Equations of state for phase transitions in the early universe

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Outline

- Background and context
  - The early universe and Higgs mechanism
  - Phase transitions and gravitational waves
- Relativistic hydrodynamics and combustion
  - Bubble walls and fluid velocity
  - Equations of state

Timeline of the Big Bang

Epoch	**Time (s)**	**T (K)**	**T (GeV)**	**Description**
Planck epoch	$< 10^{-43}$	$> 10^{32}$	$> 10^{19}$	???, quantum gravity?
Grand unification epoch	$< 10^{-36}$	$> 10^{29}$	$> 10^{16}$	Grand unified theories?
Inflationary epoch?	$< 10^{-32}$	10^{28}~10^{22}	10^{15}~10^{9}	Inflation, exact temperature range unknown
Electroweak transition	**10^{-12}**	**10^{15}**	**~150**	**Higgs mechanism**
QCD transition	10^{-12}~1	10^{15}~10^{10}	150 GeV ~ 1 MeV	Quarks form hadrons
Big bang nucleosynthesis	10~10^{3}	10^{9}~10^{7}	100 keV ~ 1 keV	Hydrogen & helium production
Recombination	18 kyr ~ 370 kyr	4000	0.4 keV	Cosmic microwave background

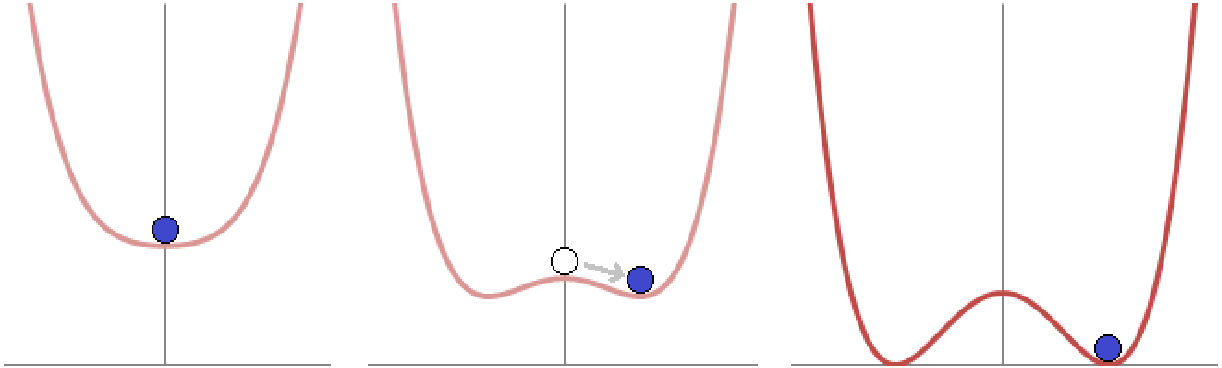
Higgs mechanism: from massless to massive

Temperature decreases

- Energetically optimal to break symmetry
- Higgs field obtains a vacuum expectation value
- Particles obtain their rest masses by the Yukawa couplings

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + |D_\mu\phi|^2 - V(\phi) \quad |D_\mu = \partial_\mu + ieA_\mu$$
$$= \dots$$
$$= -\frac{1}{4}(F_{\mu\nu})^2 + (\partial_\mu\phi)^2 + \boxed{e^2\phi^2} A_\mu A^\mu - V(\phi)$$

