The background of the slide features a vast, dense flock of birds in flight, silhouetted against a vibrant orange and yellow sunset. The birds are concentrated in the upper half of the frame, creating a sense of immense activity and complexity.

# Differentiable Agent-based Modeling

## Systems, Methods and Applications

Resources: [bit.ly/diff-abms](https://bit.ly/diff-abms)

Ayush Chopra (MIT)

Arnau Quera-Bofarull (Oxford)

Sijin Zhang (ESR, New Zealand)

# Speakers



Ayush Chopra

PhD Candidate  
MIT Media Lab



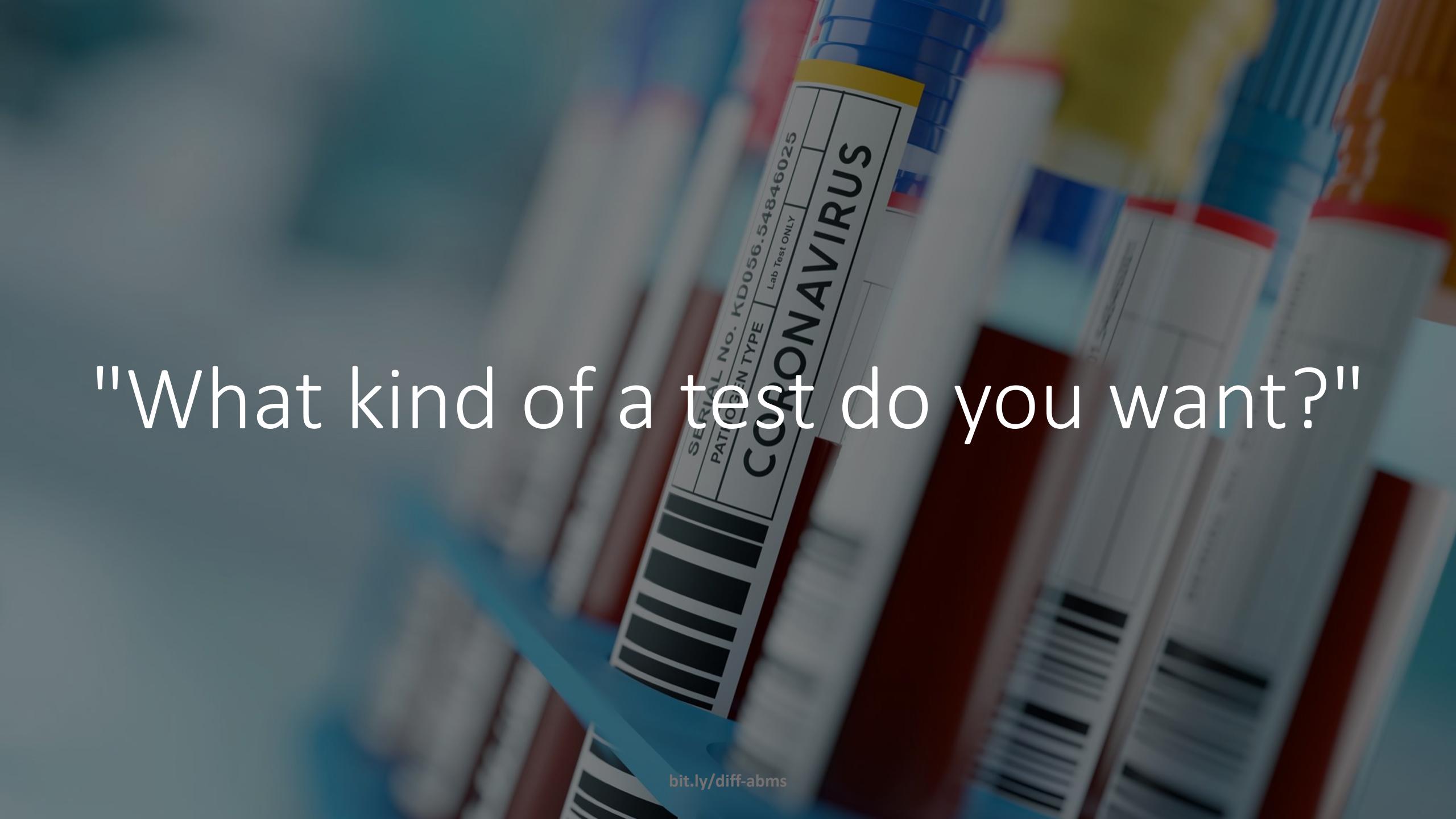
Arnaud Quera-Bofarull

Postdoc Researcher  
University of Oxford



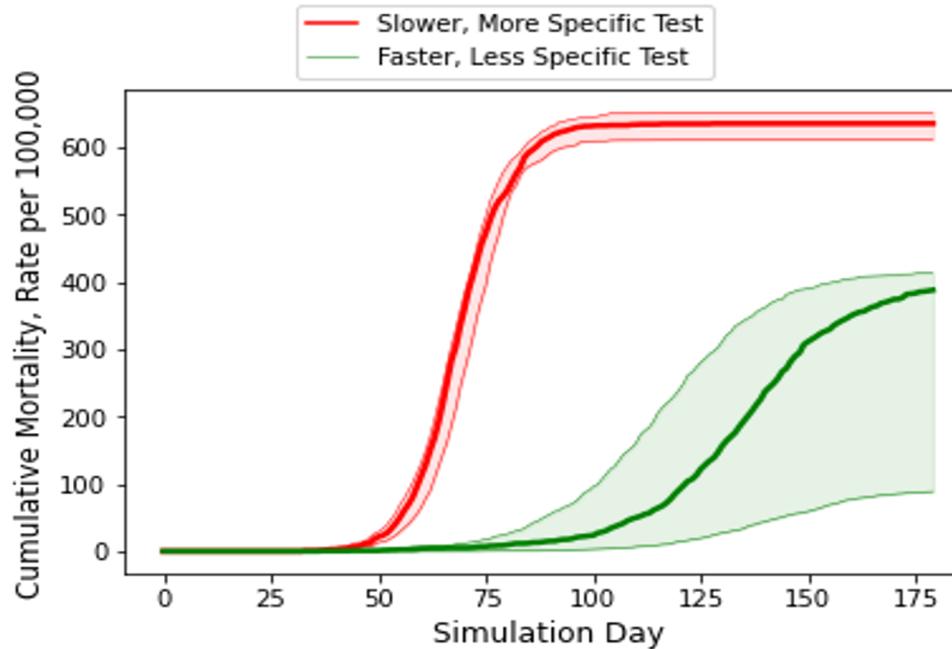
Sijin Zhang

Senior Scientist  
ESR, New Zealand

A close-up, slightly blurred photograph of several test tubes standing upright. One tube in the foreground has a white label with black text and a barcode. The text on the label includes 'SIGNAL NO. KDO56.54846025', 'PATIENT TYPE', 'Lab Test ONLY', and 'COVID-19 VIRUS'.

"What kind of a test do you want?"

collective prioritizes test speed over accuracy...



Collective outcomes can be very different from the sum of individual choices

modeling collective behavior is critical



build bridges?



[bit.ly/diff-abms](http://bit.ly/diff-abms)



[bit.ly/diff-abms](http://bit.ly/diff-abms)

# Collective behavior across scales and substrates

Cities



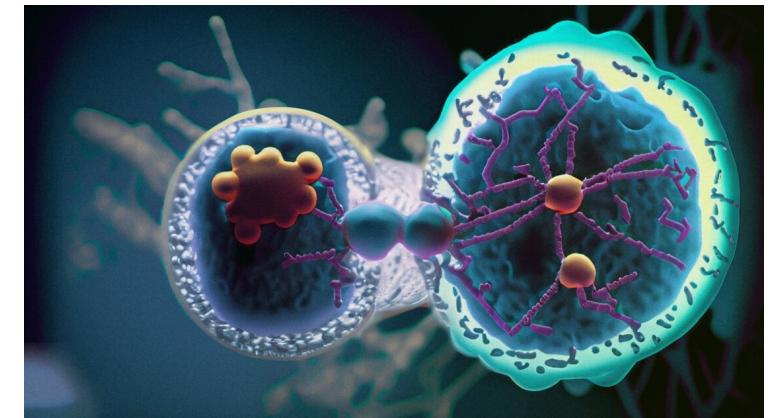
Supply Chains

Citizens



Pandemics

Cells



Morphogenesis

# Collective behavior across scales and substrates

Cities



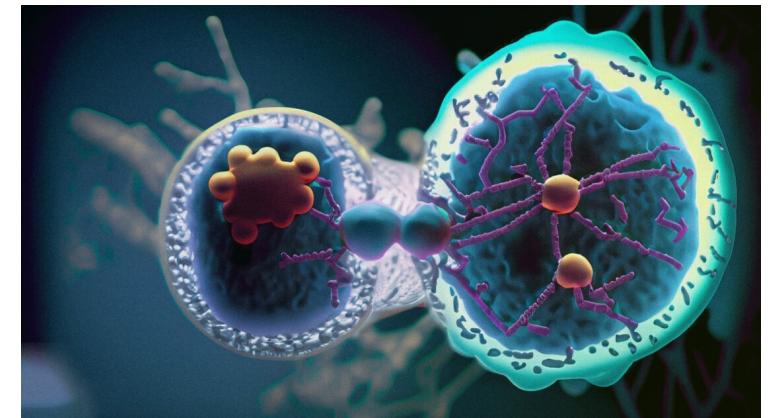
Supply Chains

Citizens



Pandemics

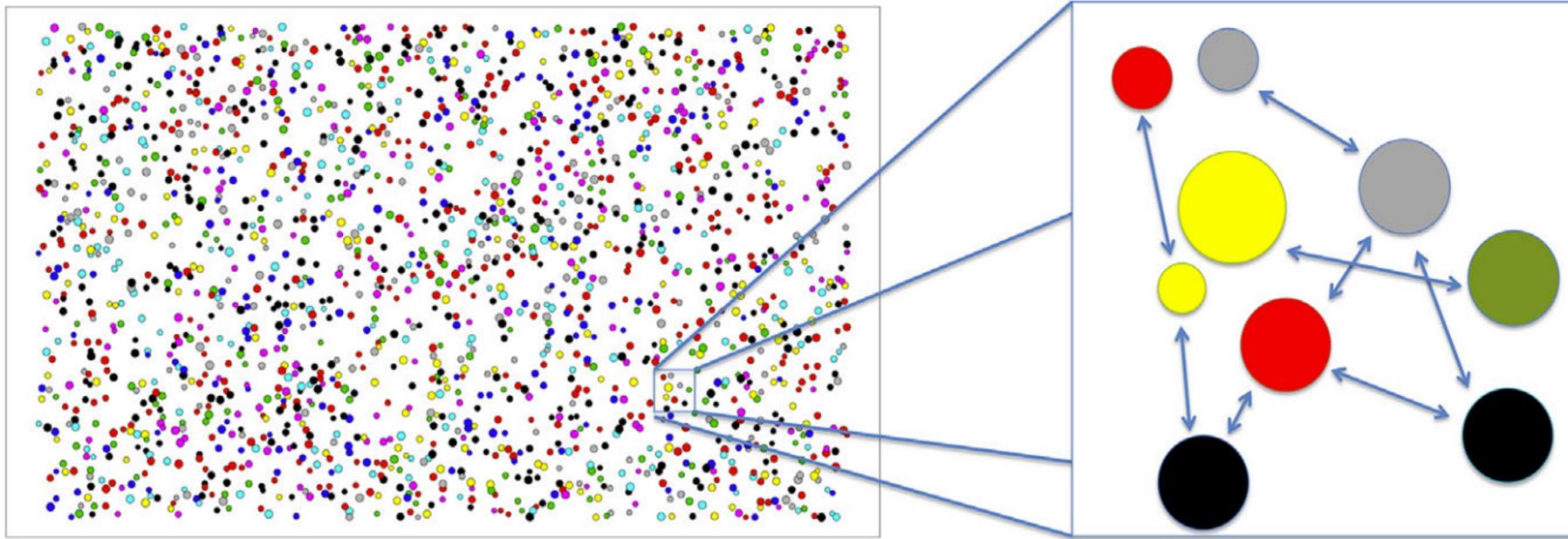
Cells



Morphogenesis

how to capture?

# Agent-based Models

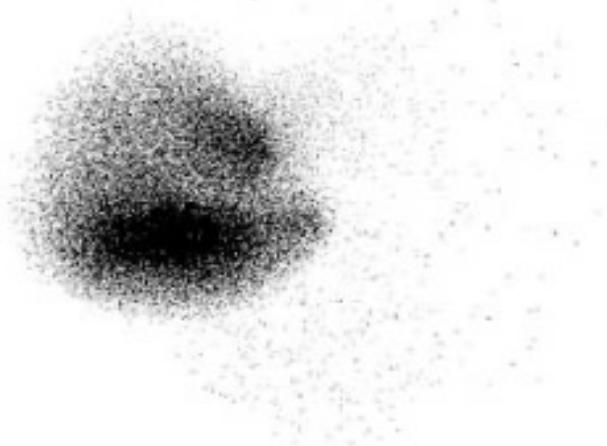


Simulate microscopic behavior and interactions in heterogeneous collectives

# ABMs vs Multi-Agent Reinforcement Learning

## ABMs

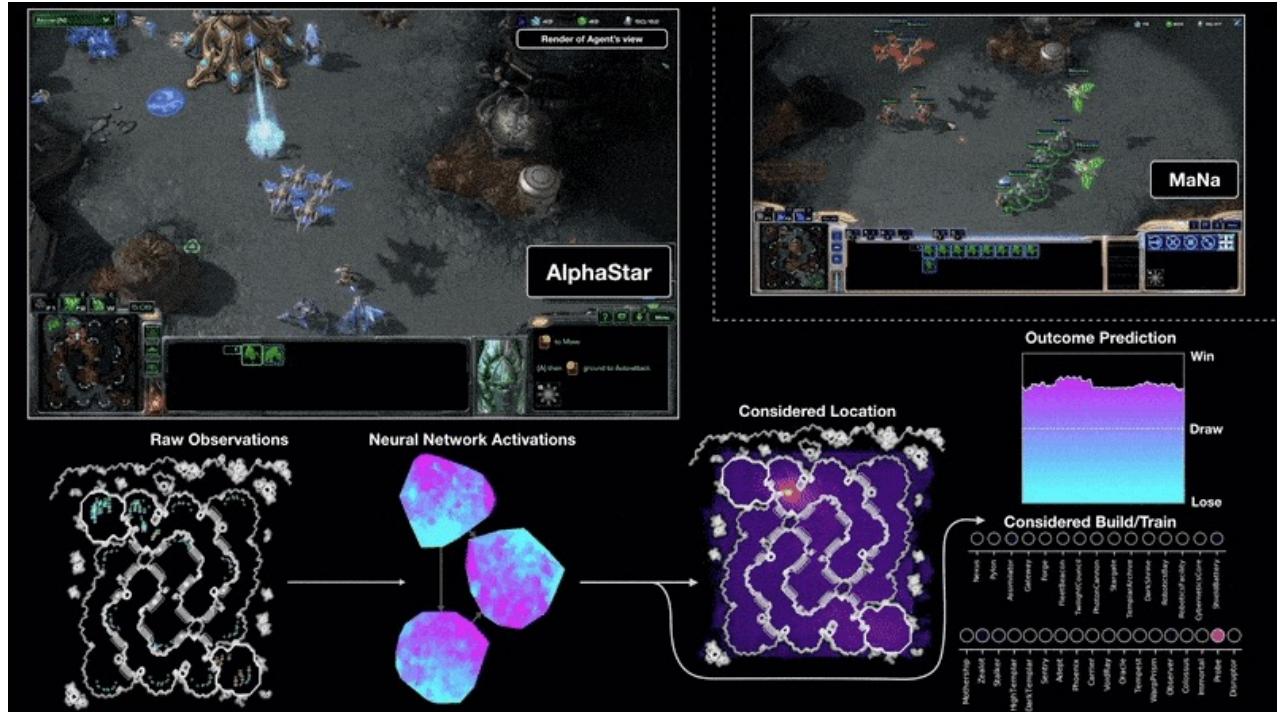
- Many agents
- Simple behavior



Flocking birds

## MARL

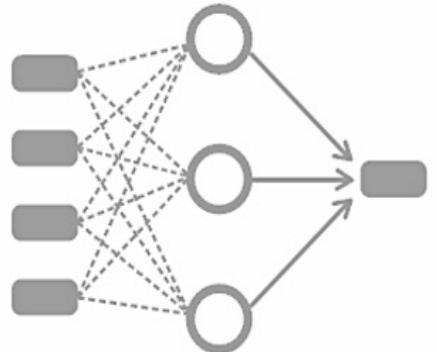
- Few agents
- Complicated behavior



[bit.ly/diff-abms](http://bit.ly/diff-abms)

Starcraft2 (AlphaStar)

# long history of research and open challenges



## Computation

Simulation?  
Calibrate?  
Analyze?



## Data

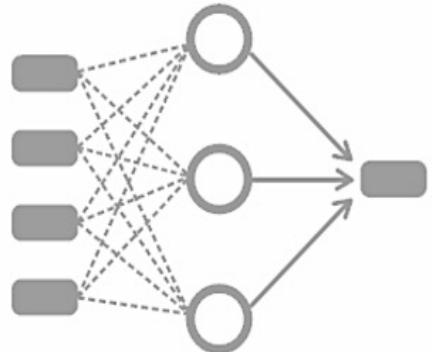
Multi-modal?  
Multi-scale?  
Distributed?



## Expressiveness

Behaviour?  
Mechanism?  
Real-world feedback?

# Proposal: Differentiable Agent-based Modeling



## Computation

Simulation?  
Calibrate?  
Analyze?



## Data

Multi-modal?  
Multi-scale?  
Distributed?



## Expressiveness

Behaviour?  
Mechanism?  
Real-world feedback?

