OpenFOAM Assignment 1

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1 Introduction

All code used for computing the results and figures listed is provided in the **OpenFOAMAssignment1.ipynb** in Jupyter Notebook format and **OpenFOAMAssignment1.py** for a simpler view. We used Python for both parsing, computing, and visualizing our results. This is functionally equivalent to using Paraview, but adds additional lines just for visualizing. To make grading easier, we have included a **ComputationsOnly.py** that contains only code relevant to the timings, shear stress, and force computations. All paths to simulation files are local, but if you must view them you can find them here on Stampede2 (permissions have been granted to all users, but if not please contact Cameron Cummins, csc3323, cameron.cummins@utexas.edu):

/work2/07644/oxygen/stampede2/COE-347-SP21/of1_group1

2 Non-Dimensionalized Form of Navier Stokes

Given steady form conditions of the Navier Stokes Equations:

ρ and μ are constants

Continuity Equation:

from the generalized form:

$$\frac{d\rho}{dt} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\frac{\delta\rho}{\delta t} + \frac{\delta(\rho u)}{\delta t} + \frac{\delta(\rho v)}{\delta t} + \frac{\delta(\rho w)}{\delta t} = 0$$

Momentum Equation:

from the generalized form:

$$\rho\frac{D\mathbf{u}}{Dt} = \rho\mathbf{g} - \nabla\mathbf{p} + \mu\nabla^2\mathbf{u}$$

x-direction:

$$\frac{\delta(\rho u)}{\delta t}+\frac{\delta(\rho u^2)}{\delta x}+\frac{\delta(\rho uv)}{\delta y}+\frac{\delta(\rho uw)}{\delta z}=\frac{-\delta\rho}{\delta x}+\mu\left(\frac{\delta u}{\delta x}+\frac{\delta u}{\delta y}+\frac{\delta u}{\delta z}\right)$$

y-direction:

$$\frac{\delta(\rho v)}{\delta t}+\frac{\delta(\rho uv)}{\delta x}+\frac{\delta(\rho v^2)}{\delta y}+\frac{\delta(\rho vw)}{\delta z}=\frac{-\delta\rho}{\delta y}+\mu\left(\frac{\delta u}{\delta x}+\frac{\delta u}{\delta y}+\frac{\delta u}{\delta z}\right)$$

Using reference quantities for length, velocity, viscosity, density, and pressure:

L = cavity wall, U = speed of lid, μ = dynamic velocity, ρ = density

Non-dimensionalized in the forms:

$$x=\tilde{x}L \qquad y=\tilde{y}L$$
$$u=\tilde{u}U \qquad v=\tilde{v}U$$

$$\tilde{p}=\frac{P}{\rho u^2} \quad\rightarrow\quad P=\tilde{p}(\rho u^2)$$
$$\tilde{t}=\frac{t}{\frac{L}{U}} \quad\rightarrow\quad t=\tilde{t}\left(\frac{L}{U}\right)$$

Non-dimensionalized form of the Continuity Equation:

$$\frac{\delta\rho}{\delta t}+\frac{\delta(\rho u)}{\delta x}+\frac{\delta(\rho v)}{\delta y}=0$$

$$\rho\frac{\delta}{\delta\tilde{t}(\frac{L}{U})}+\rho\frac{\delta(\tilde{u}U)}{\delta(\tilde{x}L)}+\rho\frac{\delta(\tilde{v}U)}{\delta(\tilde{y}L)}=0$$

$$\rho\frac{U}{L}\left(\frac{\delta}{\delta\tilde{t}}+\frac{\delta\tilde{u}}{\delta\tilde{x}}+\frac{\delta\tilde{v}}{\delta\tilde{y}}\right)=0$$

