GNSS Overview

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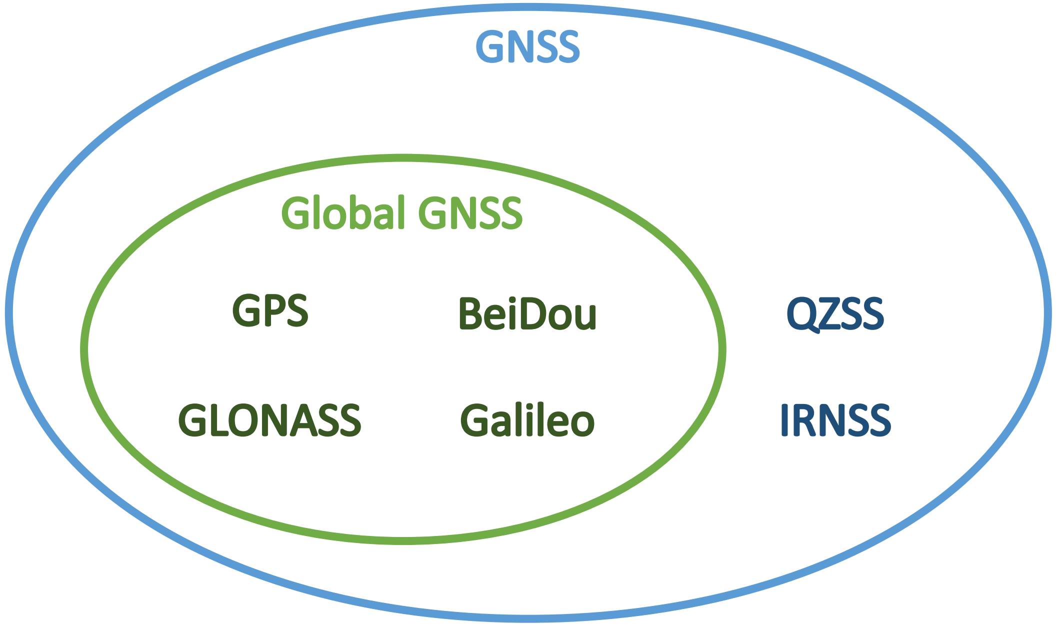
Introduction

- What is GNSS?
  - GNSS stands for **Global Navigation Satellite System**.
    Uses *satellites* to provide autonomous geo-spatial *positioning.*

- GPS vs. GNSS
  - GPS is the first GNSS launched in 1978, but there are many satellite navigation systems out there established by other organizations, either globally or regionally.



Reference:https://www.gps.gov/systems/gnss/

A Little Bit of Yesterday – Birth

- The first idea of satellite-based positioning originated during the Cold War, when two American scientists realized that they could pinpoint Soviet Union satellite along its orbit because of the Doppler effect.
- To solve the inverse problem – pinpointing user's location given the satellite's, the U.S. developed TRANSIT, providing a navigational fix about *once per hour*, becoming the first satellite navigation system.
- At 1978, the first experimental GPS satellite was launched.

Reference: https://en.wikipedia.org/wiki/Global_Positioning_System

A Little Bit of Yesterday – Evolution

- GPS does not come for free! Thus it's designed to have
  - Low accuracy for civilian use (~ 100 m), high accuracy only for military use.

- But with improved receiver design public accuracy can be ~ 20 m!
  - Nah that's too good! Let's degrade that unexpected performance :(
  - Use Selective Availability (SA) technique to deliberately introduce error (~ 100 m) for non-military users during 1990s.

- Differential GPS (DGPS) was invented to solve SA and led to even better performance (< 5 m, and < 20 cm for best cases).

Reference: https://en.wikipedia.org/wiki/Differential_GPS

A Little Bit of Yesterday – Evolution (cont'd)

- SA was turned off in 2000, partly because of the invention of DGPS.
- GPS began transmitting its second civilian signal in 2005 to achieve better accuracy :)
- Development of A-GPS for cellular phones
  - No warm up needed (no longer to wait for ~ 30 s at start up).
  - Amount of processing can be reduced with the help of server.