## Vectorization

## Gradient-based learning

## Neural Network composition

# Agent-based Model



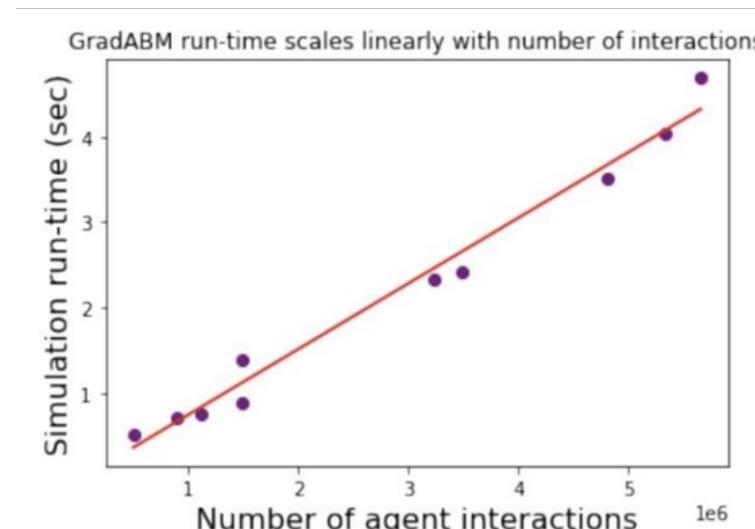
Differentiable if

$$\nabla_{\theta} \mathbb{E}[f(\theta)] \text{ exists}$$

# Why do we care about the gradient?

Simulate **country-scale ecosystems** for few hundred dollars on commodity hardware

Method	Simulation	Calibration	Analysis
ABM	50 hours	100,000 hours	5,000 hours
<b>Differentiable ABM</b>	5 minutes	20 minutes	10 seconds



# Differentiable ABMs are being deployed across domains



# Scope of tutorial

- Preliminaries
  - Background to automatic differentiation
  - Implement a differentiable ABM
- Algorithms
  - Techniques to calibrate and analyze differentiable ABMs
- Applications
  - Real-world case study in New Zealand
- Systems
  - Tooling to build and calibrate differentiable ABMs at scale

# Scope of tutorial

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# Automatic Differentiation

# Stochastic Automatic Differentiation

# Implement a Differentiable ABM

# Scope of tutorial

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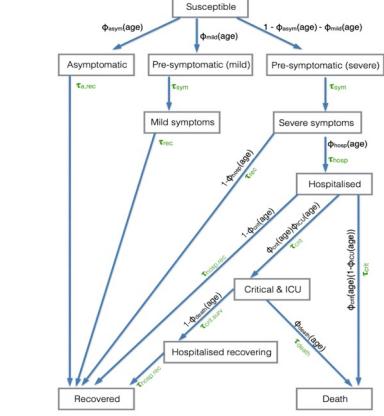
Critical Challenges  
Rapid Action  
Effective Policies



# Dynamics and Interventions



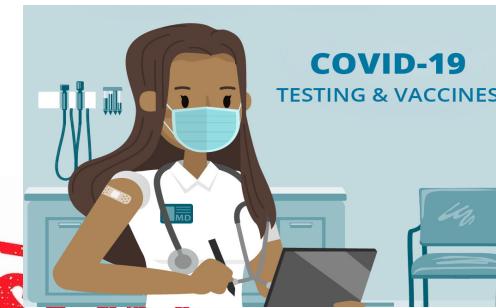
New Transmission



Disease Progression

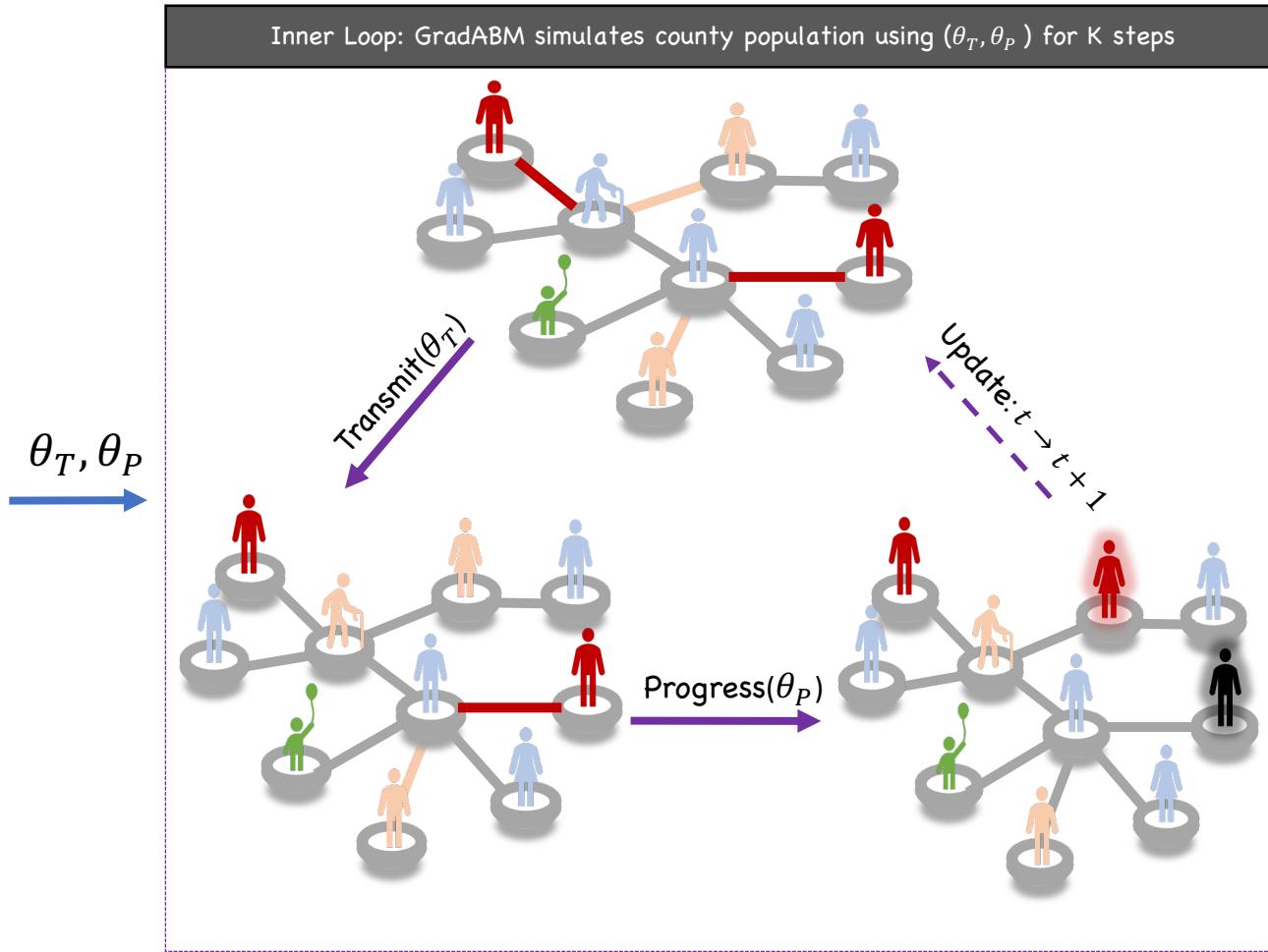


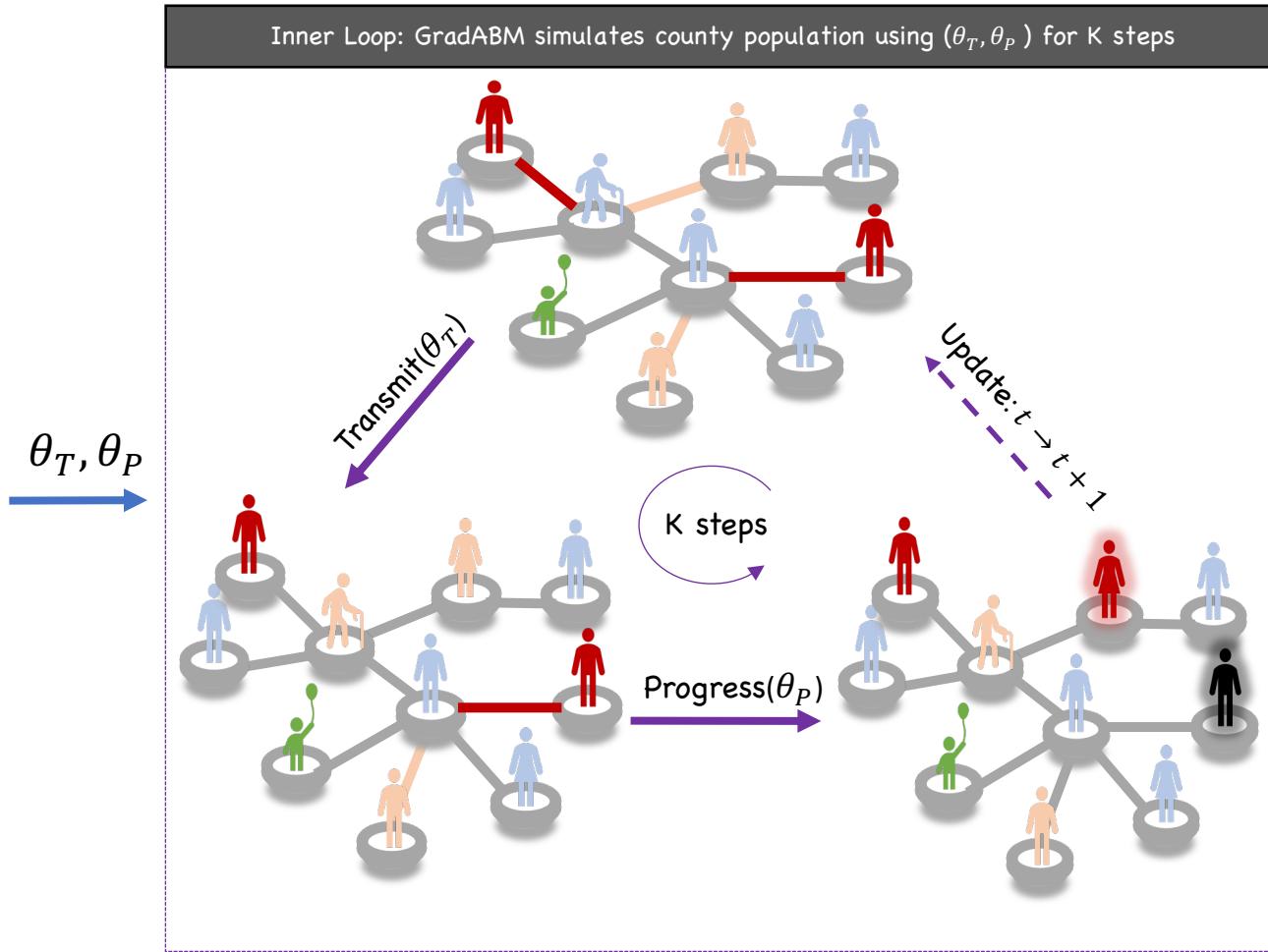
Health Interventions  
(Testing, Vaccination, Lockdowns)

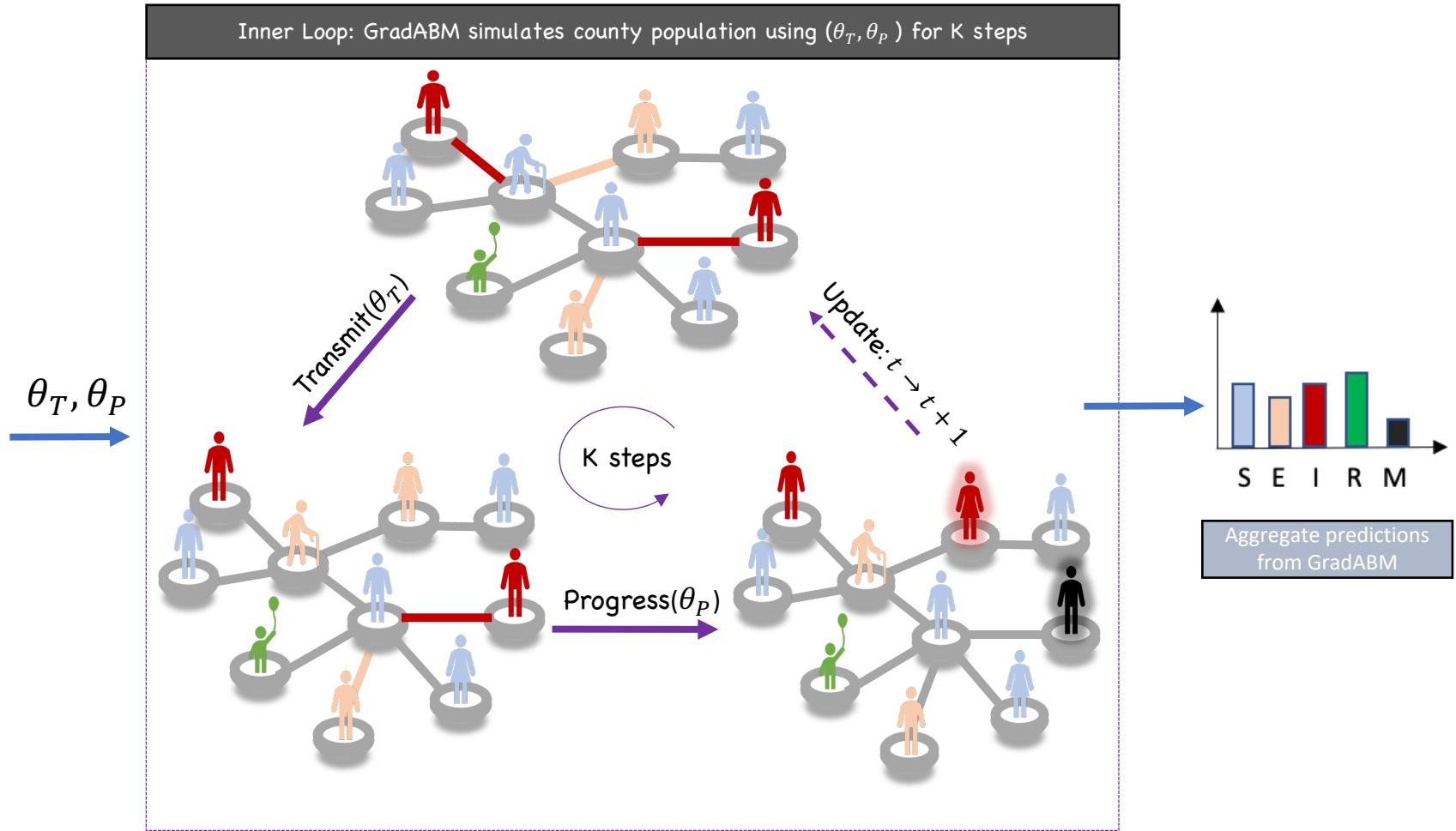


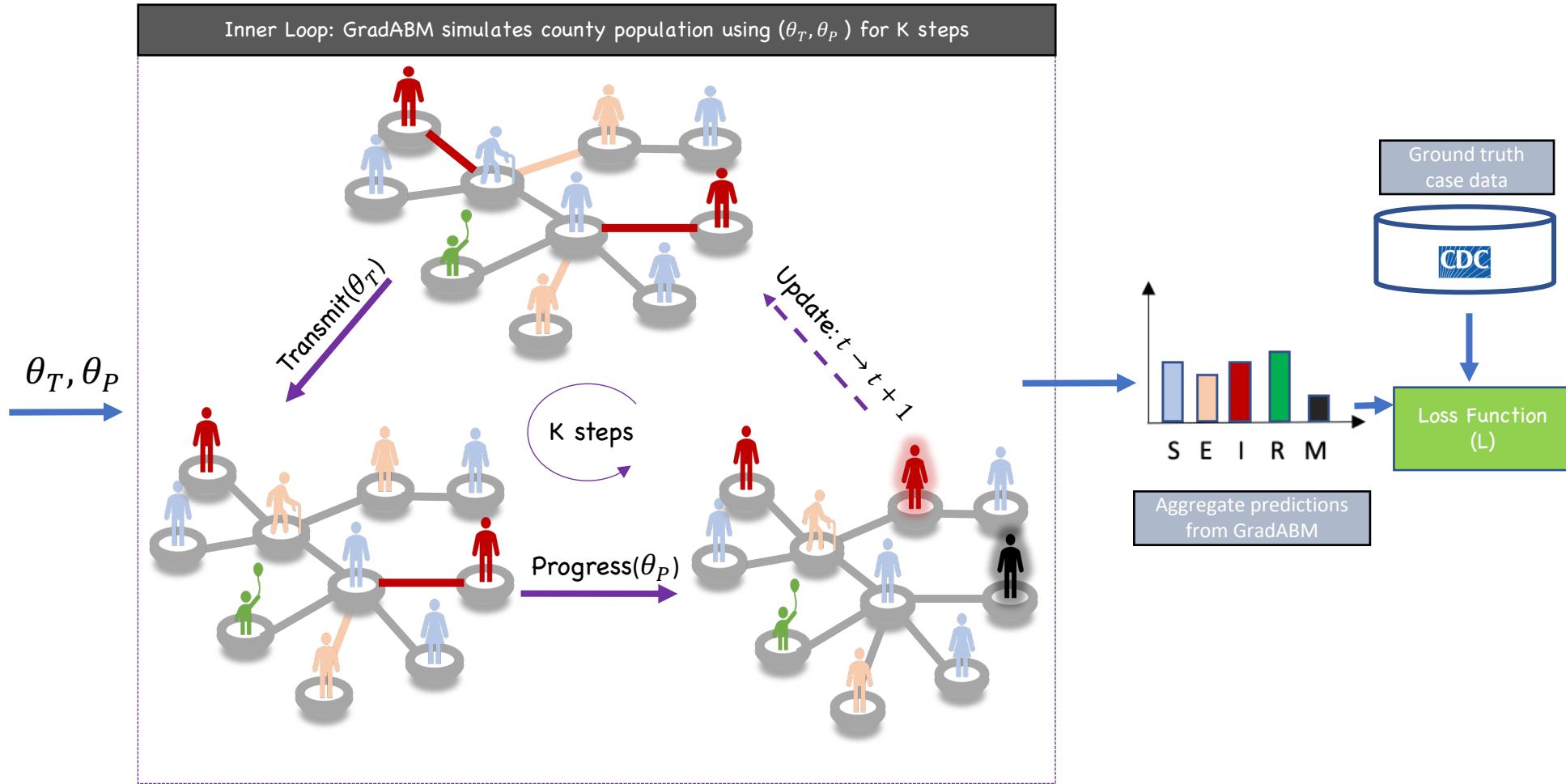
Financial Interventions  
(Stimulus, PUA, PPP, FPUC)

# Gradient-assisted calibration

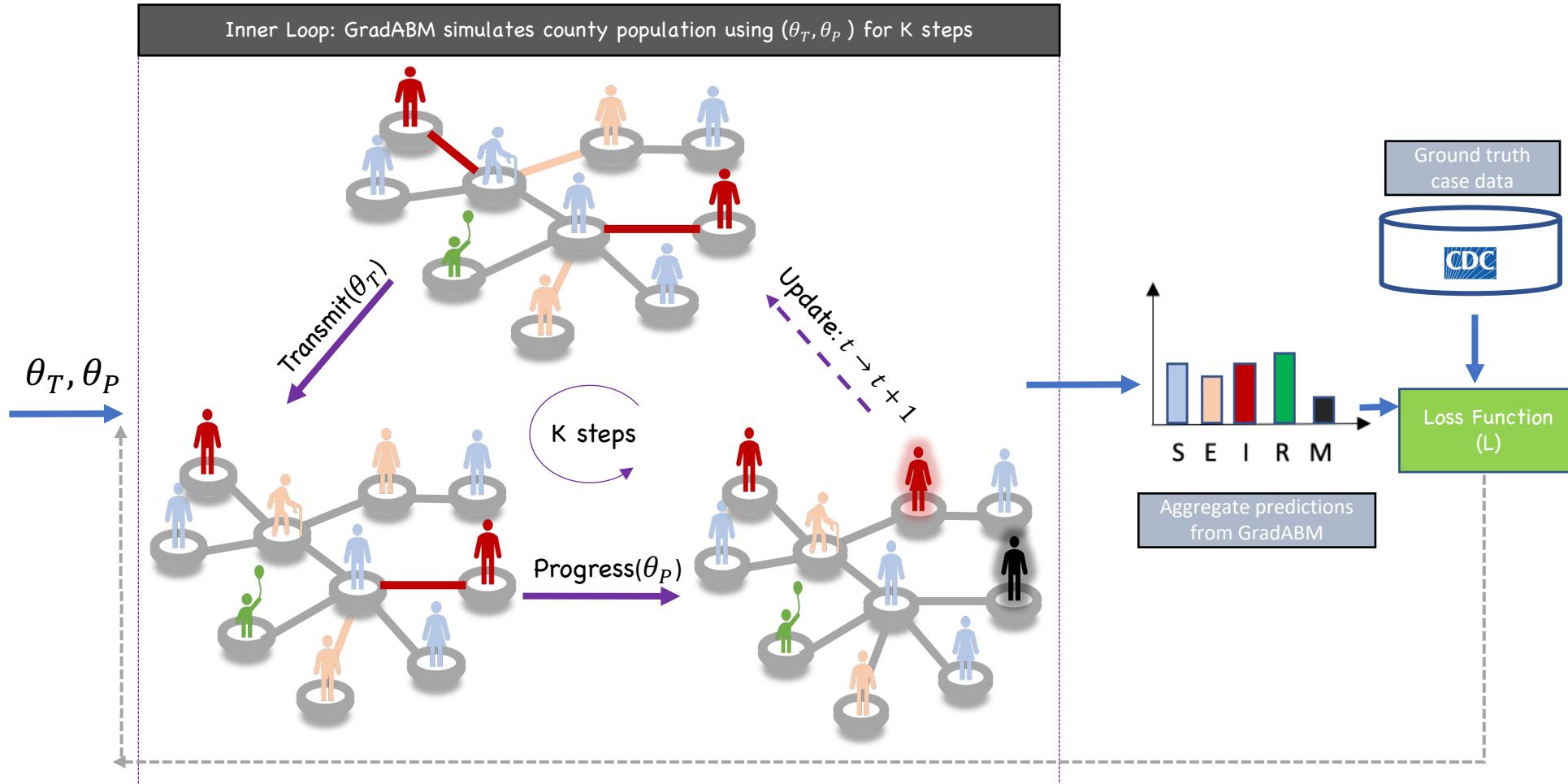








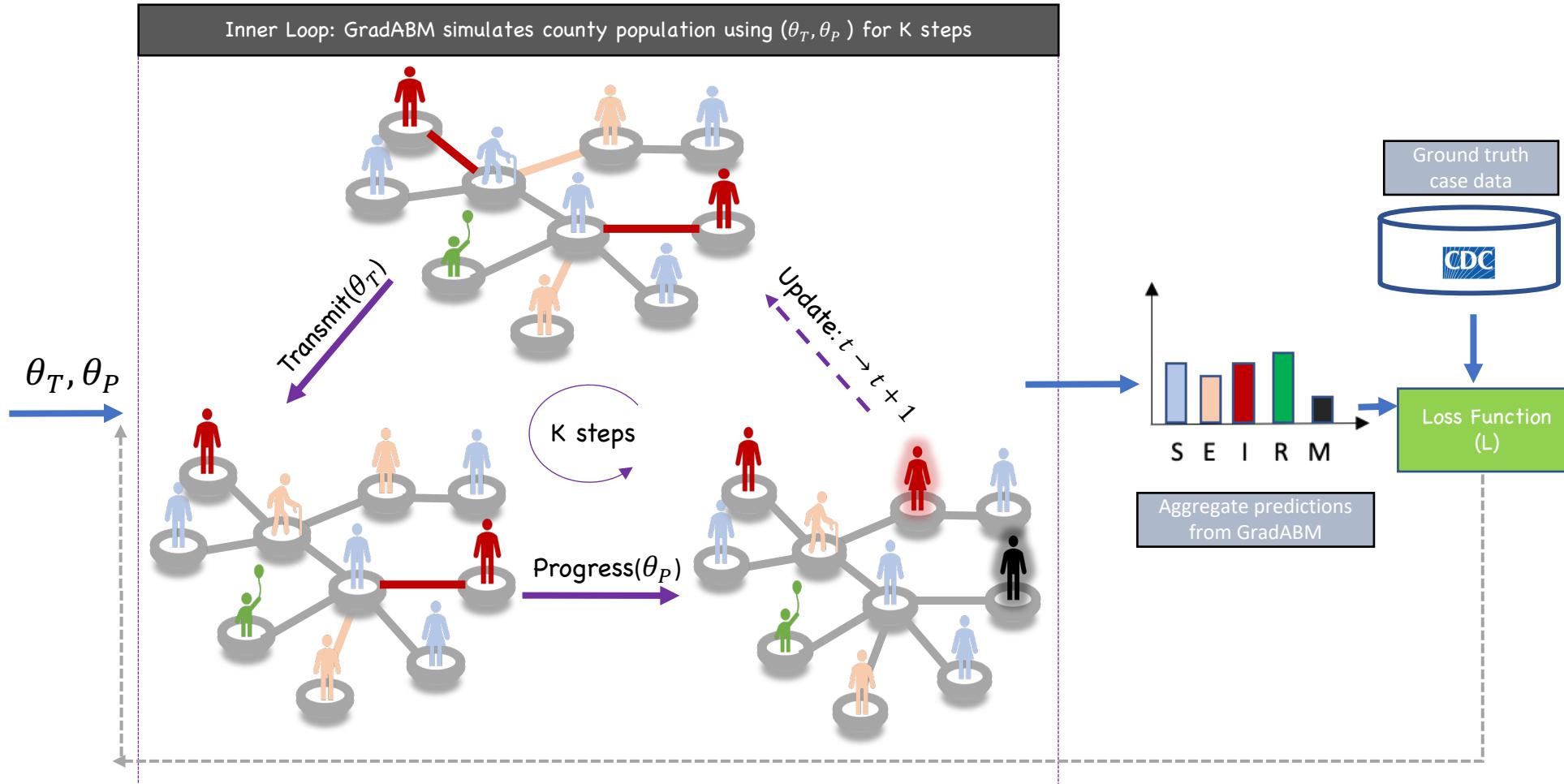
[bit.ly/diff-abms](http://bit.ly/diff-abms)



$$\theta_T = \theta_T - \alpha \frac{\partial \mathcal{L}(\hat{y}, y)}{\partial \theta_T}$$

$$\theta_P = \theta_P - \alpha \frac{\partial \mathcal{L}(\hat{y}, y)}{\partial \theta_P}$$