Non-dimensionalized form of the Momentum Equation (x-direction):

$$\rho\left(\frac{\delta u}{\delta t}+u\frac{\delta(u)}{\delta x}+v\frac{\delta u}{\delta y}\right)=\frac{-\delta P}{\delta x}+\mu\left(\frac{\delta^2 u}{\delta x^2}+\frac{\delta^2 u}{\delta y^2}\right)$$

$$\rho\left[\frac{\delta\tilde{u}U}{\delta\tilde{t}(\frac{L}{U})}+(\tilde{u}U)\frac{\delta\tilde{u}U}{\delta\tilde{x}L}+(\tilde{v}U)\frac{\delta\tilde{u}U}{\delta\tilde{y}L}\right]=\frac{-\delta\tilde{(p)}\rho u^2}{\delta\tilde{x}L}+\mu\left[\frac{\delta^2\tilde{u}U}{\delta(\tilde{x}L)^2}+\frac{\delta^2\tilde{u}U}{\delta(\tilde{y}L)^2}\right]$$

$$\frac{\rho U^2}{L}\left[\frac{\delta\tilde{u}}{\delta\tilde{t}}+(\tilde{u})\frac{\delta\tilde{u}}{\delta\tilde{x}}+(\tilde{v})\frac{\delta\tilde{u}}{\delta\tilde{y}}\right]=-\frac{\rho U^2}{L}\frac{\delta\tilde{p}}{\delta\tilde{x}}+\frac{\mu U}{L^2}\left[\frac{\delta^2\tilde{u}}{\delta(\tilde{x})^2}+\frac{\delta^2\tilde{u}}{\delta(\tilde{y})^2}\right]$$

$$\left[\frac{\delta\tilde{u}}{\delta\tilde{t}} + (\tilde{u})\frac{\delta\tilde{u}}{\delta\tilde{x}} + (\tilde{v})\frac{\delta\tilde{u}}{\delta\tilde{y}}\right] = -\frac{\delta\tilde{p}}{\delta\tilde{x}} + \frac{\mu}{\rho UL}\left[\frac{\delta^2\tilde{u}}{\delta(\tilde{x})^2} + \frac{\delta^2\tilde{u}}{\delta(\tilde{y})^2}\right]$$

$$\frac{\mu}{\rho UL} = \frac{1}{Re}$$

Non-dimensionalized form of the Momentum Equation (y-direction):

$$\rho\left(\frac{\delta v}{\delta t} + u\frac{\delta(v)}{\delta x} + v\frac{\delta v}{\delta y}\right) = \frac{-\delta P}{\delta y} + \mu\left(\frac{\delta^2 v}{\delta x^2} + \frac{\delta^2 v}{\delta y^2}\right)$$

$$\rho\left[\frac{\delta\tilde{v}U}{\delta\tilde{t}(\frac{L}{U})} + (\tilde{u}U)\frac{\delta\tilde{v}U}{\delta\tilde{x}L} + (\tilde{v}U)\frac{\delta\tilde{v}U}{\delta\tilde{y}L}\right] = \frac{-\delta\tilde{(p)}\rho u^2}{\delta\tilde{y}L} + \mu\left[\frac{\delta^2\tilde{v}U}{\delta(\tilde{x}L)^2} + \frac{\delta^2\tilde{v}U}{\delta(\tilde{y}L)^2}\right]$$

$$\frac{\rho U^2}{L}\left[\frac{\delta\tilde{v}}{\delta\tilde{t}} + (\tilde{u})\frac{\delta\tilde{v}}{\delta\tilde{x}} + (\tilde{v})\frac{\delta\tilde{v}}{\delta\tilde{y}}\right] = -\frac{\rho U^2}{L}\frac{\delta\tilde{p}}{\delta\tilde{y}} + \frac{\mu U}{L^2}\left[\frac{\delta^2\tilde{v}}{\delta(\tilde{x})^2} + \frac{\delta^2\tilde{v}}{\delta(\tilde{y})^2}\right]$$

