EE239AS.2, Spring 2017 | **Homework #1**
Department of Electrical Engineering | Prof. J.C. Kao
University of California, Los Angeles | Special Reader M. Srinivasan

Due Monday, 17 February 2017, to Gradescope.
Covers material up to Lecture 4.
100 points total.

1. (20 points) Please state whether each of the following statements is true or false. If it is false, correct the statement to receive full credit. Each statement is worth one point.

   (a) The human brain has on the order of 10^{12} neurons.

   Solution: False. There are $\mathcal{O}(10^{11})$ neurons in the brain.

   (b) The frontal lobe of the brain is posterior to the occipital lobe.

   Solution: False. The frontal lobe is anterior to the occipital lobe.

   (c) The brain has folds which increase its surface area.

   Solution: True.

   (d) The left hemisphere of the brain in general deals with the sensory and motor processing of the left side of the body (more so than the right hemisphere of the brain).

   opposite part

   Solution: False. Sensory and motor processing is more contralateral than ipsilateral.

   (e) The monosynaptic circuit involed in the knee-jerk spinal reflex does not involve the brain.

   Solution: True.

   (f) A type of glial cell, called astrocytes, produce myelin to insulate neural axons.

   Solution: False. While glia are responsible for these, the cells are called oligodendrocytes and Schwann cells.

   (g) Neurons can convey electrical signals on the order of meters.

   Solution: True.

   (h) At the synapse, neurons transmit information exclusively through electromagnetic transduction.

   Solution: False. Information is conveyed at the synapse largely through chemical signaling.

   (i) The action potential shape does not convey information.

   Solution: True.

Na^+ extracellular

(j) Sodium ions are far more concentrated within the cytoplasmic side of a neuron (as opposed to the extracellular side).

cytoplasmic

Solution: False. They are more concentrated on the extracellular side.

(k) It is possible for a neuron to fire action potentials at a rate of 10,000 Hz (10,000 in one second).

Solution: False. Action potentials have widths on the time scale of $\mathcal{O}(1ms)$.

(l) Consider measuring the membrane potential of a neuron (with the cytoplasmic side as "+" and the extracellular side as "-"). A depolarization corresponds to a more negative membrane potential.

de → negative = positive

Solution: False. A depolarization increases the membrane potential as defined.

(m) A depolarized cell increases the likelihood of an action potential.

Solution: True.

K^+ radius > Na^+ radius

(n) Na^+ ions hold water more closely than K^+ ions do.

Smaller ion → stronger E → attrack H_2O more

→ lower mobility

Solution: True.

(o) Increasing the thickness of myelin sheaths will decease action potential speed.

Solution: False. It will increase the action potential transmission speed.

(p) Multiple sclerosis is caused by the demyelination of axons.

Solution: True.

(q) Action potentials are all-or-nothing events.

Solution: True.

(r) All action potentials have the same shape.

Solution: False. Action potentials can have different shapes. **Note: we accepted "true" here as well because of HO3, slide 10.**

(s) The equation relating conductance (G) to current (I) and voltage (V) is $I = VG$.

Solution: True.

(t) Anionic proteins residing in the cytoplasm are as permeable to the membrane as sodium and potassium ions.

Solution: False. They are typically not permeable to the membrane.

2. (25 points) Consider an "alien" neuron that you have just measured as having: (1) an internal concentration of 336 mM K^+ and 50 mM Na^+, and (2) an external concentration of 42 mM

K^+ and 300 mM Na^+. Your measurements also indicate that the membrane is not permeable to any other ions.

(a) (10 points) What are the equilibrium membrane potentials E_K and E_{NA}, including the appropriate sign?

Solution: Using the Nernst equation, we have that:
$E_{K^+} = 58\ mV \log(^{42\ mM}/_{336\ mM}) = -52.4\ mV$, and
$E_{Na^+} = 58\ mV \log(^{300\ mM}/_{50\ mM}) = 45.1\ mV$.

(b) (5 points) A fellow researcher studying an identical neuron in a make-shift lab (there's a national emergency with many researchers scrambling to understand these cells because alien neurons have fallen to Earth!) comes over to you and says that she is seeing action potentials with a peak-to-peak amplitude of 110 mV. Do you say (i) "that's great, very useful information," or (ii) "I don't think so, why don't you repeat your measurement and check the calibration on your amplifier and oscilloscope"? Why?