Reference: https://en.wikipedia.org/wiki/Assisted_GPS

A Little Bit of Today

- China successfully launched the final Beidou satellite on Jun 23, 2020.
- Cost of GPS receivers has dropped dramatically — from the first commercial GPS receiver with a cost over \$100,000 and a weight of 53 pounds, down to the chips that only costs a couple of dollars nowadays.

Reference: https://onlinemasters.ohio.edu/blog/the-evolution-of-portable-gps/

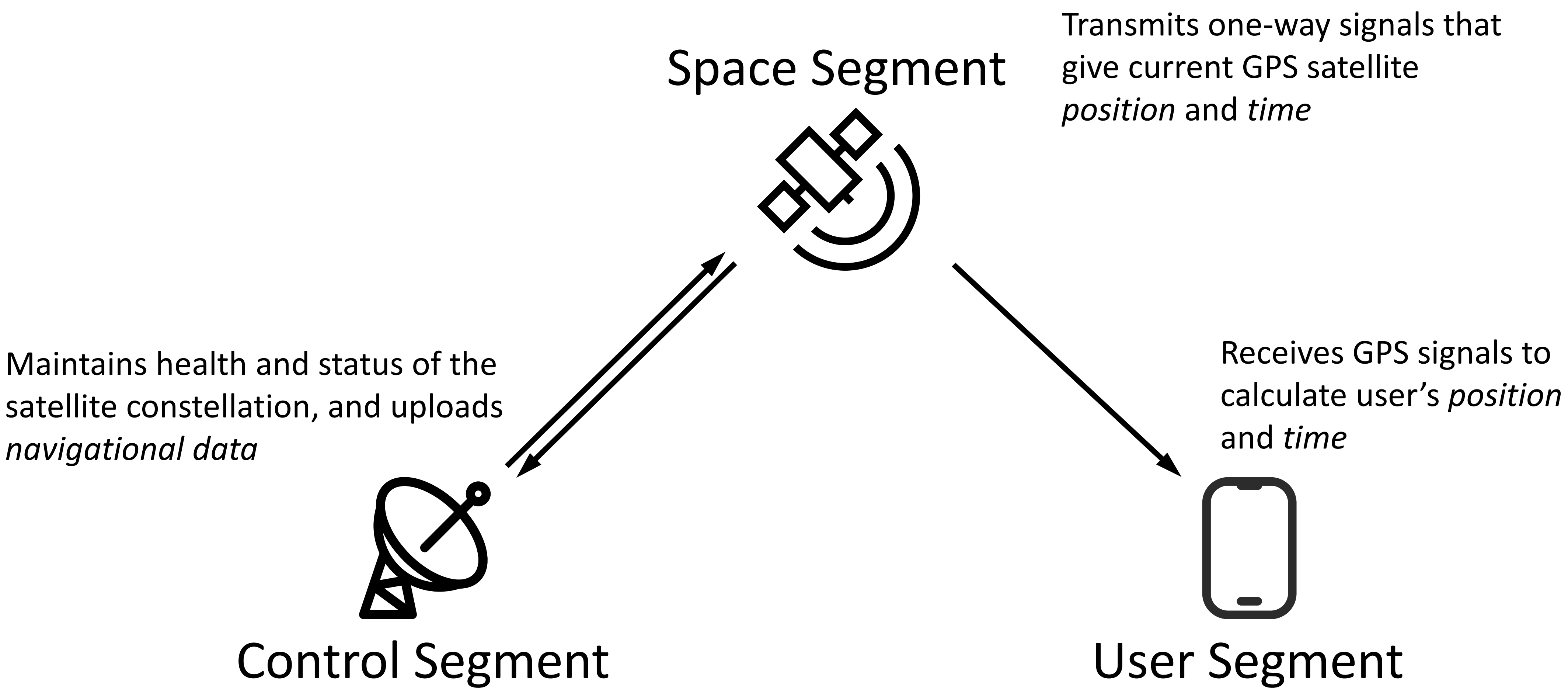
Let’s Get Started

- How to determine the user’s position?
- How to capture GPS signal, and why can’t we decode military signal?
- How did DGPS technique render Selective Availability ineffective?
- ......

Overview of GPS Positioning

First question, how to determine the user’s position?

