Tutorial 1

Formulation of Linear Programming Problem (LPP) and Graphical method

Date of the Session: ....

Learning outcomes:

- Understanding the process of formulating a given problem in linear form.
- This requires defining the decision variables of the problem, establishing inter-relationship between them and formulating the objective function and constraints.
- Understanding the process of solving a given LPP using graphs will be discussed.

1.1 PRE-TUTORIAL

1. What is Linear Programming Problem (LPP)?

Linear Programming Problem (LPP) is a mathematical technique which is used to optimize [maximize or minimize] the objective function with the limited resources.

2. Define the following terminology in LPP:

(a) Objective function

(b) Constraints

(c) Decision variables

(d) Non-Negativity Conditions

a) Objective Function: The Function that needs to be optimized.
(Maximized or Minimized)
$Z = ax + by$

b) Constraints: These are the linear Inequalities or equations that restrict the value of Decision variables.

c) Decision Variables: The Variables which we take in LPP problem to take the Decision.

d) Non-negativity Constraints: The Decision variables which are greater than or equal to zero or non-negative values.

3. Enumerate the steps involved in Formulation of LPP.

The procedure for mathematical formulation of a linear programming problem consists of the following steps.

1. Identify the decision variables

2. Identify the objective function to be maximized or minimized and express it as a linear function of decision variables.

3. Identify the set of constraint conditions and express them as linear inequalities or equations in terms of the decision variables.

4. Which type of L.P.P. can be solved using graphical method?

The Graphical Method for solving Linear Programming Problems (LPP) can only be used when the problem involves two decision variables. This method is particularly effective for visualising the feasible region, constraints and the objective function

1.2 IN-TUTORIAL

1. A hotel has requested a manufacturer to produce pants and jackets for their boys. For materials, the manufacturer has $750m^2$ of cotton textile and $1000m^2$ of silk. Every pair of pants (1 unit) needs $2m^2$ of silk and $1m^2$ of cotton. Every jacket needs $1.5m^2$ of cotton and $1m^2$ of silk. The price of the pants is fixed at \$50 and the jacket, \$40. What is the number of pants and jackets that the manufacturer must give to the hotel so that these items obtain a maximum sale? Formulate the problem using mathematical modeling of LPP and define the objective function?

Solution:

i) Decision Variables: Let x_1 and x_2 denote the number of pants and number of jackets respectively.

ii) Objective Function:

Profit on x_1 pants $= 50x_1$

Profit on x_2 jackets $= 40x_2$

Let $z = 50x_1 + 40x_2$, which is the objective function.

$\text{Max } z = 50x_1 + 40x_2$

iii) Constraints:

We make the following table from given data.

Item	Pants	Jackets	Availability
Silk	2	1	1000
Cotton	1	1.5	750
Price	50	40	

$2x_1 + x_2 \leq 1000$

$x_1 + 1.5x_2 \leq 750$

iv) Non-negative Restrictions:

$\text{Max } Z = 50x_1 + 40x_2$

Subject to Constraints: $2x_1 + x_2 \leq 1000$

$x_1 + 1.5x_2 \leq 750$

Decision Variables: $x_1, x_2 > 0$

2. A transport company has two types of trucks, Type A and Type B. Type A has refrigerated capacity of $20m^3$ and a non-refrigerated capacity of $40m^3$ while Type B has refrigerated capacity of $30m^3$ and non-refrigerated capacity of $30m^3$. A grocer needs to hire trucks for the transport of $3000m^3$ of refrigerated stock and $4000m^3$ of non-refrigerated stock. The cost per kilometer of a Type A is \$30 and \$40 for Type B. How many trucks of each type should the grocer rent to achieve the minimum total cost? Formulate the problem using mathematical modeling of LPP and define the objective function?

Solution:

i) Decision Variables: Let x_1 be the trucks of Type A and x_2 be the trucks of Type B.

ii) Objective function:

Cost per kilometer of x_1 Type-A Trucks = 30

Cost per kilometer of x_2 Type-B Trucks = 40

Let $Z = 30x_1 + 40x_2$, which is the objective function.

$$\text{Min } Z = 30x_1 + 40x_2$$

	Type-A	Type-B	
ref	20	30	3000
non-ref	40	30	4000

iii) Conditions:

$$20x_1 + 30x_2 \geq 3000 \quad \text{—} (1)$$

$$40x_1 + 30x_2 \geq 4000 \quad \text{—} (2)$$

$$x_1, x_2 \geq 0$$

iv) Non-negative Restrictions:

$$\text{Min } Z = 30x_1 + 40x_2$$

Subject to constraints:

$$20x_1 + 30x_2 \geq 3000$$

$$40x_1 + 30x_2 \geq 4000$$

$$x_1, x_2 \geq 0$$

3. A hotel has requested a manufacturer to produce pillows and blankets for their room service. For materials, the manufacturer has $750m^2$ of cotton textile and $1000m^2$ of silk. Every pillow needs $2m^2$ of cotton and $1m^2$ of silk. Every blanket needs $2m^2$ of cotton and $5m^2$ of silk. The price of the pillow is fixed at \$5 and the blanket is fixed at \$10. What is the number of pillows and blankets that the manufacturer must give to the hotel so that these items obtain a maximum sale? Formulate and Solve using Python.

Solution:

Let x_1 be pillows and x_2 be blankets.

	Pillows	Blankets	
Cotton	2	2	750
Silk	1	5	1000

$\text{Max } z = 5x_1 + 10x_2$

$2x_1 + 2x_2 \leq 750$

$x_1 + 5x_2 \leq 1000$

The objective function: $\max z = 5x_1 + 10x_2$

Subject to constraints: $2x_1 + 2x_2 \leq 750$ &

$x_1 + 5x_2 \leq 1000$

4. An Industry makes two items of P and Q by using two devices X and Y. Processing time requires 50 hrs for item P on device X and 30 hrs requires on device Y. Processing time requires 24 hrs for item Q on device X and 33 hrs requires on device Y. At starting of the current week, 30 pieces of A and 90 pieces of B are available. Processing time that is available on device X is predict to be 40 hrs and on device Y is predict to be 35 hrs. Demand for P in the current week is predict to be 75 pieces and for Q is predict to be 95 pieces. Industry policy is to maximize the combined sum of the pieces of P and the pieces of Q in stock at the end of the week. Formulate the problem of deciding how much of each item to make in the current week as a linear program. Obtain the solution using graphical method.

Solution:

$x : P$

$y : Q$

	P	Q	
X	50	24	40
Y	30	33	35

Max $z = x P y$.

$50x + 4y \leq 40$

$30x + 33y \leq 35$

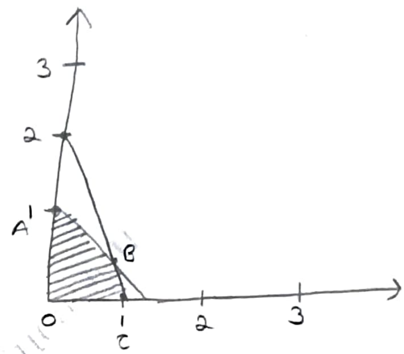
$x \leq 30$

$y \leq 90$

$x \geq 75$

$y \geq 95$

$x, y \geq 0$.



$30 \times (1)$ $1500x + 720y = 1200$

$1500x + 1650y = 1750$

$920y = 550$

$\boxed{y = 0.5}$

$\boxed{x = 0.6}$

$x = 0$ | $x = 0.8$

$y = 1.6$ | $y = 0$

$x = 0$ | $x = 1.1$

$y = 1$ | $y = 0$

$A \rightarrow (0,1) \rightarrow 1$

$B \rightarrow (0.6, 0.5) \rightarrow 1.1$ (max)

$C \rightarrow (0.8, 0) \rightarrow 0.8$

$\therefore$ The point $B(0.6, 0.5)$ has maximum value of 1.1.

5. Consider the following linear programming problem

$$\text{Maximize} \quad P = 7x + 12y$$
$$2x + 3y \leq 6$$
$$3x + 7y \leq 12$$

Obtain the solution using graphical method.

Solution:

Max $P = 7x + 12y$

$2x + 3y \leqslant 6$

$3x + 7y \leqslant 12$

$2x + 3y = 6$

$3x + 7y = 12$

① $x = 0$, $y = 2$ | $x = 3$, $y = 0$

② $x = 0$, $y = 1$ | $x = 4$, $y = 0$

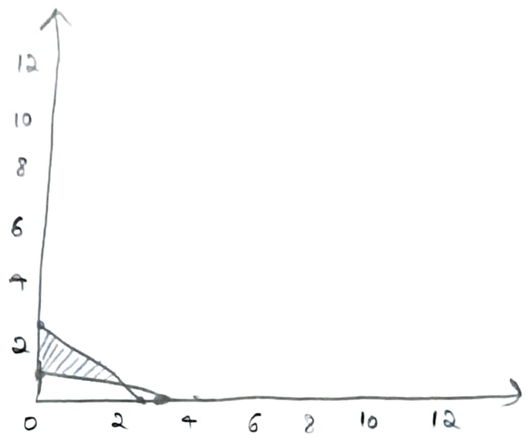
$3 \times ①$ $6x + 9y = 18$

$2 \times ②$ $6x + 14y = 24$

$5y = 6$

$\boxed{y = 1.2}$

$\boxed{x = 1.2}$



A → (0,1)

B → (1·2,1·2)

C → (3,0)

Max values are A → 12

B → 22·8(max)

C → 21·

∴ The point B(1·2,1·2) has maximum value of 22·8

.3 POST-TUTORIAL

1. What are the Advantages of Linear Programming Problem?
   Solution:

Linear Programming Problems (LPP) optimize the allocation of limited resources, helping to maximize profit or minimize costs. LPP supports informed decision-making through quantitative analysis to access the impact of changes

2. Applications of Linear Programming.
   Solution:

Engineering Industries:

Engineering Industries use linear programming to solve design and manufacturing problems and to get the maximum output from a given condition.

Manufacturing Industries:

Manufacturing Industries use linear programming to maximize the profit of the companies and to reduce the manufacturing cost.

Energy Industries:

Energy Companies use linear programming to optimize their production output.

Transportation Industries:

Linear programming is also used in transportation industries to find the path to minimize the cost of transportation.

3. A company owns two flour mills viz. A and B, which have different production capacities for high, medium and low-quality flour. The company has entered a contract to supply flour to a firm every month with at least 8, 12 and 24 quintals of high, medium and low quality respectively. It costs the company Rs.2000 and Rs.1500 per day to run mill A and B respectively. On a day, Mill A produces 6, 2 and 4 quintals of high, medium and low-quality flour. Mill B produces 2, 4 and 12 quintals of high, medium and low-quality flour respectively. How many days per month should each mill be operated in order to meet the contract order most economically? Formulate the LPP.

