# Structural Analysis HW 4

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## 1 Introduction

The finite element code for this assignement is contained in **HW4_run.py** and uses the input file **HW4_bridge.json**. The code used to generate the input file is contained in **genHW4_Bridge.py**.

## 2 Element Stiffness Contributions

Consider a three-dimensional truss element with Young's Modulus $E$ and cross-sectional area $A$. The length $L$ can be derived from the end points $x_i$, $y_i$, and $z_i$ as the following:

$$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \tag{1}$$

Thus, we can calculate the relevant cosines associated with the two end points:

$$\cos \theta_x = \frac{x_2 - x_1}{L} \tag{2}$$

$$\cos \theta_y = \frac{y_2 - y_1}{L} \tag{3}$$

$$\cos \theta_z = \frac{z_2 - z_1}{L} \tag{4}$$

$$\tag{5}$$

We can then use these cosines to transform the local stiffness matrix:

$$K_{local} = \frac{EA}{L} \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{6}$$

$$u_{local} = \begin{bmatrix} u_1^a \\ u_1^t \\ u_1^s \\ u_2^a \\ u_2^t \\ u_2^s \end{bmatrix} \tag{7}$$

$$K_{local} u_{local} = F_{local} \tag{8}$$

$$R^T K_{local} R u_{local} = K_{global} u_{global} = F_{global} \tag{9}$$

Here, $N/a$ indicates values that aren't needed for calculating the contribution to the global stiffness matrix. So, to save energy, they are not calculated.

$$u_{global} = R u_{local} = \begin{bmatrix} \frac{x_2-x_1}{L} & \frac{y_2-y_1}{L} & \frac{z_2-z_1}{L} & 0 & 0 & 0 \\ N/a & N/a & N/a & 0 & 0 & 0 \\ N/a & N/a & N/a & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{x_2-x_1}{L} & \frac{y_2-y_1}{L} & \frac{z_2-z_1}{L} \\ 0 & 0 & 0 & N/a & N/a & N/a \\ 0 & 0 & 0 & N/a & N/a & N/a \end{bmatrix} u_{local} \tag{10}$$

# 3  Structure Analysis


HW4 Structure Deformation
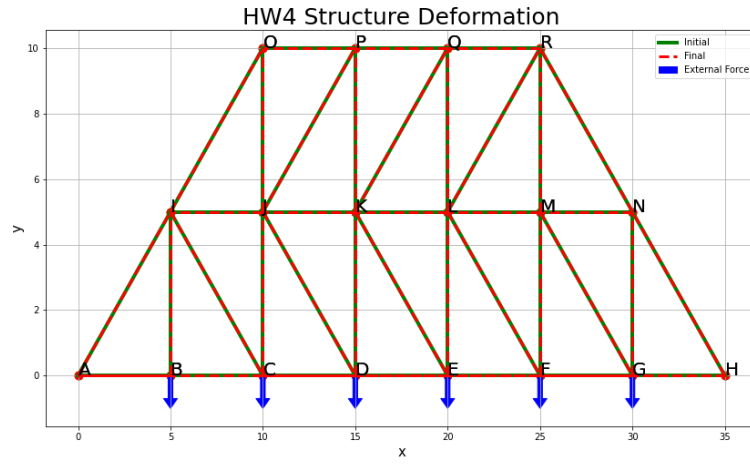
Assuming that nodes **A** and **H** are fixed and that the structure has the following values for all elements:

$$E = 210 \times 10^9 \text{ Pa} \tag{11}$$

$$A = 0.01 \text{ m} \tag{12}$$

$$P = 1 \text{ N } \textit{for scaling} \tag{13}$$

We then obtain the following results from the finite element analysis:

