Algorithmic Game Theory, Assignment 1

Agata Borkowska, UID: 1690550

February 14, 2017

1

Note: here we use notation (x,y) to mean that the payoff for player I for the given strategy is x, and the payoff for player II is y. (LateX didn't want to cooperate)

1.1

Let $b = 1$, $a = 1$, $c = -1$, $d = 1$. i.e.

	A	B
1	(1,0)	(0,1)
2	(0, 0)	(1, -1)

Then:

- If player I chooses 1, then player II would want to play B, but...
- if player II chooses B, player I will want to play 2, but...
- if player I chooses 2, player II will want to play A, but...
- if player II chooses A, player I will want to choose 1.

There is a mixed Nash equilibrium, in which each player plays each strategy with a probability $\frac{1}{2}$. The expected payoff for I is $\frac{1}{2}$, and for II is 0.

1.2

Assuming Player I has a strategy $(\frac{1}{2}, \frac{1}{2})$, the payoff for Player II if they chose B would be $\frac{1}{2} \cdot a + \frac{1}{2} \cdot c$, and the payoff if they chose A is 0.

Therefore, for Player II to always prefer A over B,

$$\frac{1}{2} \cdot a + \frac{1}{2} \cdot c < 0$$

$$\Rightarrow a + c < 0$$

1.3

Let $a, b, c, d = 0$. Then for either player, a strategy $(\lambda, 1 - \lambda)$ for any $\lambda \in [0, 1]$, the payoff is 0, and there are infinitely many such λ's.

1.4

Let $b = 1$, $a = -1$, $c = 1$, $d = 1$. I.e.

	A	B
1	(1,0)	(0,-1)
2	(0, 0)	(1, 1)

Then the game has 2 pure Nash equilibria: (1, A) and (2, B). If I chooses 1, then II choosing A always gives them a better payoff, because $a < 0$. If II chooses A, I will choose 1, because $b > 0$. Similarly for (2, B).

2

The possible choices for player I are 2, 3, and 4, and for player II are {2,3}, {2,4}, and {3,4}. Putting it in a payoff matrix (note that this is a 0-sum game, and we only need to write the payoff for player I, as player II will get 0 - that amount).

	{2,3}	{2,4}	{3,4}
2	-2	-2	0
3	-3	0	-3
4	0	-4	-4

Let player I play 2 with probability p, 3 with probability q, and 4 with probability $(1 - p - q)$. Then II's expected winnings are:

- **II plays {2,3}**: $2 \cdot p + 3 \cdot q + 0 \cdot (1 - p - q) = 2 \cdot p + 3 \cdot q$
- **II plays {2,4}**: $2 \cdot p + 0 \cdot q + 4 \cdot (1 - p - q) = (-2) \cdot p + (-4) \cdot q + 4$
- **II plays {3,4}**: $0 \cdot p + 3 \cdot q + 4 \cdot (1 - p - q) = (-4) \cdot p - q + 4$

For player II to be willing to randomise, the expected payoffs for each of those strategies should be the same, call it V.

Thus we arrive at a system of linear equations with 3 unknowns:

$$2 \cdot p + 3 \cdot q - V = 0 \tag{1}$$
$$(-2) \cdot p + (-4) \cdot q - V = -4 \tag{2}$$
$$(-4) \cdot p - q - V = -4 \tag{3}$$

After solving it, we get:

$$p = \frac{6}{13} \approx 0.462$$
$$q = \frac{4}{13} \approx 0.307$$
$$(1 - p - q) = \frac{3}{13} \approx 0.231$$

So player I's strategy is $(\frac{6}{13}, \frac{4}{13}, \frac{3}{13})$.

Now let's assume player II picks {2,3} with probability s, {2,4} with probability t, and {3,4} with probability $1 - s - t$. Let the winnings for player I be W. The system of linear equations is now:

$$-2 \cdot s - 2 \cdot t - W = 0 \tag{4}$$
$$3 \cdot t - W = 3 \tag{5}$$
$$4 \cdot s - W = 4 \tag{6}$$

after solving it, we get:

$$s = \frac{7}{13} \approx 0.538$$
$$t = \frac{5}{13} \approx 0.385$$
$$(1 - s - t) = \frac{1}{13} \approx 0.077$$

So player II's strategy is $(\frac{7}{13}, \frac{5}{13}, \frac{1}{13})$.

As a useful check, from the above systems of linear equations, we get $V = -W = \frac{24}{13} \approx 1.846$. This is a zero sum game, and the expected payoff for both players adds up to 0.

Another way of finding Nash equilibrium would be to use linear programming, however in a simple game like this one, this method suffices.