Solution:

Step 1: Decision Variables

x be the no. of days Mill A is operated per month.

y be the no. of days Mill B is operated per month.

Step 2: Objective function

Total cost of operating Mill A, Mill B

$$\text{Min } z = 2000x + 1500y$$

Step 3: Subject to constraints

High Quality Flour: $6x + 2y \geq 8$

Medium Quality Flour: $2x + 4y \geq 12$

Low Quality Flour: $4x + 12y \geq 24$

Step 4: LPP

$$\text{Min } z = 2000x + 1500y$$

Subject to:

$$6x + 2y \geq 8$$

$$2x + 4y \geq 12$$

$$4x + 12y \geq 24$$

where $x \geq 0, y \geq 0$

4. An advertising company plans its advertising strategy in three different media- television radio and magazines. Following data have been obtained from market survey. The com-

	Television	Radio	Magazine I	Magazine II
Cost of an advertising unit	Rs. 30,000	Rs. 20,000	Rs. 15,000	Rs. 10,000
No. of potential customers reached per unit	2,00,000	6,00,000	1,50,000	1,00,000
No. of female customers reached per unit	1,50,000	4,00,000	70,000	50,000

pany wants to spend no more than Rs. 4,50,000 on advertising. Following are the set of requirements that must be met:

(a) At least 1 million exposures take place among female customers.

(b) Advertising on magazines be limited to Rs. 1,50,000.

(c) The number of advertising units on television and radio should each be between 5 and 10.

Formulate the LPP.

Solution:

Step 1: Decision Variables

x be the no. of advertising units on television.

y be the no. of advertising units on radio.

z_1 be the no. of advertising units in magazine 1.

z_2 be the no. of advertising units in magazine 2.

Step 2: Objective function

$$\text{Max } Z = 200000x + 600000y + 150000z_1 + 100000z_2$$

Step 3: Constraints.

Budget:

$$30{,}000x + 20{,}000y + 15{,}000z_1 + 10{,}000z_2 \leq 4{,}50{,}000$$

Female customer exposure:

$$1{,}50{,}000x + 1{,}30{,}000y + 70{,}000z_1 + 50{,}000z_2 \geq 1{,}00{,}000$$

Magazine Budget:

$$15{,}000 z_1 + 10{,}000 z_2 \leq 1{,}50000$$

Non-negativity constraints, No of advertising units

$$5 \leq x \leq 10, \quad 5 \leq y \leq 10$$

$$x, y, z_1, z_2 \geq 0$$

step 4: LPP

$$\text{Max } z = 2{,}00{,}000x + 6{,}00{,}000y + 1{,}50000 z_1 + 1{,}00{,}000 z_2$$

Subject to:

$$30{,}000x + 20{,}000y + 15{,}000 z_1 + 10{,}000 z_2 \leq 4{,}50{,}000$$

$$1{,}50{,}000x + 1{,}50{,}000y + 70{,}000 z_1 + 50{,}000 z_2 \geq 10{,}00{,}000$$

$$15{,}000 z_1 + 10000 z_2 \leq 1{,}50{,}000$$

$$5 \leq x \leq 10, \quad 5 \leq y \leq 10$$

$x, y, z_1, z_2 \geq 0 \Rightarrow$ Non-negative constaints

5. A cabinet maker makes benches and desks. Each bench can be sold for a profit of \$30 and each desk for a profit of \$10. The cabinetmaker can afford to spend up to 40 hrs per week working and takes 6 hrs to make a bench and 3 hrs to make a desk. Customer demand requires that he makes at least 3 times as many desks as benches. Benches take up 4 times as much storage space as desks and there is room for at most four benches each week. Formulate this problem as a linear programming problem and solve it graphically.

olution:

Step 1: Decision Variables.

x: be the number of benches to be produced per week.

y: be the number of desks to be produced per week.

Step 2: Objective Function.

$\text{Max } z = 30x + 10y$

Step 3: Constraints.

Time:

$6x + 3y \leq 40$

Demand:

$y \geq 3x$

Storage Space:

$4x + y \leq 4$

Non-negative Constraint:

$x \geq 0, y \geq 0.$

Step 4: LPP

$\text{Max } z = 30x + 10y$

Subject to:

$6x + 3y \leq 40 \rightarrow l_1$

$3x - y \geq 0 \Rightarrow y \geq 3x \rightarrow l_2$

$4x + y \leq 4 \rightarrow l_3$

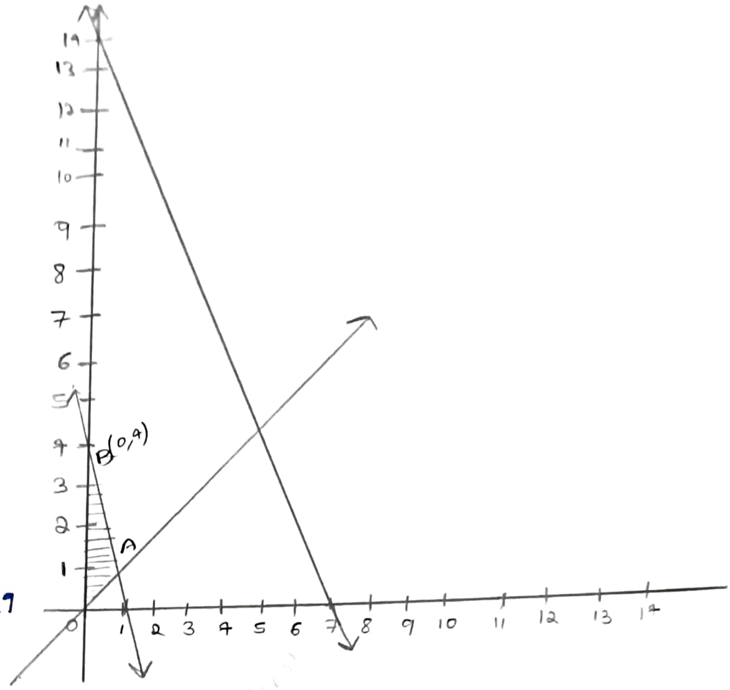
pts are

$l_1 \rightarrow (0, \frac{40}{3}), (\frac{40}{6}, 0)$

$l_2 \rightarrow (0,0), (0,0)$

$l_3 \rightarrow (0,4), (1,0)$

$A = (4/7, 12/7), B = (0,4), C = (0,0)$



$(0,0)$ $\quad 30(0)+10(0) = 0$

$(0,4)$ $\quad 30(0)+10(4) = 40$

$(4/7, 12/7)$ $\quad 30(4/7)+(12/7)10 = 34.29$

Optimum soln: $x=0, y=4$

Optimum value: $Z = 30x+10y$

$Z = 30(0)+10(4)$

$Z = 40$

For Evaluator's Use only

Evaluators Comments

Evaluator's Observation

Marks Secured ....48.. out of 50

Full Name of the Evaluator:

Signature of the Evaluator:

Date of Evaluation:

Tutorial 2

Simplex method and Principle of Duality

Date of the Session: ....

Learning outcomes:

- Understanding the key terms: feasible solution, feasible region and optimal solution.
- Understanding the limitations of graphical method and introduce the Simplex algorithm.
- Understand that an LPP consists of more number of constraints as compared to number of decision variables.
- Understanding the computational procedure can be considerably reduced by converting the LPP into a form called as DUAL and then solving it.

2.1 PRE-TUTORIAL

1. Which type of L.P.P. can be solved using Simplex method?

A) i) Linear

ii) Decision Variables

iii) Equality Constraints

iv) Feasible Starting Point

v) Bounded Feasible Region

vi) Non-negative Constraints

2. What do you mean by feasible region, feasible solution and optimal solution?

i) Feasible Region means also known as the feasible set; is the set of all possible Combinations of values for decision variables that satisfy all of problems constraints.

ii) Feasible Solution means is a specific set of values for decision variables that satisfies all the constraints of the linear programming problem, it is a point.

iii) Optimal Solution means the best possible solution within the feasible region, as determined by the obj function.

3. State the general rules for formulating a dual LPP from its primal?

i) Primal Problem

ii) Dual Variables

iii) Dual PF

iv) Dual Constraints

v) Dual Constraints computed variables

vi) Complete Dual CP problem.

2.2 IN-TUTORIAL

1. Consider the following linear programming problem

$$\text{Maximize} \quad P = 7x + 12y$$
$$2x + 3y \le 6$$
$$3x + 7y \le 12$$

Set up the Initial Simplex Tableau and obtain the solution.

Solution:

Let S_1, S_2 are slack variables.

becomes,

$\text{Max } P = 7x + 12y + 0S_1 + 0S_2$

subject to constraints:

$2x + 3y + S_1 + 0S_2 = 6$

$3x + 7y + 0S_1 + S_2 = 12$

where

$x_1, y_1, S_1, S_2 \geq 0$.