rest mass for A



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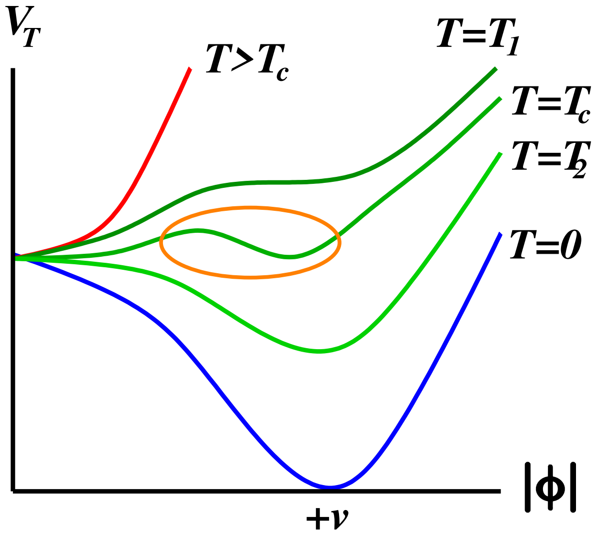
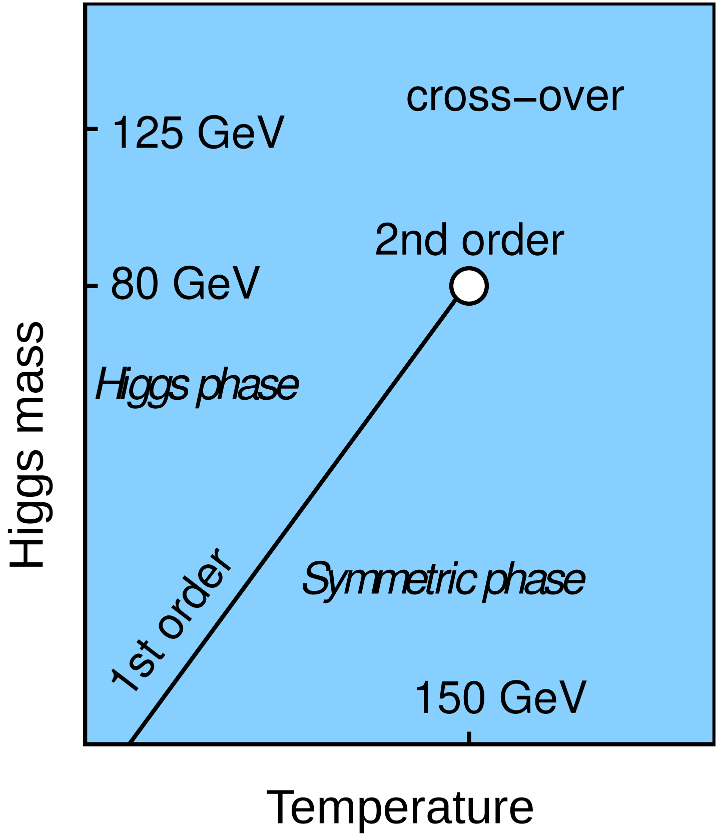


Hindmarsh et al., 2021

Many extensions of the Standard Model result in first-order phase transitions

- Can help in solving
  - Dark matter
  - Electroweak hierarchy problem:
    Why is the Higgs mass only 125 GeV?
  - Baryogenesis: matter-antimatter asymmetry
- Examples
  - Additional singlet scalar fields
  - Electroweakly charged scalar fields: two Higgs doublets etc.
  - Supersymmetry (non-minimal, e.g. minimal + singlet)
  - Extra dimensions
  - (Dark sectors)
- Change of the critical point in the phase diagram



