

# Self-similar hydrodynamics of first-order phase transitions in the early universe

Mika Mäki  
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# Hi everyone!



- Who am I?
  - 2<sup>nd</sup> year MSc student of particle physics and cosmology
  - Already a MSc (diplomi-insinööri) in applied physics from Tampere University (lasers, computational physics, software engineering, machine learning)
  - Personal interests: free software, automating everything with Python ([github.com/AgenttiX](https://github.com/AgenttiX)), RPGs (D&D etc.), video games (Factorio etc.)
- What am I doing in the CFT group?
  - Summer employee 2021: PTtools refactoring and optimization
    - Parallelisation, code quality, documentation etc.
  - Thesis: Self-similar hydrodynamics of first-order phase transitions in the early universe
    - How to model the fluid velocity profile of an expanding bubble that has an arbitrary equation of state?

# First-order phase transition proceeds by bubbles

- Sharp boundary between the phases
- In our case the transition is from a high-temperature phase to a low-temperature phase
- If strong enough, the sound waves generate gravitational waves
- "Listening to the noise of a cosmic kettle"
- <https://www.youtube.com/watch?v=mfGL8CpORPA>



# From fluid properties to GWs

- Starting point: equation of state  $p(T, \phi)$
- Equation of state  $\rightarrow$  fluid velocity profile  $\rightarrow$  GW power spectrum
  - Bubbles are spherically symmetric  $\rightarrow$  1D simulation
  - Self-similar: scaled fluid velocity profile is time-independent
  - Numerical integration, sine transform
- How to account for different models of the underlying particle physics?
  - Goal: only five key parameters

$$T_n \quad \alpha_n \quad v_{\text{wall}} \quad \beta \quad g_{\text{eff}}(T, \phi) \rightarrow p(T, \phi) \rightarrow c_s(T, \phi)$$



# Thesis seminar slides: Equations of state for phase transitions in the early universe

Mika Mäki  
2022-04-27

# Outline

- Background and context
  - The early universe and Higgs mechanism
  - Phase transitions and gravitational waves
- Relativistic hydrodynamics and combustion
  - Bubble walls and fluid velocity
  - Equations of state

# Timeline of the Big Bang

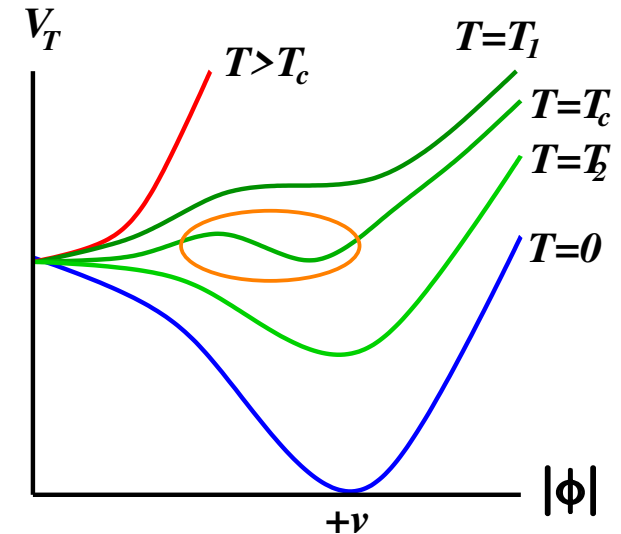
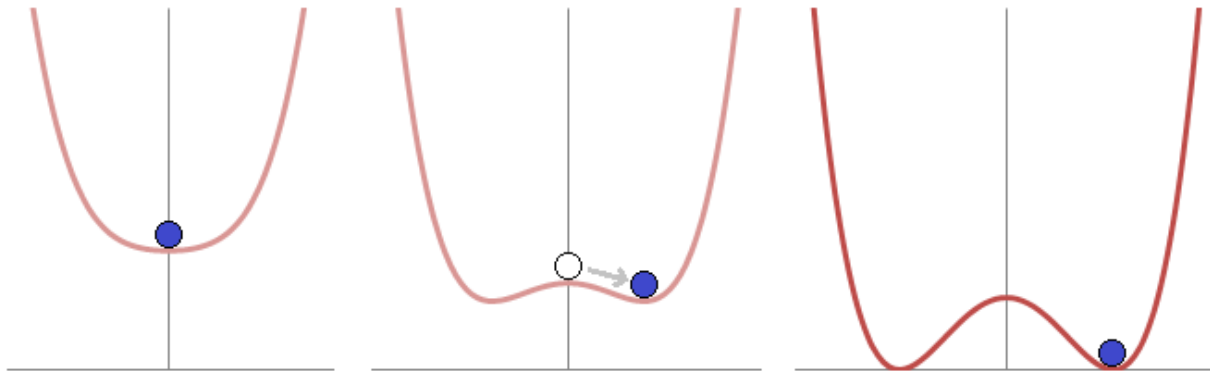
Epoch	Time (s)	T (K)	T (GeV)	Description
Planck epoch	$< 10^{-43}$	$> 10^{32}$	$> 10^{19}$	???, quantum gravity?
Grand unification epoch	$< 10^{-36}$	$> 10^{29}$	$> 10^{16}$	Grand unified theories?
Inflationary epoch?	$< 10^{-32}$	$10^{28} \sim 10^{22}$	$10^{15} \sim 10^9$	Inflation, exact temperature range unknown
<b>Electroweak transition</b>	<b><math>10^{-12}</math></b>	<b><math>10^{15}</math></b>	<b><math>\sim 150</math></b>	<b>Higgs mechanism</b>
QCD transition	$10^{-12} \sim 1$	$10^{15} \sim 10^{10}$	150 GeV $\sim$ 1 MeV	Quarks form hadrons
Big bang nucleosynthesis	$10 \sim 10^3$	$10^9 \sim 10^7$	100 keV $\sim$ 1 keV	Hydrogen & helium production
Recombination	18 kyr $\sim$ 370 kyr	4000	0.4 keV	Cosmic microwave background