Initial Simplex Table:

C_B	B_v	X_B	x	y	s_1	s_2	Ratio
		C_j	7	12	0	0	
0	s_1	6	2	3	1	0	$6/3 = 2$
0	s_2	12	3	7	0	1	$12/7 = 1.2$
		Z_j	0	0	0	0	
		$C_j - Z_j$	$-+$	-12	0	0	
	s_1	$8/7$	$5/7$	0	1	$-3/7$	
	x_2	$12/7$	$5/7$	1	0	$1/7$	
	A_j		$17/7$	0	0	$12/7$	

$x_1 = 0$

$x_1 = 12/7 = 1.7$

$s_1 = 6/7$

$s_2 = 0$

$Z = \cancel{7x} + 2y$

$= 7(0) + 2 \times \frac{12}{7}$

$= \frac{24}{7}$

$=$

2. Find the dual problem for the given LPP model

$$\begin{aligned} \text{Minimize} \quad & C = 5x_1 + 2x_2 \\ \text{Subject to} \quad & x_1 + 3x_2 \geq 15, \\ & 2x_1 + x_2 \geq 20, \\ & x_1, x_2 \geq 0 \end{aligned}$$

Solution:

$$A = \begin{bmatrix} 1 & 5 \\ 2 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$$

The Dual Problem:

$$\text{Max } z = 15y_1 + 20y_2$$

$$y_1 + 2y_2 \leq 5$$

$$3y_1 + y_2 \leq 2$$

where $y_1, y_2 \geq 0$

3. State the dual for the following LPP and hence solve LPP.

$$\begin{aligned} \text{Minimize : } & C = 21x_1 + 50x_2 \\ \text{Subject to : } & 2x_1 + 5x_2 \geq 12, \\ & 3x_1 + 7x_2 \geq 17, \\ & x_1, x_2 \geq 0 \end{aligned}$$

Solution:

$$A = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix} \qquad A^T = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$$

The Dual Function:

$$\text{Max } z = 12y_1 + 17y_2$$

$$2y_1 + 3y_2 \leq 21$$

$$5y_1 + 7y_2 \leq 50$$

where $y_1, y_2 \geq 0$

Bv	C_B	X_B	x_1	x_2	S1	S2	Ratio
S1	0	21	2	3	1	0	21/3
S2	0	50	5	7	0	1	50/7
x_2	0	7	−12 2/3 1/3	−17 1 0	0 1/3 −7/3	0 0 1	105 1/3 = 3
	A_j		−2/3	0	17/3	0	
x_2	0	5	0	1	5/3	−2	
x	0	3	1	0	−7	3	
			0	0	1	2	

$x_1 = 3$

$x = 5$

$Z = 12y_1 + 17y_2$

$= 12(3) + 17(5)$

$= 36 + 85$

$= 121$

1. A XYZ company is hired by a retailer to transport goods from its store rooms in A and B to its outlet stores in C and D. The XYZ company is contracted to deliver 30 vehicles each month to deliver goods. The company determines that it will need to send at least 12 of the vehicles to the 'C' location and at least 13 vehicles to the 'D' location. At least 15 vehicles can come from the A storeroom and at least 20 vehicles can come from the 'B'. The truck company wants to minimize the number of miles placed on its trucks. How many trucks should the send out from each location and to which outlets should they send them?

	A	B
C	24 ml	31 ml
D	20 ml	38 ml

Formulate its dual and solve the LPP.

Solution:

w is dual variable of Room A.

x Room B

y Room C

z Room D

of,

Min $z = 15w + 20x + 10y + 13z$

Constraints:

$w + y \geq 2$

$x + z \geq 20$

where $w, x, y, z \geq 0$

Let

x_{AC} : A to C = 24

x_{AD} : A to D = 20

x_{BC} : B to C = 31

x_{BD} : B to D = 38

Primal:

Min $z = 24x_{AC} + 20x_{AD} + 31x_{BC} + 38x_{BD}$

$x_{AC} + x_{BC} \geq 12$

$x_{AD} + x_{BD} \geq 13$

$x_{AC} + x_{AD} \geq 15$

$x_{BC} + x_{BD} \geq 20$

$x_{AC} + x_{AD} + x_{BC} + x_{BD} \geq 30$

$x_{AC}, x_{AD}, x_{BC}, x_{BD} \geq 0$

Dual:

Max $z = 12y_{AC} + 13y_{AD} + 15y_{BC} + 20y_{BD}$

Let $u_1 = x_{AC} + x_{BC}$

$u_2 = x_{AD} + x_{BD}$

$v_1 = x_{AC} + x_{AD}$

$v_2 = x_{BC} + x_{BD}$

$W = x_{AC} + x_{AD} + x_{BC} + x_{BD}$

$\max z = 12U_1 + 13U_2 + 15V_1 + 20V_2 - 30W$

$U_1 + V_1 \leq 24$

$U_1 + V_2 \leq 31$

$U_2 + V_1 \leq 20$

$U_2 + V_2 \leq 38$

$W, U_1, U_2, V_1, V_2 \geq 0$

Graphical method is not applicable

Simplex method is applicable

	C_j	12	13	15	20	-30		
B.V	CV	U_1	U_2	V_1	V_2	W	Sol^n	Ratio
0	S1	1	0	1	0	0	24	24
0	S2	1	0	0	1	0	31	31
0	S3	0	1	1	0	0	20	20
0	S4	0	1	0	1	0	38	38
	Z_j	0	0	0	0	0		
	$C_j - Z_j$	12	13	15	20	30		

Entering variable : W

Leaving variable : S3

2.3 POST-TUTORIAL

1. Solve LPP using the simplex method

$$\text{Maximize} \quad Z = 3x_1 + 5x_2$$

Subject to

$$3x_1 + 2x_2 \leq 18$$

$$x_1 \leq 4$$

$$2x_2 \leq 12$$

$$x_1, x_2 \geq 0$$

Solution:

$Z = 3x_1 + 5x_2 + 0S_1 + 0S_2 + 0S_3$

Subject to :

$3x_1 + 2x_2 + S1 = 18$

$x_1 + S2 = 4$

$2x_2 + S_3 = 12$

$x_1, x_2, S1, S2, S3 \geq 0$

BV	C_B	X_B	x_1	x_2	S1	S2	S3	θ
S1	0	18	3	2	1	0	0	18/2 = 9
S2	0	14	1	0	0	1	0	12/2 = 6
S3	0	12	0	2	0	0	1	12/2 = 6
	A_j	-3	-5	0	0	0		
S1	0	6	5	0	1	0	-1	
S2	0	7	1	0	0	1	0	
S3	0	6	0	1	0	0	0	
	A_j		3	0	0	0	5/2	

$x_1 = 0$

$x_2 = 6$

$S_1 = 6$

$S_2 = 0$

$S_3 = 0$

$Z = 0$

2 Find its dual and obtain the optimal solution for minimization problem

$$\begin{aligned} \text{Minimize} \quad & C = 16x_1 + 8x_2 + 4x_3 \\ \text{Subject to} \quad & 3x_1 + 2x_2 + 2x_3 \geq 16 \\ & 4x_1 + 3x_2 + x_3 \geq 14 \\ & 5x_1 + 3x_2 + x_3 \geq 12 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Solution

Primal Problem:

Min $C = 16x_1 + 8x_2 + 4x_3$

Subject to:

$3x_1 + 2x_2 + 2x_3 \geqslant 16$

$4x_1 + 3x_2 + x_3 \geqslant 14$

$5x_1 + 3x_2 + x_3 \geqslant 12$

$x_1, x_2, x_3 \geqslant 0$.

Dual Problem:

Max $C = 16y_1 + 14y_2 + 12y_3$

Subject to

$3y_1 + 4y_2 + 5y_3 \leqslant 16$

$2y_1 + 3y_2 + 3y_3 \leqslant 8$

$2y_1 + y_2 + y_3 \leqslant 4$

$x_1, x_2, x_3 \geqslant 0$.

mplex Method:

CBV	BV	x_1	x_2	x_3	S1	S2	S3	Sol^n	Ratio
	C_j	16	8	4	0	0	0		
0	S1	3	2	2	1	0	0	16	16/3
0	S2	4	3	1	0	1	0	14	14/4
0	S3	5	3	1	0	0	1	12	12/5
	Z_j	0	0	0	0	0	0	0	
	C_j-Z_j	16	8	4	0	0	0		

CBV	BV	x_1	x_2	x_3	S1	S2	S3	Sol^n	Ratio
	C_j	16	8	4	0	0	0		
0	S1	0	1/5	7/5	1	0	-3/5	44/5	6.2
0	S2	0	3/5	1/5	0	1	-4/5	22/5	22
16	x_1	1	3/5	1/5	0	0	1/5	12/5	12
	Z_j	16	48/5	16/5	0	0	16/5	192/5	
	C_j-Z_j	0	-1.6	0.8	0	0	-16/5		

CBV	B.V	x_1	x_2	x_3	S1	S2	S3	Sol^n	Ratio
	C_j	16	8	4	0	0	0		
4	x_3	0	0.1	1	5/7	0	0.4	6.2	
0	S2	0	0.01	0	0.14	1	0.1	1.8	
16	x_1	1	0.4	1	0.14	0	0.5	0.4	
	Z_j	16	8	20	5	0	2.4	31.2	
	C_j-Z_j	0	0	-16	-5	0	-2.4		

∴ The values are

$x_3 : 6.2$

$s_2 : 1.8$

$x_1 : 0.4$

$Z_j : 31.2$

3. A producer of Healthy food makes two important and secret ingredients that goes into their human food, named as a Healthy Man and Common Man. Each kg of Healthy Man contains 300 g of vitamins, 400 g of protein, and 100 g of carbs. Each kg of common man contains 100 g of vitamins, 300 g of protein, and 200 g of carbs. Guidelines for minimum nutritional that require a mixture made from these ingredients contain at least 900 g of vitamins, 2400 g of protein, and 800 g of carbs. Healthy Man costs \$2 per kg to produce and Common Man costs \$1.25 per kg to produce. Find the number of kgs of each ingredient that should be produced in order to minimize cost. Obtain its Dual and solve.

Solution:

<u>Primal:</u>

$\text{Min } C = 2x_1 + 1.25x_2$

Subject to:

$300x_1 + 100x_2 \geq 900$

$400x_1 + 300x_2 \geq 2200$

$100x_1 + 200x_2 \geq 800$

where $x_1, x_2 \geq 0.$

Dual:

$\text{Max } C = 900y_1 + 2200y_2 + 800y_3$

Subject to:

$300y_1 + 400y_2 + 100y_3 \leq 2$

$100y_1 + 300y_2 + 200y_3 \leq 1.25$

where $y_1, y_2, y_3 \geq 0.$

Simplex Method:

	C_j	300	2400	300	0	0		
CBV	BV	y_1	y_2	y_3	S1	S2	Sol^n	Ratio
0	S1	300	400	100	1	0	2	2/400
0	S2	100	300	200	0	1	1.25	1.25/300
	L_j	0	0	0	0	0	0	
	C_j-L_j	300	2400	300	0	0		

	C_j	300	2400	300	0	0		
CBV	B.V	Y_1	Y_2	y_3	S1	S2	Sol^n	Ratio
0	S1	500/300	0	500/300	1	$-1/30$	0.4	
2400	y_2	1/3	1	2/3	0	1/3	0.004	
	L_j	2400/3	2400	4800/3	0	2400/3	9.6	
	C_j-L_j	-500	0	-1300	0	$-2400/3$		

∴ The values of S1 : 0.4 & y_2 : 0.004 & L_j : 9.6

For Evaluator's Use only

Evaluators Comments	Evaluator's Observation
	Marks Secured ....48.... out of 50
	Full Name of the Evaluator:
	Signature of the Evaluator:
	Date of Evaluation:

Tutorial 3

Transportation problem

Date of the Session: ....

Learning outcomes:

- Understanding the problem of transporting/shipping the commodities from the industry to the destinations with the least possible cost while satisfying the supply and demand limits.