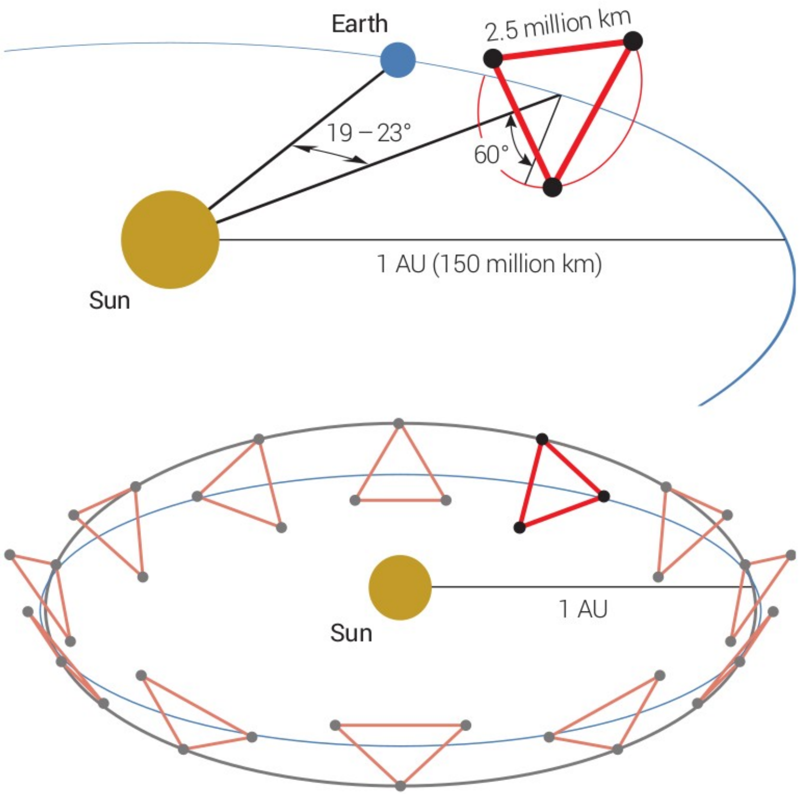
Hindmarsh et al., 2021

First-order phase transition proceeds by bubbles

- Sharp boundary
- In our case the transition is from a high-temperature phase to a low-temperature phase
- If strong enough, the sound waves generate gravitational waves
- ”Listening to the noise of a cosmic kettle”
- https://www.youtube.com/watch?v=mfGL8CpORPA

Gravitational waves

- Weak coupling with matter
  → Early universe is transparent
- Production steps in phase transitions
  - Bubble collision and merger
  - **Expansion of fluid kinetic energy shells: sound waves**
  - Turbulence: non-linearities, sound waves → shocks
- GW mathematics omitted from this presentation
- Isotropic background signal
  - The galactic white dwarf foreground etc. is expected to vary → distinguishable
- Detection: LISA in mid-2030s



Hindmarsh et al., 2021

Relativistic hydrodynamics

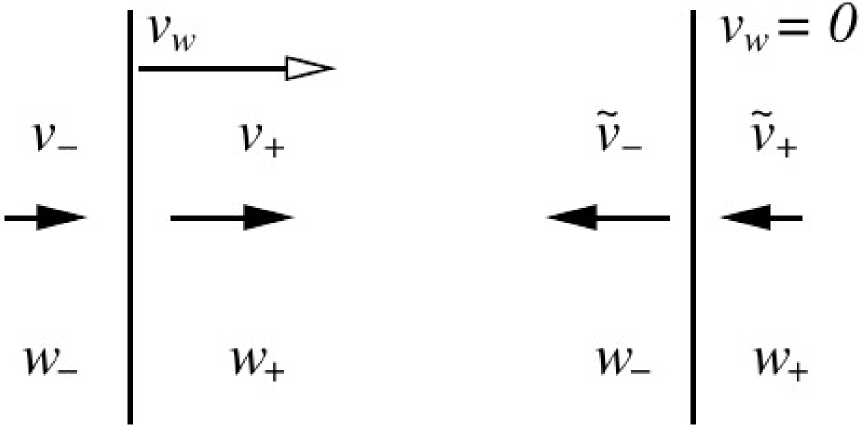
- [Ultra]relativistic plasma
- Unlike classical fluids: non-conserved particle number
- Characterised by an equation of state
- State given by the energy-momentum tensor

GW power spectrum is characterized by five key parameters

- GW power spectrum is characterized by
  - Nucleation temperature T_n
  - Phase transition strength at the nucleation temperature α_n
  - Bubble wall speed v_{wall}
  - Transition rate parameter β
  - Sound speed $c_s(T, \phi)$
- Initial analysis: simple toy models
- **Goal of the thesis: arbitrary model from particle physics parameters**

Relativistic combustion: self-similar bubbles

- Conservation of energy-momentum at the wall $\partial^\mu T_{\mu\nu} = 0$
  - Junction conditions $w_- \tilde{\gamma}_-^2 \tilde{v}_-^2 + p_- = w_+ \tilde{\gamma}_+^2 \tilde{v}_+^2 + p_+$
    $$w_- \tilde{\gamma}_-^2 \tilde{v}_- = w_+ \tilde{\gamma}_+^2 \tilde{v}_+$$
  - Change in the potential gives kinetic energy to the plasma
- Constant wall speed due to friction
- Relative shape is constant
  = self-similarity
  $$\xi = \frac{r}{t}$$

