Nonlinear Model Order Reduction using POD/DEIM for Optimal Control of Burgers' Equation

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In my Master thesis I consider the following optimal control problem,

$$\min_{u} \frac{1}{2} \int_0^T \int_0^L [y(t,x) - z(t,x)]^2 + \omega u^2(t,x) \, dx \, dt, \quad \omega \in \mathbb{R}_+, \tag{1}$$

where y is the solution of the nonlinear, unsteady Burgers' equation with homogeneous Dirichlet boundary conditions and given initial condition $y_0(x)$,

$$\begin{aligned} y_t + \left(\frac{1}{2} y^2 - \nu y_x \right)_x &= f + u, \quad (x,t) \in (0,L) \times (0,T), \\ y(t,0) = y(t,L) &= 0, \quad t \in (0,T), \\ y(0,x) &= y_0(x), \quad x \in (0,L), \end{aligned} \tag{2}$$

and u being a control such that (1) is small and, hence, the solution of Burgers' equation is close to the desired state z. This problem has been considered in [2] and the authors of [3] proposed the well-known method of Proper Orthogonal Decomposition (POD) in order to reduce the computational cost of the optimization algorithm which solves (1)-(2).

In my thesis work, the Discrete Empirical Interpolation Method (DEIM), introduced by [1] in 2010, has been used for the first time in the context of optimal control for Burgers' equation in order to further improve the computational benefit of POD and solving (1)-(2) on a POD-DEIM reduced model.

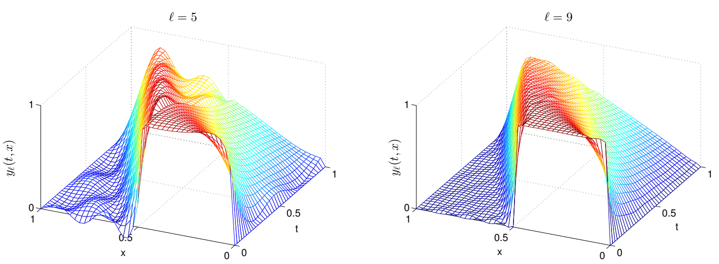

Figure 1: POD-DEIM approximations of the uncontrolled ($u \equiv 0$) Burgers' equation for two different reduced dimensions ℓ.

References

[1] S. Chaturantabut and D. Sorensen. Nonlinear Model Reduction via Discrete Empirical Interpolation. *SIAM J. Sci. Comput.*, 32:2737–2764, 2010.

[2] M. Heinkenschloss. Numerical solution of implicitly constrained optimization problems. Technical report, Department of Computational and Applied Mathematics, Rice University, 2008.

[3] K. Kunisch and S. Volkwein. Control of the Burgers Equation by a Reduced-Order Approach Using Proper Orthogonal Decomposition. *Journal of Optimization Theory and Applications*, 102:345–371, 1999.