GPS Segments



Reference: https://www.gps.gov/systems/gps/, Icon: https://www.iconfont.cn/

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GPS Signals to Distance

- After user successfully decodes the information of GPS signals sent by each visible satellite, it will have
  - Three-dimensional position of each satellite p^s
  - Time t^s when GPS signal is transmitted

- Satellites are way above the sky (~ 20,200 km), thus when GPS signal being received, it would be delayed by some time, determined by the range it travelled across space (i.e. distance of satellite and user)
  - Time t_r when GPS signal is received, read from user clock
  - Distance $R = c(t_r - t^s)$, where c is speed of light (GPS signal speed)

Reference: https://www.gps.gov/systems/gps/space/#:~:text=Constellation%20Arrangement,the%20Earth%20twice%20a%20day.

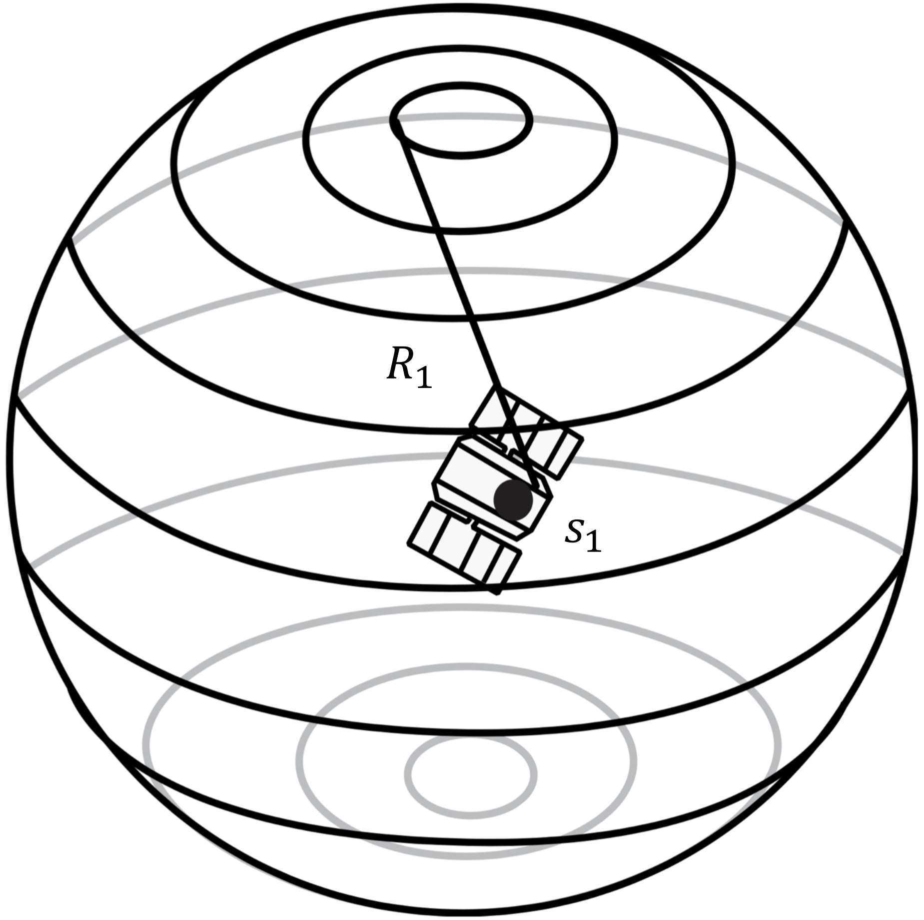
Trilateration

- Now let's consider a simple case when user has received GPS signals from three satellites and calculated the distance between each satellite
  - Distance R_1, R_2, R_3 with regard to satellite s_1, s_2, s_3

- Then it's able to solve user's three-dimensional position $p_r = (x_r, y_r, z_r)$ because there are three knowns R_1, R_2, R_3 and three unknowns.