# Higgs mechanism: from massless to massive

Temperature decreases

- Energetically optimal to break symmetry
- Higgs field obtains a vacuum expectation value
- Particles obtain their rest masses by the Yukawa couplings

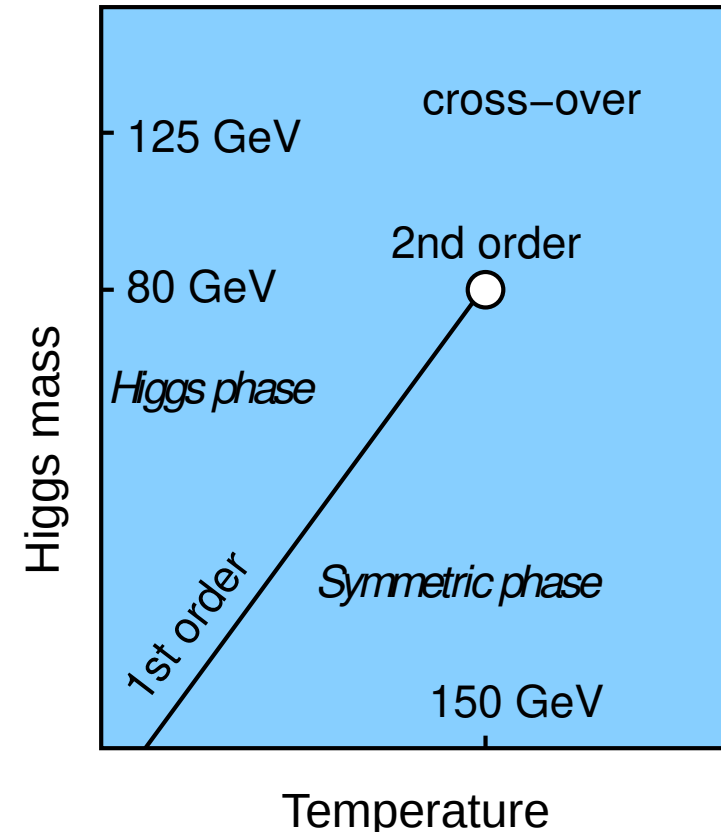
$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}(F_{\mu\nu})^2 + |D_\mu\phi|^2 - V(\phi) \quad |D_\mu = \partial_\mu + ieA_\mu \\ &= \dots \\ &= -\frac{1}{4}(F_{\mu\nu})^2 + (\partial_\mu\phi)^2 + \boxed{e^2\phi^2}A_\mu A^\mu - V(\phi) \\ &\quad \text{rest mass for } A\end{aligned}$$