$$\left[\frac{\delta\tilde{v}}{\delta\tilde{t}} + (\tilde{u})\frac{\delta\tilde{v}}{\delta\tilde{x}} + (\tilde{v})\frac{\delta\tilde{v}}{\delta\tilde{y}}\right] = -\frac{\delta\tilde{p}}{\delta\tilde{y}} + \frac{\mu}{\rho UL}\left[\frac{\delta^2\tilde{v}}{\delta(\tilde{x})^2} + \frac{\delta^2\tilde{v}}{\delta(\tilde{y})^2}\right]$$

$$\frac{\mu}{\rho UL} = \frac{1}{Re}$$

The only non-dimensional number in the momentum equations is the Reynolds Number (Re). As this number approaches ∞, the diffusive term of the equation will decrease approaching a number close to zero. As the Reynolds Number approaches zero, the diffusive term becomes larger approaching ∞.

3 Description of the flow for $Re = 10$

In figure 1, we plot multiple iterations of the same openFOAM simulation, but with different time steps and spatial resolutions. Note that here we use a scaling of 1 instead of 1.0 due to a typo in our code. Notably, the u-component shows a negative sink near the center, a positive source near the lid of the cavity, and is asymmetrical overall. In contrast, the v-component is symmetric through $x = 0.5$ and polarized, with a positive source on the right and a negative sink on the left, though the features themselves are not very symmetrical in appearance, with a greater spatial derivative with respect to x and than with respect to y. The higher resolution runs show greater fidelity around features, such as the sink in the u-component and the opposing sink and source in the v-component. The times for each run are displayed in the table below along with the number of steps and the time per step (C). The velocity components through the center cavity for the original 20x20 simulation are seen in figure 2.

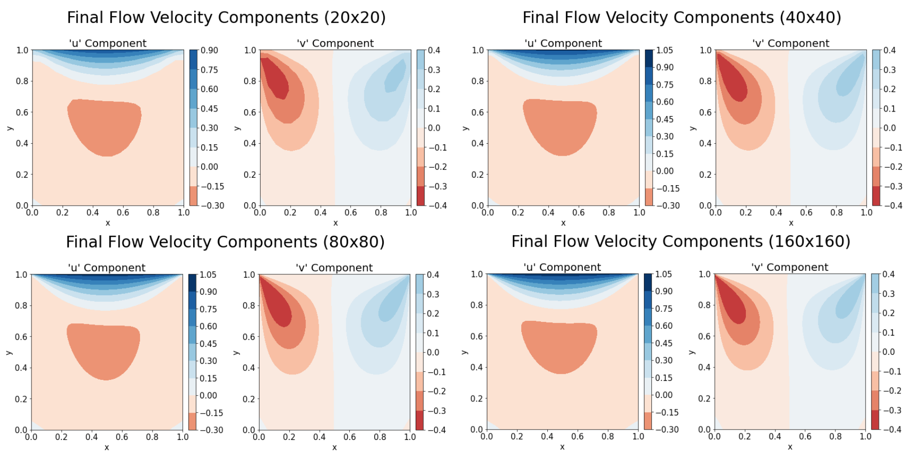

Figure 1: Contour map of 2D velocity components for the original 20x20 grid and all subsequent refinements (higher resolutions).

4 Refining The Solution

The times to compute the OpenFOAM simulation for different grid sizes and time steps are listed below:

Grid Dim.	Time Step	Wallclock	Steps	Time Per Step (C)
20x20	0.005s	0.68s	100	0.0068s
40x40	0.00125s	5.04s	400	0.0126s
80x80	0.0003125s	67.73s	1600	0.0423s
160x160	0.000078125s	1417.45s	6400	0.2215s