Solution: An action potential cannot go higher than E_{Na^+} and cannot go lower than E_{K^+} which, for this neuron, means that the peak-to-peak voltage cannot be greater than $45.1 - (-52.4) = 97.5$ mV. Thus a report of seeing 110 mV action potentials can't be right; probably an amplifier's gain was incorrectly set/read or an oscilloscope's scale was incorrectly read. NOTE: for this part there is no reason to believe that YOUR concentration measurement(s) are wrong.

(c) (10 points) Your colleague returns and she says, "Dear Moron – I've checked my experimental setup and even repeated the measurement on another setup, and these neurons have action potentials with a peak-to-peak voltage of 110 mV." You sheepishly remember that when you were originally measuring the cell concentrations you dropped your meter on your Double Double burger from In-N-Out (thereby getting that really yummy pinkish sauce all over your meter) just before measuring the external Na^+ concentration. You measured the other three concentrations before this little mishap. Before re-measuring the external Na^+ concentration you think a moment and then say either (i) that it will be higher than your original (and incorrect) measurement or (ii) lower than your original (and incorrect) measurement? Why?

we expect a higher external Na^+ concentration

Solution: To account for a 110 mV peak-to-peak voltage, you need a more positive E_{Na^+}. Thus the external [Na] would have to be higher than your original (and incorrect) measurement.

3. (20 points) After bonding over your practice of eating In-N-Out while conducting experiments, you two decide to team up to further characterize the same alien neuron. Your colleague has made some measurements, but needs you to help her understand aspects of the neuron's signaling time course. Assume the properties of the alien neuron from question (1) continue to hold. An additional property your colleague has noticed is that these alien neurons are shaped like spheres, and that their radius is 10 μm.

(a) (5 points) Your colleague reports that she's measured the change in voltage across the membrane when she injects current into Na^+ and K^+ ion channels. She observes the relationship is linear, so that these ion channels behave like resistors. When she blocks K^+ channels using tetraethylammonium and injects 1 pA of current into the neuron, the membrane voltage changes by 45 mV. When she blocks Na^+ channels using tetrodotoxin and injects 1 pA of current into the neuron, the membrane voltage changes by 50 mV. Assume that the Na^+ ion channels only allow through Na^+ ions, and that the K^+ ion channels only allow through K^+ ions. What are the Na^+ and K^+ ion channel conductances?

Solution: The conductances are given by $g = {}^{I}/_{V}$. Thus we have that $g_{Na^+} = {}^{1\times10^{-12}}/_{45\times10^{-3}} = 2.22\times10^{-11}S = 22.2pS$. Similarly, we have that $g_{K^+} = {}^{1\times10^{-12}}/_{50\times10^{-3}} = 2 \times 10^{-11}S = 20pS$.

(b) (5 points) What is the membrane's input resistance? (Hint: Assume no cytoplasmic resistance. Also recall that at DC steady state, a capacitor looks like an open circuit.)

Solution: Because there is no cytoplasmic resistance, and we treat the capacitor as an open circuit in steady state when injecting a DC current, the membrane's input resistance is simply the parallel combination, $R_{Na^+}||R_{K^+}$. Thus,

$$R_{\text{in}} = \frac{1}{\frac{1}{R_{Na^+}} + \frac{1}{R_K^+}} = 23.7\ G\Omega.$$

(c) (5 points) Assume that the membrane is well-modeled by a parallel plate capacitor, with ends separated by a distance of 100 pm. What is the membrane's input capacitance? (Please use the permittivity of free space for this calculation.)

Solution: The membrane's input capacitance is given by $C_{\text{in}} = C_m 4\pi a^2$. Here, $C_m = {}^{\epsilon}/_{d} = 0.089\ F/m$. With $a = 10\ \mu m$, we have that $C_{\text{in}} = 0.11nF$.

(d) (5 points) If the input voltage ΔV_m and threshold to trigger a spike were the same for the alien neuron and a human neuron (which have membrane potential time constants of 20-50ms), will the alien neuron fire a spike more quickly or slowly than a human neuron?

Solution: The time constant of the membrane potential is $\tau = R_{\text{in}}C_{\text{in}} = 2.8\ s$. So this alien neuron is remarkably slow to integrate incoming signals, and certainly slower than a human neuron.