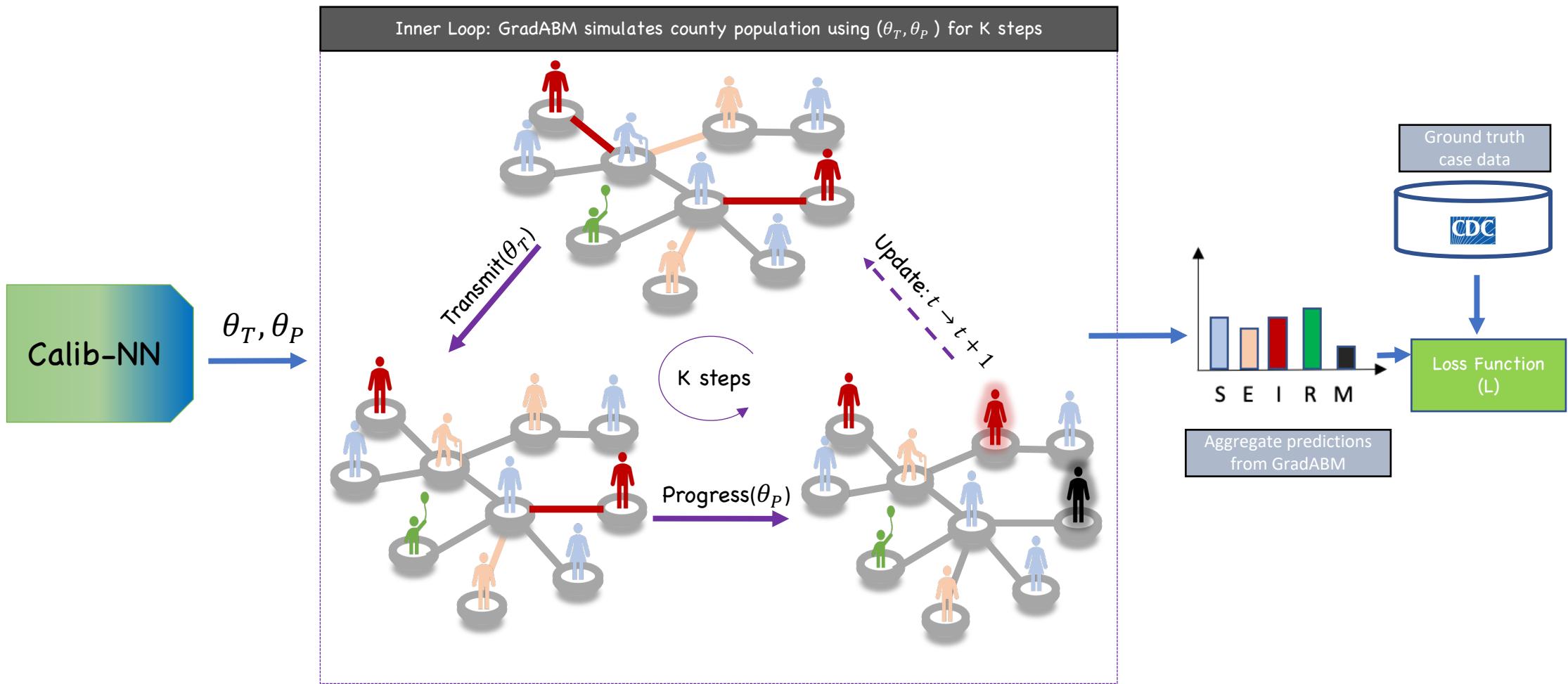
[bit.ly/diff-abms](http://bit.ly/diff-abms)

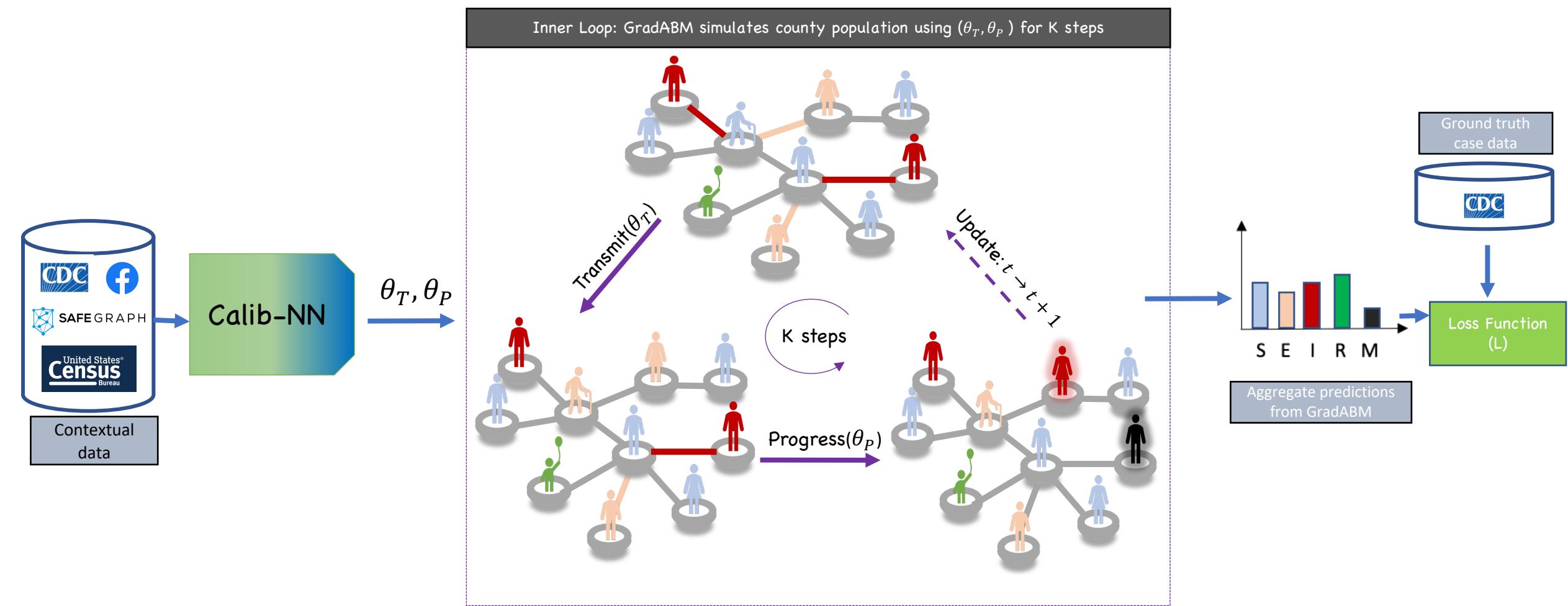


$$\theta_T = \theta_T - \alpha \frac{\partial \mathcal{L}(\hat{y}, y)}{\partial \theta_T}$$

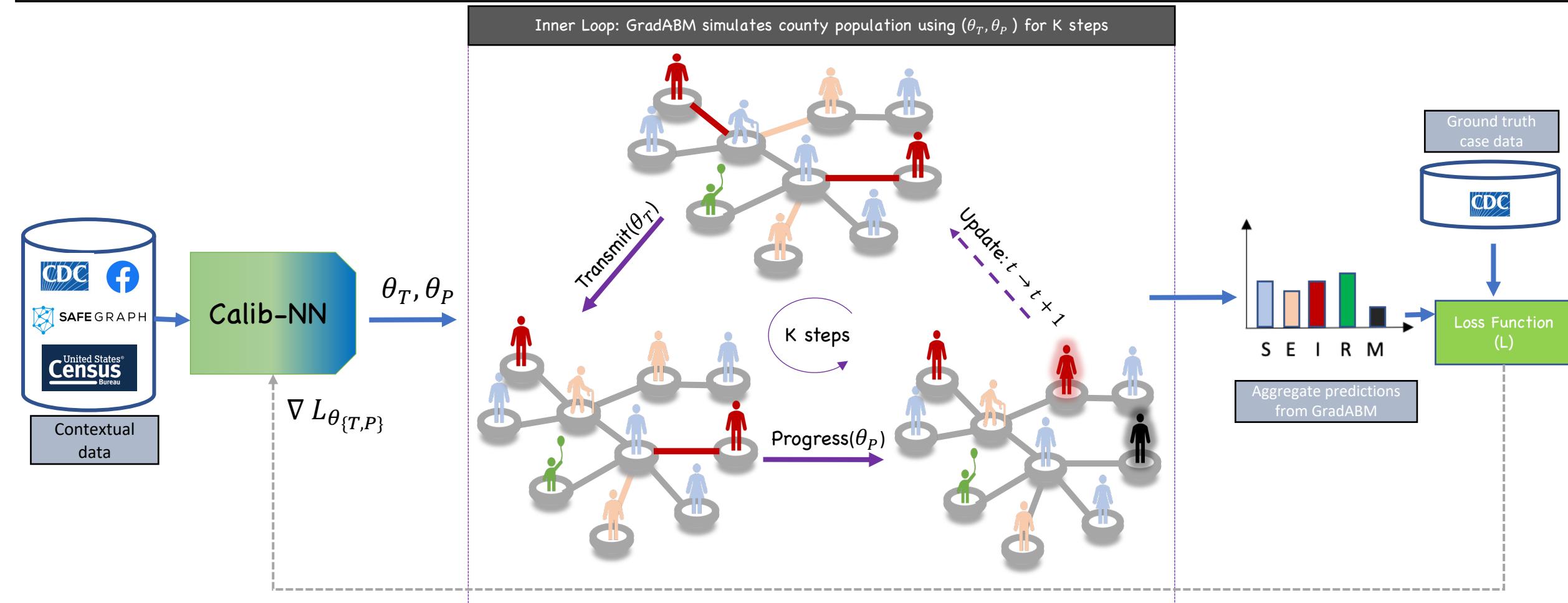
$$\theta_P = \theta_P - \alpha \frac{\partial \mathcal{L}(\hat{y}, y)}{\partial \theta_P}$$

Mode 1: Calibrate **parameters** with gradient descent (c-GRADABM)



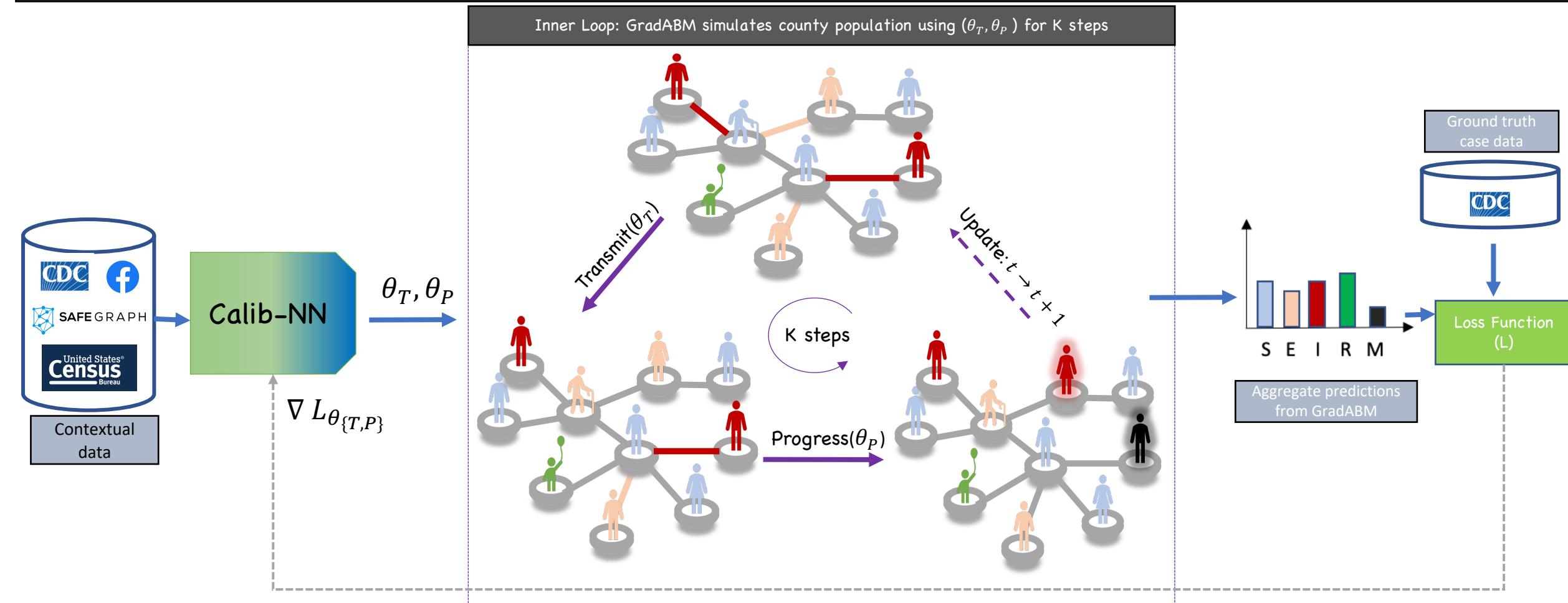


Outer Loop: Calib-NN predict infection parameters ( $\theta_T, \theta_P$ ) for county population used in *differentiable* GradABM and is optimized using end-to-end gradient flow



$$\phi = \phi - \alpha \frac{\partial \mathcal{L}(\hat{y}, y; (\theta_T^t, \theta_P^t))}{\partial \phi},$$

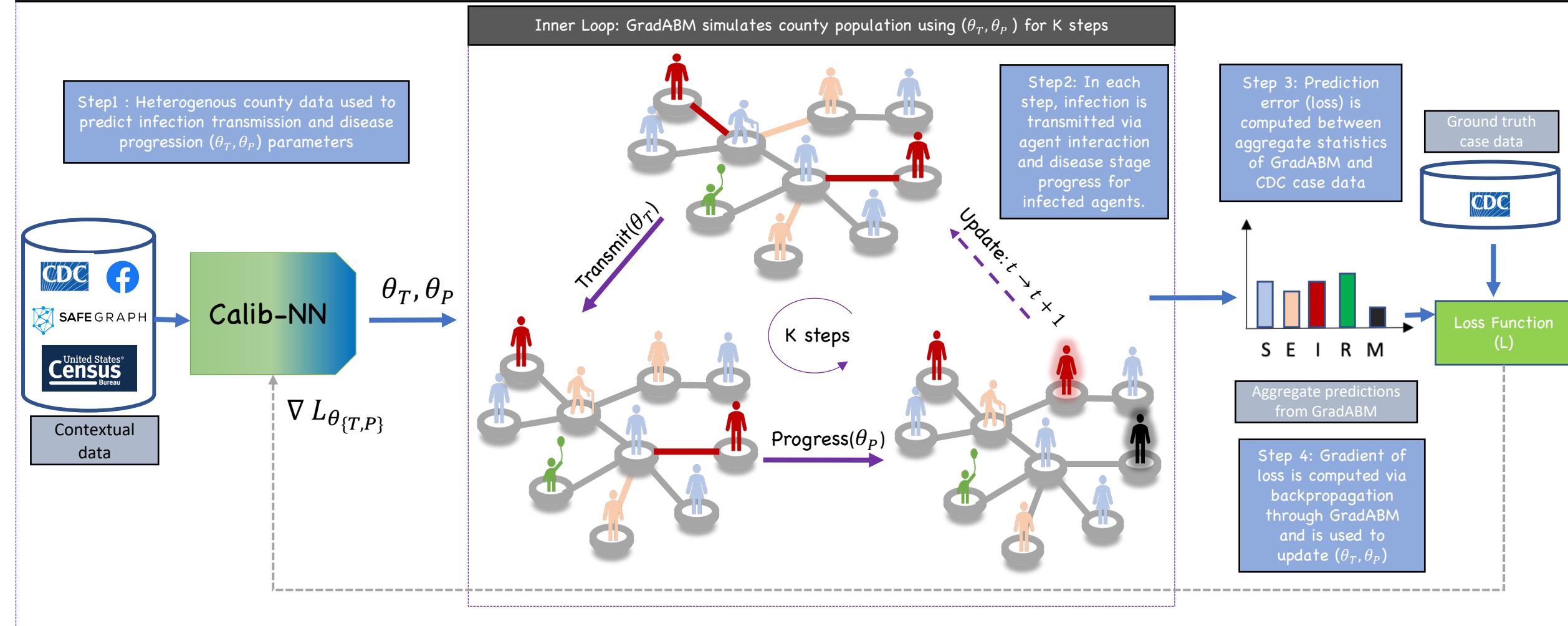
Outer Loop: Calib-NN predict infection parameters ( $\theta_T, \theta_P$ ) for county population used in *differentiable* GradABM and is optimized using end-to-end gradient flow