From the table, it is clear that with larger grids and smaller time steps, the wallclock time increases. It is also worthy to note that the time per step increases as well. This indicates that scaling both the grid size and time step together will not increase the time to compute linearly.
We define C as the following:

$$C = \frac{\text{Total Wallclock Time}}{\#\text{ of Steps}} \tag{1}$$

In figure 3 we plot the linear regression of C versus the number of grid points N.
The linear regression uses the following format:

$$C \approx mN + b \tag{2}$$

Where m is the slope and b is the y-intercept. We can apply a linear transfor-

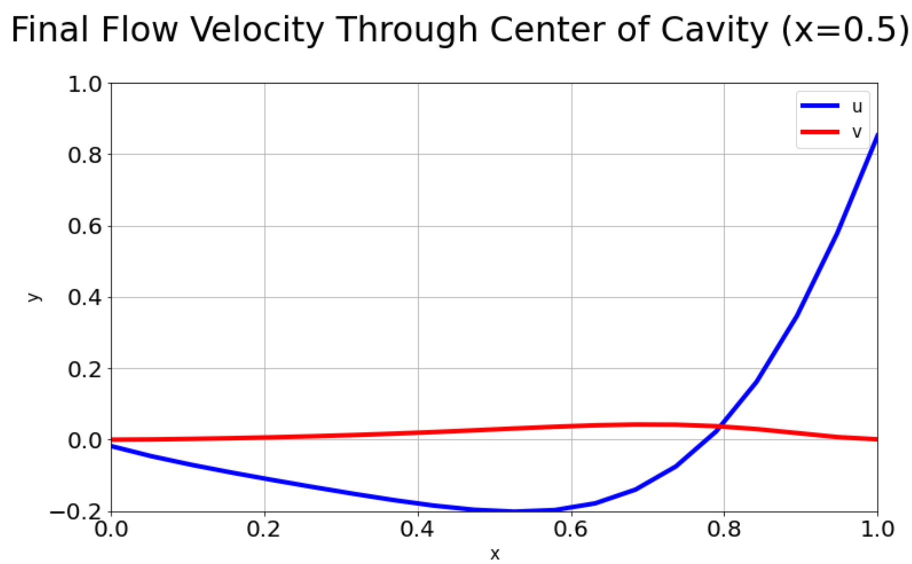

Figure 2: Plot of velocities along center of cavity at $x = 0.5$.

mation to convert to the form we want.

$$\begin{aligned} \ln(C) &= \ln(\beta N^{\alpha}) && (3) \\ &= \ln(\beta) + \ln(N) * \ln(\alpha) && (4) \\ &= b + mN && (5) \end{aligned}$$

and thus,

$$\alpha \approx 1.1587 \tag{6}$$

Thus, because $\alpha > 0$, the time to compute each time step relative to the number of points increases exponentially, not linearly nor constantly with additional points.

5 Force on the Lid

In figure 4, we use a line sampling to measure the change in the u-component of velocity for the original 20x20 grid openFOAM simulation, and then 40x40 grid simulations with Reynolds number (Re) equal to 100, 200, 400, and 500. We can then approximate the derivative at each pair of points $(y_i, u(x, y_i))$ using a polynomial fit function. In figure 5, we approximate the results of these with a fixed-end polynomial fit function to force $u(x, y = 1) = 1$ for the original 20x20 Re=10 simulation. This produces an estimate of the u-component velocity as a function of y approaching the boundary $y = 1$. We can the take the derivative of this polynomial to approximate the shear stress τ:

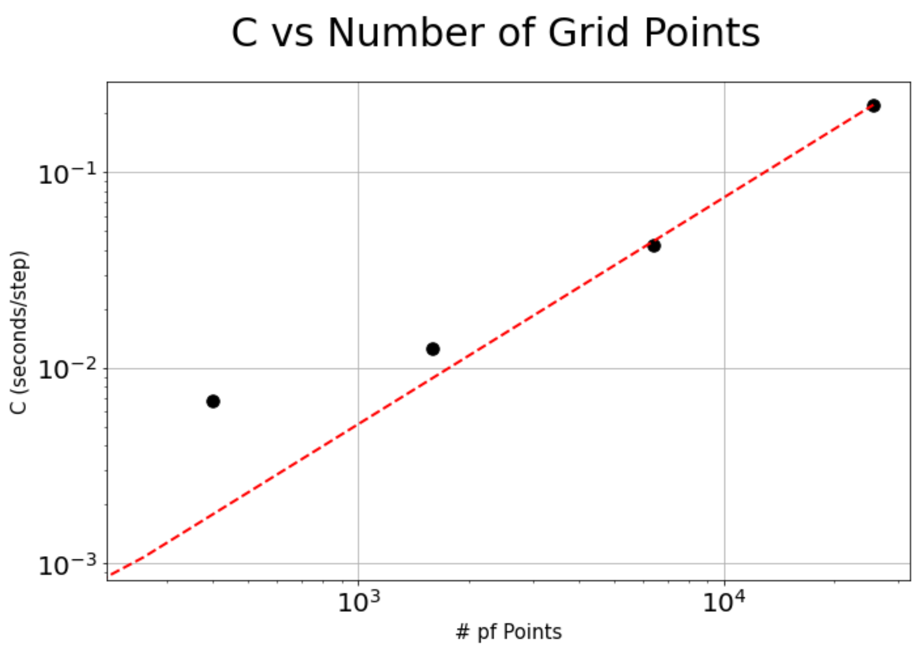

Figure 3: Plot of C relative to the number of grid points with a linear regression.

$$u(y) \approx 2320.48y^2 - 374.38y + 15.23 \tag{7}$$

$$\rightarrow \frac{\delta u}{\delta y} \approx 4640.96y - 374.38 \tag{8}$$

$$\approx \tau \tag{9}$$

This approximation is calculated and evaluated at $y = 1$ for simulations with the Reynolds number equal to 100, 200, 400, and 500 in figure 6. In both cases, the shear stress and force on the lid increases with higher Reynolds numbers.