4. (15 points) Consider a neuron with (1) a (non-standard) membrane that is only permeable to one ion, (2) the permeable ion has a concentration gradient, and (3) the overall system eventually reaches an equilibrium where drift current and diffusion current are equal and opposite.

Now, in contrast, consider a neuron with (1) a standard membrane that is permeable to multiple ionic species, (2) each ion has a concentration gradient, and (3) the overall system eventually reaches steady state.

(a) (10 points) In this multiple-species case, are the drift and diffusion currents for each individual ionic species equal and opposite? Why or why not?

Solution: In the single-ion permeable membrane case, the membrane voltage changes until drift current and diffusion currents are equal and opposite. In the multiple-ion permeable membrane case, there can still only be a single membrane voltage. Thus it is not possible, in general, for all ionic species to have their drift and diffusion currents be exactly equal and opposite. Therefore, at least one ion will have a net nonzero current. (Barring the trivial case where all gradients are zero.)

(b) (5 points) What do neurons do to maintain their concentration gradients?

Solution: Energy-consuming ion pumps transport ions across the membrane, and against their concentration gradient, to counteract the net current flow (which acts to deplete the concentration gradient)

5. (20 points) You are comparing the axons of neurons taken from two different species. In species A, the cytoplasm has radius r = 490μm and the membrane is t = 10μm thick. In species B, the cytoplasm has radius r = 250μm and the membrane is surrounded by a thick layer of myelin, so that the total thickness of the membrane and myelin is t = 250μm. The two neurons are otherwise identical in all other ways (e.g., ion concentrations, membrane conductance, conductance of the axon's intracellular fluid.)

(a) (10 points) You stimulate each neuron and measure the speed of the resulting action potential. Which axon would you expect to have faster action potential conduction? Please show calculations. (Hint: Ignore membrane resistance. A simple RC model is sufficient.)

Solution: Ignoring membrane resistance, delay per length L is proportional to the RC delay of the axial resistance and the membrane capacitance. (See Lecture 5, around slide 5 and 11). There are two models to get at this.

Model 1:

$$\begin{aligned} R_{\text{axial}} C_{\text{membrane}} &= \frac{\rho L}{A} \frac{\epsilon A}{d} \\ &= \frac{\rho L}{\pi r^2} \frac{\epsilon 2 \pi r L}{t} \\ &= 2\rho\epsilon L^2 \frac{1}{rt} \\ &= k\frac{1}{rt} \end{aligned}$$

If we plug in r and t for each species, we find that Species B is about 12.8 times faster.

Model 2: Outer plate (e.g., what if the area for the capacitor should use R = r + t to be more accurate?)

$$R_{\text{axial}} C_{\text{membrane}} = \frac{\rho L}{A} \frac{\epsilon A}{d}$$

$$= \frac{\rho L}{\pi r^2}\frac{\epsilon 2\pi RL}{t}$$
$$= 2\rho\epsilon RL^2\frac{1}{r^2 t}$$
$$= k_2\frac{1}{r^2 t}$$

If we plug in r and t for each species, we find that Species B is about 6.5 times faster. BOTH model 1 and 2 say NEURON B is faster.

(b) (5 points) Myelin is widely known to increase speed of conduction, but it takes up room. In a situation where many neurons must be packed into a limited space, is it worth it? (Hint: The overall diameter for species A and B is the same.) Why (1 or 2 succinct sentences is sufficient)?

Solution: Yes, myelin still helps. Myelin decreases membrane capacitance more than it increases axial resistance by consuming space that could be used for wider axons.

(c) (5 points) Species B has Nodes of Ranvier. What is the purpose of these nodes, and how do they function? Why (1 or 2 succinct sentences is sufficient)?

Solution: Nodes regenerate action potential to counteract signal attenuation. The have a high density of voltage-gated Na^+ channels, so that even a just-barely-above-threshold depolarizing membrane potential can cause huge Na^+ influx thereby restoring action potential amplitude.

Lecture 3-4 15 P.

Speed of AP $\propto \frac{1}{r_\alpha C_m}$

r_α resister $\propto \frac{1}{\text{diameter}^2}$

C_m capacity $\propto \frac{1}{\text{thickness}}$