3.1 PRE-TUTORIAL

1. List out different types of transportation methods.

→ North-west Corner

→ Least Cost

→ Row Minimum

→ Column Minimum

→ Vogel's Approximation.

2. List out the steps involved in solving the Transportation problem using North-West corner rule?

→ Setup problem

→ Initialize variables

→ Repeat Allocation

→ Calculate Total Cost.

→ Optimality.

3. List out the steps involved in solving the Transportation problem using Row Minimum method.

→ Problem setup.

→ Initialization

→ Allocation

→ Iterate

→ Total Cost

→ Optimality.

3.2 IN-TUTORIAL

1. Luminous lamps have three factories - F1, F2, and F3 with production capacity 30, 50, and 20 units per week respectively. These units are to be shipped to four warehouses W1, W2, W3, and W4 with requirement of 20, 40, 30, and 10 units per week respectively. The transportation costs (in Rs.) per unit between factories and warehouses are given below. Solve Transportation problem using NW corner rule.

Factory	Warehouse W1	W2	W3	W4	Supply
F1	1	2	1	4	30
F2	3	3	2	1	50
F3	4	2	5	9	20
Demand	20	40	30	10	

Solution:

$\boxed{20}$ 1	$\boxed{10}$ 2	1	4	~~30~~ ~~10~~ 0
3	$\boxed{30}$ 3	$\boxed{20}$ 2	1	~~50~~ ~~20~~ 0
4	2	$\boxed{10}$ 5	$\boxed{10}$ 9	~~20~~ ~~10~~ 0
~~20~~ 0	~~40~~ ~~30~~ 0	~~30~~ ~~10~~ 0	~~10~~ 0	100

$1\times 20 + 2\times 10 + 3\times 30 + 2\times 20 + 5\times 10 + 9\times 10$

$= 20 + 20 + 90 + 40 + 50 + 90$

$= 310$

2 The Ushodaya departmental store has three plants located throughout a state with production capacity 80, 60 and 70 kilo grams of rice. Each day the firm must furnish its four retail shops R1, R2, R3, R4 with at least 40, 60, 50, and 60 gallons respectively. The transportation costs (in Rs.) are given below. Solve Transportation problem using Row Minimum method.

Store	Retailshop				Supply
	1	2	3	4	
1	3	5	7	6	80
2	2	5	8	2	60
3	3	6	9	2	70
Demand	40	60	50	60	

Solution:

$\boxed{40}$ 3	$\boxed{40}$ 5	7	6	~~80~~ ~~40~~ 0
2	$\boxed{20}$ 5	$\boxed{40}$ 8	2	~~60~~ ~~40~~ 0
3	6	$\boxed{10}$ 9	$\boxed{60}$ 2	~~70~~ ~~60~~ 0
~~40~~ 0	~~60~~ ~~20~~ 0	~~50~~ ~~10~~ 0	~~60~~ 0	210

$3\times40+5\times40+5\times20+8\times40+9\times10+2\times60$

[40] 3	[40] 5	7	6	~~80~~ ~~40~~ 0
2	5	8	[60] 2	~~60~~ 0
3	[20] 6	[50] 9	2	~~70~~ ~~50~~ 0
~~40~~ 0	~~60~~ ~~20~~ 0	~~50~~ 0	~~60~~ 0	

$3 \times 40 + 5 \times 40 + 2 \times 60 + 6 \times 20 + 9 \times 50$

$= 120 + 200 + 120 + 120 + 450$

$= 910$

3.3 POST-TUTORIAL

1. KL Universitybranches located at Vijayawada, Hyderabad, and Chennai. KL University provides course material in printed form at these locations with capacities 15, 30 and 20 units at Vijayawada, Hyderabad, and Chennai respectively. The university distributes the course material to students located at three locations Bangalore, Hyderabad and Coimbatore. The demand of the students is 5, 20 and 40 units for Bangalore, Hyderabad and Coimbatore respectively.The cost of transportation per unit varies between different supply points and destination points. The transportation costs are given in the table. The management of KL University would like to determine minimum transportation cost.Solve Transportation problem using Column Minimum method in Linear Programming using python.

U/S	BGR	HYD	CON	Supply
BZA	15	60	35	15
HYD	45	30	60	30
CHE	30	90	20	20
Demand	5	20	40	

Solution:

X	[5] 15	60	[10] 35	15 10 0
X	45	[20] 30	[10] 60	30 10 0
X	30	90	[20] 20	20 0
	5 0	20 0	40 20 10 0	65

15×5 + 30×20 + 20×20 + 35×10 + 60×10

= 75 + 600 + 400 + 350 + 600

= 1600 + 425

= 2025

350
75
425

1600
425
2025

2. The distribution manager of a company needs to minimize global transport costs between a set of three factories (supply points) S1, S2, and S3, and a set of four distributors (demand points) D1, D2, D3, and D4. The following table shows the transportation cost from each supply point to every demand point, the supply of the product at the supply points, and the demand of the product at the demand points. Solve Transportation problem using Column Minimum method in Linear Programming.

F/D	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	34

Solution:

$\boxed{5}$ 19	30	50	$\boxed{2}$ 10	~~7~~ ~~2~~ 0
70	30	$\boxed{7}$ 40	$\boxed{2}$ 60	~~9~~ ~~2~~ 0
40	$\boxed{8}$ 8	70	$\boxed{10}$ 20	~~18~~ ~~10~~ 0
~~5~~ 0	~~8~~ 0	~~7~~ 0	~~14~~ ~~2~~ ~~12~~ 0	34

$19 \times 5 + 8 \times 8 + 40 \times 7 + 10 \times 2 + 20 \times 10 + 60 \times 2$

$= 95 + 64 + 280 + 20 + 200 + 120$

$= 300 + 320 + 95 + 64$

$= 620 + 159$

$= 779$

For Evaluator's Use only

Evaluators Comments	Evaluator's Observation
	Marks Secured ....49.... out of 50
	Full Name of the Evaluator:
	Signature of the Evaluator:
	Date of Evaluation:

Tutorial 4

Gomory's Cut-Plane Method

Date of the Session: ....

<u>**Learning outcomes**</u>:

- If the optimal solution is integers, then problem is solved. Otherwise, add Gomory's constraint (cut) is added to optimal solution Now new problem is solved using dual simplex method The method terminates as soon as optimal solution become integers.

4.1 PRE-TUTORIAL

1. Enumerate the steps involved in Gomory's cutting plane method.

Step 1: Build the Simplex Table

Step 2: check out the conditions & build it.

Step 3: Initially follow the simplex.

Step 4: Get the corresponding values.

Step 5: If values are integer then stop next all steps.

Step 6: If not by considering the max fractional value from it.

Step 7: Generate the source row.

Step 8: Update the table.

Step 9: Solve it.

Step 10: We'll get corresponding values.

4.2 IN-TUTORIAL

1. Find solution using integer simplex method (Gomory's cutting plane method)

$$\text{Maximize} Z = x_1 + x_2$$

subject to

$$3x_1 + 2x_2 \leq 5$$

$$x_2 \leq 2$$

and x_1, x_2 are non-negative integers

Solution:

$\text{Max } z = x_1 + x_2$

$3x_1 + 2x_2 \leq 5$

$x_2 \leq 2.$

$\text{Max}(z) = x_1 + x_2 + 0S1 + 0S2.$

$\Rightarrow 3x_1 + 2x_2 + S1 = 5$

$0x_1 + x_2 + S2 = 2$

Initial Simplex Table

C_B	C_j	1	1	0	0	Sol	Ratio
	B.V	x_1	x_2	S1	S2		
0	S1	3	2	1	0	5	5/3
0	S2	0	1	0	0	2	2/0 = ∞
	Z_j	0	0	0	0		
	$C_j - Z_j$	1	1	0	0		

C_B	C_j	1	1	0	0		
	B.V	x_1	x_2	S1	S2	Sol	Ratio
1	x_1	1	2/3	1/3	0	5/3	$\frac{5}{3}\times\frac{3}{2}=\frac{5}{2}$
0	S2	0	1	0	0	2	2
	Z_j	1	2/3	1/3	0		
	$C_j - Z_j$	0	1/3	−1/3	0		

C_B	C_j	1	1	0	0		
	B.V	x_1	x_2	S1	S2	Sol	Ratio
1	x1	1	0	$\frac{1}{3}$	0	1/3	
1	x2	0	1	0	0	2	
	Z_j	1	1	$\frac{1}{3}$	0		
	$C_j - Z_j$	0	0	$\frac{-1}{3}$	0		

$x_1 = 0+\frac{1}{3} = \frac{1}{3}$, $x_2 = 2+0 = 2$

To obtain an optimum integer we have to add gomory constant.

$\text{Max}\left(\frac{1}{3}, 0\right) = \frac{1}{3}$

x is source row.

$\frac{1}{3} = x_1 + 0x_2 + \frac{1}{3}s1 + \frac{0}{3}s2$.

Fractional cut constraint

$\frac{s1}{3} + \frac{0s2}{3} \geq \frac{1}{3}$

$-\frac{s1}{3} - \frac{0s2}{3} + G_1 = \frac{-1}{3}$

C_B	C_j	1	1	0	0	0	Sol	Ratio
	BV	x_1	x_2	S1	S2	G1		
1	x_1	1	0	$\frac{1}{3}$	0	0	$1/3$	1
1	x_2	0	1	0	0	0	2	∞
0	G1	0	0	$\frac{-1}{3}$	0	1	$-1/3$	1
	Z_j	1	1	$\frac{1}{3}$	0	0		
	$C_j - Z_j$	0	0	$\frac{-1}{3}$	0	0		

C_B	C_j	1	1	0	0	0	Sol
	B.V	x_1	x_2	S1	S2	G1	
1	x_1	1	0	0	0	1	0
1	x_2	0	1	0	0	0	2
0	S1	0	0	1	0	-3	1
	Z_j	1	1	0	0	1	
	$C_j - Z_j$	0	0	0	0	-1	

$\therefore x_1 = 0, x_2 = 2$ Max $= 2$

4.3 POST-TUTORIAL

1. Consider the following integer programming problem

$$\text{Maximize } Z = 14x_1 + 16x_2$$

Subject to the constraints

$$4x_1 + 3x_2 \le 12$$
$$6x_1 + 8x_2 \le 24$$

and $x_1, x_2 \ge 0$ and are integers

Solve the problem by Gomory's cutting plane method

$Z = 14x_1 + 16x_2 + 0s1 + 0s2$

$4x_1 + 3x_2 + s1 = 12$

$6x_1 + 8x_2 + s2 = 12$

C_B	C_j	14	16	0	0		
	B.V	x_1	x_2	s1	s2	Sol	Ratio
0	s1	4	3	1	0	12	4
0	s2	3	4	0	1	12	3
	Z_j	0	0	0	0		
	$C_j - Z_j$	14	16	0	0		