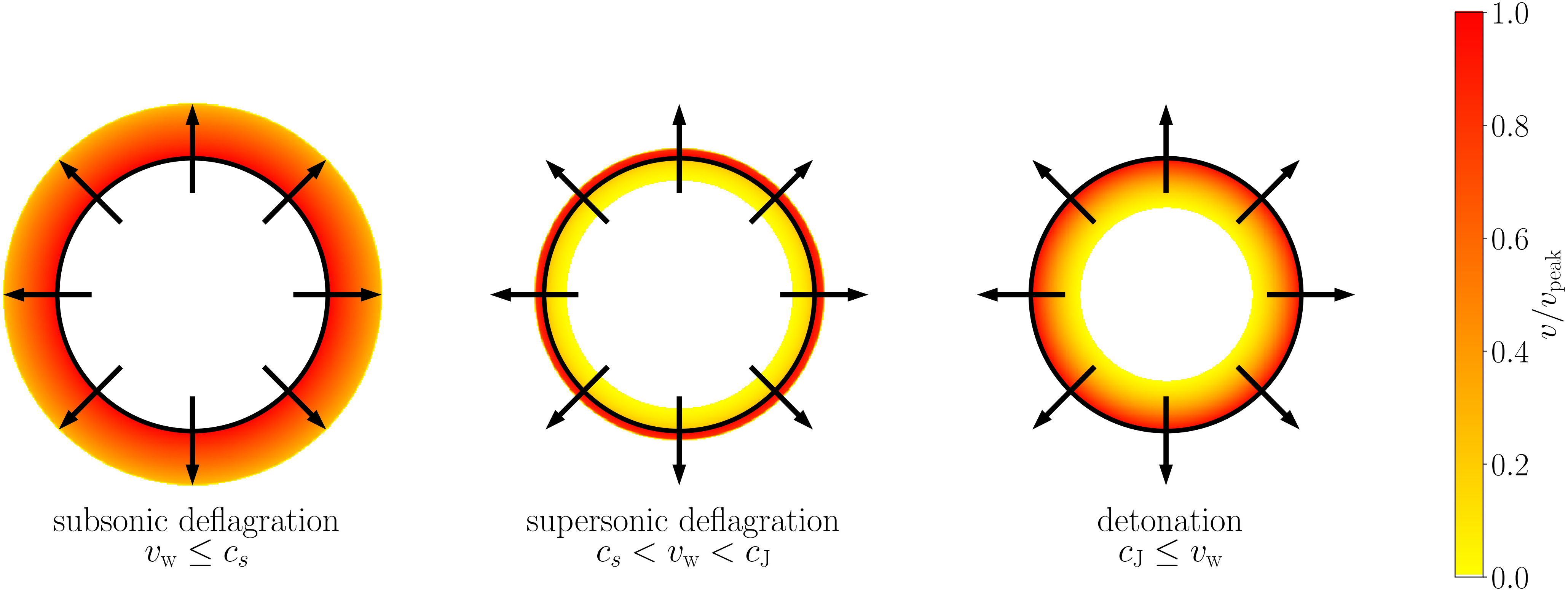


Universe/plasma rest frame

Wall frame

Hindmarsh et al., 2021

Types of relativistic combustion



Black: bubble wall / phase boundary
Color: velocity of moving plasma

Hindmarsh et al., 2021

Bag model: the simplest model

- Equation of state: $p(T, \phi)$

  $$p_s = a_s T^4 - V_s$$

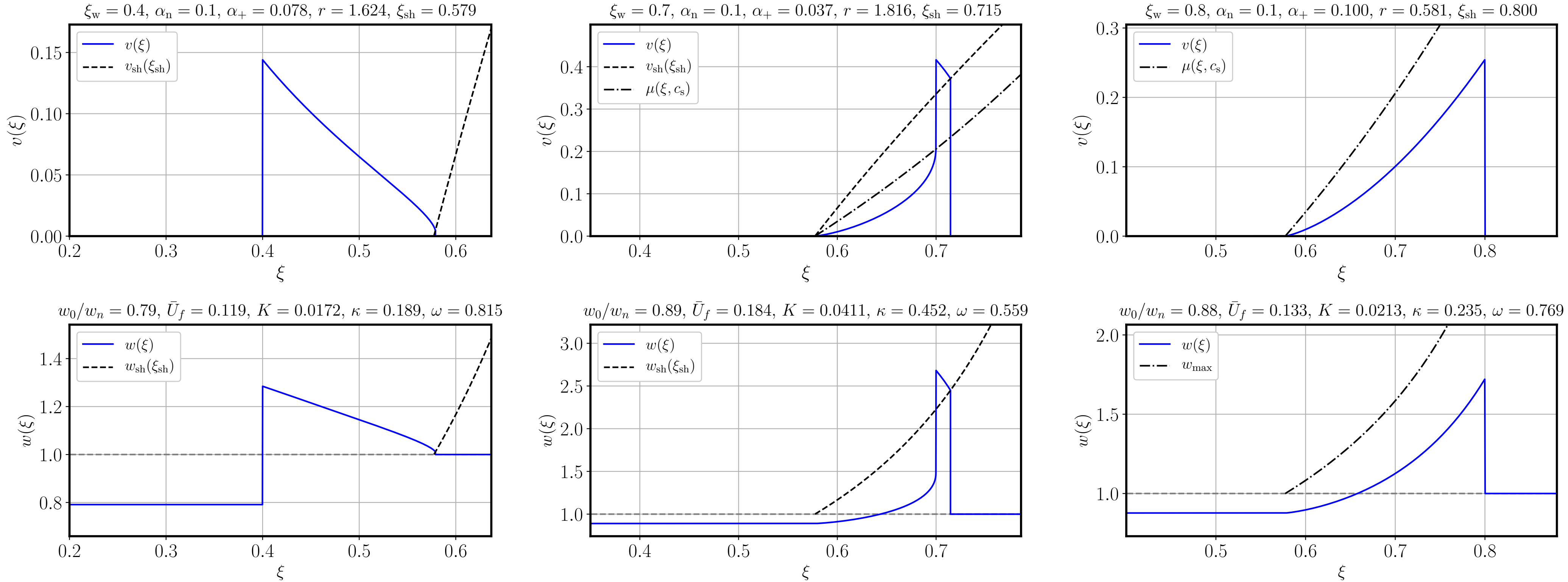
  $$p_b = a_b T^4$$

- The rest can be deduced with thermodynamics