# Many extensions of the Standard Model result in first-order phase transitions

- Can help in solving
  - Dark matter
  - Electroweak hierarchy problem: Why is the Higgs mass only 125 GeV?
  - Baryogenesis: matter-antimatter asymmetry
- Examples
  - Additional singlet scalar fields
  - Electroweakly charged scalar fields: two Higgs doublets etc.
  - Supersymmetry (non-minimal, e.g. minimal + singlet)
  - Extra dimensions
  - (Dark sectors)
- Change of the critical point in the phase diagram



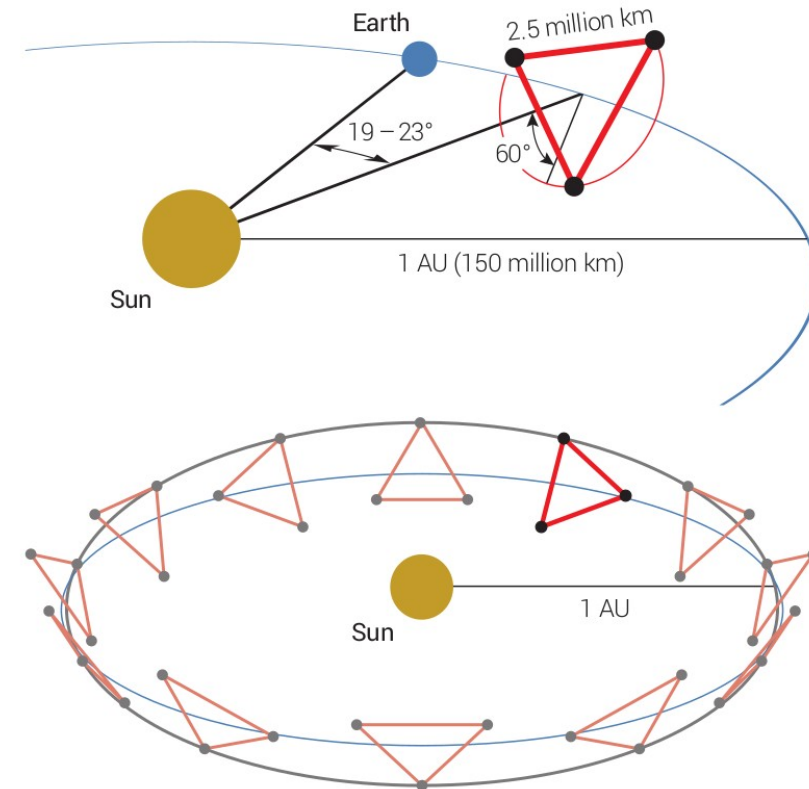
# First-order phase transition proceeds by bubbles

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# Gravitational waves

- Weak coupling with matter
  - Early universe is transparent
- Production steps in phase transitions
  - Bubble collision and merger
  - **Expansion of fluid kinetic energy shells: sound waves**
  - Turbulence: non-linearities, sound waves → shocks
- GW mathematics omitted from this presentation
- Isotropic background signal
  - The galactic white dwarf foreground etc. is expected to vary → distinguishable
- Detection: LISA in mid-2030s



# Relativistic hydrodynamics

- [Ultra]relativistic plasma
- Unlike classical fluids: non-conserved particle number
- Characterised by an equation of state
- State given by the energy-momentum tensor

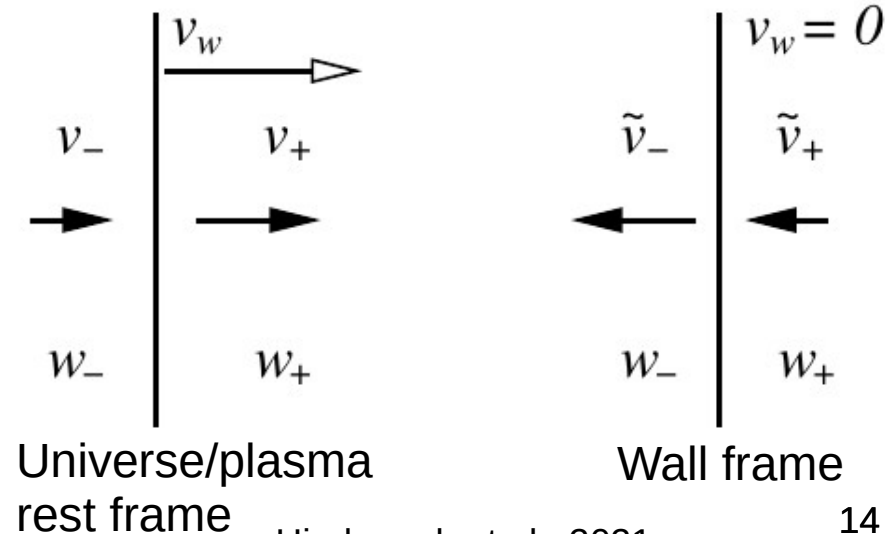
# GW power spectrum is characterized by five key parameters