Trilateration (cont'd)

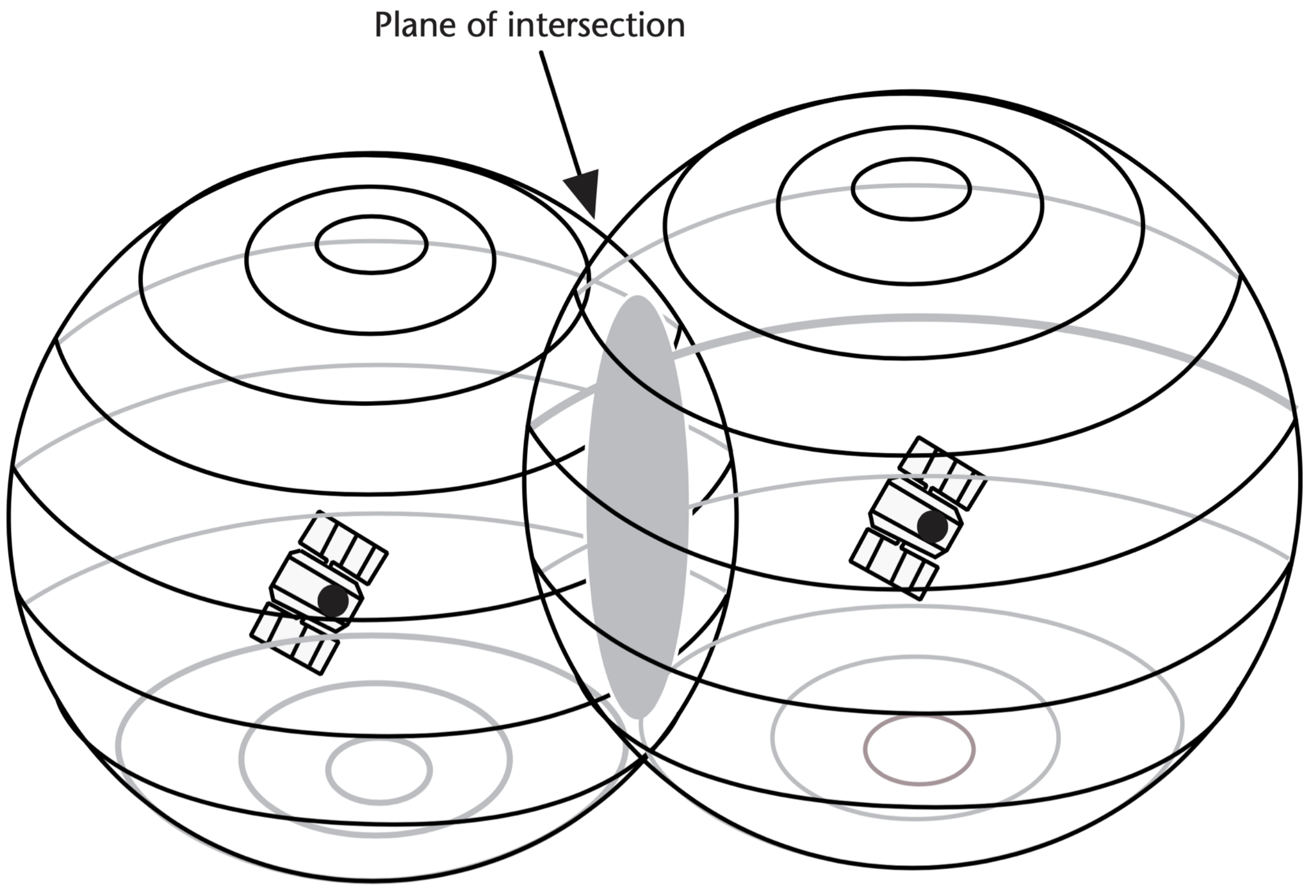
If there is only one satellite, then user's location couldn't be determined, but it should locate somewhere on the surface of sphere centered at the satellite s_1 with radius R_1.



Reference: Elliott D. Kaplan. Understanding GPS : Principles and Applications. Boston :Artech House, 1996.