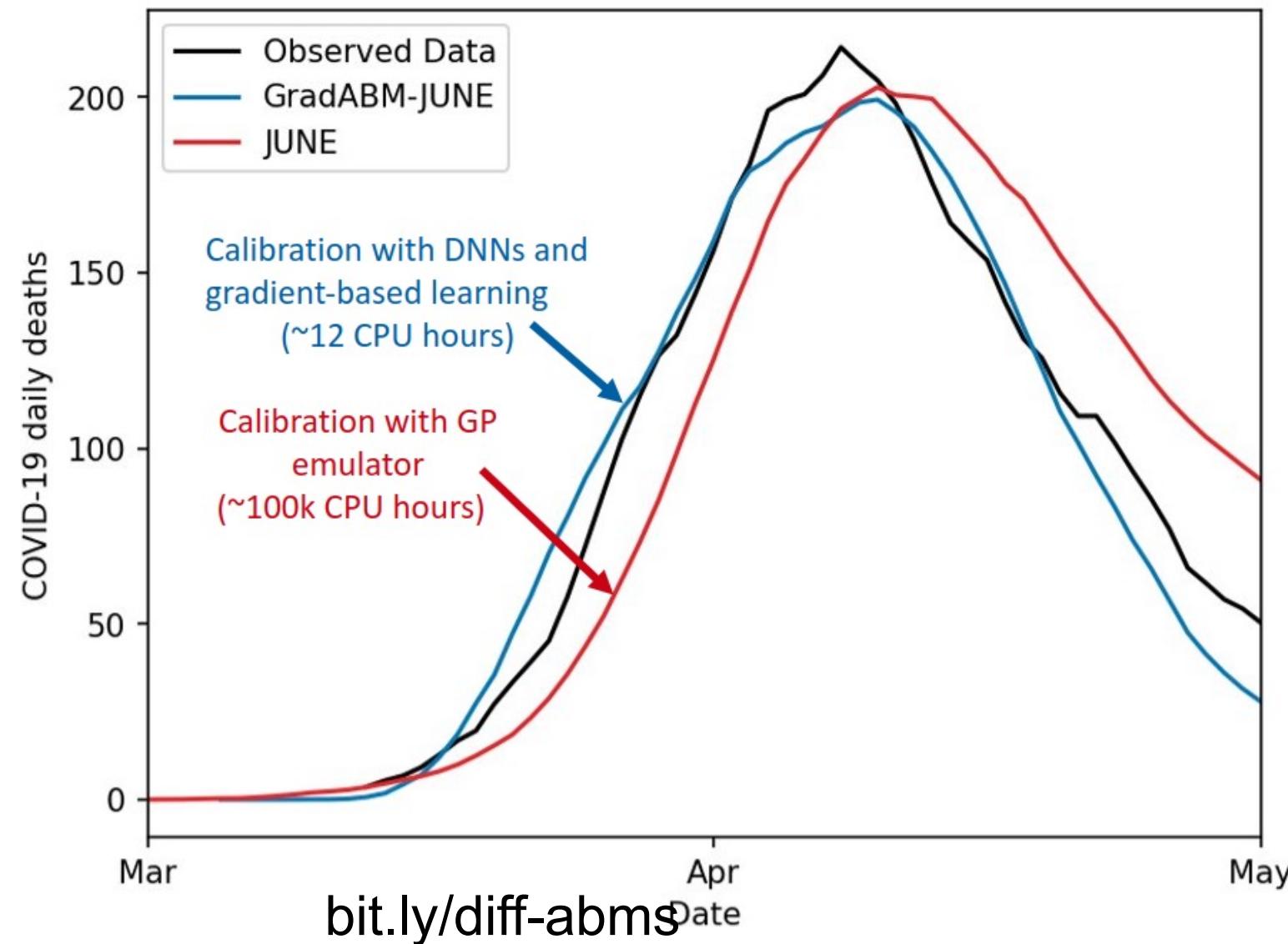
Mode 2: Calibrate generator function with gradient descent (dc-GRADABM)

$$\phi = \phi - \alpha \frac{\partial \mathcal{L}(\hat{y}, y; (\theta_T^t, \theta_P^t))}{\partial \phi},$$

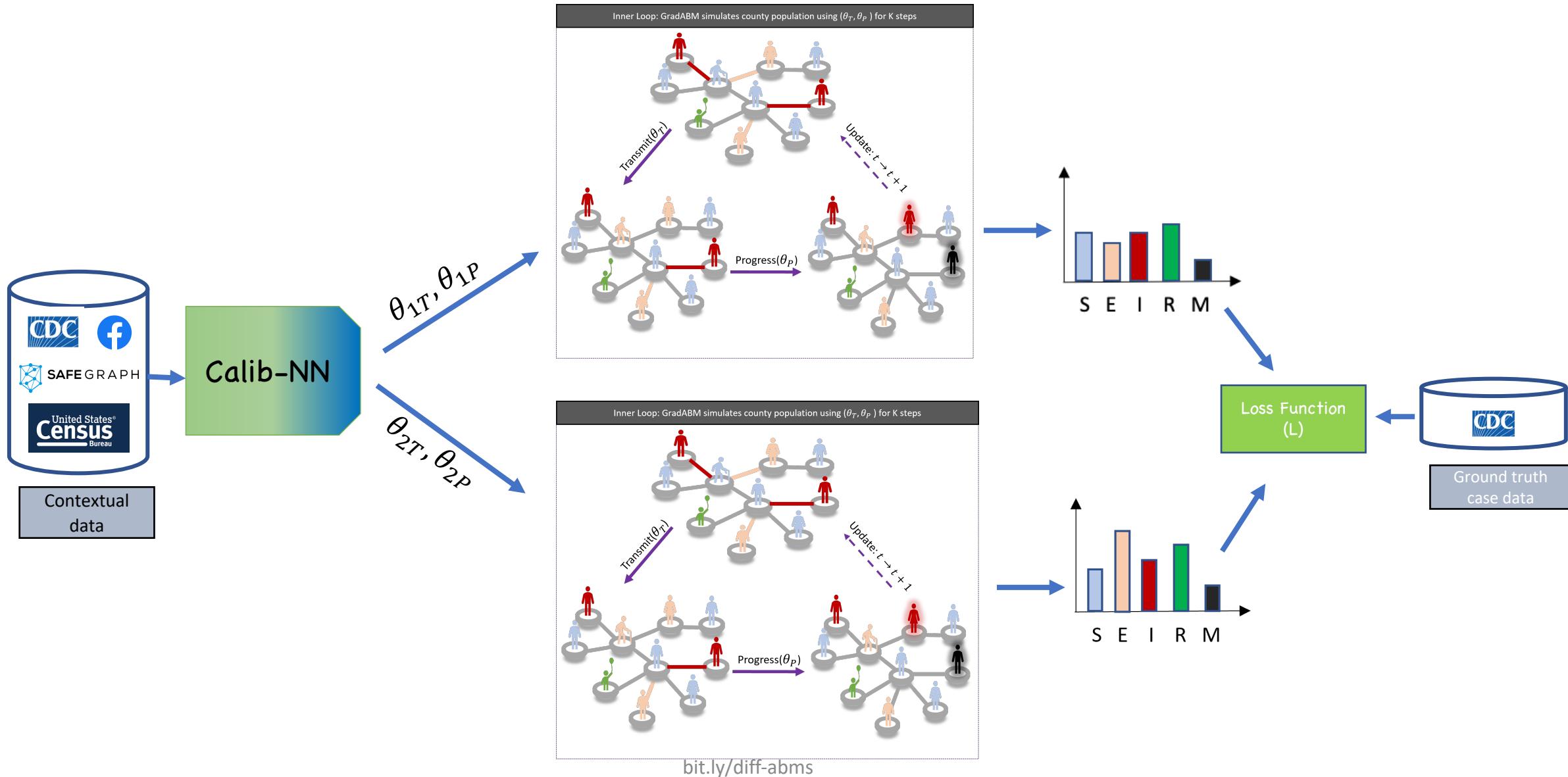
Outer Loop: Calib-NN predict infection parameters ( $\theta_T, \theta_P$ ) for county population used in *differentiable* GradABM and is optimized using end-to-end gradient flow



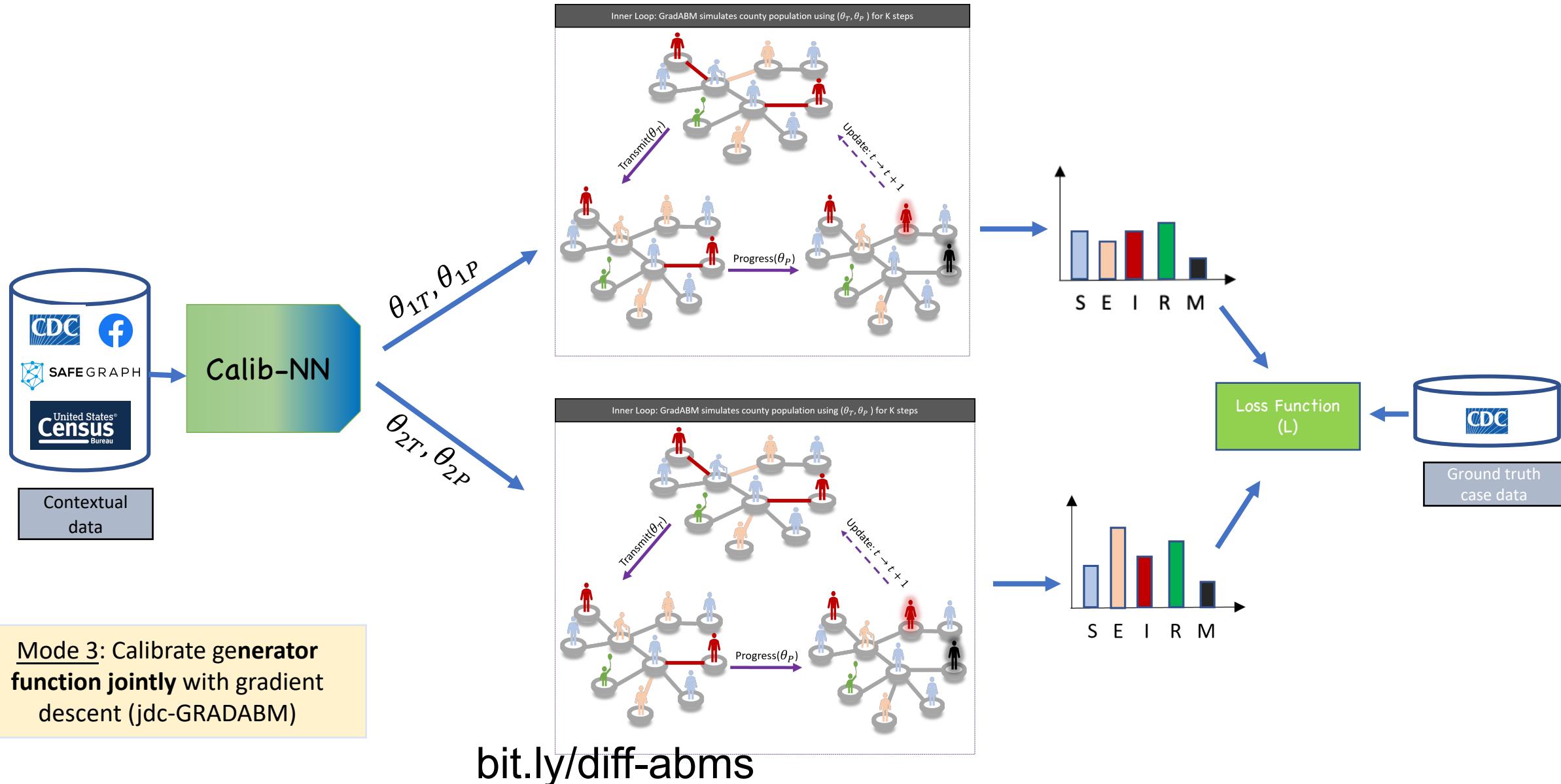
# Gradients enable fast calibration over emulators: 100k to 12 CPU hours



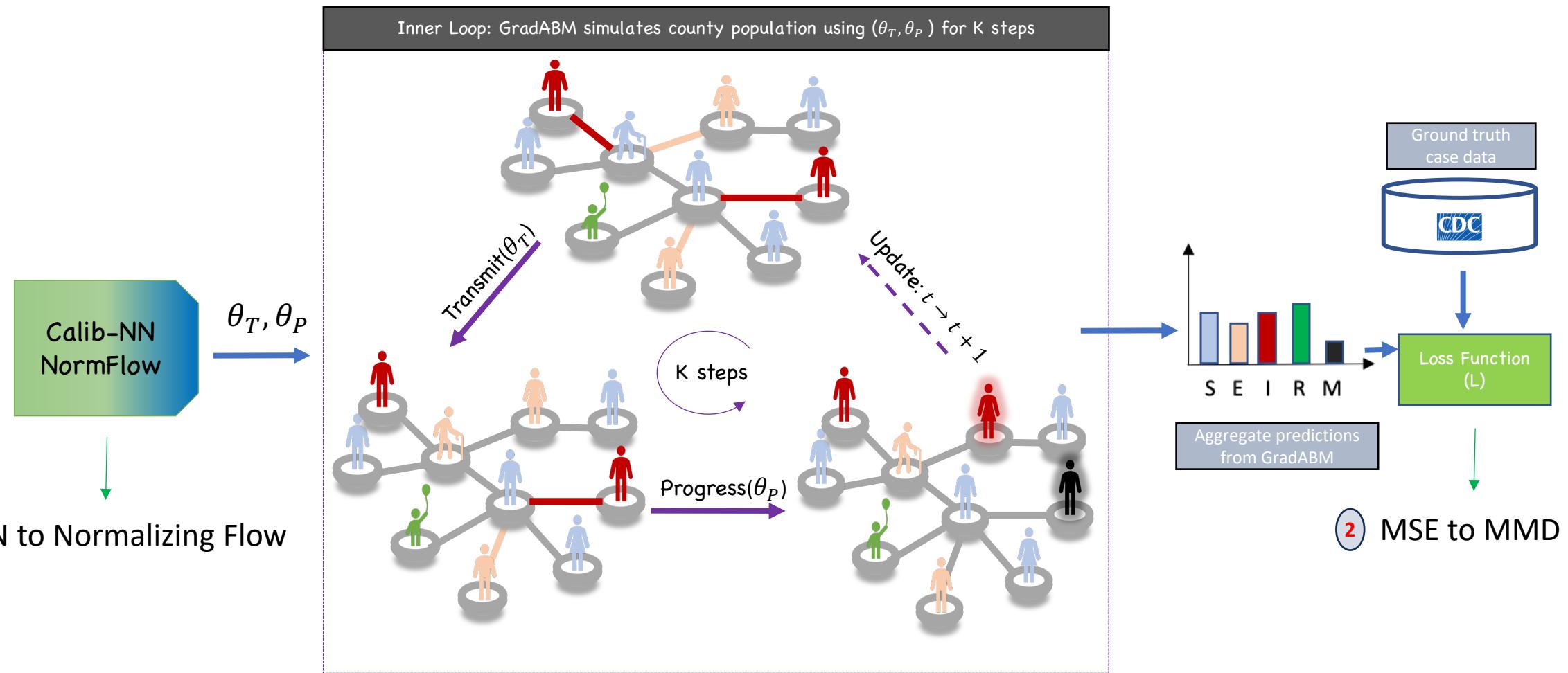
# Calibrate with ensemble learning to reduce overfitting



# Calibrate with ensemble learning to reduce overfitting



# Calibrate posteriors with variational inference



Mode 4: Calibrate with uncertainty quantification (dc-GRADABM)

# Gradient-assisted sensitivity analysis

# Sensitivity Analysis is critical for validation

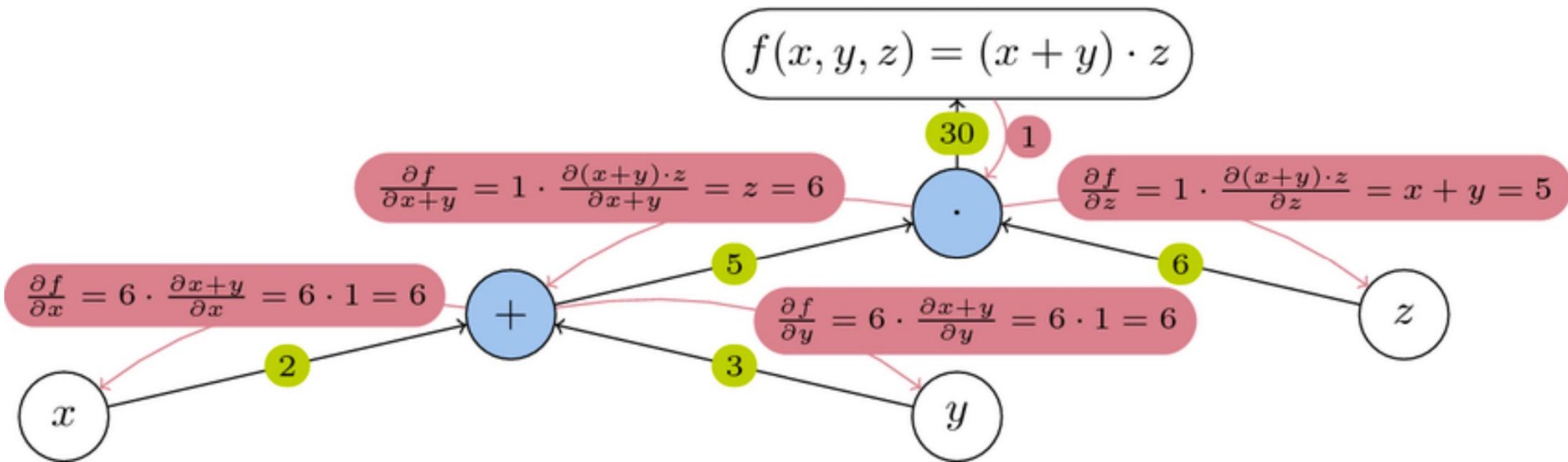
## The impact of uncertainty on predictions of the CovidSim epidemiological code

Wouter Edeling<sup>1</sup>, Hamid Arabnejad<sup>ID 2</sup>, Robbie Sinclair<sup>3</sup>, Diana Suleimenova<sup>2</sup>, Krishnakumar Gopalakrishnan<sup>ID 3</sup>, Bartosz Bosak<sup>4</sup>, Derek Groen<sup>2</sup>, Imran Mahmood<sup>2</sup>, Daan Crommelin<sup>1,5</sup> and Peter V. Coveney<sup>ID 3,6 ✉</sup>

Epidemiological modelling has assisted in identifying interventions that reduce the impact of COVID-19. The UK government relied, in part, on the CovidSim model to guide its policy to contain the rapid spread of the COVID-19 pandemic during March and April 2020; however, CovidSim contains several sources of uncertainty that affect the quality of its predictions: parametric uncertainty, model structure uncertainty and scenario uncertainty. Here we report on parametric sensitivity analysis and uncertainty quantification of the code. From the 940 parameters used as input into CovidSim, we find a subset of 19 to which the code output is most sensitive—imperfect knowledge of these inputs is magnified in the outputs by up to 300%. The model displays substantial bias with respect to observed data, failing to describe validation data well. Quantifying parametric input uncertainty is therefore not sufficient: the effect of model structure and scenario uncertainty must also be properly understood.