- GW power spectrum is characterized by
  - Nucleation temperature  $T_n$
  - Phase transition strength at the nucleation temperature  $\alpha_n$
  - Bubble wall speed  $v_{\text{wall}}$
  - Transition rate parameter  $\beta$
  - Sound speed  $c_s(T, \phi)$
- Initial analysis: simple toy models
- **Goal of the thesis: arbitrary model from particle physics parameters**

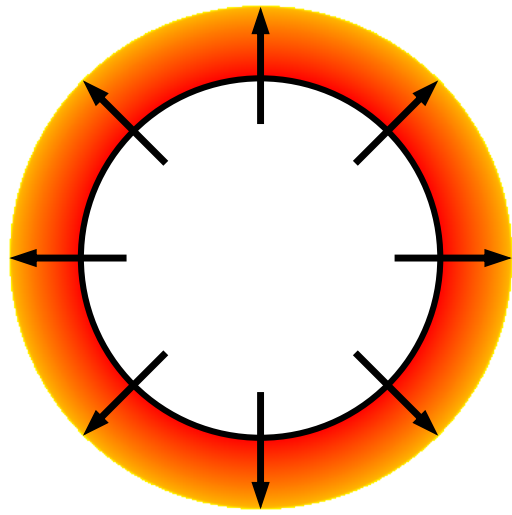
# Relativistic combustion: self-similar bubbles

- Conservation of energy-momentum at the wall  $\partial^\mu T_{\mu\nu} = 0$ 
  - Junction conditions  $w_- \tilde{\gamma}_-^2 \tilde{v}_-^2 + p_- = w_+ \tilde{\gamma}_+^2 \tilde{v}_+^2 + p_+$   
 $w_- \tilde{\gamma}_-^2 \tilde{v}_- = w_+ \tilde{\gamma}_+^2 \tilde{v}_+$
  - Change in the potential gives kinetic energy to the plasma
- Constant wall speed due to friction
- Relative shape is constant  
= self-similarity