| Element | Internal Force (N) | $\nabla$ Length (m) | Strain | Stress $\frac{N}{m^2}$ |
|---------|--------------------|--------------------|--------|------------------------|
| A-H | 0.00e+00 | 0.00e+00 | 0.00e+00 | 0.00e+00 |
| B-A | -1.46e-10 | -7.29e-10 | -1.46e-10 | -1.46e-08 |
| C-B | -1.46e-10 | -7.29e-10 | -1.46e-10 | -1.46e-08 |
| D-C | 7.08e-11 | 3.54e-10 | 7.08e-11 | 7.08e-09 |
| E-D | 2.00e-10 | 1.00e-09 | 2.00e-10 | 2.00e-08 |
| F-E | 1.56e-10 | 7.82e-10 | 1.56e-10 | 1.56e-08 |
| G-F | 1.00e-11 | 5.02e-11 | 1.00e-11 | 1.00e-09 |
| H-G | -1.46e-10 | -7.29e-10 | -1.46e-10 | -1.46e-08 |
| J-I | -9.09e-10 | -4.55e-09 | -9.09e-10 | -9.09e-08 |
| K-J | -6.92e-10 | -3.46e-09 | -6.92e-10 | -6.92e-08 |
| L-K | -6.04e-10 | -3.02e-09 | -6.04e-10 | -6.04e-08 |
| M-L | -7.88e-10 | -3.94e-09 | -7.88e-10 | -7.88e-08 |
| N-M | -6.32e-10 | -3.16e-09 | -6.32e-10 | -6.32e-08 |
| P-O | -7.36e-10 | -3.68e-09 | -7.36e-10 | -7.36e-08 |
| Q-P | -1.08e-09 | -5.41e-09 | -1.08e-09 | -1.08e-07 |
| R-Q | -1.13e-09 | -5.63e-09 | -1.13e-09 | -1.13e-07 |
| B-I | 4.76e-10 | 2.38e-09 | 4.76e-10 | 4.76e-08 |
| C-J | 2.60e-10 | 1.30e-09 | 2.60e-10 | 2.60e-08 |
| D-K | 3.47e-10 | 1.74e-09 | 3.47e-10 | 3.47e-08 |
| E-L | 5.20e-10 | 2.60e-09 | 5.20e-10 | 5.20e-08 |
| F-M | 6.23e-10 | 3.11e-09 | 6.23e-10 | 6.23e-08 |
| G-N | 6.32e-10 | 3.16e-09 | 6.32e-10 | 6.32e-08 |
| J-O | 7.36e-10 | 3.68e-09 | 7.36e-10 | 7.36e-08 |
| K-P | 3.47e-10 | 1.74e-09 | 3.47e-10 | 3.47e-08 |
| L-Q | 4.36e-11 | 2.18e-10 | 4.36e-11 | 4.36e-09 |
| M-R | 4.67e-10 | 2.33e-09 | 4.67e-10 | 4.67e-08 |
| I-C | 3.06e-10 | 2.17e-09 | 3.06e-10 | 3.06e-08 |
| J-D | 1.83e-10 | 1.29e-09 | 1.83e-10 | 1.83e-08 |
| K-E | -6.16e-11 | -4.36e-10 | -6.16e-11 | -6.16e-09 |
| L-F | -2.07e-10 | -1.46e-09 | -2.07e-10 | -2.07e-08 |
| M-G | -2.20e-10 | -1.56e-09 | -2.20e-10 | -2.20e-08 |
| J-P | -4.91e-10 | -3.47e-09 | -4.91e-10 | -4.91e-08 |
| K-Q | -6.16e-11 | -4.36e-10 | -6.16e-11 | -6.16e-09 |
| L-R | 4.66e-10 | 3.30e-09 | 4.66e-10 | 4.66e-08 |
| A-I | -2.02e-09 | -1.43e-08 | -2.02e-09 | -2.02e-07 |
| I-O | -1.04e-09 | -7.36e-09 | -1.04e-09 | -1.04e-07 |
| H-N | -2.02e-09 | -1.43e-08 | -2.02e-09 | -2.02e-07 |
| N-R | -1.13e-09 | -7.97e-09 | -1.13e-09 | -1.13e-07 |

Looking at the results, we identify the element in maximum tension as **J-O** and the element in maximum compression as **H-N**. Euler's buckling equation is as follows:

$$F_{critical} = \frac{\pi^2 EI}{(cL)^2} \tag{14}$$

Here, $F_{critical}$ refers to the force required until the element buckles, $E$ is Young's modulus of elasticity, $I$ is the smallest area moment of inertia, $c$ is the column effective length factor, and $L$ is the length of the element without loading. Since all the elements are pinned at both ends, $c = 1$. We can calculate the $L$ and $I$ for elements **J-O** and **H-N** by assuming the beams are uniform along their length and have a square cross-sections:

$$L = \left[ (x_2 - x_1)^2 + (y_2 - y_1)^2 \right]^{\frac{1}{2}} \tag{15}$$

$$I = I_x = I_y = \frac{s^4}{12} = \frac{A^2}{12} \tag{16}$$

We can then calculate $F_{critical}$ for each element:

$$F_{critical}^{JO} = \frac{\pi^2 E \frac{A^2}{12}}{5^2} \approx 6.91 \times 10^5 \text{ N} \tag{17}$$

$$F_{critical}^{HN} = \frac{\pi^2 E \frac{A^2}{12}}{\sqrt{5^2 + 5^2}} \approx 2.44 \times 10^6 \text{ N} \tag{18}$$