C_B	C_j	14	16	0	0		
	B.V	x_1	x_2	s1	s2	Sol	Ratio
0	s1	7/4	0	1	-3/4	3	12/7
16	x_2	3/4	1	0	1/4	3	12/3
	Z_j	12	16	0	4		
	$C_j - Z_j$	2	0	0	-4		

C_B	C_j	14	16	0	0	Sol	Ratio
	B.V	x_1	x_2	S_1	S_2		
14	x_1	1	0	$4/7$	$-3/7$	$12/7$	
16	x_2	0	1	$-3/7$	$4/7$	$12/7$	
	Z_j	14	16	2	3		
	$C_j - Z_j$	0	0	-2	-3		

$x_1 = \frac{12}{7}$, $x_2 = \frac{12}{7}$

$Max\left(\frac{12}{7}, \frac{12}{7}\right)$ = I'm considering x_2 is the source row.

$$\frac{-3}{7} = -Z + \frac{11}{7}$$

$$\frac{12}{7} = x_2 - 2S_1 + \frac{11}{7}S_1 + \frac{4}{7}S_2$$

fractional cut constraint

$$\frac{11}{7}S_1 + \frac{4}{7}S_2 \geq \frac{12}{7}$$

$$-\frac{11}{7}S_1 - \frac{4}{7}S_2 \leq -\frac{12}{7}$$

$$-\frac{11}{7}S_1 - \frac{4}{7}S_2 + G_1 = \frac{-12}{7}$$

C_B	C_j	14	16	0	0	0		
	B.V	x_1	x_2	S1	S2	G1	Sol	Ratio
14	x_1	1	0	$-4/7$	$-3/7$	0	$12/7$	3
16	x_2	0	1	$-3/7$	$4/7$	0	$12/7$	-4
0	G1	0	0	$-11/7$	$-11/7$	1	$12/7$	
	Z_j	14	16	2	3	0		
	$C_j - Z_j$	0	0	-2	-3	0		

C_B	C_j	14	16	0	0	0	
	B.V	x_1	x_2	S1	S2	G1	Sol
14	x_1	1	0	0	$-\frac{7}{11}$	$\frac{21}{11}$	2
16	x_2	0	0	0	$\frac{4}{11}$	$\frac{7}{11}$	1
0	G1	0	0	1	$4/11$	$-7/11$	$12/11$

Max. $x_1 = 2$, $x_2 = 1$

$14x_1 + 16x_2$

$28 + 16 = 44$

For Evaluator's Use only

Evaluators Comments	Evaluator's Observation
	Marks Secured ...49... ~~out of~~ 50
	Full Name of the Evaluator:
	Signature of the Evaluator:
	Date of Evaluation:

Tutorial 5

Branch and Bound Method

Date of the Session: ....

Learning outcomes:

- Understanding to do the process of solving branch and bound method.
- Understanding to do the process of Gomory's cutting plane method.

5.1 PRE-TUTORIAL

1. What is branch and bound Technique?

The branch and bound Technique systematically explains solution spaces by branching into sub-problems & pruning paths that exceed known bounds to find optimal solution efficiently.

2. Which strategy can be used to solve branch and bound problem?

We can use simplex method & graphical method. Graphical method is suitable for only two variables. But where as simplex-method for 2 & more variables.

5.2 IN-TUTORIAL

1 Discrete Optimization using Cutting Plane method Solve the integer programming problem

$$\text{Maximize} \quad Z = 3x_1 + x_2 + 3x_3$$

Subject to:

$$-x_1 + 2x_2 + x_3 \leq 4$$

$$2x_2 - \frac{3}{2}x_3 \leq 1$$

$$x_1 - 3x_2 + 2x_3 \leq 3$$

Where $x_1, x_2, x_3 \geq 0$ and integer. Get the optimal solution as an integer value using Gomory's cutting plane method.

Solution:

		Cj	3	1	3	0	0	0	
CB	B.V	XB	x_1	x_2	x_3	S1	S2	S3	Ratio
0	S1	7.5	0	0	2.25	1	0.5	1	3.35
1	x_2	0.5	0	1	-0.75	0	0.5	0	–
3	x_1	4.5	1	0	-0.25	0	1.5	1	–
	Zj		3	1	1.5	0	5	3	
	Cj-Zj		0	0	-4.5	0	5	3	

		Cj	3	1	3	0	0	0	
CB	B.V	XB	x_1	x_2	x_3	S1	S2	S3	Ratio
3	x_3	3.33	0	0	1	0.44	0.22	0.44	
1	x_2	3	0	1	0	0.33	0.66	0.33	
3	x_1	5.33	1	0	0	0.11	1.55	0.11	
	Zj		3	1	3	2	6	5	
	Cj-Zj		0	0	0	2	6	5	

C_B	B.V	C_j / X_B	3 / x_1	1 / x_2	3 / x_3	0 / S1	0 / S2	0 / S3	Ratio
3	x_3	3.33	0	0	1	0.44	0.22	0.44	
1	x_2	3	0	1	0	0.33	0.66	0.33	
3	x_1	5.33	1	0	0	0.11	1.56	1.11	
		Z_j	3	1	3	2	6	5	
		$C_j - Z_j$	0	0	0	2	6	5	

C_B	B.V	X_B	x_1	x_2	x_3	S1	S2	S3	G1	G2
3	x_3	3	0	0	1	0	0	0	1	0
1	x_2	2.75	0	1	0	0	0.5	0	0.75	0
3	x_1	5.25	1	0	0	0	1.5	1	0.25	0
0	S1	0.75	0	0	0	1	0.5	1	-2.25	0
0	S2	-0.75	0	0	0	0	-0.5	0	-0.75	1
	Z_j		3	1	3	0	5	3	4.5	0
	$C_j - Z_j$		0	0	0	0	5	3	4.5	0
	Ratio		–	–	–	–	–	–	–	–

C_B	B.V	X_B	X_1	X_2	X_3	S1	S2	S3	G1
3	x_3	2	0	0	1	0	-0.667	0	0
1	x_2	2	0	1	0	0	0	0	0
3	x_1	5	1	0	0	0	1.33	1	0
0	S1	3	0	0	0	1	2	1	0
0	G1	1	0	0	0	1	0.66	0	1
	Z_j		3		03	0	2	3	0
	$C_j - Z_j$		0	0	0	0	2	3	0
	Ratio		–	–	–	–	–	–	–

Since all $Z_j - C_j = 0$

$x_1 = 5, x_2 = 2, x_3 = 2$ $\quad$ Max $Z = 23$

2. Use Branch and Bound method to

$$\text{Maximize} \quad Z = 3x_1 + 5x_2$$

Subject to

$$2x_1 + 4x_2 \leq 25$$
$$x_1 \leq 8$$
$$2x_2 \leq 10$$

Where x_1, x_2 are non-negative integers.

Solution:

Given,

$2x_1 + 4x_2 \leq 25$

$x_1 + 2x_2 \leq 18$

& $\max Z = 3x_1 + 5x_2$

$2x_1 + 4x_2 = 25$

$x_1 = 0, x_2 = \frac{25}{4}$

$4x_2 = 25$

$x_2 = \frac{25}{4}$

$x_1 = \frac{25}{2}, x_2 = 0$

$2x_1 = 25$

$x_1 = \frac{25}{2}$

Points are

$\left(0, \frac{25}{4}\right) \left(\frac{25}{2}, 0\right)$

$x_1 + 2x_2 = 18$

$x_1 = 0, x_2 = 9$

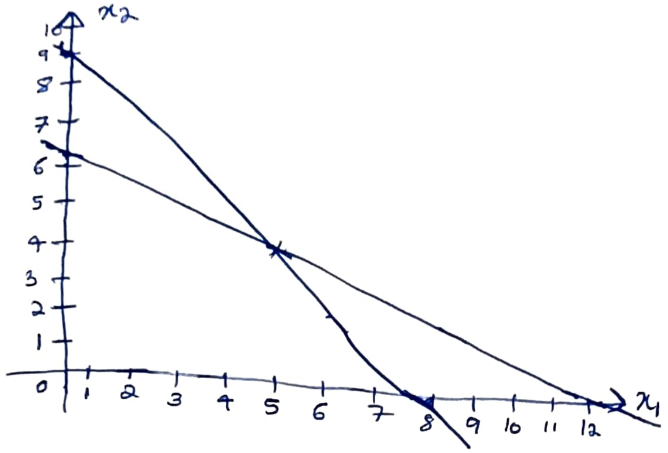
$2x_2 = 18$

$x_2 = 9$

$x_1 = 18, x_2 = 0$

$x_1 = 8$

$(0, 9), (18, 0)$



By graphical method

$\therefore$ Intersection point is (5,4).

Max $z = 3(5)+5(4)$

$= 35$

$\therefore$ $\boxed{\text{Max } z = 35}$

5.3 POST-TUTORIAL

1. Explain Gomory's method for solving an Integer Programming Problem and hence solve the following

$$\text{Maximize} \quad Z = 2x_1 + 6x_2$$
Subject to:
$$3x_1 + x_2 < 5$$
$$4x_1 + x_2 < 9$$

Where $x_1, x_2 \geq 0$ and are integers.

Solution:

$Z = 2x_1 + 6x_2 + 0s_1 + 0s_2$

$3x_1 + x_2 + s_1 = 5$, $4x_1 + x_2 + s_2 = 9$

C_B	C_j	2	6	0	0		
	B.V	x_1	x_2	S1	S2	Sol	Ratio
0	S1	3	1	1	0	5	$5/1 = 5$
0	S2	4	1	0	1	9	$9/1 = 9$
	Z_j	0	0	0	0		
	$C_j - Z_j$	2	6	0	0		

C_B	C_j	2	6	0	0		
	B.V	x_1	x_2	S1	S2	Sol	Ratio
6	x2	3	1	1	0	5	
0	S2	1	0	-1	1	7	
	Z_j	18	6	6	0		
	$C_j - Z_j$	-16	0	-6	0		

$\therefore x_1 = 4,\ x_2 = 5$

$Z = 2(4) + 6(5)$

$= 8 + 30$

$= 38$

For Evaluator's Use only

Evaluators Comments	Evaluator's Observation
	Marks Secured ...49... out of 50 Full Name of the Evaluator: Signature of the Evaluator: Date of Evaluation:

Tutorial 6

Dynamic Programming

Date of the Session: ....

