$  is checked if  $q_s$  is in the free configuration space  $\mathcal{C}_f$ . These sampled points are added to the roadmap  $G$ . Then the algorithm tries to connect  $q_s$  to atmost  $k$  nearest existing nodes in the graph which is within some predefined distance  $\text{maxDist}$ . It is ensured that  $q_s$  and potential nodes to which it is to be connected do not lie in the same connected component. This prevents cycles in the graph and reduces the number of edges. A sample roadmap is shown in Fig. 2 for one robot.

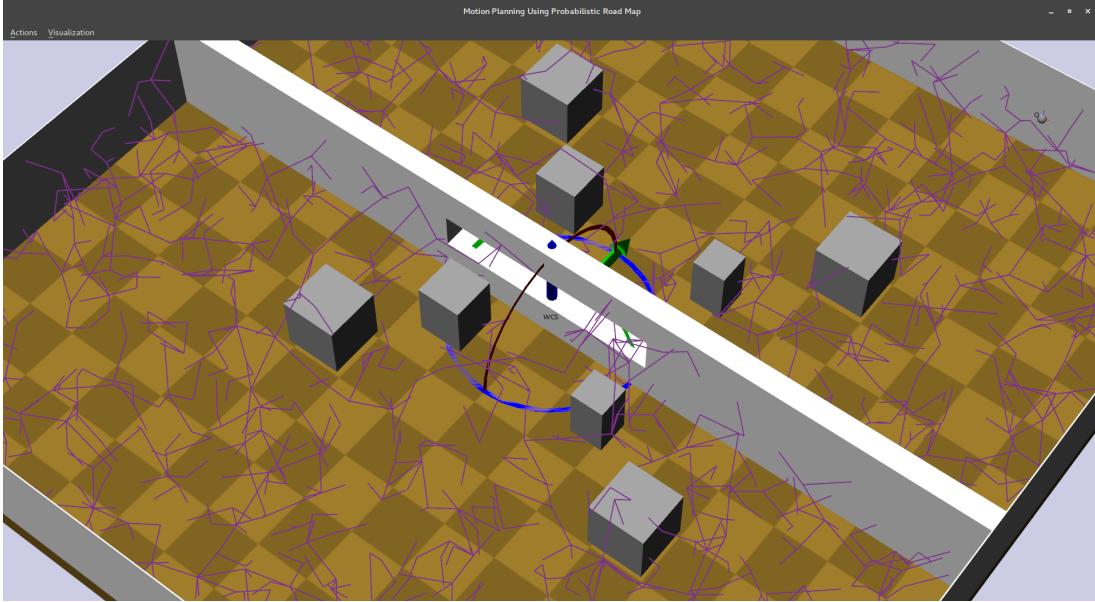


Fig. 2. Roadmap generated using PRM for one robot. The roadmap has 2000 nodes and 1980 edges. There are 5 connected components with more than one node. There are 19 nodes which remain disconnected.

*2) The Expansion Step:* In the expansion step, the connectivity of the graph is improved. This is done by locating connected components which are ‘difficult’ based on some heuristic. It can correspond to regions of narrow passages. The size of the connected component is used as a simple heuristic for the project. Once such components are identified, nodes at random are selected from these components. Then a random walk is implemented. The system of robots move in a random direction for some distance or until hit by an object. Then the direction is changed. The random walk is run for a specified duration. A new node is added corresponding to the random walk. The path taken by the system of robots need to be added to as it is obtained in a non-deterministic way.

#### E. Query Phase

In the query phase, an initial configuration  $q_i$  and a goal configuration  $q_g$  are provided. The algorithm then tries to connect to the closest node in the roadmap for each of these configurations. If the two nodes can be connected to the roadmap, shortest path is found between the connected nodes in the roadmap.

The path obtained can be improved upon by applying a smoothing procedure. Two techniques are used in this project:

*1) Greedy Approach:* First it is checked if  $q_i$  and  $q_g$  can be connected directly. If such a path exists, the function simply returns the direct path.

If the configurations cannot be directly connected, a greedy approach is employed to connect first node to the end node, if successful the path is shortened by directly joining these nodes. If not the next node is checked with the last node and so on. This greatly reduces the path taken by the system of robots.

*2) Random Approach:* A random approach is augmented with the greedy approach. Two nodes in the shortest path are checked if they can be directly connected. If a collision-free path exists, the nodes are *short-circuited*. This approach has been found to be effective for several environments.