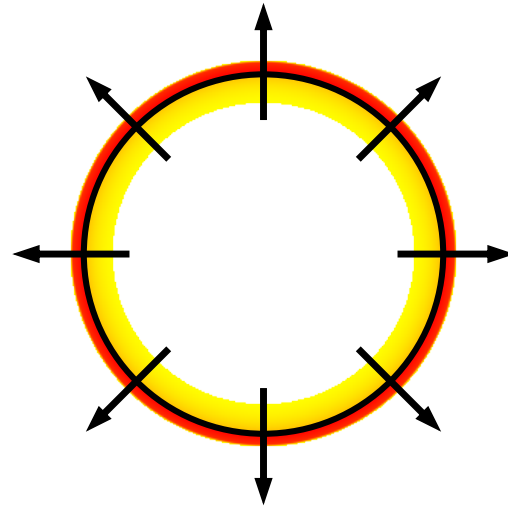
$$\xi = \frac{r}{t}$$



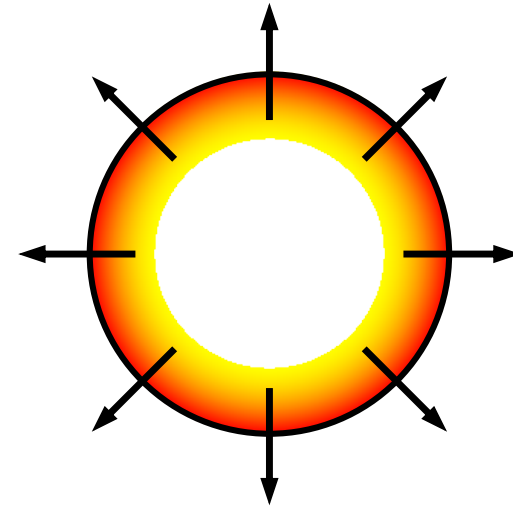
# Types of relativistic combustion



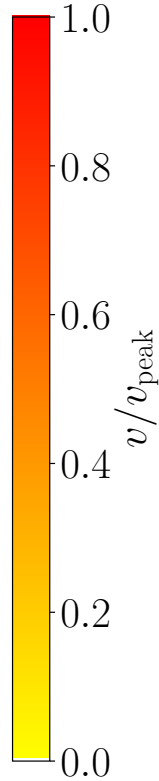
subsonic deflagration  
 $v_w \leq c_s$



supersonic deflagration  
 $c_s < v_w < c_J$



detonation  
 $c_J \leq v_w$



Black: bubble wall / phase boundary  
Colour: velocity of moving plasma

Hindmarsh et al., 2021

# Bag model: the simplest model

- Equation of state:  $p(T, \phi)$

$$p_s = a_s T^4 - V_s$$

$$p_b = a_b T^4$$

- The rest can be deduced with thermodynamics

- Enthalpy density  $w$

- Energy density  $e$

- Entropy density  $s$

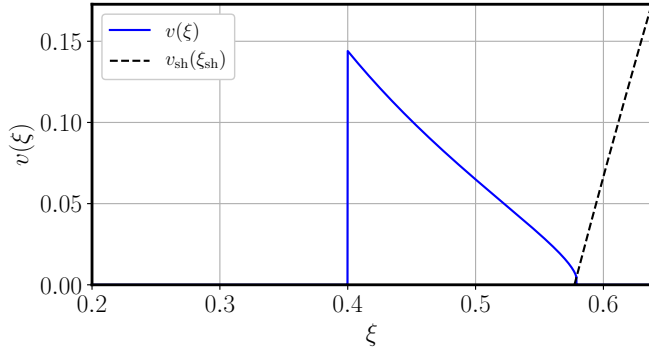
- Sound speed  $c_s$

$$\begin{aligned} w &\equiv T \frac{\partial p}{\partial T} & c_s^2 &\equiv \left( \frac{\partial p}{\partial e} \right)_s = \frac{1}{3} \\ &= e + p \\ &= T s \end{aligned}$$

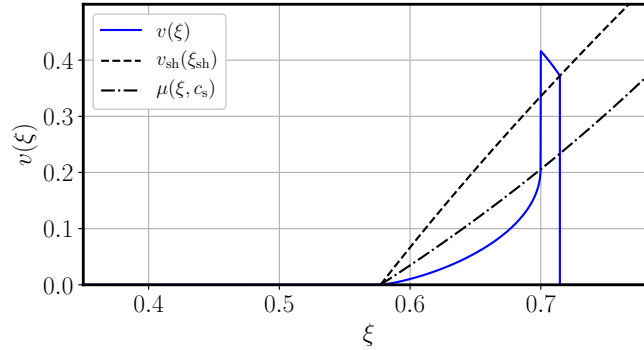


# Velocity and enthalpy profiles are different for each type of combustion

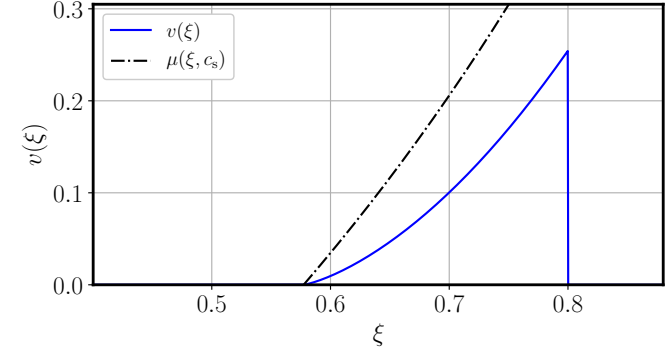
$\xi_w = 0.4, \alpha_n = 0.1, \alpha_+ = 0.078, r = 1.624, \xi_{sh} = 0.579$



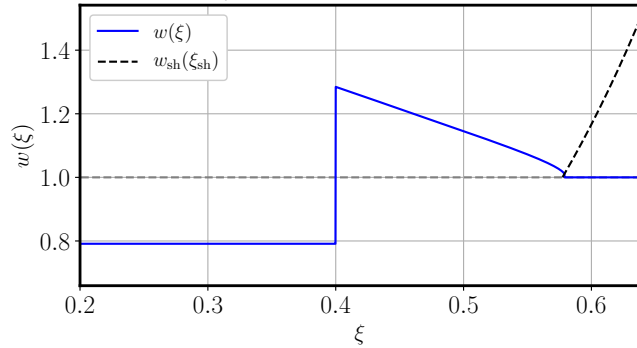
$\xi_w = 0.7, \alpha_n = 0.1, \alpha_+ = 0.037, r = 1.816, \xi_{sh} = 0.715$



