Structural Analysis HW 4

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1 Element Stiffness Contributions

Consider a three-dimensional truss element with Young's Modulus E and cross-sectional area A. The length L can be derived from the end points x_i, y_i, and z_i as the following:

$$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \tag{1}$$

Thus, we can calculate the relevant cosines associated with the two end points:

$$\cos\theta_x = \frac{x_2 - x_1}{L} \tag{2}$$
$$\cos\theta_y = \frac{y_2 - y_1}{L} \tag{3}$$
$$\cos\theta_z = \frac{z_2 - z_1}{L} \tag{4}$$
$$\tag{5}$$

We can then use these cosines to transform the local stiffness matrix:

$$K_{local} = \frac{EA}{L}\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{6}$$

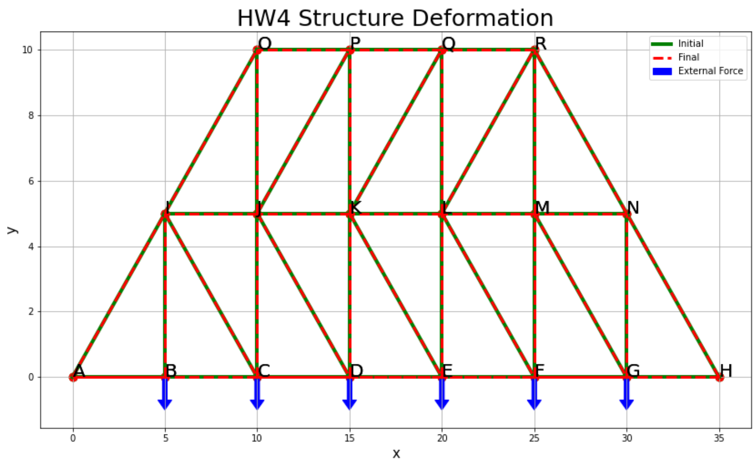
$$u_{local} = \begin{bmatrix} u_1^a \\ u_1^t \\ u_1^s \\ u_2^a \\ u_2^t \\ u_2^s \end{bmatrix} \tag{7}$$

$$K_{local}u_{local} = F_{local} \tag{8}$$
$$R^T K_{local} R u_{local} = K_{global}u_{global} = F_{global} \tag{9}$$

Here, N/a indicates values that aren't needed for calculating the contribution to the global stiffness matrix. So, to save energy, they are not calculated.

$$u_{global} = Ru_{local} = \begin{bmatrix} \frac{x_2-x_1}{L} & \frac{y_2-y_1}{L} & \frac{z_2-z_1}{L} & 0 & 0 & 0 \\ N/a & N/a & N/a & 0 & 0 & 0 \\ N/a & N/a & N/a & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{x_2-x_1}{L} & \frac{y_2-y_1}{L} & \frac{z_2-z_1}{L} \\ 0 & 0 & 0 & N/a & N/a & N/a \\ 0 & 0 & 0 & N/a & N/a & N/a \end{bmatrix} u_{local} \tag{10}$$

2 Structure Analysis



Assuming that nodes **A** and **H** are fixed and that the structure has the following values for all elements:

$$E = 210 \times 10^9 \text{ Pa} \tag{11}$$

$$A = 0.01 \text{ m} \tag{12}$$

$$P = 1 \text{ N } \textit{for scaling} \tag{13}$$

We then obtain the following results from the finite element analysis:

Element	Internal Force (N)	∇ Length (m)	Strain	Stress $\frac{N}{m^2}$
A-H	0.00e+00	0.00e+00	0.00e+00	0.00e+00
B-A	-1.46e-10	-7.29e-10	-1.46e-10	-1.46e-08
C-B	-1.46e-10	-7.29e-10	-1.46e-10	-1.46e-08
D-C	7.08e-11	3.54e-10	7.08e-11	7.08e-09
E-D	2.00e-10	1.00e-09	2.00e-10	2.00e-08
F-E	1.56e-10	7.82e-10	1.56e-10	1.56e-08
G-F	1.00e-11	5.02e-11	1.00e-11	1.00e-09
H-G	-1.46e-10	-7.29e-10	-1.46e-10	-1.46e-08
J-I	-9.09e-10	-4.55e-09	-9.09e-10	-9.09e-08
K-J	-6.92e-10	-3.46e-09	-6.92e-10	-6.92e-08
L-K	-6.04e-10	-3.02e-09	-6.04e-10	-6.04e-08
M-L	-7.88e-10	-3.94e-09	-7.88e-10	-7.88e-08
N-M	-6.32e-10	-3.16e-09	-6.32e-10	-6.32e-08
P-O	-7.36e-10	-3.68e-09	-7.36e-10	-7.36e-08
Q-P	-1.08e-09	-5.41e-09	-1.08e-09	-1.08e-07
R-Q	-1.13e-09	-5.63e-09	-1.13e-09	-1.13e-07
B-I	4.76e-10	2.38e-09	4.76e-10	4.76e-08
C-J	2.60e-10	1.30e-09	2.60e-10	2.60e-08
D-K	3.47e-10	1.74e-09	3.47e-10	3.47e-08
E-L	5.20e-10	2.60e-09	5.20e-10	5.20e-08
F-M	6.23e-10	3.11e-09	6.23e-10	6.23e-08
G-N	6.32e-10	3.16e-09	6.32e-10	6.32e-08
J-O	7.36e-10	3.68e-09	7.36e-10	7.36e-08
K-P	3.47e-10	1.74e-09	3.47e-10	3.47e-08
L-Q	4.36e-11	2.18e-10	4.36e-11	4.36e-09
M-R	4.67e-10	2.33e-09	4.67e-10	4.67e-08
I-C	3.06e-10	2.17e-09	3.06e-10	3.06e-08
J-D	1.83e-10	1.29e-09	1.83e-10	1.83e-08
K-E	-6.16e-11	-4.36e-10	-6.16e-11	-6.16e-09
L-F	-2.07e-10	-1.46e-09	-2.07e-10	-2.07e-08
M-G	-2.20e-10	-1.56e-09	-2.20e-10	-2.20e-08
J-P	-4.91e-10	-3.47e-09	-4.91e-10	-4.91e-08
K-Q	-6.16e-11	-4.36e-10	-6.16e-11	-6.16e-09
L-R	4.66e-10	3.30e-09	4.66e-10	4.66e-08
A-I	-2.02e-09	-1.43e-08	-2.02e-09	-2.02e-07
I-O	-1.04e-09	-7.36e-09	-1.04e-09	-1.04e-07
H-N	-2.02e-09	-1.43e-08	-2.02e-09	-2.02e-07
N-R	-1.13e-09	-7.97e-09	-1.13e-09	-1.13e-07

Looking at the results, we identify the element in maximum tension as **J-O** and the element in maximum compression as **H-N**. Euler's buckling equation is as follows:

$$F_{critical} = \frac{\pi^2 EI}{(cL)^2} \tag{14}$$

Here, $F_{critical}$ refers to the force required until the element buckles, E is Young's modulus of elasticity, I is the smallest area moment of inertia, c is the column effective length factor, and L is the length of the element without loading. Since all the elements are pinned at both ends, $c = 1$. We can calculate the L and I for elements **J-O** and **H-N** by assuming the beams are uniform along their length and have a square cross-sections:

$$L = \left[(x_2 - x_1)^2 + (y_2 - y_1)^2\right]^{\frac{1}{2}} \tag{15}$$

$$I = I_x = I_y = \frac{s^4}{12} = \frac{A^2}{12} \tag{16}$$

We can then calculate $F_{critical}$ for each element:

$$F_{critical}^{JO} = \frac{\pi^2 E \frac{A^2}{12}}{5^2} \approx 6.91 \times 10^5 \text{ N} \tag{17}$$

$$F_{critical}^{HN} = \frac{\pi^2 E \frac{A^2}{12}}{\sqrt{5^2 + 5^2}} \approx 2.44 \times 10^6 \text{ N} \tag{18}$$