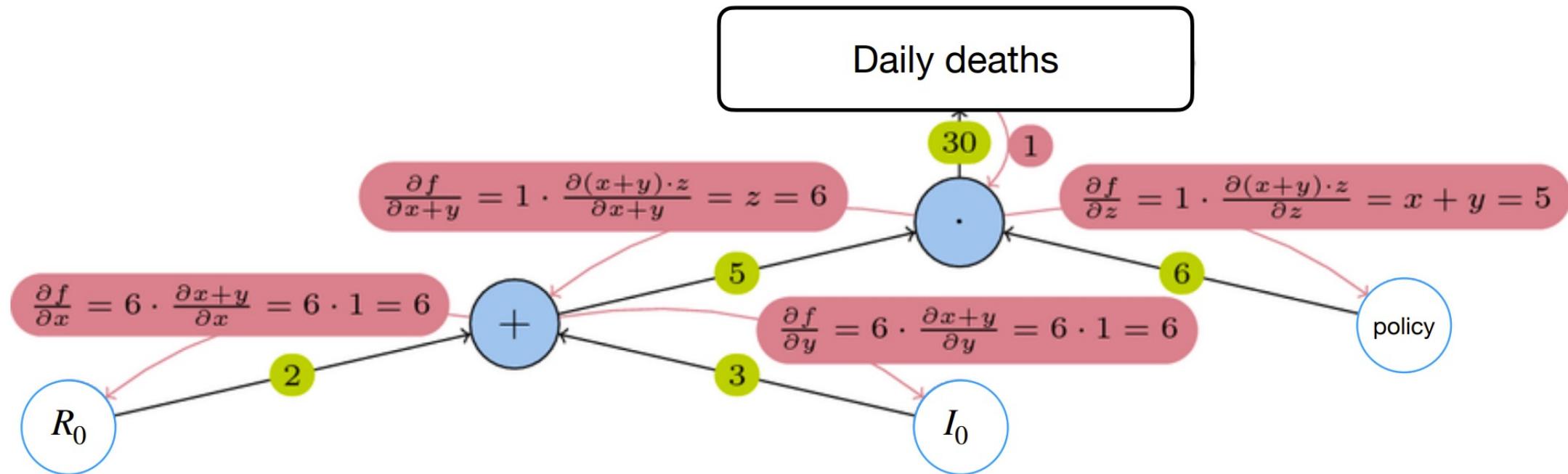
*Ensemble execution.* Consequently, through the use of adaptive methods we make the uncertainty analysis of CovidSim tractable, but our analysis nevertheless required us to perform thousands of runs, each with its own unique set of input parameters. Specifically, we used the Eagle supercomputer at the Posnan

# Recap: Reverse-mode automatic differentiation



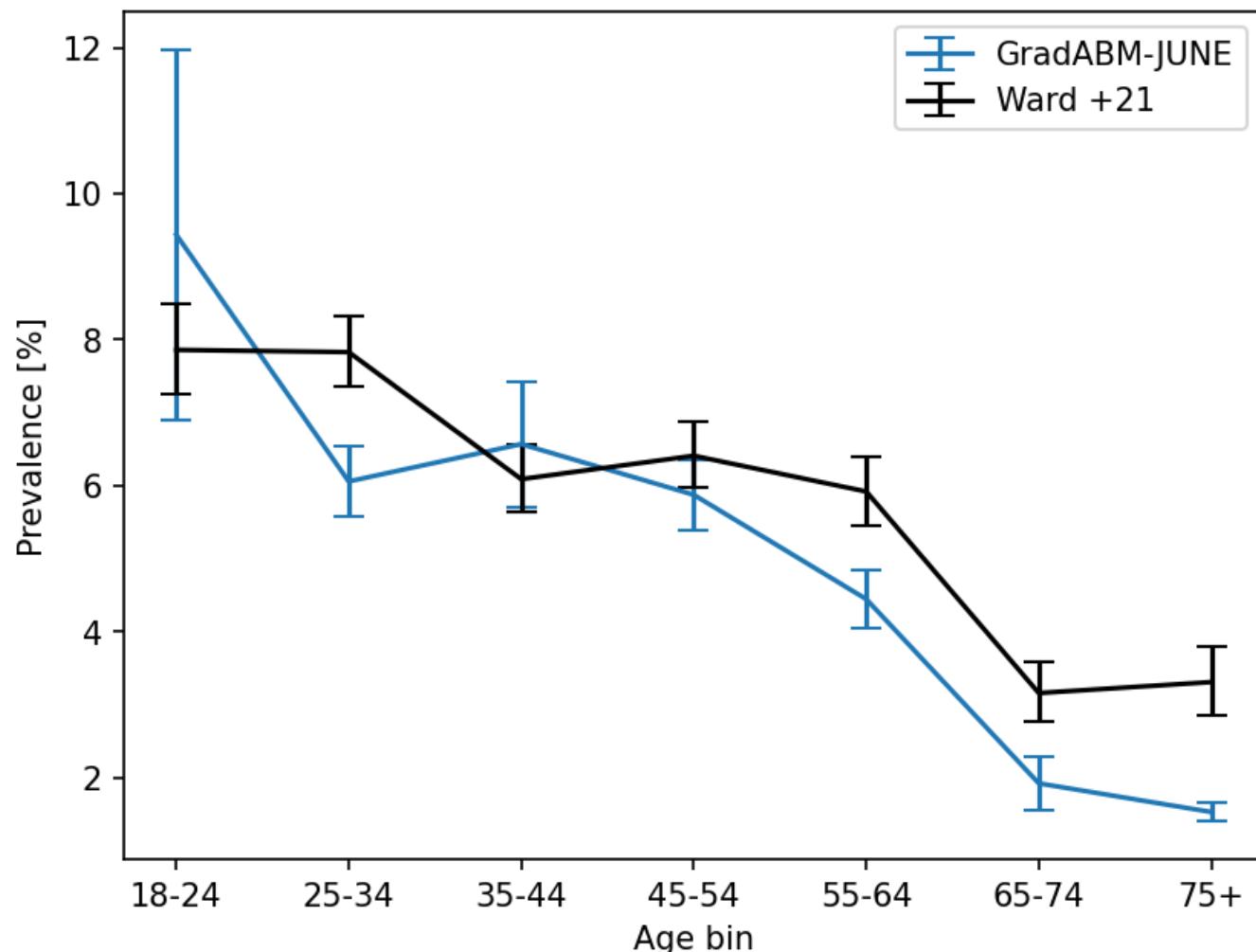
# Sensitivity analysis via reverse-mode automatic differentiation

Reverse-mode automatic differentiation is independent of the number of parameters!!



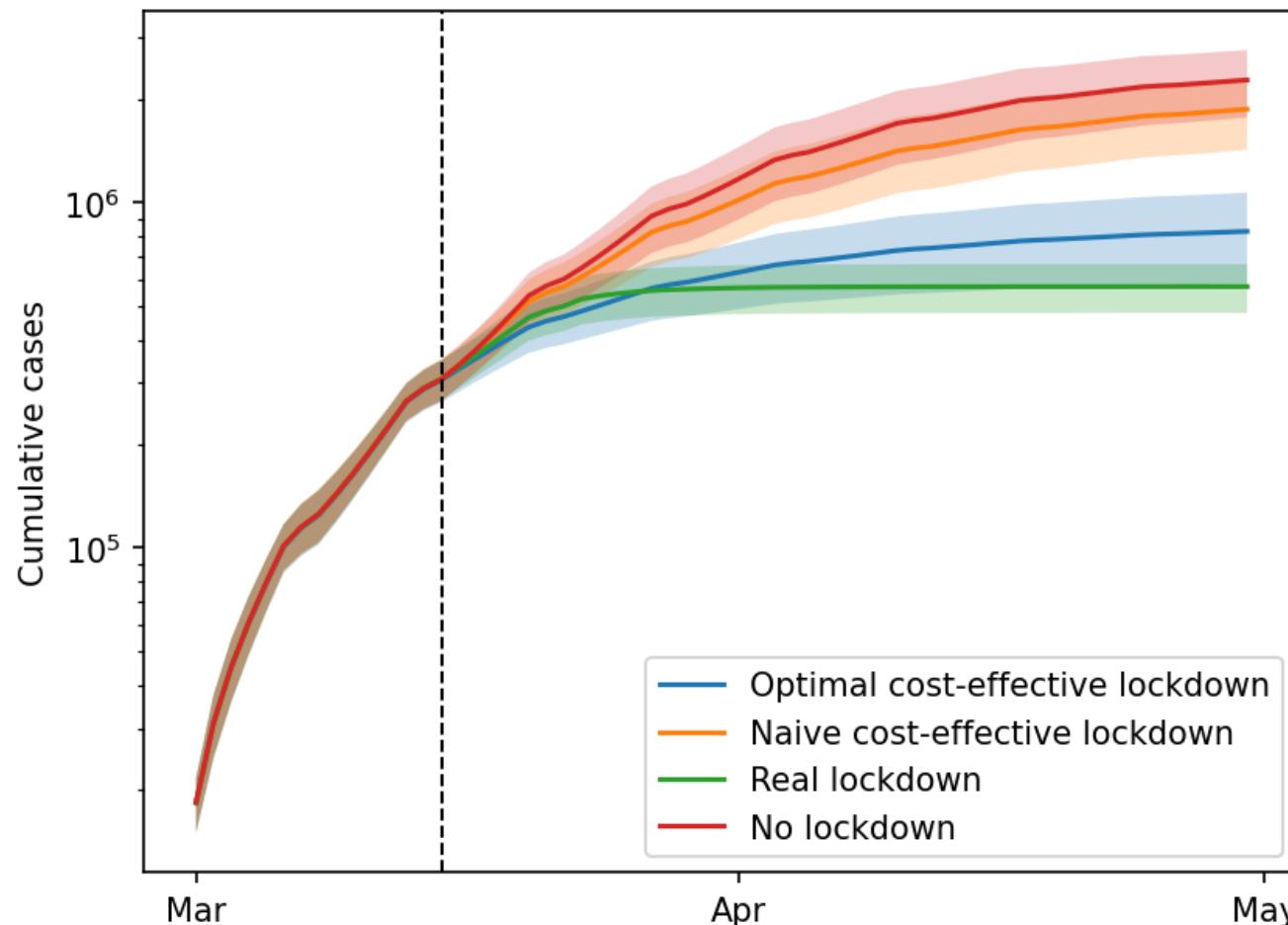
# How effective *really* were lockdown policies?

Analyze retrospective decisions by reproducing seroprevalence studies in-silico



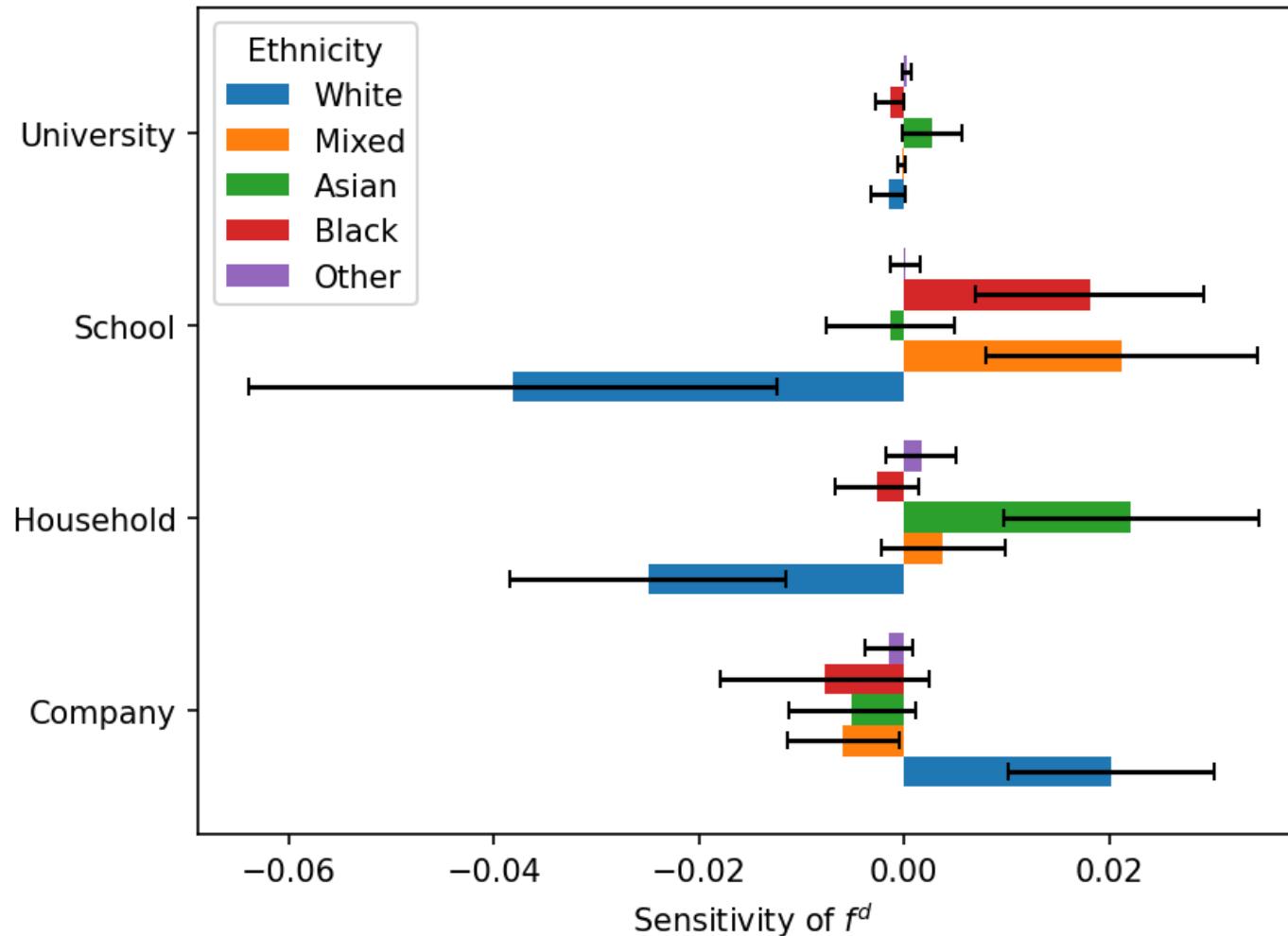
# What could we have done differently?

Design counterfactual lockdown policies with multiple constraints in-silico!



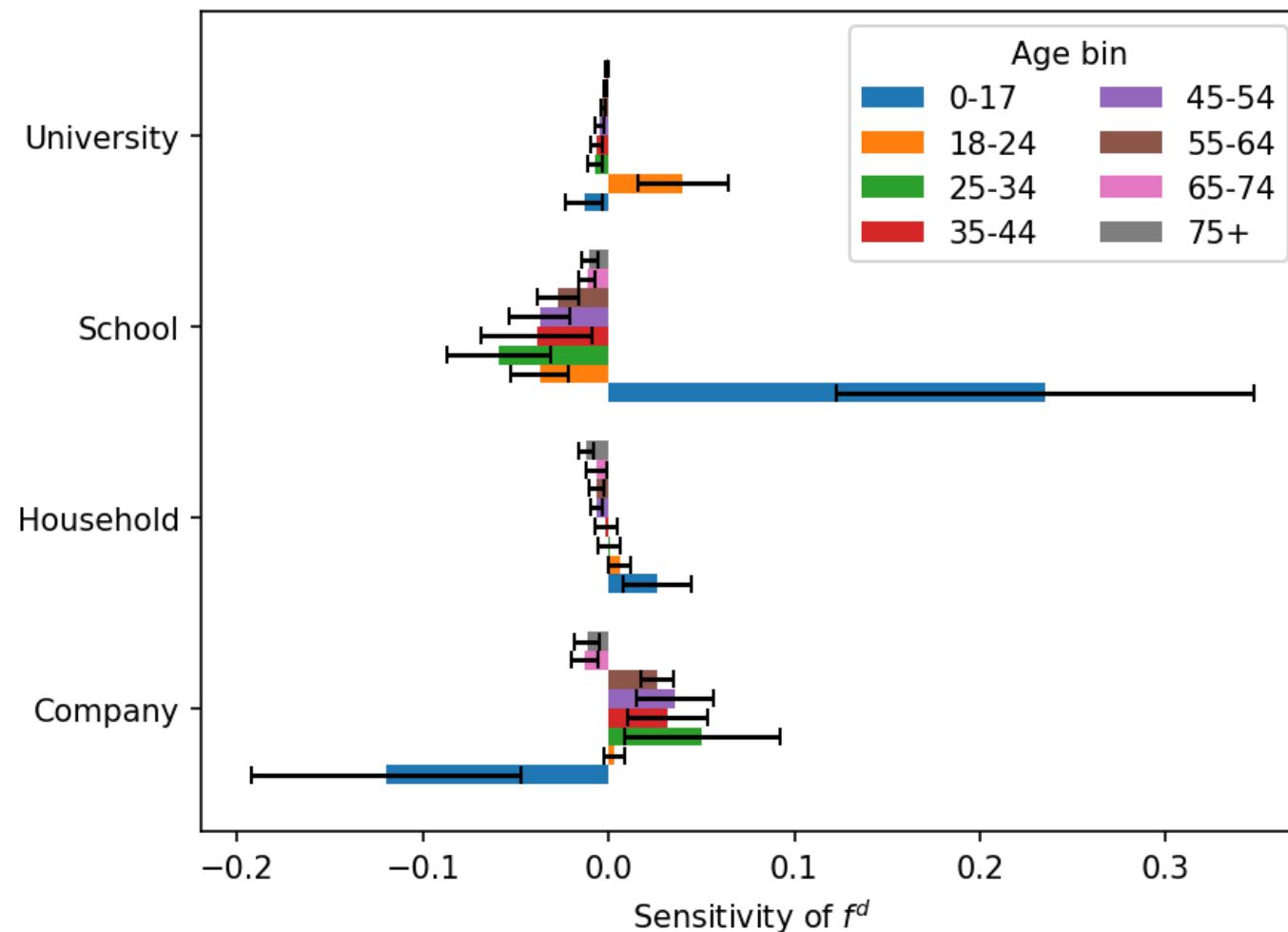
# How sensitive was infection to ethnicity?

More infection among South Asians through households in contrast to white British people



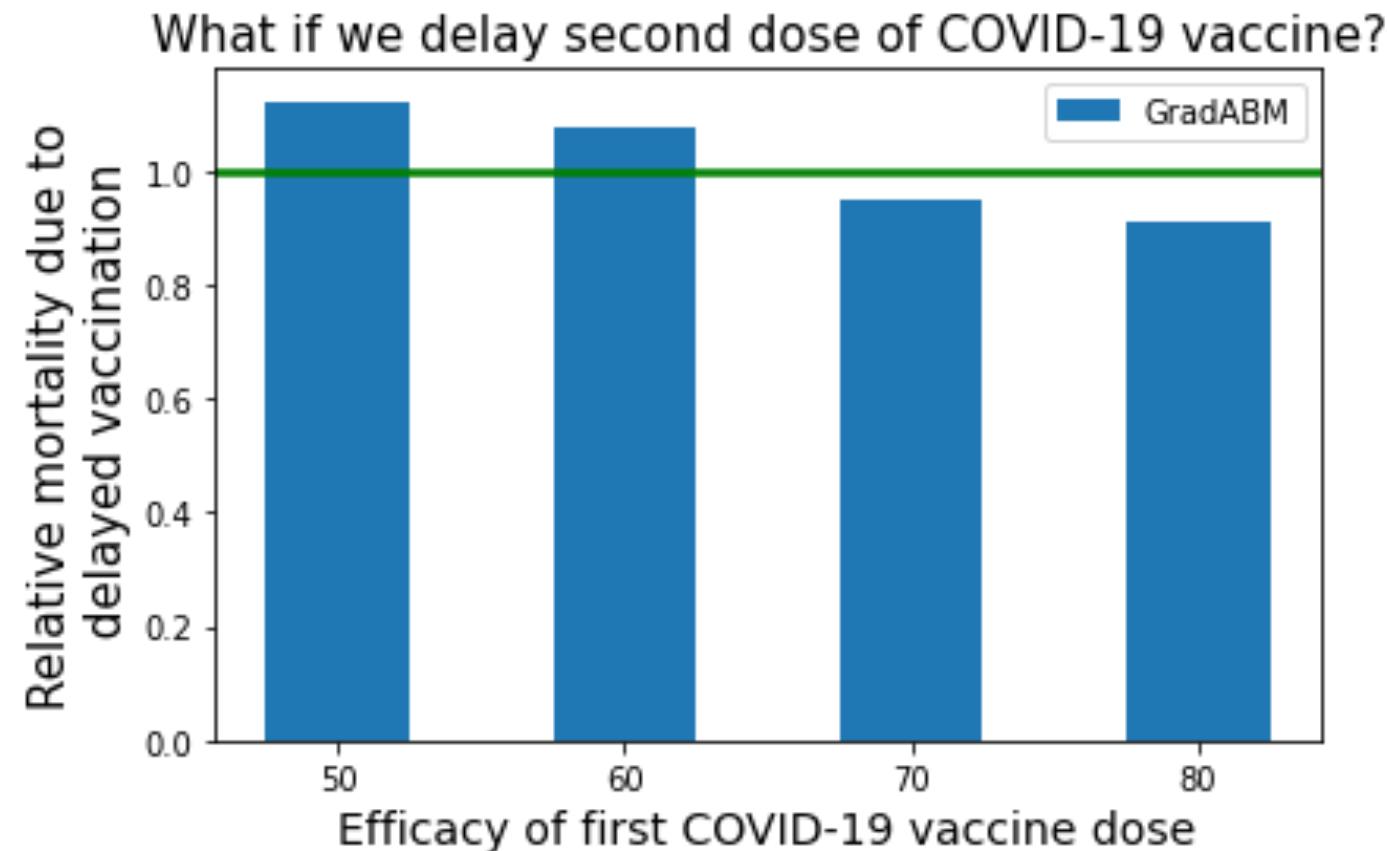
# How sensitive was infection to age?