Trilateration (cont'd)

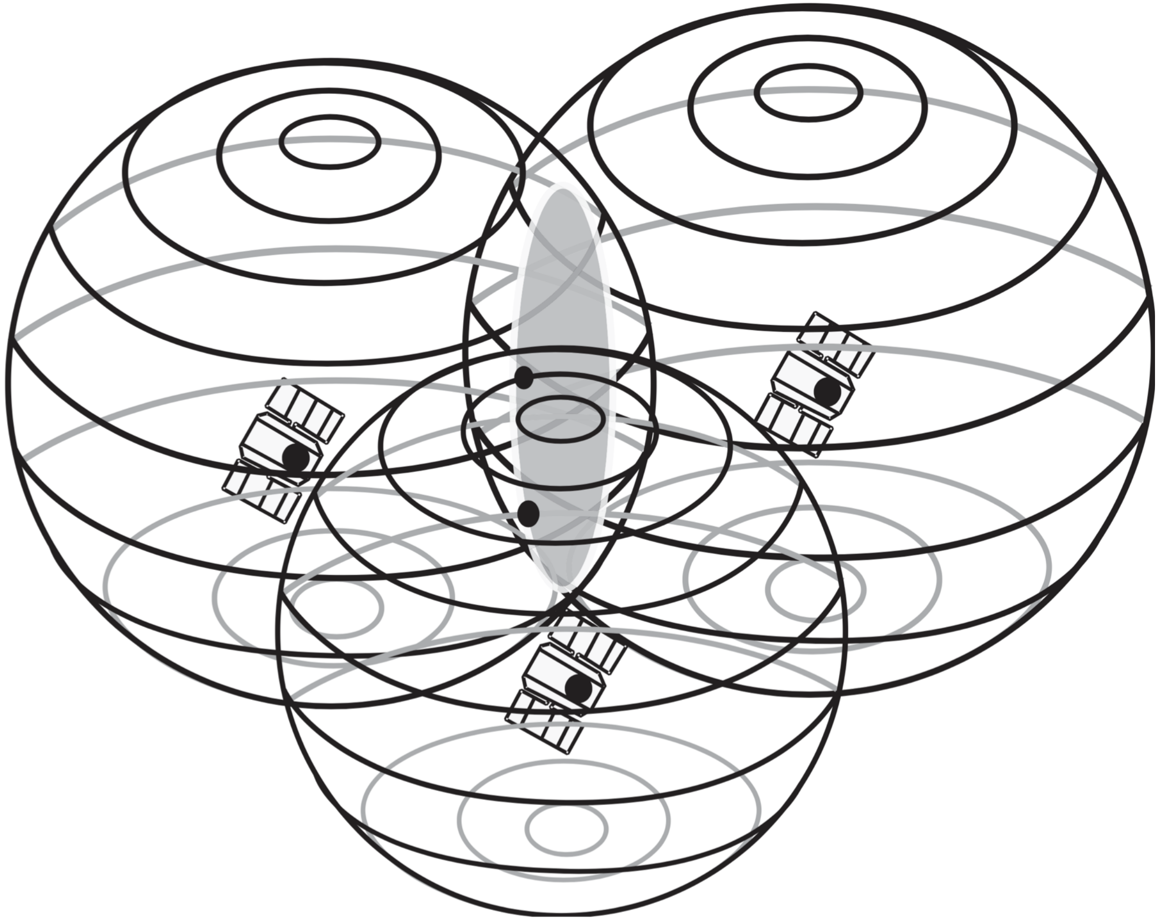
If two satellites, the user would then be somewhere on the surface of both spheres, which is the perimeter of plane of intersection denoted by the shaded circle in the figure.



Reference: Elliott D. Kaplan. Understanding GPS : Principles and Applications. Boston :Artech House, 1996.

Trilateration (cont'd)

With three satellites, the user is at the intersection of the perimeter of the circle and the surface of the third sphere. This third sphere intersects the shaded circle perimeter at two points. For a user on Earth's surface, the lower point should be true position (the other one is mirrored up above the plane decided by the three satellites).



Reference: Elliott D. Kaplan. Understanding GPS : Principles and Applications. Boston :Artech House, 1996.

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But That's Not All

- However user clock time may not be accurate enough
  - Remember the distance is calculated by time delay multiplied by speed of light c?
  - If user clock is not accurate, let's say it has an offset of only 1 us compared with true time, then an error of 300 m would be introduced when calculating distance between user and satellite (suppose satellite time is accurate).

- To solve this problem, we have to know user clock offset with regard to true time (or satellite time).