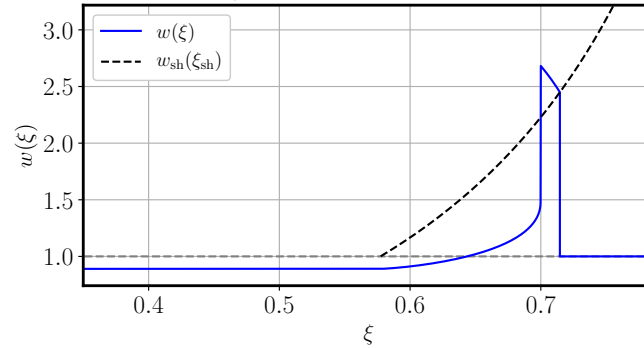
$\xi_w = 0.8, \alpha_n = 0.1, \alpha_+ = 0.100, r = 0.581, \xi_{sh} = 0.800$



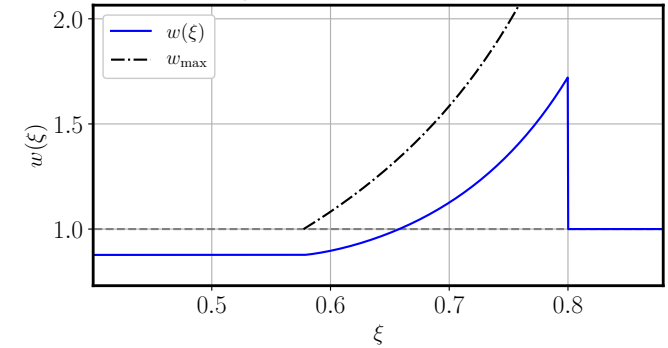
$w_0/w_n = 0.79, \bar{U}_f = 0.119, K = 0.0172, \kappa = 0.189, \omega = 0.815$



$w_0/w_n = 0.89, \bar{U}_f = 0.184, K = 0.0411, \kappa = 0.452, \omega = 0.559$



$w_0/w_n = 0.88, \bar{U}_f = 0.133, K = 0.0213, \kappa = 0.235, \omega = 0.769$



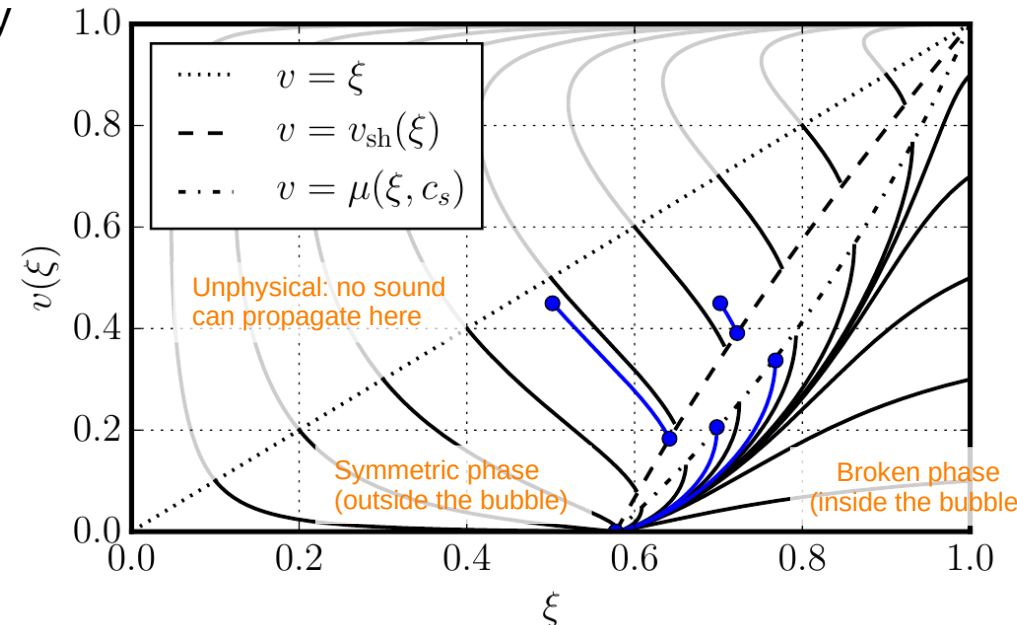
Subsonic deflagration:  
movement only outside