Dominant infection spread through schools for 0-17 and university for 18-24



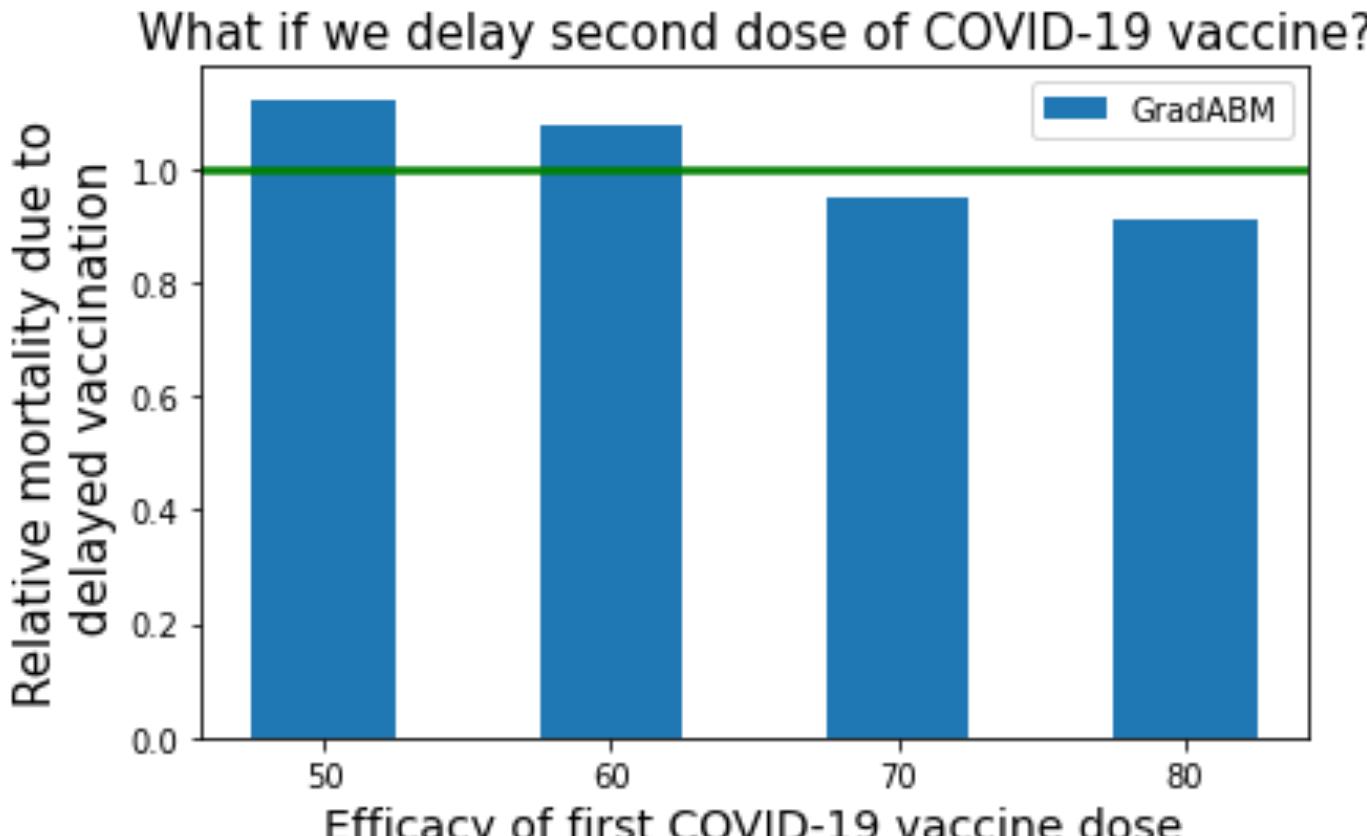
# What if we delay second dose of the COVID-19 vaccine?

Supply chain limitations and population behavior to design immunization policies



# What if we delay second dose of the COVID-19 vaccine?

Consider supply chain limitations and population behavior to design immunization policies



was higher under the 12-week strategy than the 3-week strategy. For this period, we estimated that delaying the interval between the first and second COVID-19 vaccine doses from 3 to 12 weeks averted a median (calculated as the median of the posterior sample) of 58 000 COVID-19 hospital admissions (291 000 cumulative hospitalisations [95% credible interval 275 000–319 000] under the 3-week strategy vs 233 000 [229 000–238 000] under the 12-week strategy) and 10 100 deaths (64 800 deaths [60 200–68 900] vs 54 700 [52 800–55 600]). Similarly, we estimated that the

Retrospective impact of delaying 2<sup>nd</sup> covid-19 dose in England  
(Lancet '23)

More details: Jade Room 3 on Friday at 10 am

- Composing with neural networks
- Using LLM as agents for million-scale simulations
- Modeling with private and distributed data
- Generating diverse simulation scenarios

# Scope of tutorial

- Preliminaries
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# Differentiable ABMs in action: Case Study of New Zealand

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# Variational Inference with Blackbirds

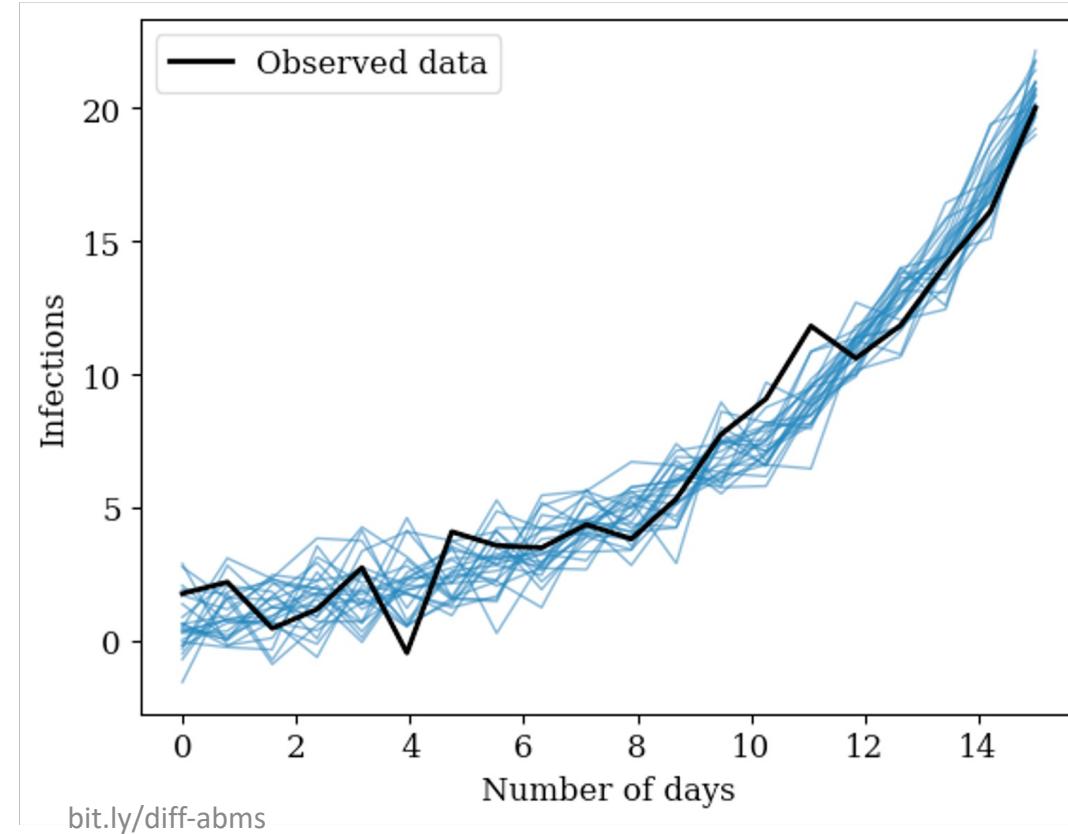
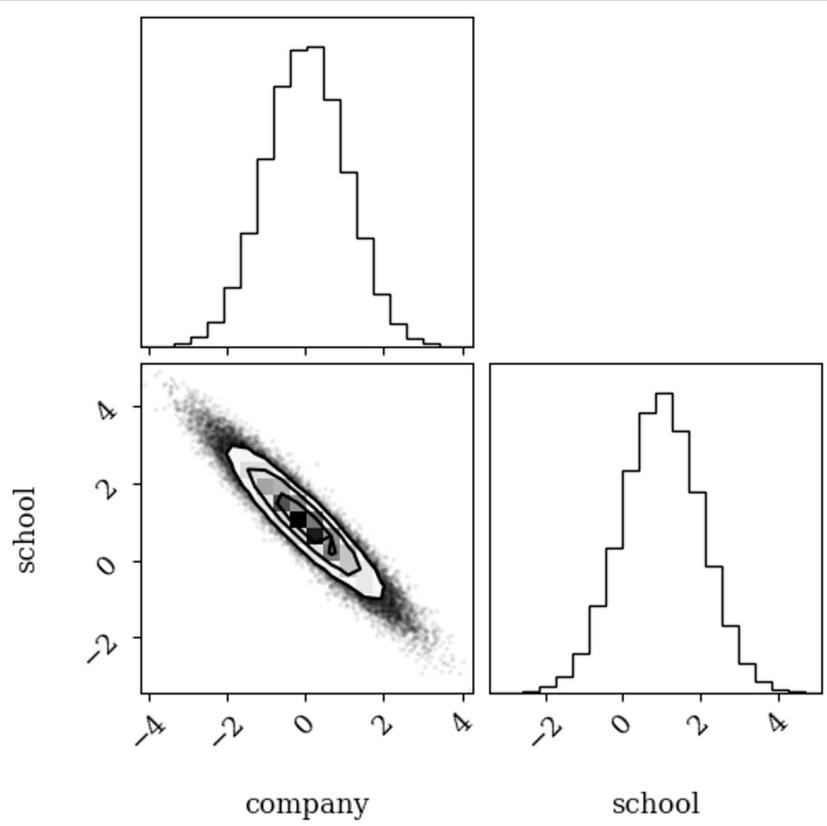
[github.com/arnauqb/blackbirds](https://github.com/arnauqb/blackbirds)

**pip install blackbirds**

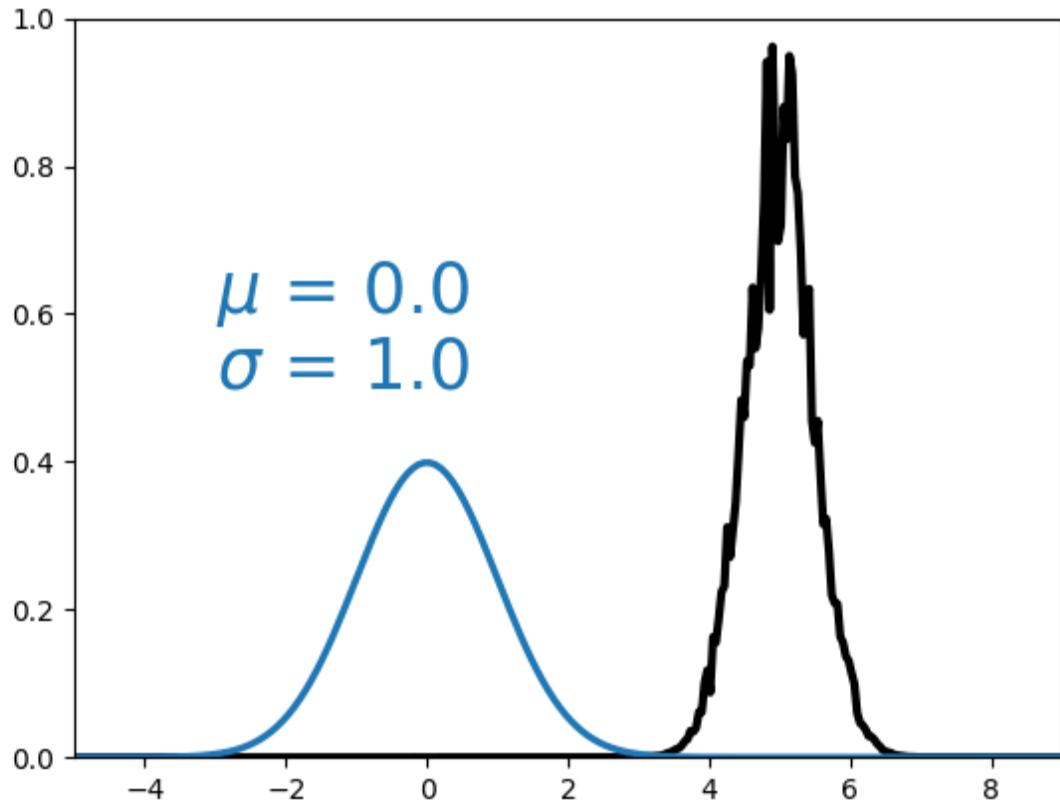
# Bayesian inference

$$p(\theta|x) \propto p(x|\theta)p(\theta)$$

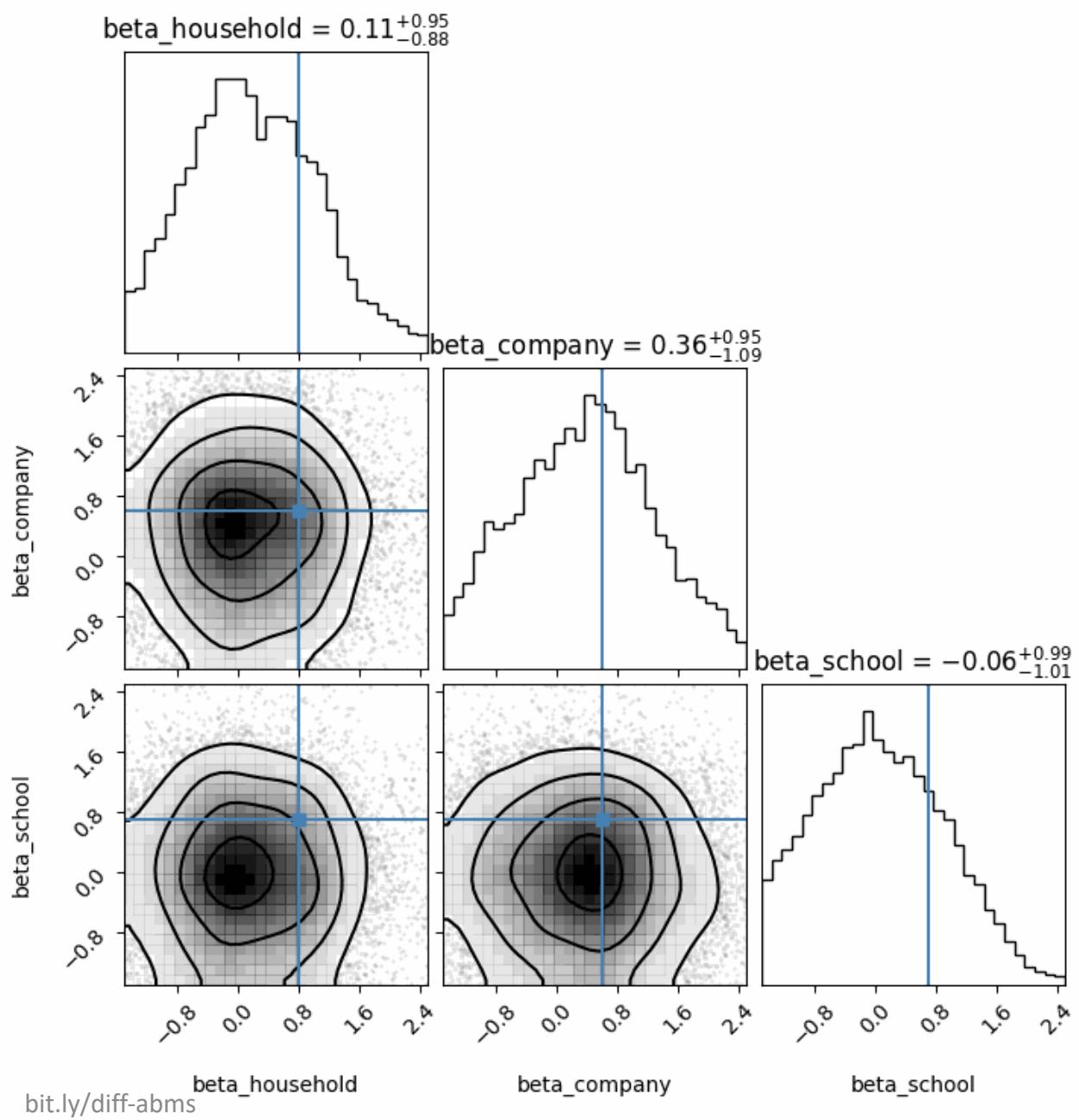
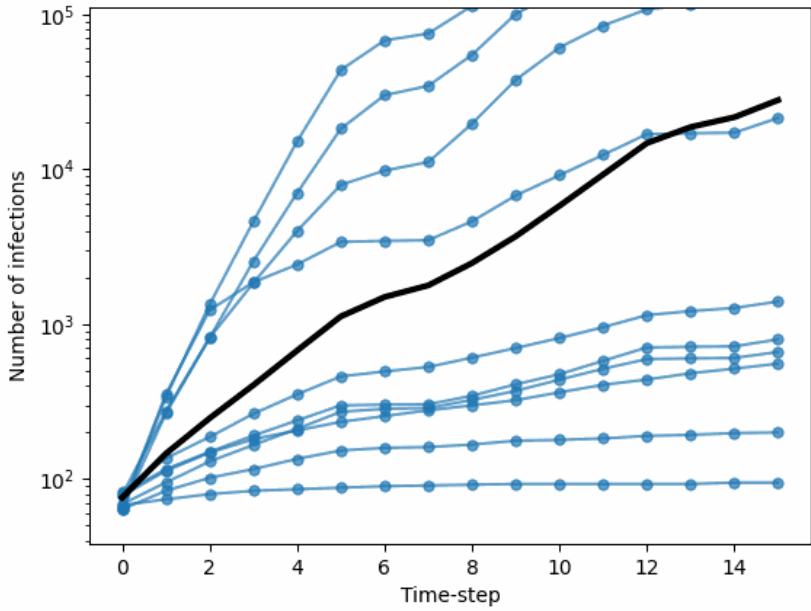
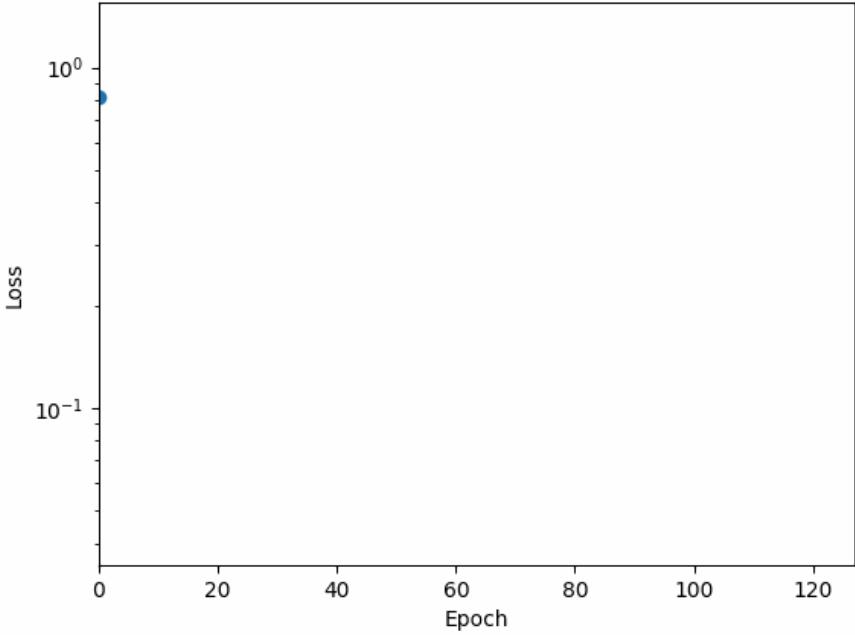
posterior      likelihood      prior



# Variational Inference: Bayesian Inference as an optimization problem



1. Assume posterior can be approximated by a family of distributions
2. Optimise for optimal parameters



# Build your own Differentiable ABMs with AgentTorch

[github.com/AgentTorch/AgentTorch](https://github.com/AgentTorch/AgentTorch)

**pip install agent-torch**

# Using the AgentTorch API

execute simulation

talk to your simulation

customize agents (eg: LLM as agent)

customize population (eg: NZ -> NYC)

# Execute a simulation with AgentTorch

Simple Python API. Get started in 3 lines of code. Massive Acceleration. "AI Compatible"



```
from AgentTorch.models import disease
from AgentTorch.populations import new_zealand

from AgentTorch.execute import Executor

simulation = Executor(disease, new_zealand)
simulation.execute()
```

# Gradient-based learning with AgentTorch

Pytorch compatible. Optimize parameters. Compose with Neural Networks



```
from torch.optim import SGD

optimizer = SGD(simulation.parameters())
for i in range(episodes):
    optimizer.zero_grad()
    simulation.execute()
    optimizer.step()
    simulation.reset()
```

# Visualize your simulation with AgentTorch

Interactive geo-plots and natural language interface



```
from AgentTorch.visualize import Visualizer
from AgentTorch.LLM.qa import load_state_trace

state_trace = load_state_trace(simulation)
visualizer = Visualizer(state_trace)

visualizer.plot('agent_behavior')
```

# Talk to your AgentTorch simulation

Understand the past. "brainstorm" for the future. Verify the data. Speculate reliably.



```
from AgentTorch.LLM.qa import SimulationAnalysisAgent  
  
analyzer = SimulationAnalysisAgent(simulation, state_trace)  
  
analyzer.query("Which age group has lowest median income, how  
much is it?")  
  
analyser.query("how are stimulus payments affecting disease?")
```