Equation for Trilateration

- Let's introduce a little bit of mathematics for better understanding
  - Consider a simple trilateration case:

$$R_1 = c(t_r - t^{s_1}) = \|p_r - p^{s_1}\|$$
$$R_2 = c(t_r - t^{s_2}) = \|p_r - p^{s_2}\|$$
$$R_3 = c(t_r - t^{s_3}) = \|p_r - p^{s_3}\|$$

where $\|p_r - p^s\|$ denotes the distance between p_r and p^s. Signal reception time t_r is known from the reading of current user clock time, satellite position p^{s_1} , p^{s_2} , p^{s_3} can be obtained from GPS signal, thus there are only three unknowns $p_r = (x_r, y_r, z_r)$.

Estimate Clock Offset

- Add one more variable, user clock offset dt, into user time and rewrite as

$$R = c(t_r + dt - t^s) = \|p_r - p^s\|$$

- Since there are four unknowns instead of three, then why not add one more satellite?

$$R_1 = c(t_r + dt - t^{s_1}) = \|p_r - p^{s_1}\|$$
$$R_2 = c(t_r + dt - t^{s_2}) = \|p_r - p^{s_2}\|$$
$$R_3 = c(t_r + dt - t^{s_3}) = \|p_r - p^{s_3}\|$$
$$R_4 = c(t_r + dt - t^{s_4}) = \|p_r - p^{s_4}\|$$

Problem Solved!

Estimate Clock Offset

- Add one more variable, user clock offset dt, into user time and rewrite as
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$$R_3 = c(t_r + dt - t^{s_3}) = \|p_r - p^{s_3}\|$$
$$R_4 = c(t_r + dt - t^{s_4}) = \|p_r - p^{s_4}\|$$

A minimum of four satellites is needed to determine user's location.

What if More Satellites

- What if we have five or more satellites?

$$R_1 = c(t_r + dt - t^{s_1}) = \|p_r - p^{s_1}\|$$
$$\vdots$$
$$\vdots$$
$$R_i = c(t_r + dt - t^{s_i}) = \|p_r - p^{s_i}\|$$

can be formulated as a non-linear least square problem and solved by the Gauss-Newton algorithm.

- But do we actually need more? Aren't four satellites enough?

Measurement Errors

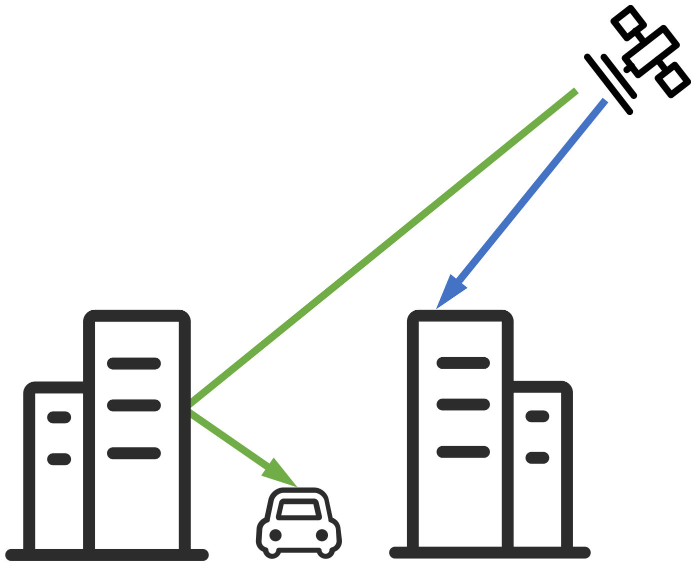
- It’s a long journey for GPS signal to travel through space, thus there are a lot of errors when measuring the distance.

- Atmospheric effects: error introduced when travelling across earth’s atmosphere
  - Ionospheric delay, caused by travelling across ionized atmosphere, frequency dependent
  - Delay caused by humidity at troposphere
  - Atmospheric pressure delay

Reference: https://en.wikipedia.org/wiki/Error_analysis_for_the_Global_Positioning_System Icon: https://www.iconfont.cn/

Measurement Errors (cont'd)

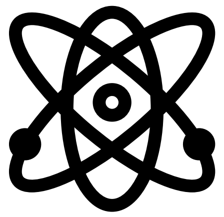
- Multipath effects
  - GPS signals can be blocked by surrounding terrain: trees, buildings, bridges, etc.
  - GPS signals can be reflected by surrounding terrain: buildings, canyon walls, hard ground, etc.
  - When direct signal is blocked but reflected signal is received, then the distance calculated will be larger than true distance .