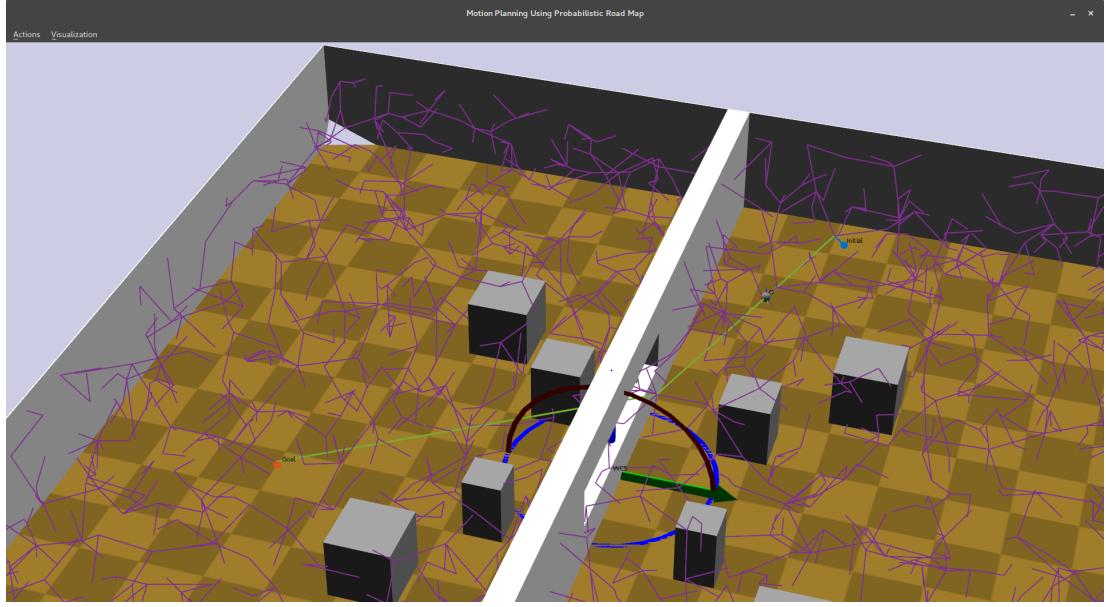


Fig. 3. A sample path for a single robot. The initial configuration is shown in blue and the final configuration in red. Smoothing procedure has been applied to the path to obtain shorter paths.

