EE239AS.2, Spring 2017
Department of Electrical Engineering
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Homework #3 Solutions
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Due Friday, 05 May 2017, uploaded to Gradescope.
Covers material up to Poisson Processes III.
100 points total.

1. (13 points) You are recording the activity of a neuron, which is spiking according to a Poisson process with rate λ. At some point during your experiment, the recording equipment breaks down and begins dropping spikes randomly with probability p.

   (a) (10 points) Let the random variable M be the number of recorded spikes with the broken equipment. Show that the distribution of M is Poisson($(1-p)\lambda s$). (Hint: If N is a random variable denoting the number of actual spikes, what is $\mathbf{Pr}(M = m|N = n)$?)

   (b) (1 points) What is the rate of the Poisson process in part (a)?

   (c) (2 points) What is the distribution on the number of spikes dropped within a τ second interval?

2. (35 points) Homogeneous Poisson process
   We will consider a simulated neuron that has a cosine tuning curve described in equation (1.15) in *TN*[1]:

$$\lambda(s) = r_0 + (r_{\max} - r_0)\cos(s - s_{\max}), \tag{1}$$

   where λ is the firing rate (in spikes per second), s is the reaching angle of the arm, $s_{\max}$ is the reaching angle associated with the maximum response $r_{\max}$, and r_0 is an offset that shifts the tuning curve up from the zero axis. Let $r_0 = 35$, $r_{\max} = 60$, and $s_{\max} = \pi/2$.

   (a) (6 points) Spike trains
   For each of the following reaching angles ($s = k \cdot \pi/4$, where $k = 0, 1, \ldots, 7$), generate 100 spike trains according to a homogeneous Poisson process. Each spike train should have a duration of 1 second. Plot 5 spike trains for each reaching angle in the same format as shown in Figure 1.6(A) in *TN*. (To do this, make a subplot of dimension 5×3 and populate the appropriate subplots) (To further simplify things, we have also provided the helper function `PlotSpikeRaster.m`; this will likelys simplify your raster plotting.)

   (b) (5 points) Spike histogram
   For each reaching angle, find the spike histogram by taking spike counts in non-overlapping 20 ms bins, then averaging across the 100 trials. Plot the 8 resulting spike histograms around a circle, as in part (a). The spike histograms should have firing rate (in spikes / second) as the vertical axis and time (in msec, not time bin index) as the horizontal axis. The `bar` command in Matlab can be used to plot histograms.

   (c) (4 points) Tuning curve
   For each trial, count the number of spikes across the entire trial. Plots these points on

[1] *TN* refers to *Theoretical Neuroscience* by Dayan and Abbott.

the axes like shown in Figure 1.6(B) in *TN*, where the x-axis is reach angle and the y-axis is firing rate. There should be 800 points in the plot (but some points may be on top of each other due to the discrete nature of spike counts). For each reaching angle, find the mean firing rate across the 100 trials, and plot the mean firing rate using a red point on the same plot. Now, plot the tuning curve (defined in (1)) of this neuron in green on the same plot. Do the mean firing rates lie near the tuning curve?

(d) (6 points) Count distribution
For each reaching angle, plot the *normalized* distribution (i.e., normalized so that the area under the distribution equals one) of spike counts (using the same counts from part (c)). Plot the 8 distributions around a circle, as in part (a). Fit a Poisson distribution to each empirical distribution and plot it on top of the corresponding empirical distribution. Are the empirical distributions well-fit by Poisson distributions?

(e) (4 points) Fano factor
For each reaching angle, find the mean and variance of the spike counts across the 100 trials (using the same spike counts from part (c)). Plot the obtained mean and variance on the axes shown in Figure 1.14(A) in *TN*. There should be 8 points in this plot – one per reaching angle. Do these points lie near the 45 deg diagonal, as would be expected of a Poisson distribution?

(f) (5 points) Interspike interval (ISI) distribution
For each reaching angle, plot the normalized distribution of ISIs. Plot the 8 distributions around a circle, as in part (a). Fit an exponential distribution to each empirical distribution and plot it on top of the corresponding empirical distribution. Are the empirical distributions well-fit by exponential distributions?

(g) (5 points) Coefficient of variation (C_V)
For each reaching angle, find the average ISI and C_V of the ISIs. Plot the resulting values on the axes shown in Figure 1.16 in *TN*. There should be 8 points in this plot. Do the C_V values lie near unity, as would be expected of a Poisson process?

3. (22 points) Inhomogeneous Poisson process
In this problem, we will use the same simulated neuron as in Problem 2, but now the reaching angle s will be time-dependent with the following form:

$$s(t) = t^2 \cdot \pi, \tag{2}$$

where t ranges between 0 and 1 second.

(a) (6 points) Spike trains
Generate 100 spike trains, each 1 second in duration, according to an inhomogeneous Poisson process with a firing rate profile defined by (1) and (2). Plot 5 of the generated spike trains.