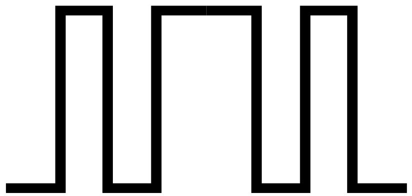
Reference: https://en.wikipedia.org/wiki/Error_analysis_for_the_Global_Positioning_System Icon: https://www.iconfont.cn/

Measurement Errors (cont'd)

- Satellite ephemeris and clock errors
  - Satellite ephemeris may be up to two hours old, lead to wrong satellite position
  - Satellites' atomic clocks experience noise and clock drift errors, and might not be able to be fully corrected.



- Signal arrival time measurement
  - User can measure signal delay as accurate as ~ 10 ns, representing an error of ~ 3m.



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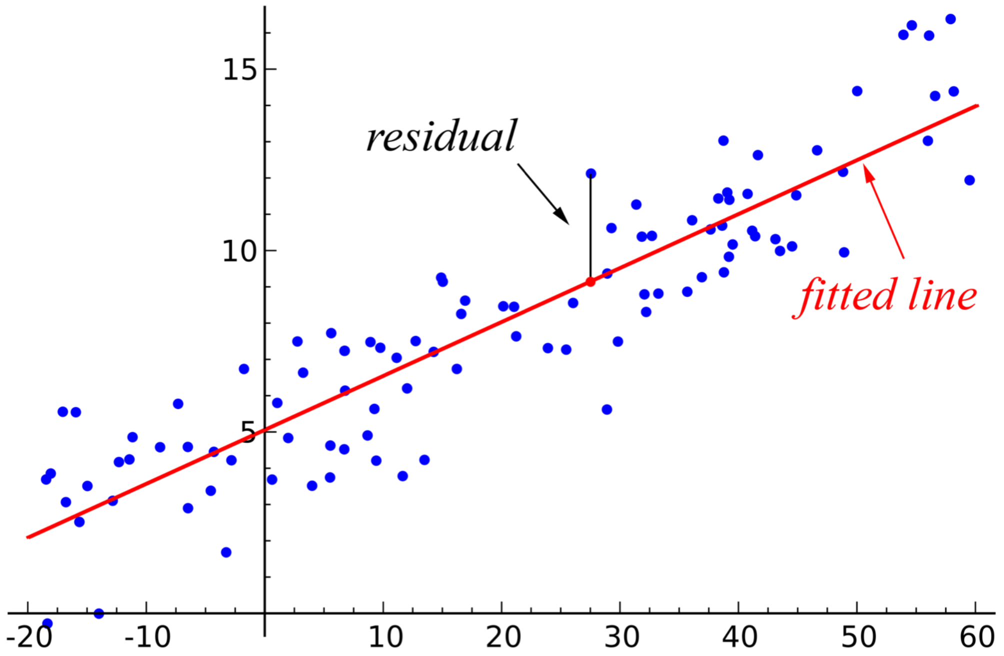
Position Uncertainty

- Distance measurements would not be perfect due to these errors.
- If we only have four measurements, which is the bare minimum required to determine user's location, the errors would be fully reflected in the solution.
- If we have more measurements, error could be partly consumed by residuals after least square estimation, thus rendering a more accurate solution with less error.

The more, the Better

- Take linear regression as an example
  - With enough measurements we can fit a line best describing the linear model
  - Residual is the difference of one measurement (blue dot) with its predicted value (red dot)
  - If the fitted line is 100% accurate, then measurement error = residual



- More measurements lead to more accurate fitted result, and same for non-linear cases