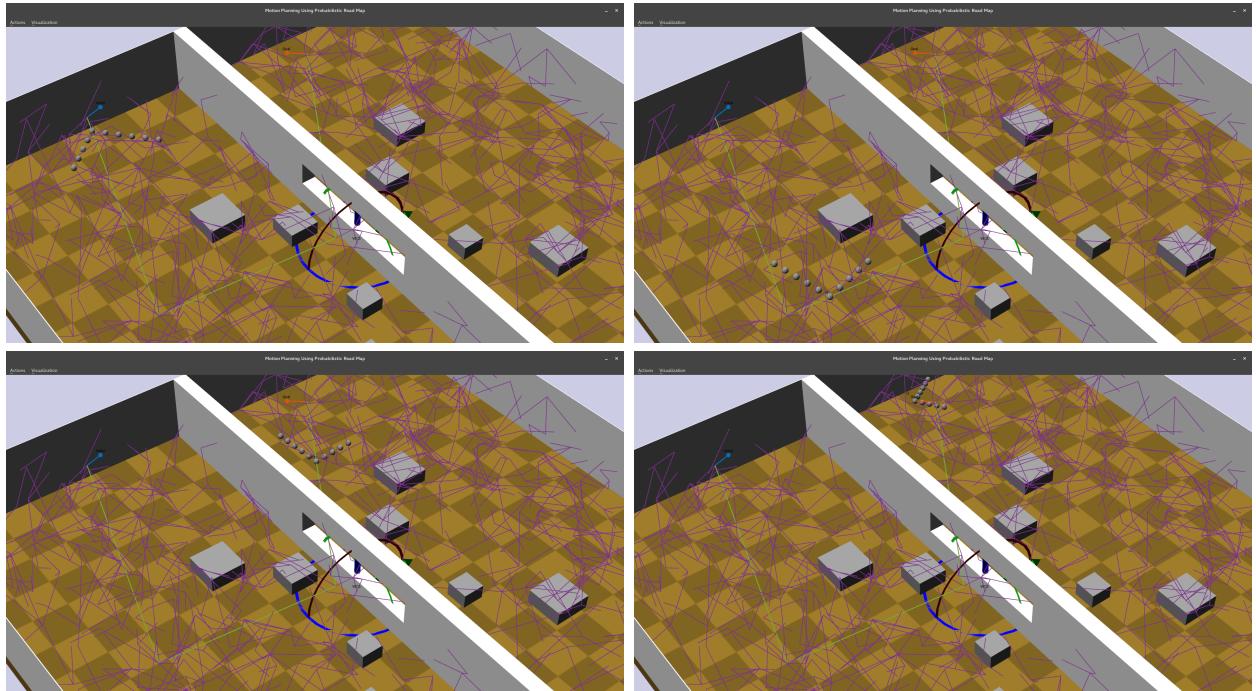


Fig. 4. Path shown for 10 robots. The initial configuration is marked in blue and the final configuration is marked in red. The blue and red lines show the path joining the respective configurations to the roadmap. The final path is smoothed. Note that the scale and orientation of the formation is also changing.

#### IV. EXPERIMENTS

Running time experiments were performed to understand the variation in running time with the number of robots (1–10). The environment for the experiment is shown in Fig. 5. The number of samples were fixed to 2000. The maximum number of nodes to connect from a new sample was set to 15. The neighbor hood distance was 0.75 times the number of robots. The sampling for collision detection was set to a distance of 0.04. The query time was averaged over 100 random queries. There were at the most 1 failure. It can be seen from the plots that the running time in learning phase increases with increase in the number of robots. However, the increase is not very steep. The number of nodes not connected to any other node also increases with number of robots. This is because the size of the formation increases and it becomes more difficult to connect configurations using collision-free path.

One interesting plot is of running time of query phase. The running time first increases with the number of robots and then decreases. The running time is more for 3–6 number of robots. This is because of the neighborhood distance is small which causes the shortest path composed of several very small segments. The complexity of the smoothing process is proportional to the number of segments in the shortest path. Hence, the running time increases for smaller number of robots. The running time increases for 1–4 robots because the collision detection is implemented to ensure the paths are collision free.