  - Enthalpy density w
  - Energy density e
  - Entropy density s
  - Sound speed c_s

$$w \equiv T \frac{\partial p}{\partial T} = e + p = Ts$$

$$c_s^2 \equiv \left(\frac{\partial p}{\partial e} \right)_s = \frac{1}{3}$$

Velocity and enthalpy profiles are different for each type of combustion



Subsonic deflagration: movement only outside

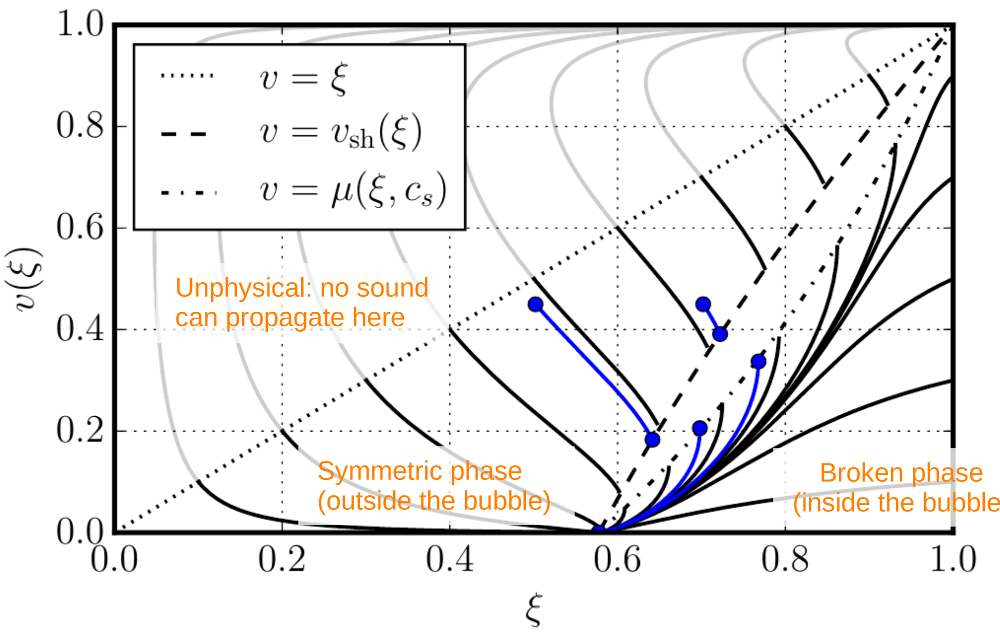
Supersonic deflagration (hybrid): movement on both sides