(b) (5 points) Spike histogram
Plot the spike histogram by taking spike counts in non-overlapping 20 ms bins, then averaging across the 100 trials. The spike histogram should have firing rate (in spikes / second) as the vertical axis and time (in msec, not time bin index) as the horizontal axis. Plot the expected firing rate profile defined by equations (1) and (2) on the same plot. Does the spike histogram agree with the expected firing rate profile?

(c) (6 points) Count distribution
For each trial, count the number of spikes across the entire trial. Plot the normalized distribution of spike counts. Fit a Poisson distribution to this empirical distribution and plot it on top of the empirical distribution. Should we expect the spike counts to be Poisson-distributed?

(d) (5 points) ISI distribution
Plot the normalized distribution of ISIs. Fit an exponential distribution to the empirical distribution and plot it on top of the empirical distribution. Should we expect the ISIs to be exponentially-distributed?

4. (30 points) Real neural data
We will analyze real neural data recorded using a 100-electrode array in premotor cortex of a macaque monkey[2]. The dataset can be found on CCLE as `'ps3_data.mat'`.

The following describes the data format. The .mat file has a single variable named `trial`, which is a structure of dimensions (182 trials) $\times$ (8 reaching angles). The structure contains spike trains recorded from a single neuron while the monkey reached 182 times along each of 8 different reaching angles (where the trials of different reaching angles were interleaved). The spike train for the `n`th trial of the `k`th reaching angle is contained in `trial(n,k).spikes`, where $\texttt{n} = 1, \ldots, 182$ and $\texttt{k} = 1, \ldots, 8$. The indices $\texttt{k} = 1, \ldots, 8$ correspond to reaching angles $\frac{30}{180}\pi$, $\frac{70}{180}\pi$, $\frac{110}{180}\pi$, $\frac{150}{180}\pi$, $\frac{190}{180}\pi$, $\frac{230}{180}\pi$, $\frac{310}{180}\pi$, $\frac{350}{180}\pi$, respectively. The reaching angles are not evenly spaced around the circle due to experimental constraints that are beyond the scope of this homework.

A spike train is represented as a sequence of zeros and ones, where time is discretized in 1 ms steps. A zero indicates that the neuron did not spike in the 1 ms bin, whereas a one indicates that the neuron spiked once in the 1 ms bin. Due to the refractory period, it is not possible for a neuron to spike more than once within a 1 ms bin. Each spike train is 500 ms long and is, thus, represented by a 1×500 vector.

(a) (6 points) Spike trains
Plot 5 spike trains for each reaching angle in the same format as shown in Figure 1.6(A) in *TN*.

(b) (5 points) Spike histogram
For each reaching angle, find the spike histogram by taking spike counts in non-overlapping 20 ms bins, then averaging across the 182 trials. The spike histograms should have firing rate (in spikes / second) as the vertical axis and time (in msec, not time bin index) as the horizontal axis. Plot the histogram for 500ms worth of data. Plot the 8 resulting spike histograms around a circle, as in part (a).

(c) (4 points) Tuning curve
For each trial, count the number of spikes across the entire trial. Plots these points on the axes shown in Figure 1.6(B) in *TN*. There should be $182 \cdot 8$ points in the plot (but some points may be on top of each other due to the discrete nature of spike counts). For each reaching angle, find the mean firing rate across the 182 trials, and plot the mean firing rate using a red point on the same plot.

[2]The neural data have been generously provided by the laboratory of Prof. Krishna Shenoy at Stanford University. The data are to be used exclusively for educational purposes in this course.

Then, fit the cosine tuning curve (1) to the 8 red points by minimizing the sum of squared errors

$$\sum_{i=1}^{8} \big(\lambda(s_i) - r_0 - (r_{\max} - r_0)\cos(s_i - s_{\max})\big)^2$$

with respect to the parameters r_0, $r_{\max}$, and $s_{\max}$. (Hint: this can be done using linear regression; refer to Homework # 2.) Plot the resulting tuning curve of this neuron in green on the same plot.

(d) (6 points) Count distribution
For each reaching angle, plot the normalized distribution of spike counts (using the same counts from part (c)). Plot the 8 distributions around a circle, as in part (a). Fit a Poisson distribution to each empirical distribution and plot it on top of the corresponding empirical distribution. Why might the empirical distributions differ from the idealized Poisson distributions?

(e) (4 points) Fano factor
For each reaching angle, find the mean and variance of the spike counts across the 182 trials (using the same spike counts from part (c)). Plot the obtained mean and variance on the axes shown in Figure 1.14(A) in *TN*. There should be 8 points in this plot – one per reaching angle. Do these points lie near the 45 deg diagonal, as would be expected of a Poisson distribution?

(f) (5 points) Interspike interval (ISI) distribution
For each reaching angle, plot the normalized distribution of ISIs. Plot the 8 distributions around a circle, as in part (a). Fit an exponential distribution to each empirical distribution and plot it on top of the corresponding empirical distribution. Why might the empirical distributions differ from the idealized exponential distributions?