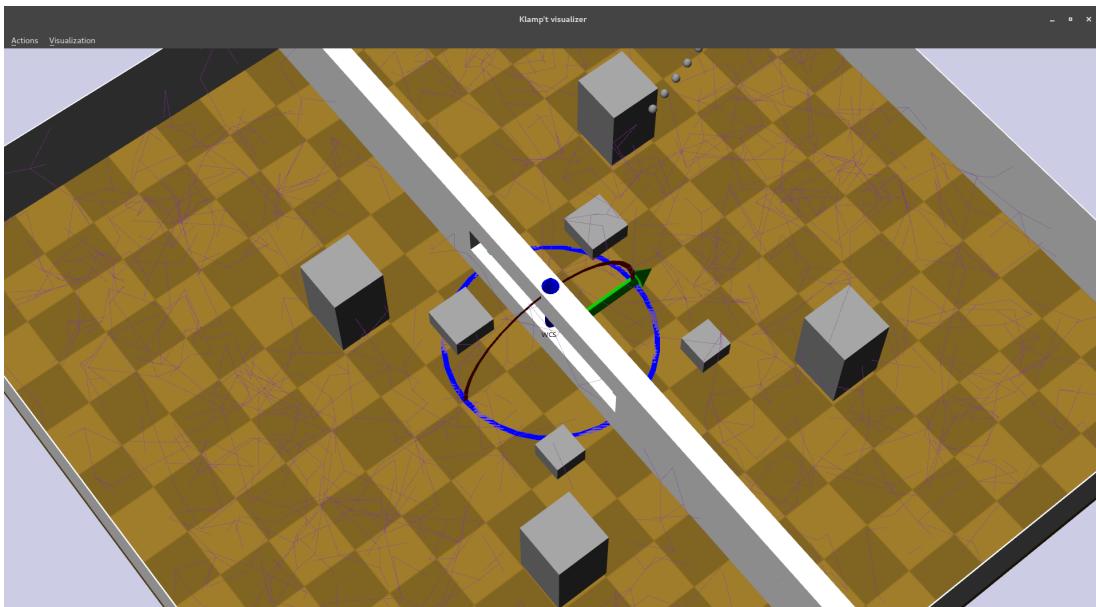


Fig. 5. Environment setup for the experiments

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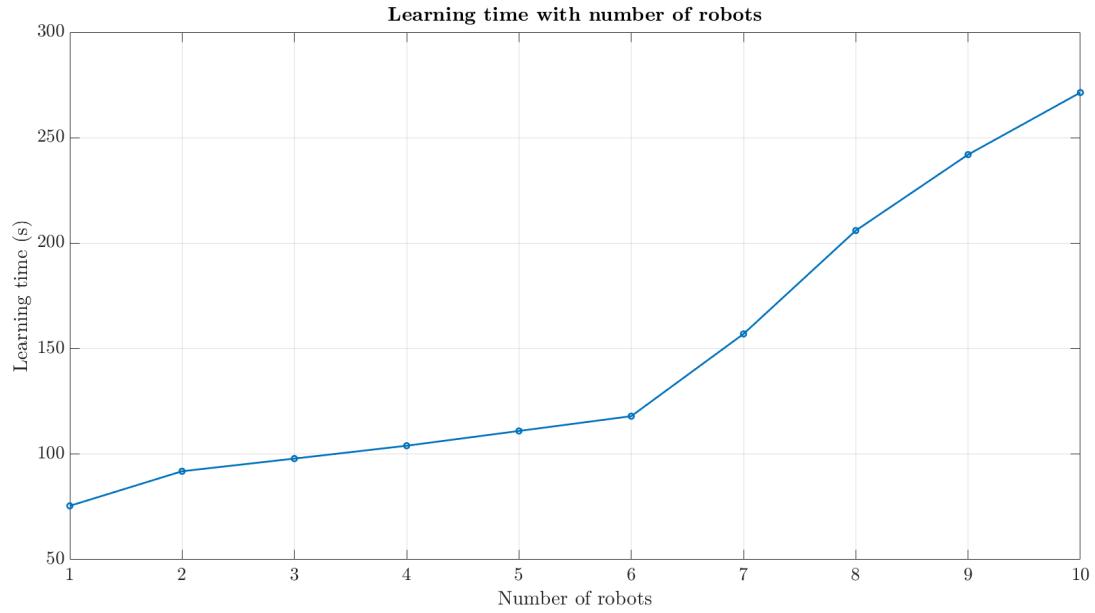


Fig. 6. Variation of running time of learning phase with number of robots.

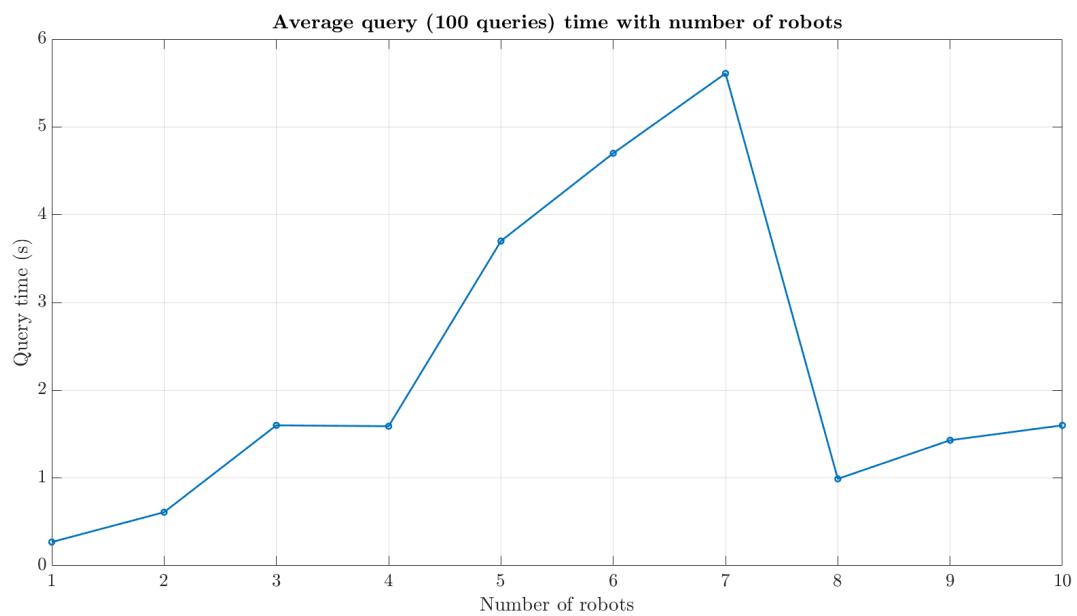


Fig. 7. Variation of running time of query phase with number of robots.