6.1 PRE-TUTORIAL

1. What is Dynamic Programming?

Dynamic Programming is a technique that breaks the problems into sub-problems & save the result for future purposes. So that we do not need to compute the result again.

• The main use of dynamic programming is to solve optimization.

Limitations: The method is applicable to only those problems which pos.

2. State the different types of Dynamic Programming?

The different types of dynamic programming are knapsack & travelling Salesman.

6.2 IN-TUTORIAL

1. Maximize the profit for the 0/1 knapsack problem using dynamic programming when W = 10

Profit	Weight
10	5
40	4
30	6
50	3

Given $n=4$; $w=10$

$S^{i+1} = S^{i} \cup S^{i}_{1}$

$S^{1} = S^{0} \cup S^{0}_{1}$

$S^{0} = \{(0,0)\}$

$S^{0}_{1} = \{(10,5)\}$

$S^{1} = \{(0,0),(10,5)\}$

$S^{2} = S^{1} \cup S^{1}_{1} \Rightarrow S^{1} = \{(0,0),(10,5)\}$

$S^{1}_{1} = \{(40,4),(50,9)\}$

$S^{3} = S^{2} \cup S^{2}_{1}$

$S^{3} = \{(30,6),(40,11),(40,4),(50,9),(30,6),(70,10)\}$

$S^{4} = S^{3} \cup S^{3}_{1}$

$S^{4} = \{(0,0),(10,5),(40,4),(50,9),(30,6),(70,10),(50,3),(60,8),(90,7)\}$

$x^{4}=1$ $S^{4} \in (90,7)$ ss $S^{3} \notin (90,7)$

$x^{3}=0$ $S^{3} \in (40,4)$ ss $S^{2} \in (40,4)$

$x^{2}=1$ $S^{2} \in (40,4)$ ss $S^{1} \notin (40,4)$

$x^{1}=0$ $S^{1} \in (0,0)$ ss $S^{0} \in (0,0)$

$40+50=90$

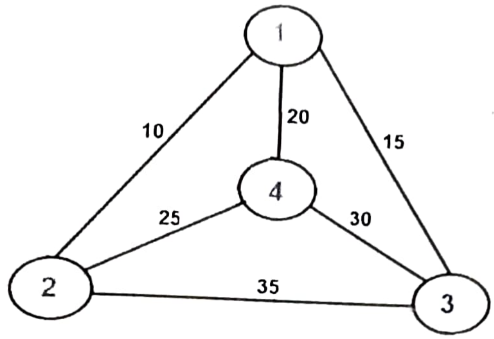
6.3 POST-TUTORIAL

1. Explain the concept of travelling salesman problem with real-time example

Solution

The travelling salesman problem (TSO) is a classic optimization problem in computer science & operations research. The goal is to find the shortest possible route that allows a salesman to visit a set of cities, exactly & return to the standing city.

2. Given a set of cities and the distance between every pair of cities, the problem is to find the shortest possible route that visits every city exactly once and returns to the starting point



Solution:

$$C_0(V,E) = \begin{cases} 0 & \text{if } i=j \\ c_{ij} & \text{if } (i,j) \end{cases}$$

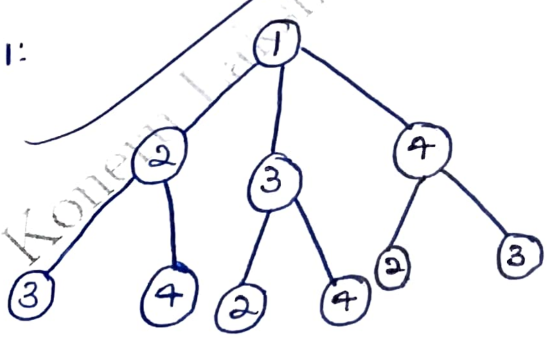
$$C(i,V) = \min\{d[i,j] + C(j, V-\{j\})\}$$

$$i \in V \ \& \ j \in V$$

Sol·i) First of all we will split the Graph ① into sub-problem.

We are choosing node-1 initial node.

step 1:



Step 2:

$$C_0\{2,\{3\},1\} = \min[d[2,3] + C(3,\phi,1)]$$

$$\min\{9+6\} = 15.$$

$$C_0\{2,\{4\},1\} = \min\{d[2,4] + C(4,\phi,1)\}$$

$$= \min\{10+8\} = 18$$

$$Co\{3,\{2\},1\} = \min\{d[3,2] + c\{2,\phi,1\}\}$$
$$= \{13+5\} = 18$$

$$Co\{3,\{4\},1\} = \min\{d[3,4] + c[4,\phi,1]\}$$
$$= 9+8 = 17$$

$$Co\{4,\{2\},1\} = \min\{d[4,2] + c[2,\phi,1]\}$$
$$= 8+5 = 13$$

$$Co\{4,\{3\},1\} = \min\{d[4,3] + c\{3,\phi,1\}\}$$
$$= 12+6$$
$$= 18$$

step 3:

$$c\{2,\{3,4\},1\} = \min\{d[2,3] + c\{2,\{3\},1\}, d[2,4] + c[2,\{4\},1]\}$$
$$= \min\{9+15, 10+18\} = 24$$

$$c\{3,\{2,4\},1\} = \min\{d[3,2] + c\{3,\{2\},1\}, d[3,4] + c\{3,\{4\},1\}\}$$
$$= \min\{8+18, 9+17\} = 26.$$

$$c\{4,\{2,3\},1\} = \min\{d[4,2] + c(4,\{2\},1), d[4,3] + c(4,\{3\},1)\}$$
$$= \min\{8+13, 9+18\} = 21$$

$$c\{1,\{2,3,4\},1\} = \min\{d[1,2] + c\{2,\{3,4\},1\}, d[1,3] + c[3,\{2,4\},1], d[1,4] + c[4,\{2,3\},1]\}$$

$$= \min\{10+24, 15+26, 20+21\}$$
$$= \min\{34, 41, 41\} = 36.$$

∴ min = 36

∴ The path is 1-2-3-4-1

For Evaluator's Use only	
Evaluators Comments	**Evaluator's Observation** **Marks Secured** ..49.. **out of 50** **Full Name of the Evaluator:** **Signature of the Evaluator:** **Date of Evaluation:**

Tutorial 7

Quadratic Optimization using Wolfe's Method

Date of the Session: ....

Learning outcomes:

- Understanding conic duality and finding the duality of given problem.
- Understanding Lagrange function.
- Understanding the process of Wolfe's methods.

7.1 PRE-TUTORIAL

1. Outline the steps involved in Wolfe's method.

* formulate the Lagrangian
* Apply KKT conditions
* Active set selection,
* Solve Quadratic subproblem
* update Active set.

7.2 IN-TUTORIAL

1. Use Wolfe's method for solving quadratic programming problem.

Maximize: $Z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

Subject to:

$x_1 + 2x_2 \leq 2$

$x_1, x_2 \geq 0$

Solution:

Given

$$z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$$

s.t

$$x_1 + 2x_2^2 + s_1^2 = 2$$

$$-x_1 + s_2^2 = 0$$

$$-x_2 + s_3^2 = 0$$

$$L = (4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2) - d_1(x_1 + 2x_2^2 + s_1^2 - 2)$$
$$-d_2(-x_1 + s_2^2) - d_3(-x_2 + s_3^2)$$

$$\frac{\partial L}{\partial x_1} = 0 \quad (4 - 4x - 2x_2 - d_1 + d_2)$$

$$= 2x_2 + 4x_1 + d_1 - d_2 = 4$$

$$4x_1 + 2x_2 + d_1 - d_2 = 4$$

$$\frac{\partial L}{\partial x_2} = 2x_1 + 4x_2 + 2d_1 - d_3 = 6$$

$$\frac{\partial L}{\partial d_1} = x_1 + 2x_2 + s_1^2 = 2$$

$$\frac{\partial L}{\partial d_2} = -x_1 + s_2^2 = 0 \qquad \frac{\partial L}{\partial d_3} = +x_2 - s_3^2 = 0$$

Using Artificial variable

$$\max z = -A_1 - A_2$$

$$4x_1 + 2x_2 + d_1 - d_2 + A_1 = 4$$

$$2x_1 + 4x_2 + 2d_1 - d_3 + A_2 = 6$$

$$x_1 + 2x_2 + s_1^2 = 2$$

C_B	BV	x_B	x_1	x_2	d_1	d_2	d_3	A_1	A_2	s_1^2	θ
−1	A_1	4	4	2	1	−1	0	1	0	0	1, 3, 2
−1	A_2	6	2	4	2	0	−1	0	1	0	
0	s_1^2	2	1	2	0	0	0	0	0	1	1.
	z_j	−10	−6	−6	−3	1	1	−1	−1	0	
	$z_j - c_j$		−6	−6	−3	1	1	0	0	0	

x_1 enters A_1 leaves

C_B	BV	x_B	x_1	x_2	d_1	d_2	d_3	A_1	A_2	s_1^2	θ
0	x_1	1/3	1	0	0	−1/3	1/6	0	1/3	−1/6	
0	d_1	1	0	0	1	0	−1/2	−1	0	1/2	
0	x_2	5/6	0	1	0	1/6	−1/12	−1/2	−1/6	1/12	
	z_j	0	0	0	0	0	0	0	0	0	
	$z_j - c_j$	0	0	0	0	0	0	0	1	1	

$\therefore$ $x_1 = 1/3$, $x_2 = 5/6$

$$z = \frac{4}{3} + 5 - \frac{2}{9} - \frac{5}{9} - 2\left(\frac{25}{36}\right)$$

$$= \frac{4}{3} + 5 - \frac{2}{9} - \frac{5}{9} - \frac{25}{18} = \frac{75}{18} \qquad \boxed{z = \frac{25}{6}}$$

7.3 POST-TUTORIAL

1. Illustrate the complexity of the steps involved in Wolfe's method.

Solution:

→ Formulate to Lagrangian:-

$$L(x, \lambda) = \frac{1}{2} x^T Q x + C^T x + \lambda^T (Ax - b)$$

→ Karush- Kuhn- Tuker:-

Stationary: $\nabla_x L(x, \lambda) = Qx + C + A^T x = 0$

- Primal feasibility :- $Ax \leq b$
- Dual feasibility :- $\lambda \geq 0$
- Complementry slackness:- $\lambda_i(A_i x - b_i) = 0$ for each i

For Evaluator's Use only

Evaluators Comments	Evaluator's Observation
	Marks Secured .... out of 50 Full Name of the Evaluator: Signature of the Evaluator:

Tutorial 8

Quadratic Optimization using Beale's Method

Date of the Session: ....

