Computer Vision. Sheet 1

Svetlana Seliunina, Aleksei Zhuravlev

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1 Convolution theorem

$$(f*(g*h))(t) = \int_{-\infty}^{\infty} f(t_1)(g*h)(t-t_1)dt_1 =$$

$$= \int_{-\infty}^{\infty} f(t_1)(\int_{-\infty}^{\infty} g(t_2)h(t-t_1-t_2)dt_2)dt_1 =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t_1)g(t_2)h((t-t_1)-t_2)dt_2dt_1 =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t_1)g((t_1+t_2)-t_1)h(t-(t_1+t_2))dt_2dt_1 =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t_1)g(t_3-t_1)h(t-t_3)dt_3dt_1 =$$

$$= \int_{-\infty}^{\infty}(\int_{-\infty}^{\infty} f(t_1)g(t_3-t_1)dt_1)(h(t-t_3)dt_3 =$$

$$= \int_{-\infty}^{\infty}(f*g)(t_3)(h(t-t_3))dt_3 =$$

$$((f*g)*h)(t)$$

6) Prove that convolution 2 times with Gauss kernel σ is the same as convolution once with Gauss kernel $\sigma\sqrt{2}$

Fourier transform of a Gaussian, $f_x(x) = \mathcal{N}(x, \mu_x, \sigma_x^2)$, :

$$\mathcal{F}\{f_x\} = F_x(\omega) = \exp[-j\omega\mu_x] \exp\left[-\frac{\sigma_x^2\omega^2}{2}\right]$$

By the convolution theorem:

$$f_z(z) = (f_x * f_y)(z) = \mathcal{F}^{-1}\{\mathcal{F}\{f_x\} \cdot \mathcal{F}\{f_y\}\} =$$

$$= \mathcal{F}^{-1}\left\{\exp[-j\omega\mu_x] \exp\left[-\frac{\sigma_x^2\omega^2}{2}\right] \exp[-j\omega\mu_y] \exp\left[-\frac{\sigma_y^2\omega^2}{2}\right]\right\} =$$

$$= \mathcal{F}^{-1}\left\{\exp[-j\omega(\mu_x + \mu_y)] \exp\left[-\frac{(\sigma_x^2 + \sigma_y^2)\omega^2}{2}\right]\right\}$$

$$= \mathcal{N}(z, \mu_x + \mu_y, \sigma_x^2 + \sigma_y^2)$$

In our case, $\mu_x = \mu_y = 0$, $\sigma_x = \sigma_y = \sigma$, $f_z(z) = (f_\sigma * f_\sigma)(z) = \mathcal{N}(z, 0, 2\sigma^2)$,

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