Supersonic deflagration (hybrid):  
movement on both sides

Detonation: movement  
only inside

# Mathematics: numerical integration

- Compute the velocity profile
  - Start from known boundary conditions
    - $v = 0$  and  $T = T_{\text{nucleation}}$  far away
    - $v = 0$  at the center of the bubble
  - Integrate  $(v, w, \xi)$  numerically
- Use the **Sound Shell Model** to convert the velocity profile to GW power spectrum
  - Sine transform (a bit like Fourier, numerical integration)



# Beyond the bag model

- Assumptions broken
  - Different equations for the phases
  - Different sound speeds, possibly temperature-dependent:  $c_s(T, \phi)$
- Computing the velocity profile becomes more difficult
  - Sound speed may change at each point
  - May require nested numerical integration
- Next approximation: Constant sound speed model
  - Different but constant speed of sound in each phase

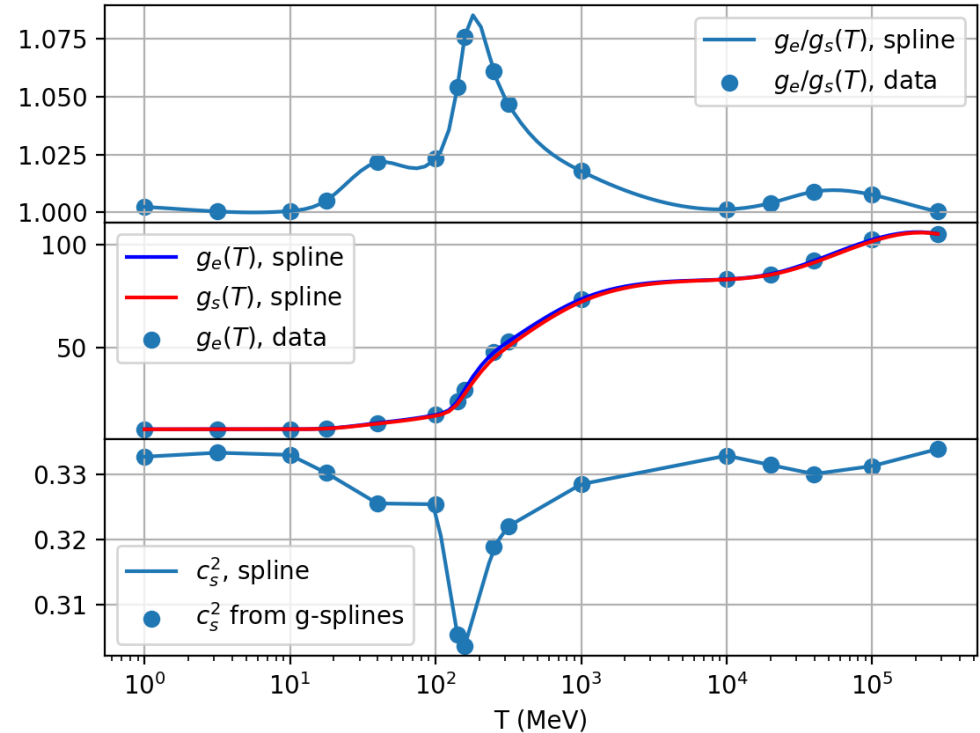
$$p_s = a_s T^\mu - V_s \quad \mu = 1 + \frac{1}{c_{s,s}^2}$$
$$p_b = a_b T^\nu \quad \nu = 1 + \frac{1}{c_{s,b}^2}$$

# Goal: Equation of state from an arbitrary model

- Example: Standard Model
- Fluid properties depend on
  - Temperature  $T$
  - Phase  $\phi$
- Arbitrary models can be tested, when  $g_{\text{eff}}(T, \phi)$  is given

$$e(T, \phi) = \frac{\pi^2}{30} g_e(T, \phi) T^4$$

$$s(T, \phi) = \frac{2\pi^2}{45} g_s(T, \phi) T^3$$



Data from Borsanyi et al., 2016

# Summary

- Many extensions of the Standard model result in first-order phase transitions
  - Gravitational waves
- GW power spectrum
  - Velocity profile of bubbles
  - Underlying physics
- Were there first-order phase transitions in the early universe?  
We will know in the 2030s when LISA is launched.
- If yes, it's a sign of new physics!

# Sources

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