Detonation: movement only inside

Hindmarsh et al., 2021

Mathematics: numerical integration

- Compute the velocity profile
  - Start from known boundary conditions
    - $v = 0$ and $T = T_{\mathrm{nucleation}}$ far away
    - $v = 0$ at the center of the bubble
  - Integrate (v, w, ξ) numerically
- Use the **Sound Shell Model** to convert the velocity profile to GW power spectrum
  - Sine transform (a bit like Fourier, numerical integration)



Hindmarsh and Hijazi, 2019

Beyond the bag model

- Assumptions broken
  - Different equations for the phases
  - Different sound speeds, possibly temperature-dependent: $c_s(T, \phi)$
- Computing the velocity profile becomes more difficult
  - Sound speed may change at each point
  - May require nested numerical integration
- Next approximation: Constant sound speed model
  - Different but constant speed of sound in each phase

$$p_s = a_s T^\mu - V_s \qquad \mu = 1 + \frac{1}{c_{s,s}^2}$$

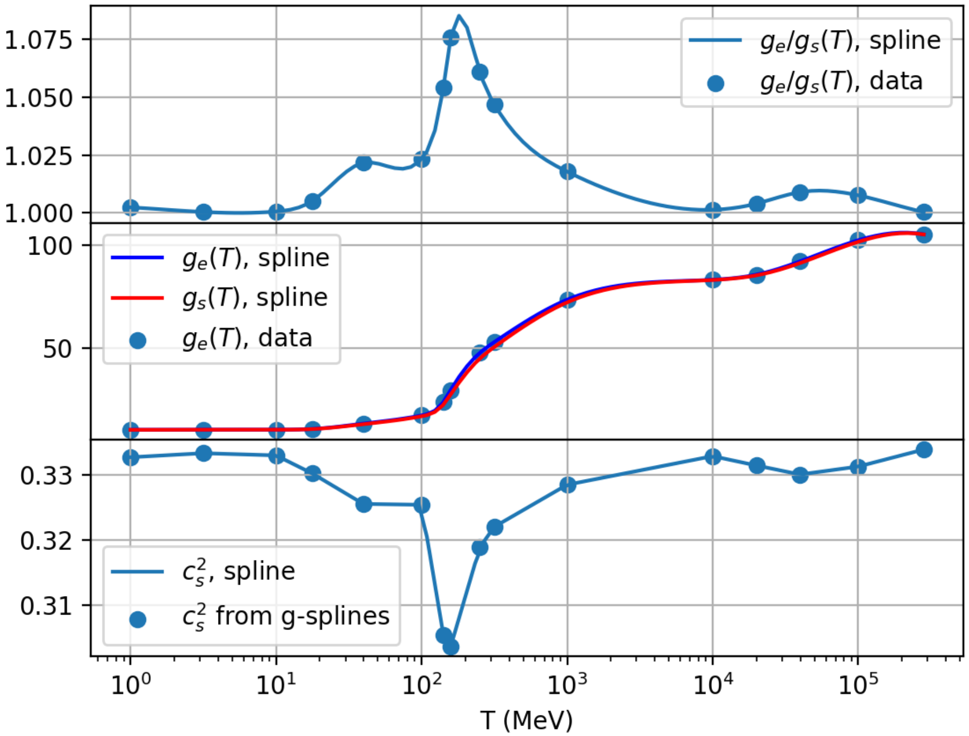
$$p_b = a_b T^\nu \qquad \nu = 1 + \frac{1}{c_{s,b}^2}$$

Goal: Equation of state from an arbitrary model

- Example: Standard Model
- Fluid properties depend on
  - Temperature T
  - Phase ϕ
- Arbitrary models can be tested, when $g_{\text{eff}}(T,\phi)$ is given

$$e(T,\phi) = \frac{\pi^2}{30} g_e(T,\phi) T^4$$

$$s(T,\phi) = \frac{2\pi^2}{45} g_s(T,\phi) T^3$$



Data from Borsanyi et al., 2016

Summary

- Many extensions of the Standard model result in first-order phase transitions
  → Gravitational waves
- GW power spectrum
  → Velocity profile of bubbles
  → Underlying physics
- Were there first-order phase transitions in the early universe? We will know in the 2030s when LISA is launched.
- If yes, it's a sign of new physics!

Sources

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