Learning outcomes:

- Understanding Kuhn tucker conditions
- Understanding the process of Beale's method

8.1 PRE-TUTORIAL

1. Outline the steps involved in Beale's method.

Step-1:- formulate lagrangian

Step-2:- compute Gradient

$\nabla_x L(x, \lambda) = \nabla f(x) + \lambda^T \nabla h(x)$

Step-3:- set up system of equations

Step-4:- solve the linear system

Step-5:- check for convergence

8.2 IN-TUTORIAL

1. Use Beale's method for solving quadratic programming problem.

Maximize: $Z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

Subject to:

$x_1 + 2x_2 \leq 2$

$x_1, x_2 \geq 0$

Solution: $x_1 + 2x_2 + S_1 = 2$

Iteration 1:- $x_1 = 2 - 2x_2 - S_1$

max $f(x_2) = 4(2-2x_2-S_1) + 6x_2 - 2(2-2x_2-S_1)^2$

$= -2(2-2x_2-S_1)x_2 - 2x_2^2$

simplyfing :-

$\Rightarrow -6x_1^2 - 2S_1^2 + 10x_2 + 4S_1 - 6S_1x_2$

$\frac{df}{dx_2} = -12x_2 + 10 - 6S_1$ $\frac{df}{dS_1} = -4S_1 + 4 - 6x_2$

$\boxed{S_1 = 0}$ $x_1 = 2 - 2x_2$

$\frac{\partial f}{\partial x_2} = -12x_2 + 10 - 6S_1 = 0$ $x_2 = \min(1, \frac{5}{6})$

$12x_2 = 10$

$x_2 = \frac{10}{12}$ $\boxed{= \frac{5}{6}}$

Iteration-2:- $x_2 = 1 - \frac{S_1}{2} - \frac{x_1}{2}$

$$\max(f_1) = 4x_1 + 6\left(1 - \frac{S_1}{2} - \frac{x_1}{2}\right) - 2x_1^2 - 2x_1\left(1 - \frac{S_1}{2} - \frac{x_1}{2}\right)$$
$$-2\left(1 - \frac{S_1}{2} - \frac{x_1}{2}\right)^2$$

$$\therefore \quad -\frac{3x_1^2}{2} - \frac{S_1^2}{2} + x_1 + S_1 + 4$$

$$\frac{\partial f}{\partial x_1} = 1 - 3x_1 \qquad \frac{\partial f}{\partial S_1} = -S_1 - 1$$

when $S_1 = 0$

$3x_1 = 1$

$\boxed{x_1 = \frac{1}{3}}$ $\min(2, \frac{1}{3}) = 3$

$\boxed{x_1 = \frac{1}{3}}$ $\boxed{x_2 = \frac{5}{6}}$

optimal value:-

$$4\left(\frac{1}{3}\right) + 5 - \frac{2}{9} - \frac{5}{9} - \frac{25}{18}$$

$$\therefore 5 + \frac{24 - 4 - 10 - 25}{18}$$

$$= 5 - \frac{15}{8}$$

$$= \frac{75}{16}$$

$$\boxed{z = \frac{25}{6}}$$

8.3 POST-TUTORIAL

1. Compute the first iteration and identify the entering and leaving variables:

Minimize: $Z = -4x_1 - 6x_2 + x_1^2 + 2x_2^4$

Subject to:

$$-x_1 - 2x_2 \geq -4$$

$$x_1, x_2 \geq 0$$

Solution: constrains:- $-x_1 - 2x_2 \geq -4$

Step-1:- $-x_1 - 2x_2 - S = -4$

Step-2:-

Set up the siplex table:-

Step-1:- Reformulate problem

$$Z = -4x_1 - 6x_2 + x_1^2 + 2x_2^4$$

S.t

$$-x_1 - 2x_2 \geq -4$$

$$x_1 + 2x_2 \leq 4$$

Step-2:- Introduce slack variables:-

$$x_1 + 2x_2 + s = 4$$

Step-3:- Gradient of objective function:-

$$\frac{dz}{dx_1} = -4 + 2x_1$$

$$\frac{dz}{dx_2} = -6 + 8x_2^3$$

Step-4:- Identify the Entering variable:-

For x_1:-

$$\frac{dz}{dx_1} = -4 + 2x_1$$

if $x_1 = 0$ then $\frac{dz}{dx_1} = -4$

for x_2:- $\frac{dz}{dx_2} = -6 + 8x_2^3$

$x_2 = 0$ then $\frac{dz}{dx_2} = -6$

step-5:- Identify leaving variable

$x_1+2x_2+s=4$

If $x_2=0$ then $s=4-x_1$

If we increase x_2 until $x=0$ $x_1+2x_2=4$

$x_1=0$

$0+2x_2=4$ $x_2=2$

$\therefore$ Entering variable:- x_2

leaving variable:- s

For Evaluator's Use only

Evaluators Comments	Evaluator's Observation
	Marks Secured .... out of 50
	Full Name of the Evaluator:

Tutorial 9

Geometric Programming

Date of the Session: ....

Learning outcomes:

1. Introduce Posynomials, and arithmetic mean - geometric mean inequality.

2. Teach optimization of posynomial objective functions using Geometric Programming.

3. Introduce the concept of degree of difficulty.

4. Sketch the method of solving problem with one degree of difficulty.

9.1 PRE-TUTORIAL

1. How to compute the degree of difficulty of Geometric programming problem?

Problem structure

No of variables & constants

Form of constraint

Condition of data

Solution Technique

Duality

Numerical stability

2. List the areas of application of Geometric programming.

Power Control

Optimal Doping Profile

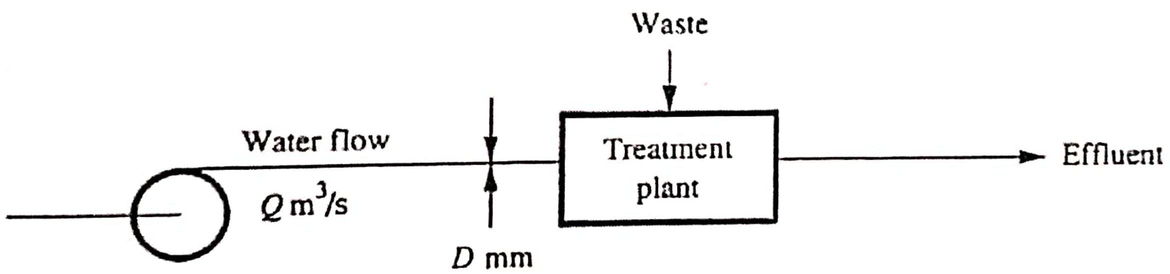
Flaor Planning

Digital Gate Sizing

9.2 IN-TUTORIAL

1. Determine the optimum pipe diameter which results in minimum first plus operating cost for 100 m of pipe. The objective function is to determine the optimum pipe diameter for highest discharge.

$$y = 160D + \frac{32 * 10^{12}}{D^5}$$



Solution: The given equation $y = 160D + \frac{32 \times 10^{12}}{D^5}$

step-1:- set the objective function

$$y(D) = 160D + \frac{32 \times 10^{12}}{D^5}$$

step-2

Take the derivative of y wrt D

$$\frac{dy}{dD} = 160 - 5 \times \frac{32 \times 10^{12}}{D^6}$$

step-3

Set the derivate equal to zero to find critical points

$$160 - \frac{160 \times 10^{12}}{D^6} = 0$$

solving for D:

$$160 = \frac{160 \times 10^{12}}{D^6}$$

$$D^6 = 10^{12}$$

$$D = (10^{12})^{1/6}$$

$$D = 10^2 = 100mm$$

S-4

Second derivative for minimum

$$\frac{d^2y}{dD^2} = \frac{960 \times 10^{12}}{D^7}$$

for $D = 100mm$

∴ Optimum pipe diameter $D = 100mm$

9.3 POST-TUTORIAL

1. Using the concept of Geometric Programming

$$\text{Minimize} f(x) = 5x_1 + 20x_2 + 10x_1^{-1}x_2^{-1},$$

where $x_1, x_2 \geq 0$

Solution:

Given Min $f(x) = 5x_1 + 20x_2 + 10x_1^{-1}x_2^{-1}$

$x_1, x_2 \geq 0$

S-1: The objective function is

$f(x_1, x_2) = 5x_1 + 20x_2 + 10x_1^{-1}x_2^{-1}$

where $5x_1$ (monomial in x_1)

$20x_2$ = (manmial in x_2)

$10x_1^{-1}x_2^{-1}$ (monomial in x_1^{-1} & x_2^{-1}

S-2:

Give constraints are:

$x_1 \geq 0, x_2 \geq 0$

Step-3

Logarithm transformation

$y_1 = \log x_1$ and $y_2 = \log x_2$

The monomial terms transform as follows

$5x_1$ becames $5e^{y_1}$

$20x_1$ becames $20e^{y_2}$

$10x_1^{-1} x_2^{-2}$ becames $10e^{-y_1 - y_2}$

$\min f(y_1, y_2) = 5e^{y_1} + 20e^{y_2} + 10e^{-y_1 - y_2}$

Step-4

Using an optimization algorithm

$x_1 = 2\text{-}00$

$x_2 = 0\text{-}50$

Verification

$5x_1 = 5x_1 = 10$

$20x_2 = 20 \times 0.5 = 10$

$10x_1^{-1} x_2^{-1} = 10 \times \frac{1}{2} \times \frac{1}{0.5} = 10$

$f(x_1, x_2) = 10 + 10 + 10 = 30$

The optimum solution $x_1 = 2$, $x_2 = 0.5$

$\min z = 30$

For Evaluator's Use only

Evaluators Comments	Evaluator's Observation
	Marks Secured ...50... out of 50 Full Name of the Evaluator: VVS Signature of the Evaluator: Date of Evaluation 26/11/24

Tutorial 10

Ant Colony Optimization (ACO)

Date of the Session: ....

Learning outcomes:

- Understanding different approaches for solving problems of Ant Colony Optimization (ACO)
- Applying various approach to solve Ant Colony Optimization (ACO).

10.1 PRE-TUTORIAL

1. What is swarm intelligence?

Swarm intelligence in mathematical programming refers to optimization techniques inspired by the collective behaviour of social organisms, like ants and birds

2. What is meant by Ant Colony Optimization?

Ant Colony optimization (ACO) is a bio-inspired algorithm that solves optimization problems by mimicking and foraging behaviour.