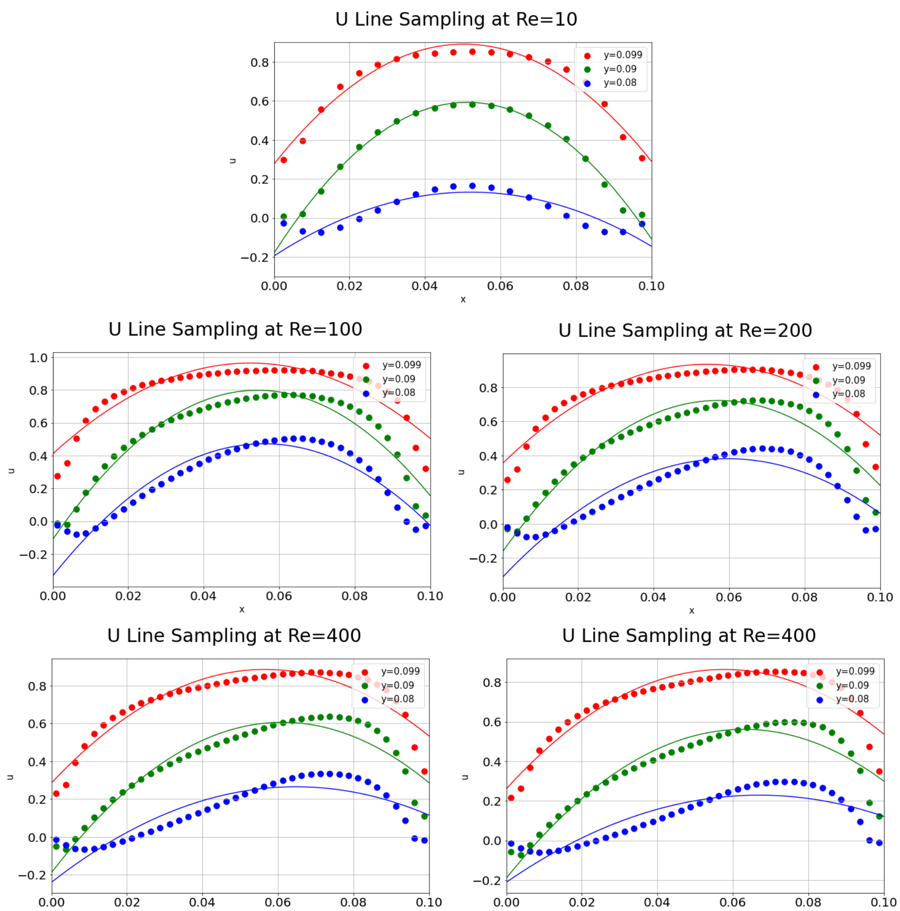

Figure 4: Results from sampling the cavity using a horizontal line at different values of y for simulations with different Reynolds numbers.

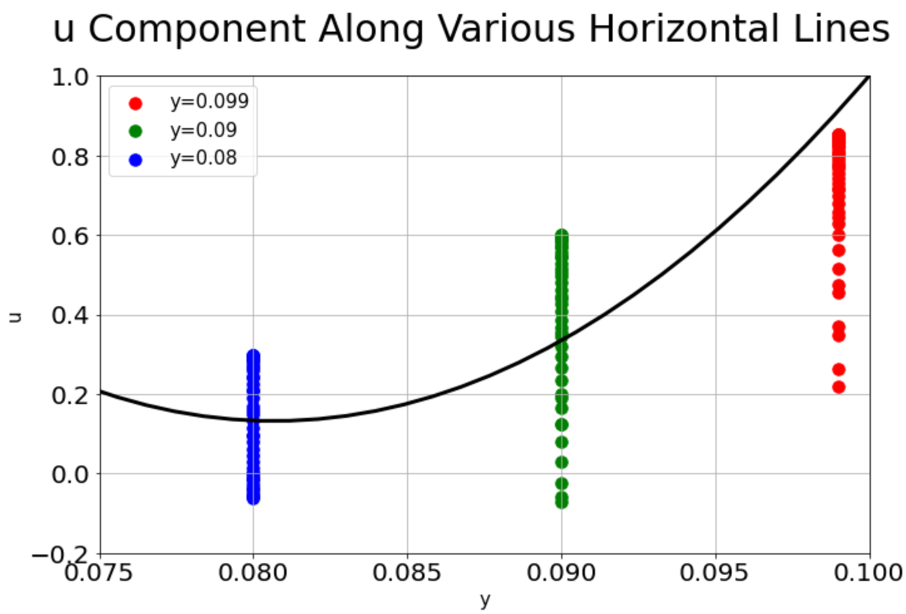

Figure 5: Results of fitting the data from figure 4 for Re=10 using a fixed end.

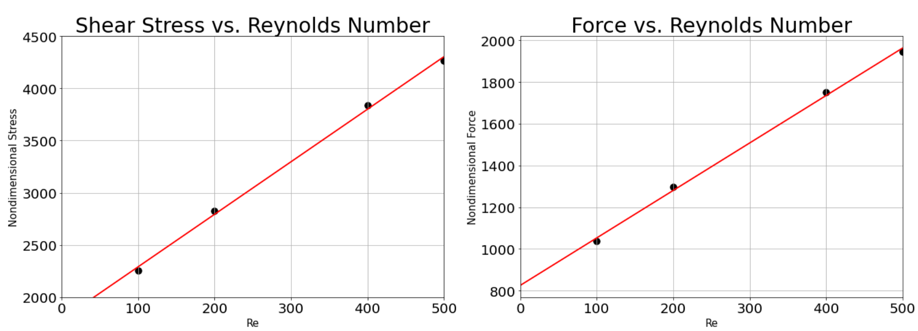

Figure 6: Plot showing non-dimensional shear stress and force compared against Reynolds number. The points were calculated using the procedure outlined in conjunction with figure 5.