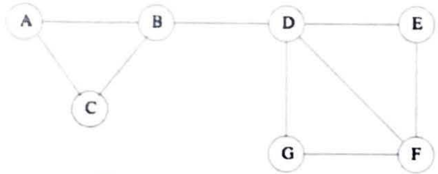
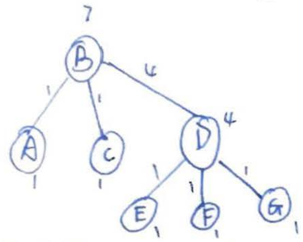
Quiz 10: Graphs; ~~Total 30pts~~ – 10 pts ~~are bonus~~
Name: ______________________ ID: ________________

1) (3pts) For node B, use the Girvan-Newman algorithm to calculate the betweeness of each edge (do this for node B ONLY). You need to show the steps of your calculation.

steps (1pt).




(2pts)

1. Run BFS on B as root.
2. Assign 1 to every leaf node; A C E F G
3. Compute the credit of D:
   1 + credit of DAG edges entering from below.
   1 + 3 = 4
4. Same computation for the credit of B: 1+1+4+1 = 7.

2) (3pt) To compute "modularity" taught in class, we need to construct a rewired network first. In the rewired graph, the expected number of edges between nodes $\boldsymbol{i}$ and $\boldsymbol{j}$ of degrees $\boldsymbol{k_i}$ and $\boldsymbol{k_j}$ equals to: $\boldsymbol{k_i} \cdot \frac{\boldsymbol{k_j}}{\boldsymbol{2m}} = \frac{\boldsymbol{k_i k_j}}{\boldsymbol{2m}}$ What is the total expected number of edges in the rewired graph? You need to show your calculation using $\frac{\boldsymbol{k_i k_j}}{\boldsymbol{2m}}$

$$= \frac{1}{2}\sum_{i\in N}\sum_{j\in N}\frac{k_i k_j}{2m}$$

$$= \frac{1}{2}\cdot\frac{1}{2m}\sum_{i\in N}k_i\left(\sum_{j\in N}k_j\right) \quad (2pts)$$

$$= \frac{1}{4m}\cdot 2m\cdot 2m$$

$$= m. \quad (1pt)$$

3) (4pts) Derive $\sum_{(i,j)\in E}\left(x_i - x_j\right)^2$ from $\mathbf{x^T L x}$

$$x^T L x = \sum_{i,j=1}^{n} L_{ij}x_i x_j = \sum_{i,j=1}^{n}(D_{ij} - A_{ij})\,x_i x_j \quad (2pts)$$

$$= \sum_i d_i x_i^2 - \sum_{(i,j)\in E} 2x_i x_j$$

$$= \sum_{(i,j)\in E}(x_i^2 + x_j^2 - 2x_i x_j)$$

$$= \sum_{(i,j)\in E}(x_i - x_j)^2$$

(2pt)