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Technical Neural Networks Assignment Sheet-1

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Assignment 1

1. A lot of simple, interconnected processing units (neurons).
2. Modifiable, plastic connections (synapses, synaptic weights).
3. Mapping from an input vector to a scalar output value.
4. Nonlinear transfer function.

Assignment 2

1. Beginning: Early days when basic neural models and learning rules inspired by biological systems were discovered.
2. Quite Years: Lack of funding triggered by the proof that perceptron type network cannot learn Boolean function XOR.
3. Renaissance: New multi-layer models and new learning rules caused rise of interest.
4. Consolidation: Neural network become widely used and accepted.
5. Second start-up: Discoveries of modern paradigms and applications.

Assignment 3

$$\Delta w_{ji} = \alpha (t_j - y_j) g'(h_j) x_i$$

where

α is a small constant called learning rate

$g(x)$ is the neuron's activation function

g' is the derivative of g

t_j is the target output

h_j is the weighted sum of the neuron's inputs, $h_j = \sum x_i w_{ji}$

y_j is the actual output, $y_j = g(h_j)$

x_i is the i th input.

Assignment 4

Because layers in MLP are fully connected, number of weights for a 4-layer MLP can be calculated using the following formula:

$$|w_4| = |w_{nh1}| + |w_{h1h2}| + |w_{h2m}| = (n+1)h_1 + (h_1+1)h_2 + (h_2+1)m,$$

where +1 indicates BIAS. For 3-layer MLP we get:

$$|w_3| = |w_{nh}| + |w_{hm}| = (n+1)h + (h+1)m = h(n+m+1) + m.$$

If $|w_4| \approx |w_3$ and $n = 5, h_1 = 21, h_2 = 30, m = 4$:

$$6 \cdot 21 + 22 \cdot 30 + 31 \cdot 4 = 10h + 4$$

$$h = 90.6 \approx 91$$

Assignment 5

Handwritten derivations for the sigmoid and tanh functions:

$$f_{\text{logist}}(z) = \frac{1}{1+e^{-z}}$$

$$f'_{\text{logist}}(z) = \frac{1}{(1+e^{-z})^2} \cdot (-e^{-z}) = \frac{e^{-z}}{(1+e^{-z})^2} = \frac{1}{1+e^{-z}} \cdot \left(1 - \frac{1}{1+e^{-z}}\right) =$$

$$= f_{\text{logist}}(z) (1 - f_{\text{logist}}(z))$$

$$f_{\text{tanh}}(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}} = \frac{e^{2z} - 1}{e^{2z} + 1}$$

$$f'_{\text{tanh}}(z) = \frac{2e^{2z}(e^{2z} + 1) - 2e^{2z}(e^{2z} - 1)}{(e^{2z} + 1)^2} = \frac{4e^{2z}}{(e^{2z} + 1)^2} = 1 - \left(\frac{e^{2z} - 1}{e^{2z} + 1}\right)^2 =$$

$$= f_{\text{tanh}}(z) (1 - f_{\text{tanh}}(z))$$

Assignment 6

Proof. According to slide 12 of the second lecture, perceptron can be described as a function $x_2 = -w_1/w_2 \cdot x_1 - w_0/w_2$ separating the 2D-plane in two parts. This function is linear so we need to prove that XOR Boolean function is not linearly separable. We will rewrite this line as

$$L : a \cdot x_1 + b \cdot x_2 + c = 0.$$

If two points lie on the same side of L then $(a \cdot x_{11} + b \cdot x_{21} + c)(a \cdot x_{12} + b \cdot x_{22} + c) > 0$, and if on opposite sides then $(a \cdot x_{11} + b \cdot x_{21} + c)(a \cdot x_{12} + b \cdot x_{22} + c) < 0$.

1. Points (0, 0) and (1, 1) lie on the same side: $c(a + b + c) > 0$.
2. Points (1, 0) and (1, 1) lie on the same side: $(a + c)(a + b + c) < 0$.
3. Points (0, 1) and (1, 1) lie on the same side: $(b + c)(a + b + c) < 0$.

Now we add (2) and (3) together:

$$(a + b + c)(a + b + 2c) = (a + b + c)^2 + c(a + b + c) < 0.$$

Because $(a + b + c)^2 \geq 0$, $c(a + b + c) < 0$ which contradicts (1). L cannot separate this function and therefore, a simple perceptron without a hidden layer is not capable to implement the Boolean function XOR. \square

Assignment 7

Error minimization by gradient descent: $\Delta w_{ij} = -\mu \frac{\partial E^*(w_{ij})}{\partial w_{ij}}$ where $E^* = \frac{1}{2} \sum_{m=0}^M (\hat{y}_m - y_m)^4$. We will start with the rule for output neuron and skip computations that are similar to those given in the lecture.

$$\frac{\partial E^*(w_{hm})}{\partial w_{hm}} = \frac{\partial E^*(w_{hm})}{\partial net_m} \cdot \frac{\partial net_m}{\partial w_{hm}} = \frac{\partial E^*(w_{hm})}{\partial y_m} \cdot \frac{\partial y_m}{\partial net_m} \cdot \frac{\partial net_m}{\partial w_{hm}}$$

According to the lecture:

$$\frac{\partial y_m}{\partial net_m} = f'(net_m); \frac{\partial net_m}{\partial w_{hm}} = out_h$$

Partial derivative of the error function:

$$\frac{\partial E^*(w_{hm})}{\partial y_m} = \frac{1}{2} \cdot 4 \cdot (\hat{y}_m - y_m)^3 \cdot (-1) = -2(\hat{y}_m - y_m)^3$$

Collecting all together:

$$\begin{aligned} \Delta w_{hm} &= 2\mu(\hat{y}_m - y_m)^3 f'(net_m) out_h \\ \delta_m &= -\frac{\partial E^*(w_{hm})}{\partial net_m} = 2(\hat{y}_m - y_m)^3 f'(net_m) \\ \Delta w_{hm} &= \mu \delta_m out_h \end{aligned}$$

Rule for hidden neuron:

$$\begin{aligned} \Delta w_{gh} &= \mu \delta_h out_g \\ \delta_h &= -\frac{\partial E^*(w_{gh})}{\partial net_h} = -\frac{\partial E^*(w_{gh})}{\partial out_h} \cdot \frac{\partial out_h}{\partial net_h} = -\frac{\partial E^*(w_{gh})}{\partial out_h} f'(net_h) = \\ &= \sum_{k=1}^K \delta_k w_{hk} f'(net_h) \end{aligned}$$

Derivation is the same as for Backpropagation of Error because it does not directly depend on partial derivative of E .

Assignment 8

Programming assignment:

<https://colab.research.google.com/drive/1P6k0W6upviiZiNSYfYHwvJtnJvXHpsyz?usp=sharing>