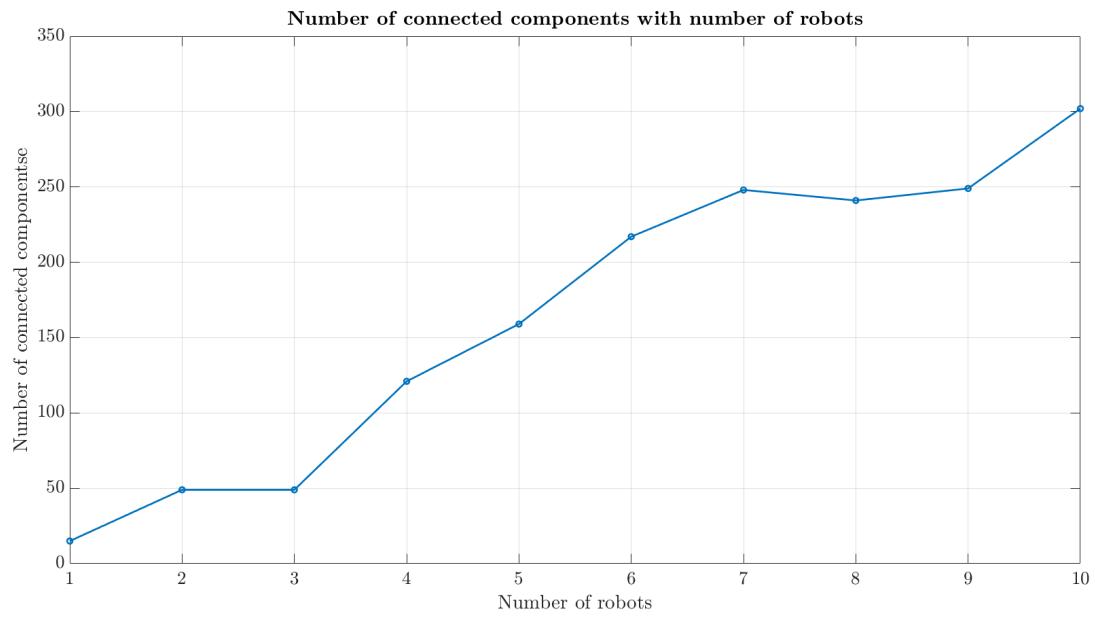


Fig. 8. Variation of number of connected components with number of robots.

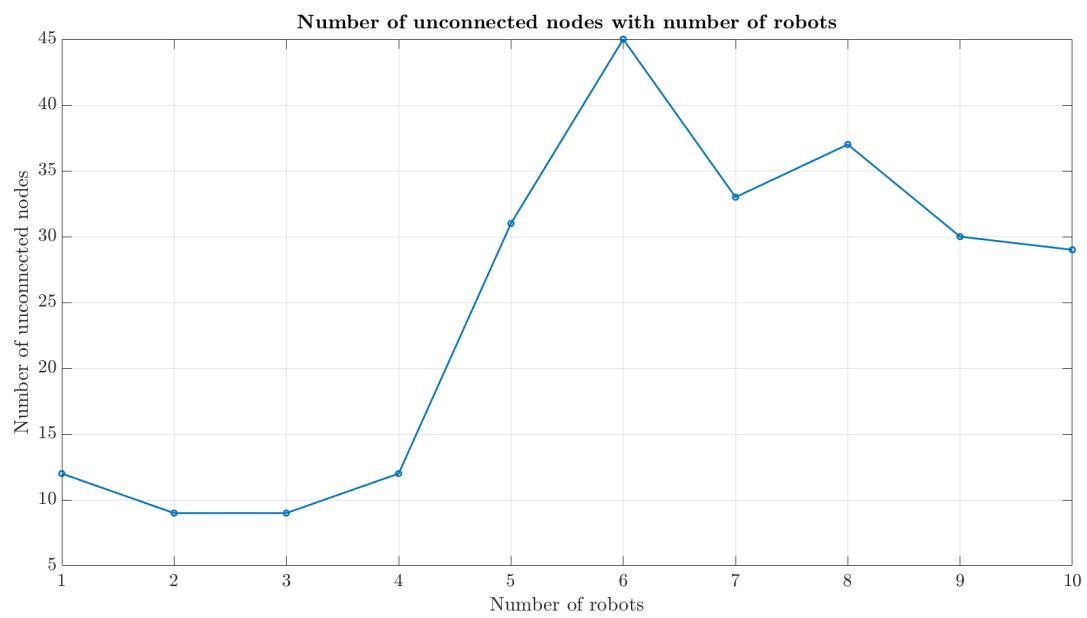


Fig. 9. Variation of number of unconnected nodes with number of robots.