3. State the Merits and demerits of Ant Colony Optimization.

Merits: ACO is robust, adaptable, and performs well in dynamic environments with distributed computation

Demerits :- It can be computationally expensive and slow to converge for large-scale problems.

10.2 IN-TUTORIAL

1. An ANT is at a distance of 5m from the TREE 15m from CAR and 4m from a DOLL, the distance between TREE and the CAR is 4m, CAR and DOLL is 1m, DOLL and TREE is 8m. Solve using Ant Colony Optimization to find the shortest route to travel through tree, car and doll.

Solution:

Locations & Distances

Ant → Tree : 5m

Ant → Car : 15m

Ant → Doll : 4m

Tree → Car : 4m

Car ↔ Doll : 4m

- Doll ↔ Tree : 8m

Possible routes (starting from the Ant)

- Ant → Tree → Car → Doll
- Ant → Tree → Doll → Car
- Ant → Car → Tree → Doll

- And more
- Ant -> Tree -> Car -> Doll = 10 m

Optimal Route :-

The shortest Route found is Ant -> Tree -> Car -> Doll with a total distance of 10 m

10.3 POST-TUTORIAL

1. Minimize function using Ant colony optimization.

$$\text{Minimize } f = x_1^2 + x_1x_2 + x_2^2$$
$$\text{with initial values } x_1 = \{1,2,3,4,5\}, x_2 = \{3,4,5\}$$

Solution:

To minimize of $f = x_1^2 + x_1x_2 + x_2^2$ using Act colony optimization (ACO)

(1) Initialize : start ants at random values from $x_1 \in \{1,2,3,4,5\}$ and $x_2 \in (3,4,5)$

for each, compute $f(x_1, x_2)$

Increase pheramone level to guide ants better solution

use pheramone levels to guid ant better solution.

Repeat until best (minimum) f value is found (x_1, x_2)

Tutorial 11

Particle Swarm Optimization (PSO)

Date of the Session: ....

Learning outcomes:

- Understanding the concept of Particle Swarm Optimization (PSO).

- Apply Particle Swarm Optimization in real-time situations.

11.1 PRE-TUTORIAL

1. What is a particle swarm and their behavior under the PSO algorithm?

In particle swarm optimization (PSO), particles represent potential solutions and "fly" through the solution space, influenced by their own best-known position and the swarm's best-known position.

2. **What are the variations and limitations of PSO algorithm?**

pso variations include gBest (global best) for faster convergence, lBest (local best) for better exploration, and constricted pso to balance exploration and exploration.

3. **Write down the steps involved in solving the problem using particle swarm optimization?**

step 1 :- Randomly initialize positions and velocities of particles within the solution space

step 2 :- Calculate each particle's fitness based on the objective function.

step 3 :- Track each particles best-known position and update the global ~~best~~ if any particle achieves a higher fitness

step 4 :- Adjust each particle's velocity based on it personal best, the global best, and its current velocity.

step 5 :- Return the best solution found by the swarm as the optimal result.

11.2 IN-TUTORIAL

1. Find out the maximum of

$$F(X) = -x^2 + 5x + 20 \text{ with } -10 \leq x \leq 10$$

Using the PSO algorithm. Use 9 particles with the initial positions $x_1 = -9.6$, $x_2 = -6$, $x_3 = -2.6$, $x_4 = -1.1$, $x_5 = 0.6$, $x_6 = 2.3$, $x_7 = 2.8$, $x_8 = 8.3$, $x_9 = 10$. Show the detailed computations for iterations 1,2 and 3.

Solution:

Given objective function is

$$F(x) = -x^2 + 5x + 20$$

Initial particle positions are

$x_1 = -9.6$, $x_2 = -6$, $x_3 = -2.6$, $x_4 = -1.1$,

$x_5 = 0.6$, $x_6 = 2.3$, $x_7 = 2.8$, $x_8 = 8.3$,

$x_9 = 10$

$$F(x_1) = -(9.6)^2 + 5(-9.6) + 20 = -92.16 - 48 + 20$$
$$= -120.16$$

$$F(x_2) = -(-6)^2 + 5(-6) + 20 = -36 - 30 + 20 = -46$$

$$F(x_3) = -(-2.6)^2 + 5(-2.6) + 20 = -6.76 - 13 + 20$$
$$= 0.24$$

$$F(x_4) = (-1.1)^2 + 5(-1.1) + 20 = -1.21 - 5.5 + 20$$
$$= 13.29$$

$$F(x_5) = -(0.6)^2 + 5(0.6) + 20 = -0.36 + 3 + 20$$
$$= 22.64$$

$x_{best} = x_6 = 2.3$

$F(x_{best}) = 26.21$

PSO formula :-

$x_i(t+1) = x_i(t) + v_i(t+1)$

Sum of fitness values after 3 iterations :-

$-120.16 + (-46) + 0.24 + 13.29 + 22.64 + 26.21$
$+ 26.16 + (-7.39) + (-30)$
$= -115.01$

2. Illustrate the steps involved in Particle Swarm Optimization

Solution:

Initialize particles: Randomly places particles in the search space with initial position and ve lanlties.

Evaluate fitness :- Calculate each particles fiteness based on the object.

update best positions:-

- Each particle records its best-knew posi-tions
- the best position across all partlel is set as the global best.

(4) update velocities and positions.

(5) Repeat

(6) Recall. best solution.

11.3 POST-TUTORIAL

1. Use Particle Swarm Optimization technique (PSO), and solve the following problem .

Maximize: $f(x) = x_1^2 - x_1x_2 + x_2^2 + 2x_1 + 4x_2 + 3$ with $-5 \leq x_1, x_2 \leq 5$

	x_1	x_2
1	-3	2
2	1	4
3	-2	-4
4	3	-2

Solution:

The problem in the image required using the particle swarm optimization techniques to maximize the function.

$f(x) = x_1^2 - x_1x_2 + x_2^2 + 2x_1 + 4x_2 - 13$

With the constraints

$-5 \leq x_1 x_2 < 5$

Given the initial particles with partion (x_1, x_2)

1 -3 2

2 1 4

3 -2 -4

4 3 -2

(1) Initialize particles: set initial positions and velocities for particle.

(2) update velocities and positions.

(3) Repeat until convergence

For Evaluator's Use only

Evaluators Comments	Evaluator's Observation
	Marks Secured .... out of 50
	Full Name of the Evaluator:
	Signature of the Evaluator:
	Date of Evaluation:

2. Suppose a genetic algorithm uses chromosomes of the form $x = abcdefgh$ with a fixed length of eight genes. Each gene can be any digit between 0 and 9. Let the fitness of individual x be calculated as:
$f(x) = (a + b) - (c + d) + (e + f) - (g + h)$ and let the initial population consist of four individuals with the following chromosomes:

$$x_1 = 6\,5\,4\,1\,3\,5\,3\,2$$
$$x_2 = 8\,7\,1\,2\,6\,6\,0\,1$$
$$x_3 = 2\,3\,9\,2\,1\,2\,8\,5$$
$$x_4 = 4\,1\,8\,5\,2\,0\,9\,4$$

Maximize the above function.

Solution:

The problem here requires using the particle swarm optimization (PSO) algorithm to maximize the function.

$$f(x) = -x^2 + 5x + 20 \quad \text{with to} < x < 10$$

The lock is to use 9 particles with given initial position.

$x_1 = -9.6$, $x_2 = -6$, $x_3 = -2.6$, $x_4 = -1.1$,

$x_5 = 0.41$, $x_6 = 2.3$, $x_7 = 2.8$, $x_8 = 6.3$

$x_9 = 10$

Ensure that the updated positions of the particles remain within the defined bounds of -5 to 5.

- The global best position found during the iterations will provide the optimal values for x_1 and x_2 that maximize the function $f(x)$.

12.2 IN-TUTORIAL

1. Maximize the value of the function

$$f(x) = -x^2 + 2x$$

over the range of real numbers 0 to 2, with initial population 111010, 01010, 10101, 00101 with random numbers 0.3, 0.2, 1.3, 0.9. Crossover point can be taken randomly.

Solution:

Given $f(x) = -x^2 + 2x$

- $111010 = 58$; $x = 58/63 \times 2 = 1.841$
- $01010 = 10$, $x = 10/63 \times 2 = 0.317$
- $10101 = 21$, $x = 21/63 \times 2 = 0.667$
- $00101 = 5$, $x = 5/63 \times 2 = 0.159$

- $f(x) = -x^2 + 2x$

$x = 1.841$

$f(x) = -(1.841)^2 + 2 \cdot 1.841 = 1.07$

$f(x) = -(0.317)^2 + 2 \cdot 0.317 = 0.61$

$f(x) = (0.667)^2 + 2 \cdot 0.667 = 0.89$

$f(x) = -(0.159)^2 + 2 \cdot 0.159 = 0.314$

Decimal $= 45$, $x = 45/63 \times 2 = 1.429$

$f(x) = -(1.429)^2 + 2 \cdot 1.429 = 1.02$

Decimal $= 50$, $x = 50/63 \times 2 = 1.587$

$f(x) = (1.587)^2 + 2 \cdot 1587 = 0.96$

$x = 1.841$

maximum fitness $= 1.07$

12.3 POST-TUTORIAL

1. Let us assume there are 10 objects (1 to 10) and 6 labels (A to F) with the initial state as follows: initial state =1:'A', 2: 'B', 3:'A', 4:'B', 5:'C', 6:'B', 7:'A', 8:'None', 9:'None', 10:'None' Here objects 1 to 7 are labeled while 8 to 9 are not. Solve the problem by using simulated annealing algorithm.

Solution:

Initial state :

1 : 'A', 2: 'B', 3: 'A', 4: 'B', 5: 'C', 6: 'B', 7: 'A', 8: 'None

Initial temperature T = T_0

Count of tables A = 3, B = 3, C = 1, None = 3

1. Randomly assign labels to objects 8, 9, and 10

- Assign 8: 'C', 9: 'A', and 10: 'B'

Next state :

1: 'A', 2: 'B', 3: 'A', 4: 'B', 5: 'C',

6: 'B', 7: 'A', 8: 'C', 9: 'A', 10: 'B'

count of labels A = 4, B = 4, C = 2

The process continues until the temperature is low or balanced label distribution is achieved.

For Evaluator's Use only

Evaluators Comments	Evaluator's Observation
	Marks Secured .... out of 50
	Full Name of the Evaluator:
	Signature of the Evaluator:
	Date of Evaluation: