Report

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§1 Abstract

The aim of the project was to simulate and analyze information markets using well known models, and observe the changes that occur by modifying the models in different manners. We will also be creating our own currency and facilitate transactions in this market only through our currency.

Due to the lack of literature on betting games, we aim to foster this area of study with our studies. This makes for an exciting project, combining several key areas such as game theory, statistics, programming, behavioral economics, and such.

§2 Introduction

Speculative Markets are often accurate representations of the future prices, thus containing a great deal of information. These information markets are quite useful, but anti-gambling laws prohibit the existence of most such markets.

Apart from legal issues, information market face the thin market problem. This is essentially a lack of coordinated trade since it is harder for people to coordinate on a time and price. The other problem the information markets face is irrational participation problem.

An older method of obtaining information from the market is using scoring rules, which do not face either of the issues mentioned above. On the contrary, they do face a thick market problem, wherein a single consensus cannot be reached as different people give different estimates.

A market scoring rule combines the advantages of the information markets and the scoring rules. The best way to implement them is by using an Automated Market Maker.

What is an automated market maker? Automated market makers (AMMs) are a type of decentralized exchange (DEX) that use algorithmic "money robots" to make it easy for individual traders to buy and sell crypto assets. Instead of trading directly with other people as with a traditional order book, users trade directly through the AMM.

One of the main problems AMMs face is that of thin markets. One of the key issues we will face is this thin market problem, and we will strategize in various ways to tackle this problem as a part of the project.

We will primarily be using LMSR and LS-LMSR models to analyze and construct the information market algorithms. These models can be used to create an information market, but apparently other models are used to make profitable systems, such as in the case of sports gambling.

§3 The Project Guideline

Our token is called CMI token, and this currency will have a fixed exchange rate. The reason for choosing a fixed exchange rate was that if we made it variable, which we initially planned on modeling using a bond curve, there would eventually be a lot of liquidation as the currency is inherently temporary, and then some people would face a huge loss.

This token is launched on the occasion of a small yearly fest in which many sports are played. We allow people to side-bet on these sports, facilitated through our currency. The betting reveals information about the market and the value of the opposing teams. We also allow people to buy exclusive product only through our currency.

- Make two Betting algorithms in Python, LMSR and LS-LMSR. Create a register for the bets and an interface for the bets. Also create a variable transaction cost that you can modify for experimentation.

  Note: We later imported our code to Excel since people wanted more transparency in the bets that were made, and the return that our algorithm provided.

- Make a login interface on the website on which people can create username and password, and then create a payment system with proper security measures, for people to buy your coins. For our purposes, the selling of the tokens was done manually by the clients pursuing us and telling us how many tokens they want to stake or sell to purchase a commodity.

- Create a SQL database that can store all the transactions, and attach a time frame to it to do any possible time series analysis. Finally, we host our website online.

- We buy exclusive products not generally available through the audience and sell them at the market retail price through our tokens. Furthermore we make posters and attach them in the main areas of the campus. We actively promoted our token to build trust in the community.

- As the side betting starts, possibly change the algorithm after a certain game ends to experiment with the conditions.

§4 The Algorithm

§4.1 Betting

On all the events, we allow side betting to allow players to bet on who will win. We are using Logarithmic Market Scoring Rule (LMSR) to figure out the cost function, and the change of odds and probabilities. Every single bet is recorded and financial analysis will be done from this data. There is a 5 percent transaction fee on all betting transactions to avoid any scope of arbitrage.

The codes are attached and one can analyze how the model works.

Mathematically,

The core of the LMSR mechanism is its **cost function**, which determines the price of an outcome based on the current bets. The cost function is given by:

$$C(\mathbf{q}) = b \cdot \ln \left(\sum_{i=1}^{n} e^{\frac{q_i}{b}} \right)$$

where $\mathbf{q} = (q_1, q_2, \ldots, q_n)$ is the vector of cumulative bets on all outcomes, b is the liquidity parameter, controlling how sensitive the market prices are to changes in bets, and n is the number of possible outcomes.

The cost function ensures that prices are continuous and arbitrage-free.

The **price** of an outcome i is calculated as the partial derivative of the cost function with respect to q_i:

$$p_i = \frac{\partial C(\mathbf{q})}{\partial q_i} = \frac{e^{\frac{q_i}{b}}}{\sum_{j=1}^{n} e^{\frac{q_j}{b}}}$$

This represents the probability assigned to outcome i based on the market's current state.

§4.2 Placing a Bet

When a bettor places a bet of size Δq on outcome i, **cost to the bettor** is the change in the marginal cost function:

$$\text{Price Paid} = C(\mathbf{q} + \Delta\mathbf{q}) - C(\mathbf{q})$$

where $\Delta\mathbf{q}$ is a vector with Δq added to q_i and 0 elsewhere. This, essentially, when divided by the new odd gives you the "number of shares".

§4.3 Transaction Cost

In this implementation, a **fixed transaction fee** τ is applied to each bet. The effective amount of the bet is reduced by:

$$\text{Transaction Fee} = \tau \cdot \Delta q$$

The **effective amount** used in the LMSR mechanism is:

$$\Delta q_{\text{effective}} = \Delta q - (\tau \cdot \Delta q)$$

This ensures the bettor pays the fee but only the effective amount impacts the market. Note that this section here is unnecessary, as we can simply consider the price bet to be the effective price bet (which is the price bet minus the transaction fee.)

§4.4 Payout

If outcome i is correct, the bettor's payout is proportional to the number of shares purchased:

$$\text{Payout} = \Delta q$$

No additional costs are deducted from the payout.

§5 References

Here are the references for our project (joint work by Agastya Agrawal, Sahil Lakhmani, Arjun Agarwal):

- Logarithmic Market Scoring Rules for Modular Combinatorial Information Aggregation by Robin Hanson.
- Automated Market Makers for Decentralized Finance (DeFi) by Yongge Wang
- Combinatorial Information Market Design by Robin Hanson
- Hanson's Market Scoring rules
- Online Learning in Betting Markets: Profit versus Prediction by Zhu et al.