# Customize agents in AgentTorch

Agents can be heuristic, LLMs or neural networks

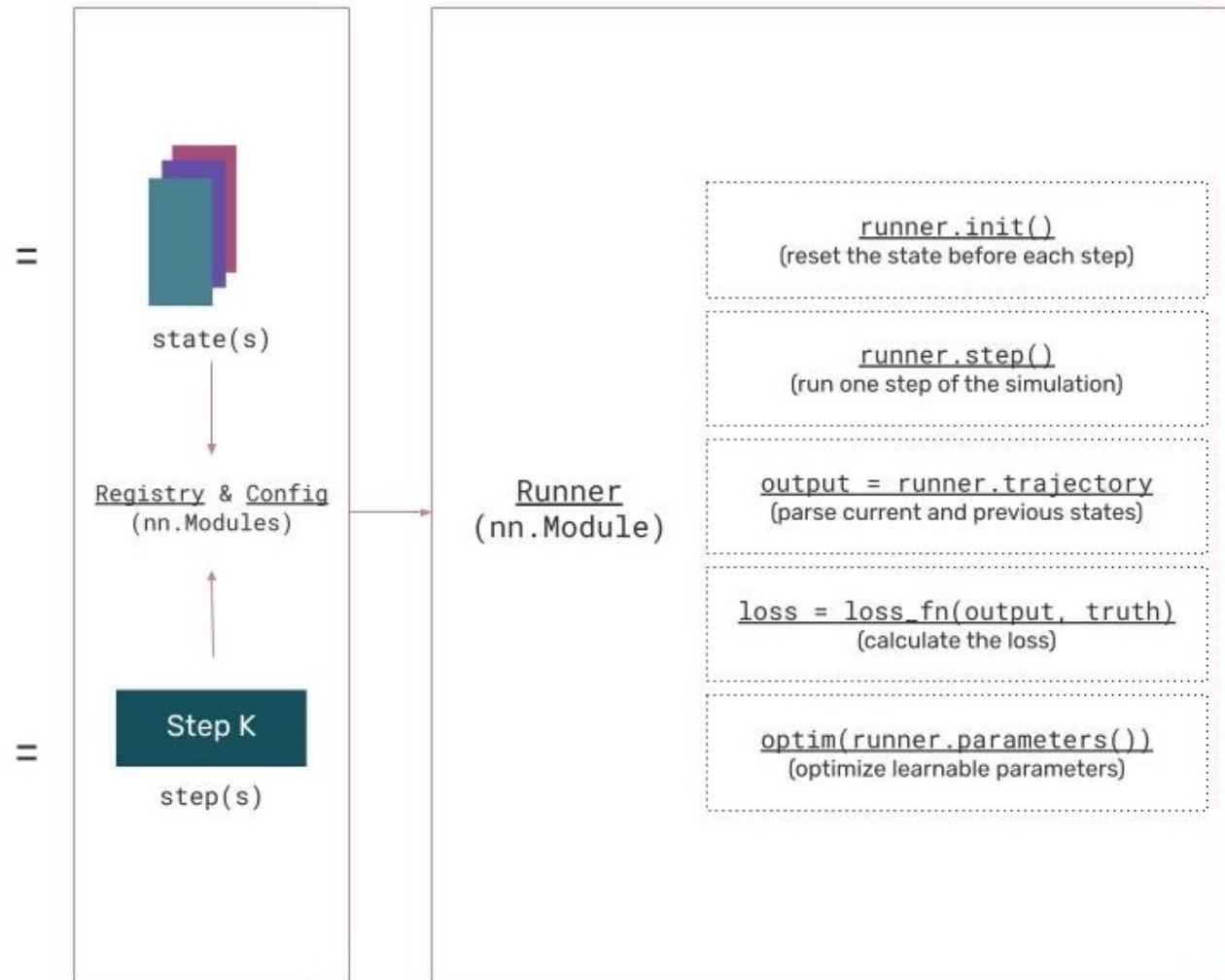
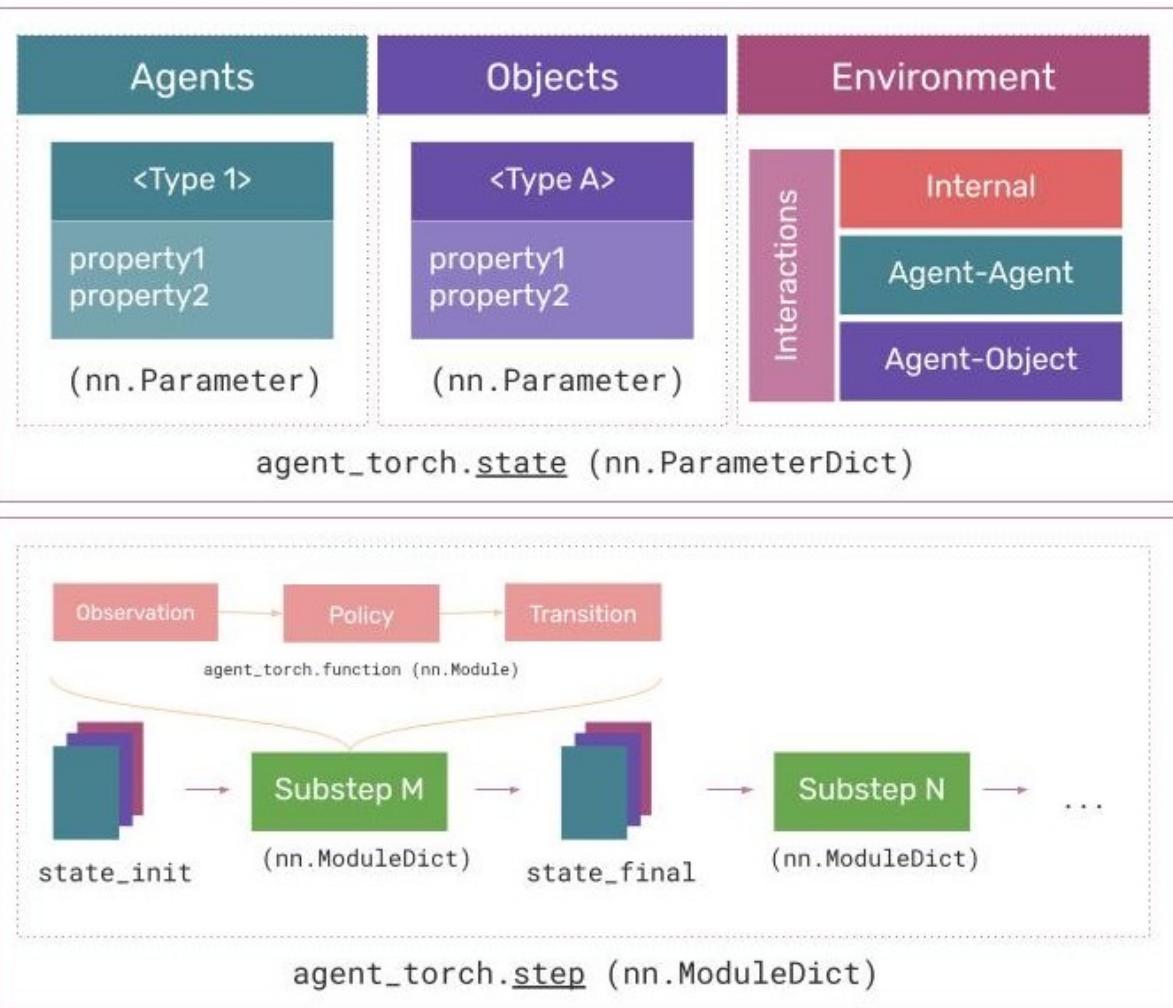


```
from AgentTorch.dataloader import DataLoader

dataloader = DataLoader(new_zealand)
dataloader._set_config_attribute('use_llm_agent', True)
dataloader._set_config_attribute('prompt', AGENT_PROMPT)

llm_simulation = Executor(disease, dataloader)
llm_simulation.execute()
```

Build a new simulator:  
Predator prey model



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# Questions and Discussion

# References

## Systems

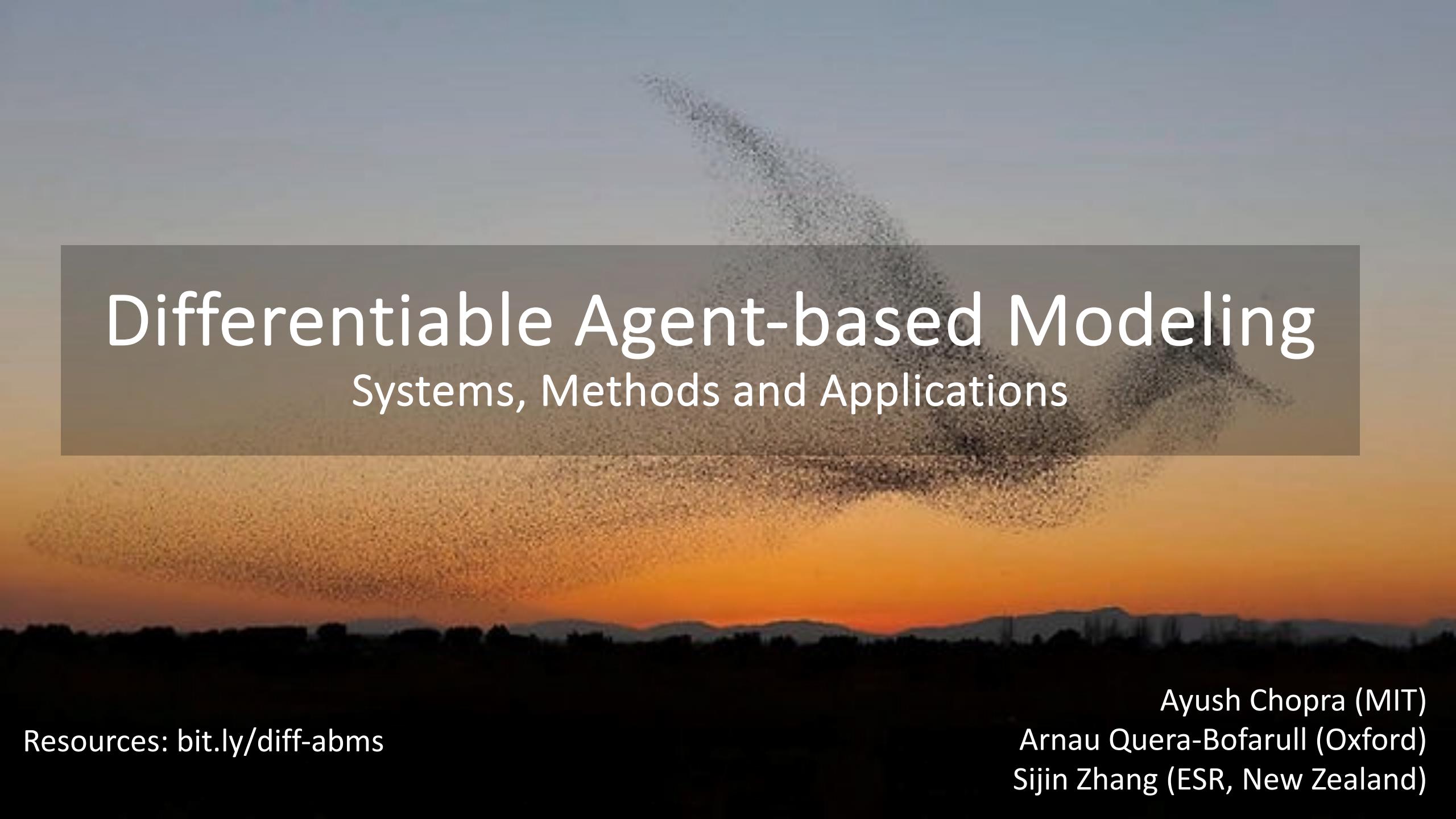
- A framework for learning in Agent-based Models (AAMAS 2024)
- BlackBIRDS: Black-Box Inference foR Differentiable Simulators (JOSS 2023)

## Methods

- Differentiable Agent-based Epidemiology (AAMAS 2023)
- Don't Simulate Twice: One-Shot Sensitivity Analyzes via Automatic Differentiation (AAMAS 2023)
- Private Agent-based Modeling (AAMAS 2024)
- Population synthesis as scenario generation for simulation-based planning under uncertainty (AAMAS 2024)

## Applications

- Public health impact of delaying second dose of BNT162b2 or mRNA-1273 covid-19 vaccine (BMJ 2021)
- Composing and evaluating interventions with ABM (AAMAS 2024, Best Student Paper Award Finalist!)

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# Differentiable Agent-based Modeling

## Systems, Methods and Applications

Resources: [bit.ly/diff-abms](https://bit.ly/diff-abms)

Ayush Chopra (MIT)

Arnau Quera-Bofarull (Oxford)

Sijin Zhang (ESR, New Zealand)

# Private Agent-based Modeling



What about the data?

ABM still rely on stale, coarse-grained data



limited granularity due to privacy concerns not scarcity of data.

## Thousands of New Zealanders being contacted after personal details leaked in Covid-19 data breach

## Illinois Bought Invasive Phone Location Data From Safegraph

## Report: Indonesian Government's Covid-19 App Accidentally Exposes Over 1 Million People in Massive Data Leak

## T-Mobile 'Put My Life in Danger' Says Woman Stalked With Black Market Location Data

Telecom giants are giving up customers' real-time location data to stalkers and bounty hunters. Now, Motherboard speaks to a victim.

**Thousands of New Zealanders being contacted after personal details leaked in Covid-19 data breach**

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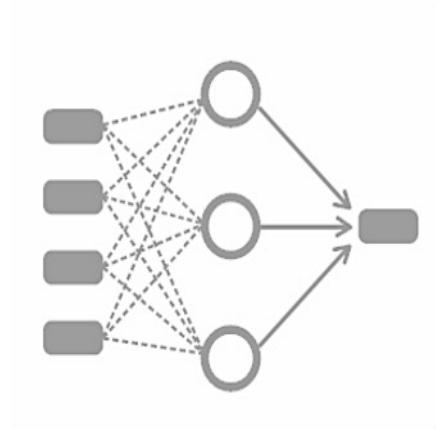
**T-Mobile 'Put My Life in Danger' Says Woman Stalked With Black Market Location Data**

Telecom giants are giving up customers' real-time location data to stalkers and bounty hunters. Now, Motherboard speaks to a victim.



how can we access high-resolution data, *securely*?

# Conventional focus on "de-sensitizing the data" for simulation



Synthetic generation

Learn distribution  
and re-sample

high sim2real gap



Differential Privacy

Add noise and  
release

bad privacy-utility trade-off

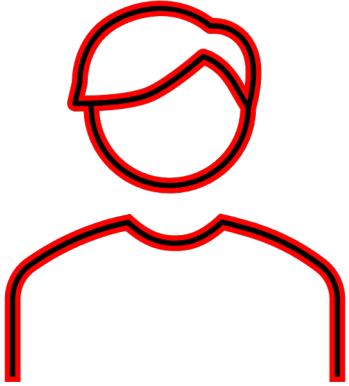
need low sim2real gap + perfect privacy!

decentralize simulation >> centralize data

rethinking the paradigm of agent-based modeling!

