University of California, Davis **Spring 2023**
Department of Statistics

Instructor: Dr. Sebastian Kühnert
TA: Mingshuo Liu

STA 106 – Analysis of Variance

(Homework 6: due Sa May 27, 2023, 11:59 AM)

1. **(28 points in total)** A study was conducted to compare the effect of four blood pressure lowering tablets P_1, P_2, P_3, P_4 on individuals having a high blood pressure after daily intake over one year. The tablets were randomly assigned to 32 individuals, and their blood pressure decrease after one year was measured by the systolic blood pressure (first value of the blood pressure). We know SSE = 180, and we assume the ANOVA model in sense of a cell means model. The following values have been raised:

	P_1	P_2	P_3	P_4
Sample mean	12.750	9.200	11.667	15.500
Sample size	8	5	9	10

   (a) **(8 points)** Calculate the confidence interval for μ_2 with significance level $\alpha = 0.1$, and use it to draw the conclusion for the test $H_0 \colon \mu_2 = 9.5$ vs. $H_a \colon \mu_2 \neq 9.5$ at significance level $\alpha = 0.1$.

   (b) **(10 points)** Use Scheffe's procedure to test, if both μ_1 is equal to $\frac{\mu_3+\mu_4}{2}$ and μ_1 is equal to $\frac{\mu_2+\mu_3+\mu_4}{3}$ (at the same time) with family-wise significance level $\alpha = 0.1$. State the null hypothesis, the alternative hypothesis, the test-statistic, the decision rule and draw the conclusion.

   (c) **(10 points)** Use Bonferroni's procedure to test, if $L_1 = \mu_1 - (2\mu_1 - \mu_4) = 0, L_2 = \mu_2 - (3\mu_3 - 2\mu_4) = 0$ and $L_3 = \mu_1 - (3\mu_3 - 2\mu_4 - \mu_2) = 0$ (at the same time) with family-wise significance level $\alpha = 0.05$. State the null hypotheses, the alternative hypotheses, the test statistic, the decision rule and draw the conclusion.

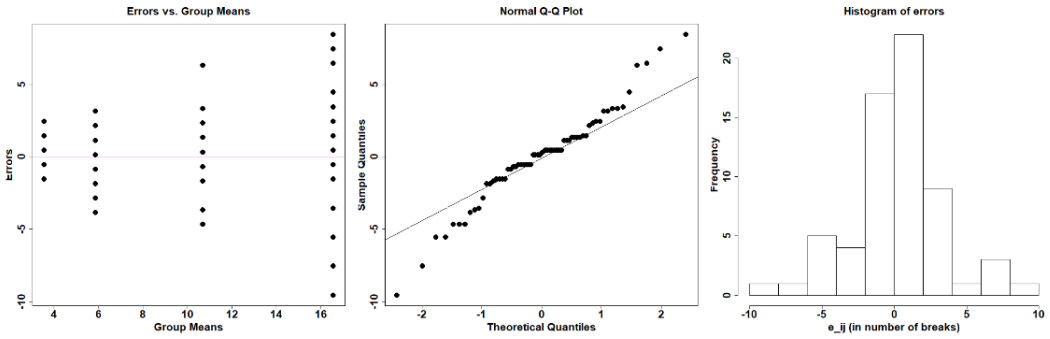
2. **(16 points in total)** Suppose that a randomly selected group of 4-year-old children were asked to complete a puzzle, which were rewarded if they found a correct piece. Not all children were rewarded each time. The percentages how they were rewarded were as follows: $I = 100\%$, $II = 66\%$, $III = 33\%$, $IV = 0\%$. The number of tries it took to complete the puzzle was measured, and we obtained the following summarized measurements:

	I	II	III	IV
Sample mean	12	11	16	17
Sample std. dev.	3.96	1.67	4.30	5.94
Sample size	10	10	10	10

   (a) **(8 points)** At first, by using the Hartley test, test with significance level $\alpha = 0.05$ if all variances of the different groups are equal or not. State the null hypotheses, the alternative hypotheses, the test statistic, the decision rule, and draw your conclusion.

   (b) **(8 points)** Now, by using weighted least squares, test with significance level $\alpha = 0.01$ whether all factor level means are equal or not. We know $SSE_w(R) = 149.37$. State the null hypotheses, the alternative hypotheses, the test statistic, the decision rule, and draw your conclusion.

3. **(6 points in total)** We have the following diagnostic plots for an ANOVA model with $n_T = 36$ total observations and $r = 4$ factor levels. One of the plots is a Q-Q-plot of the errors of one specific group, another plot illustrates the errors to the sample means in each of the $r = 4$ factor levels, and another plot is simply a histogramm of the errors.



(a) **(3 points)** Does it appear that the errors are normally distributed? Please explain.

(b) **(3 points)** Does it appear that the error's variances in each group are the same? Please explain.

4* **(5 Bonus points)** We randomly select a group of 4-year-old children to complete a puzzle again, which were rewarded if they found a correct piece. They were also not rewarded each time, where the percentages how they were rewarded were as follows: $I = 100\%$, $II = 50\%$, $III = 0\%$. We collected the following numbers of tries it took to complete the puzzle of 15 children in total:

$j \backslash i$	1 (group I)	2 (group II)	3 (group III)
1	11	13	18
2	12	15	16
3	13	15	17
4	11	14	17
5	12	14	–
6	11	–	–

We want to avoid assuming normally distributed errors here, but we want to know if the factor level means are equal or not to the significance level $\alpha = 0.01$. Use the non-parametric rank F-test to test whether the factor level means are equal or not. State the null hypothesis, the alternative hypothesis, the test-statistic, the decision rule, and draw your conclusion.