Reference: https://en.wikipedia.org/wiki/Regression_analysis

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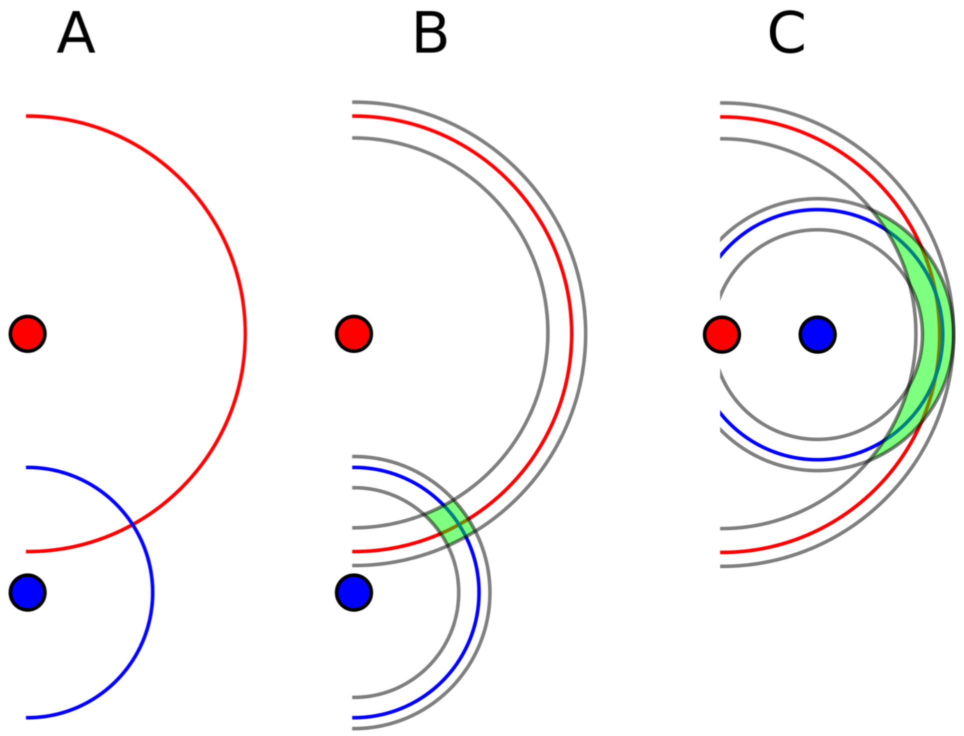
Dilution of Precision (DOP)

- Considering 2D case
  - **A** someone has measured the distance to two landmarks, and plotted their point as the intersection of two circles with the measured radius.
  - **B** the measurement has some error bounds, and their true location will lie anywhere in the green area.
  - **C** the measurement error is the same, but the error on their position has grown considerably due to the arrangement of the landmarks.



Reference: https://en.wikipedia.org/wiki/Dilution_of_precision_(navigation)

Dilution of Precision (DOP) (cont'd)

- Arrangement of satellites makes a difference
  - We hope the satellites can spread evenly throughout the space

- Notice that for a user located at earth's surface, the received satellite signals can not come below horizon
  - This is why position given by GPS will have larger vertical error compared with horizontal error

Pseudorange

- Distance R calculated based on time delay is not the geometric range between user and satellite, but contains measurement errors, thus technically it's called *pseudorange*, formulated as

$$R = c[t_r(T_2) - t^s(T_1)\ ]$$

where $t^s(T_1)$ is signal transmission time given by satellite clock s;
$t_r(T_2)$ is signal reception time given by user clock r.

- Equation can be re-written in another way, using geometric range ρ_r^s between user and satellite plus measurement errors.

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Pseudorange (cont'd)

$$
\begin{aligned}
R &= c[t_r(T_2) - t^s(T_1)\ ] \\
&= c[(\ t_r + dt_r) - (t^s + dt^s)] + \varepsilon_p \\
&= c(t_r - t^s) + c(dt_r - dt^s) + \varepsilon_p \\
&= (\rho_r^s + I_r^s + T_r^s) + c(dt_r - dt^s) + \varepsilon_p \\
&= \rho_r^s + c(dt_r - dt^s) + I_r^s + T_r^s + \varepsilon_p
\end{aligned}
$$

where t^s and t_r are the true time of satellite and user clock, dt^s and dt_r are the clock offsets, ε_p is the non-atmospheric measurement error (e.g., multipath), I_r^s and T_r^s are atmospheric error caused by Ionosphere and Troposphere.

Some Takeaways

- Principle idea behind GPS localization is simple – trilateration.
- Due to clock offset, a minimum of four satellites is needed to determine user’s location.
- Pseudorange, calculated based on time delay, is the distance from satellite to user including measurement errors.
- Horizontal accuracy is better compared with vertical due to DOP.