secure multi-party computation

Alice



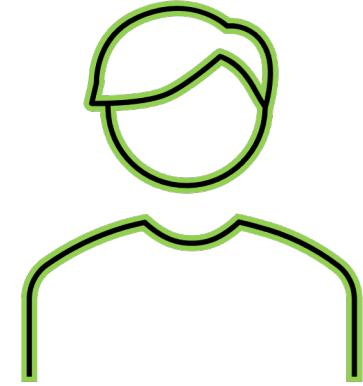
2

Bob



3

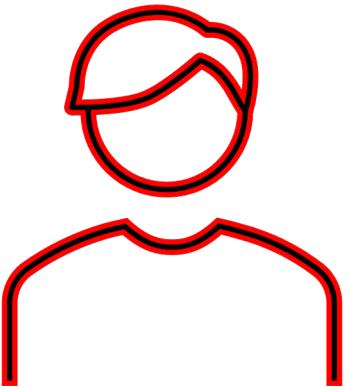
Charlie



5

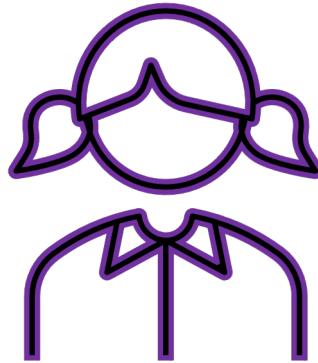
$n=11$

Alice



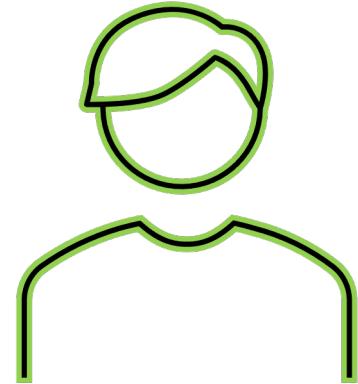
2

Bob



3

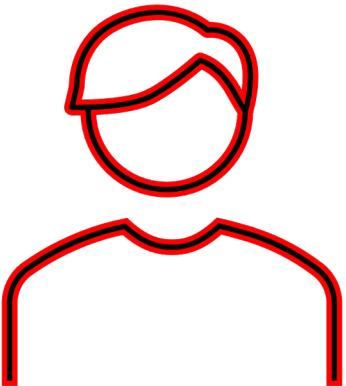
Charlie



5

$n=11$

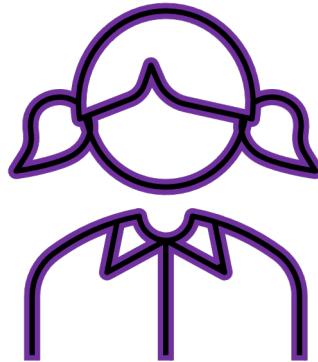
Alice



2

$7 + 5 + 1$

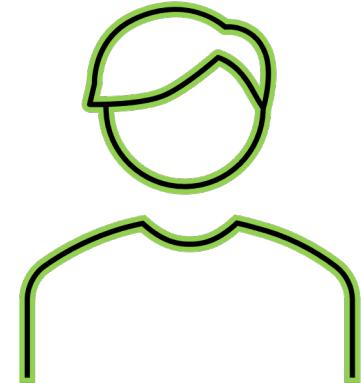
Bob



3

$2 + 0 + 1$

Charlie

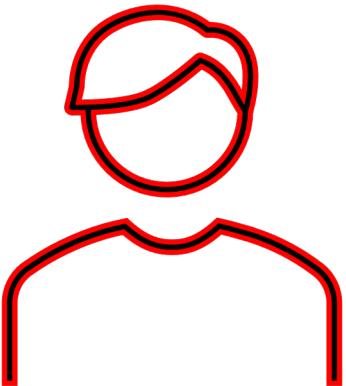


5

$3 + 1 + 1$

$n=11$

Alice

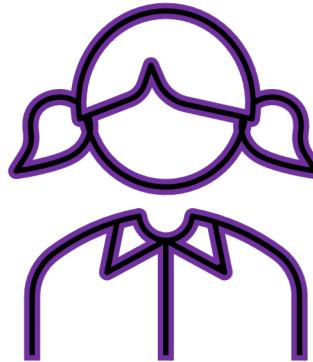


2

$7 + 5 + 1$

$7 + 2 + 3$

Bob

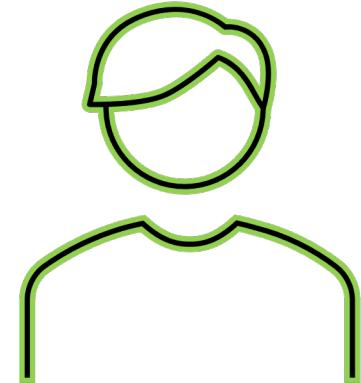


3

$2 + 0 + 1$

$5 + 0 + 1$

Charlie



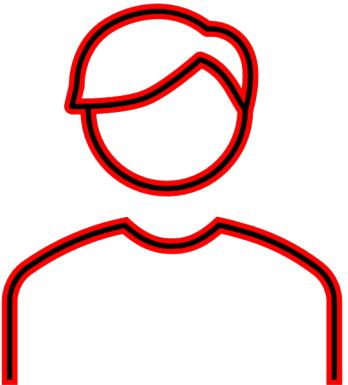
5

$3 + 1 + 1$

$1 + 1 + 1$

$n=11$

Alice



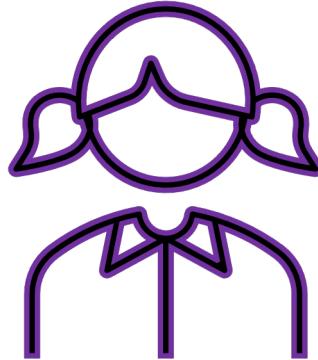
2

$7 + 5 + 1$

$7 + 2 + 3$

1

Bob



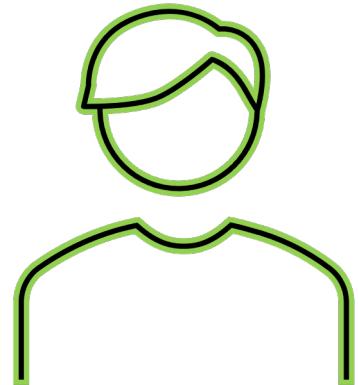
3

$2 + 0 + 1$

$5 + 0 + 1$

6

Charlie



5

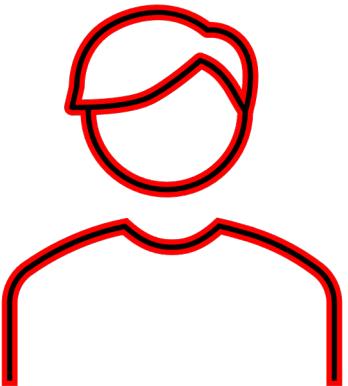
$3 + 1 + 1$

$1 + 1 + 1$

3

$n=11$

Alice



2

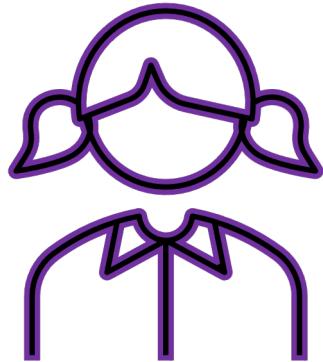
$7 + 5 + 1$

$7 + 2 + 3$

1

$1 + 6 + 3$

Bob



3

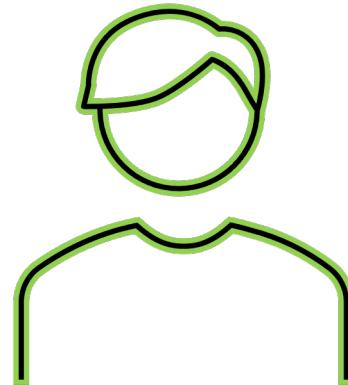
$2 + 0 + 1$

$5 + 0 + 1$

6

$1 + 6 + 3$

Charlie



5

$3 + 1 + 1$

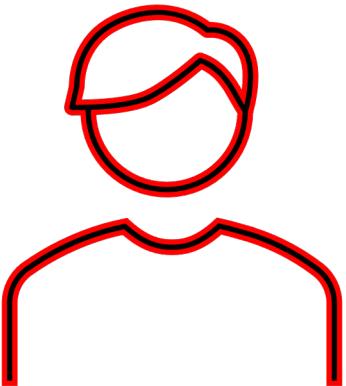
$1 + 1 + 1$

3

$1 + 6 + 3$

n=11

Alice



2

$$7 + 5 + 1$$

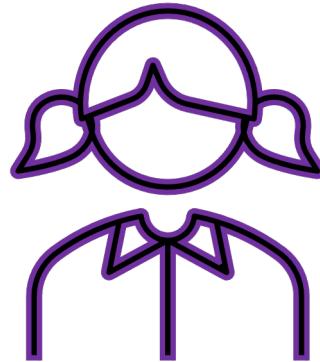
$$7 + 2 + 3$$

1

answer

10

Bob



3

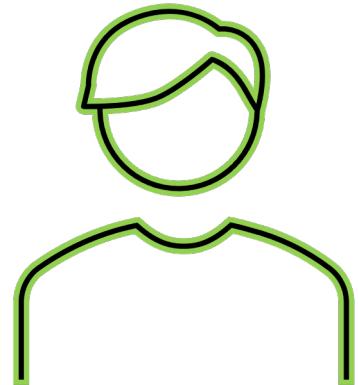
$$2 + 0 + 1$$

$$5 + 0 + 1$$

6

10

Charlie



5

$$3 + 1 + 1$$

$$1 + 1 + 1$$

3

10

# SPMC in action



Settle transactions



Trace infections



Pick auction winners

Critical Challenges  
Rapid Action  
Effective Policies



$$p_{\text{inf}}^{(i)}(t) = 1 - \exp\left(-\frac{\beta\,S_i\Delta t}{n_i}\sum_{j\in\mathcal{N}(i)}I_j(t)\right)$$

$$p_{\text{inf}}^{(i)}(t) = 1 - \exp \left( -\frac{\beta S_i \Delta t}{n_i} \sum_{j \in \mathcal{N}(i)} I_j(t) \right)$$

Annotations pointing to components of the equation:

- "disease parameter" points to the term  $\beta S_i \Delta t / n_i$ .
- "susceptibility of agent i" points to the term  $S_i$ .
- "transmissibility of neighbor j" points to the term  $I_j(t)$ .
- "neighborhood of agent i" points to the term  $\sum_{j \in \mathcal{N}(i)}$ .

Labels below the equation:

- "Infection probability for agent i" is positioned below the leftmost bracket.

# Privacy definition

Secure agent disease ( $S_i$ ), demographics ( $I_j$ ) and mobility trace ( $N_i$ ) data

$$p_{\text{inf}}^{(i)}(t) = 1 - \exp \left( -\frac{\beta S_i \Delta t}{n_i} \sum_{j \in \mathcal{N}(i)} I_j(t) \right)$$

Aggregate total transmissibility over all neighbors using additive secret sharing

$$p_{\text{inf}}^{(i)}(t) = 1 - \exp \left( -\frac{\beta S_i \Delta t}{n_i} \sum_{j \in \mathcal{N}(i)} I_j(t) \right)$$

Additive secret sharing

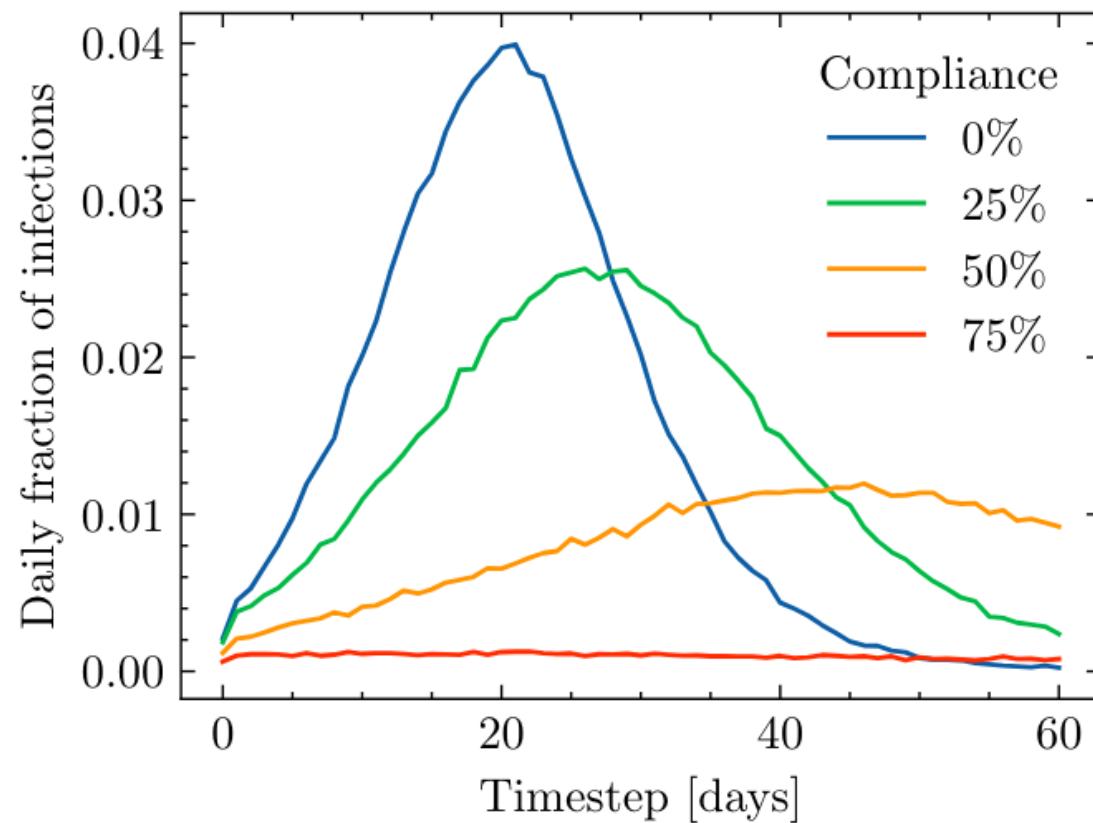
# Secure Simulation

Compliance probability  
of neighbor j

$$p_{\text{inf}}^{(i)}(t) = 1 - \exp \left( -\frac{\beta S_i \Delta_t}{n_i} \sum_{j \in \mathcal{N}(i)} I_j(t)(1 - c_j) \right)$$

# How effective will lockdown policies be?

Evaluate interventions without leaking individual disease status or compliance preference



# Secure Calibration

Calibrate the disease parameter to total infections at each time step

$x$  = number of infections

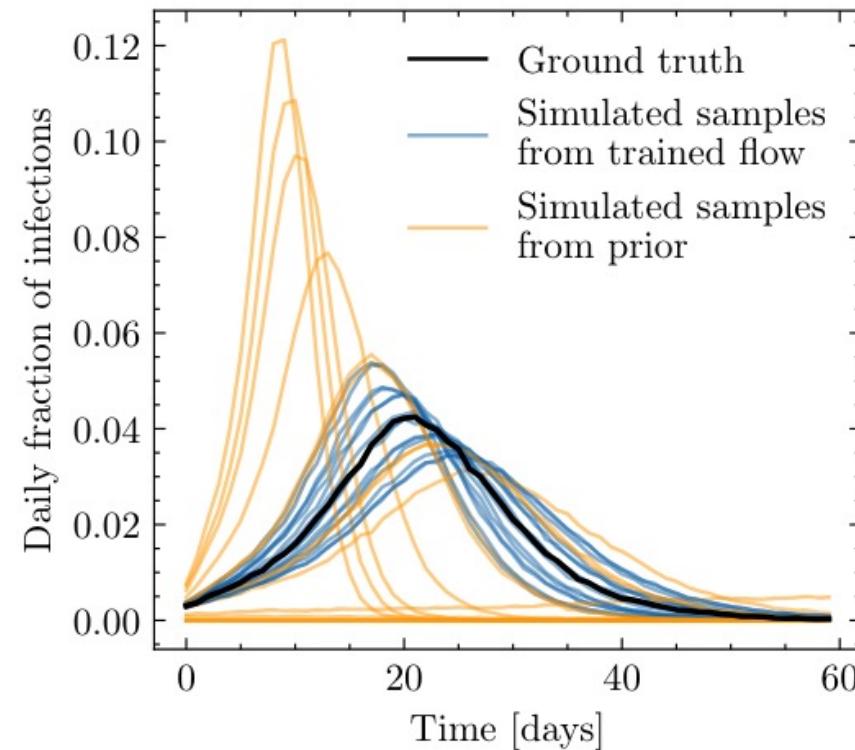
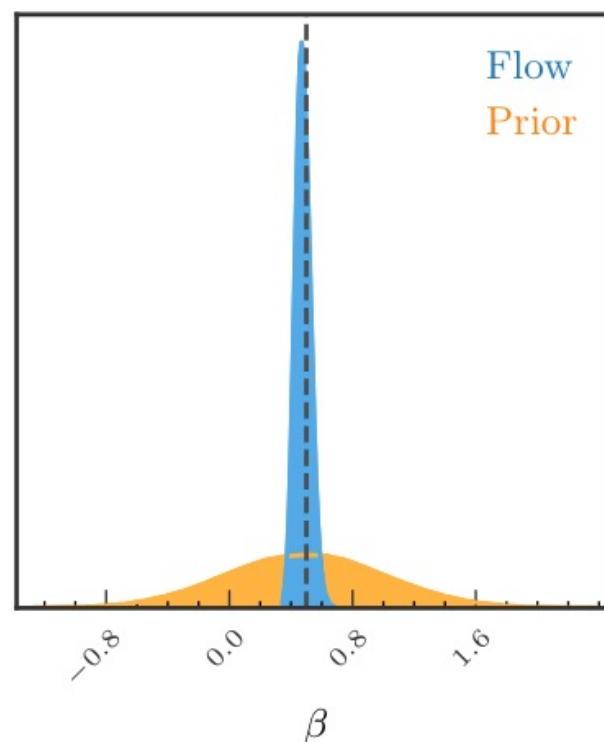
$p_i$  = prob agent  $i$  is infected

$$\frac{\partial x}{\partial \beta} = \sum_i \frac{\partial}{\partial \beta} \text{Bernoulli}(p_i) \approx \sum_i \frac{\partial p_i}{\partial \beta}$$

We can approximate the total gradient by summing the individual infection gradients (local and private).

# Calibrate simulation parameter $\beta$

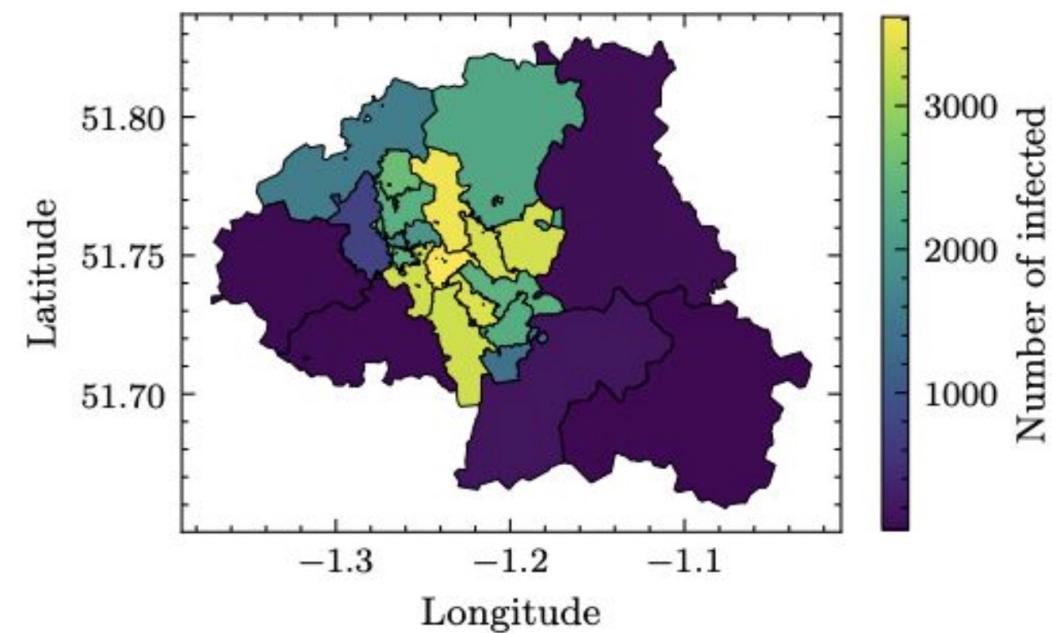
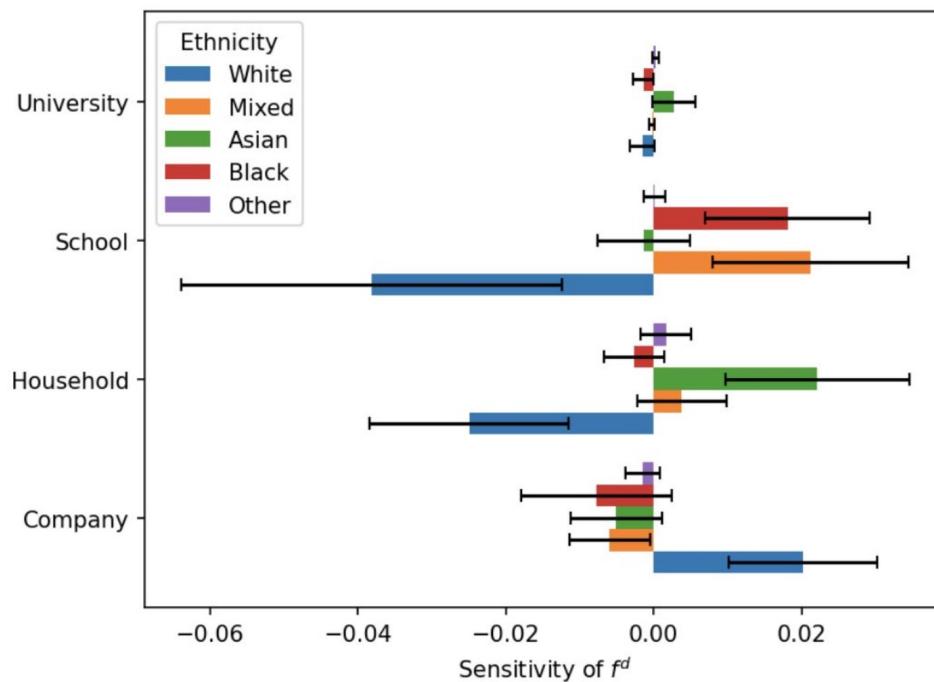
Calibrate disease parameters without leaking an agent's state or interaction trace



# Secure Analysis

# How does infection spread across age group and geography?

Analyze dynamics without leaking individual disease, demographic or geo-location

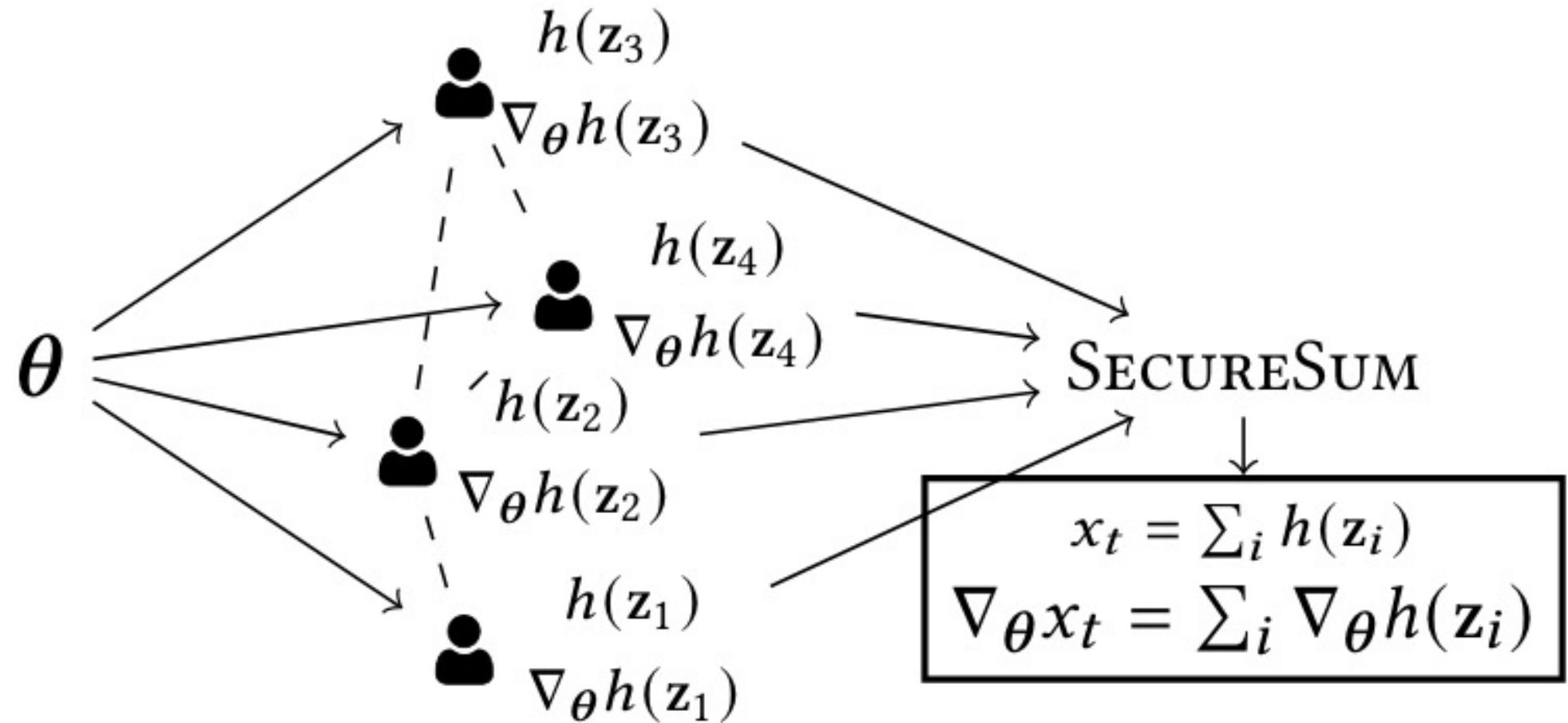


Protocol generalizes to any ABM with  
"permutation-invariant" message aggregation

$$\mathbf{z}_i(t+1) = f \left( \mathbf{z}_i(t), \bigoplus_{j \in \mathcal{N}_i(t)} M_{ij}(t), \theta \right)$$

See Section 2 in the paper (<https://arxiv.org/pdf/2404.12983>)

# SMPC to Aggregate Message and Calibration Gradient



# Growing trend of decentralized protocols across the world!



Financial networks



Supply chain networks



mobility networks

# Differentiable and Private Agent-based Models

[github.com/AgentTorch/AgentTorch](https://github.com/AgentTorch/AgentTorch)

Collaborate

[ayushc@mit.edu](mailto:ayushc@mit.edu)